Week --3 -Applications of L.T in electronic circuits

DAWAH-2025-2026

Circuit Analysis using Laplace Transform

Laplace Transform is a strong mathematical tool to solve the complex circuit problems. It converts the time domain circuit to the frequency domain for easy analysis. To solve the circuit using Laplace Transform, we follow the following steps:

1\Write the differential equation of the given circuit.

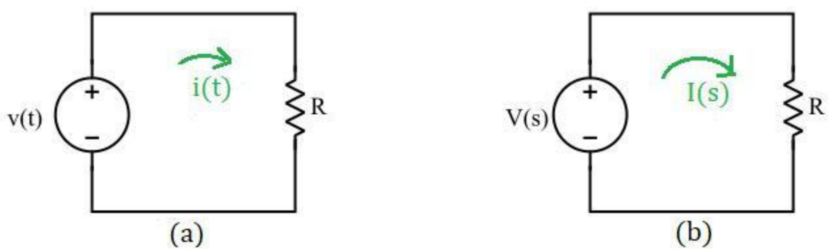
2\Take the Laplace transform of the equation written.

3\Analysis of the s-domain equations.

An electrical circuit may have three important components, i.e., Resistor (R), Conductor (L), Capacitor (C). We will see the analysis of the circuit having these components using Laplace Transform.

1. Pure Resistive Circuit

The below given image represents a pure resistive circuit.



Applying KVL in figure 1(a)

$v(t) = i(t)R$ ------ (1)

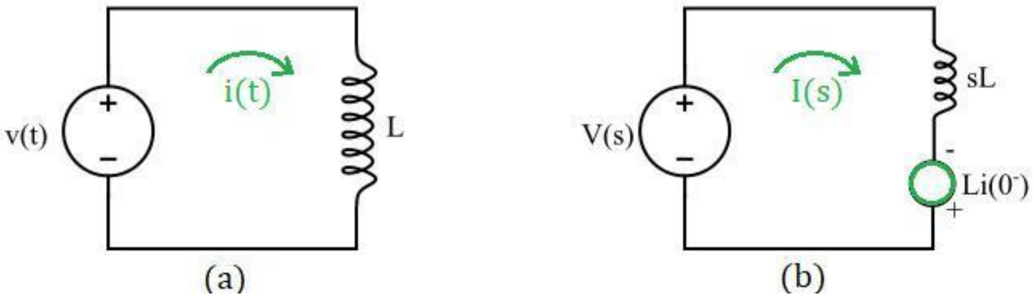
Taking the Laplace transform of equation 1

$V(s) = I(s)R$ ------ (2)

The equation 2 represents the Laplace Transform of equation 1

2. Pure Inductive Circuit

The below given image represents a pure inductive circuit



Applying KVL in figure 2(a)

$$v(t) = L\frac{di(t)}{dt} \quad \text{------ (1)}$$

Taking the Laplace transform of equation 1:

$$V(s)=[sI(s)-i(0-)]L \quad \text{------ (2)}$$

Here, $i(0-)$ is the initial current flowing through the conductor. The equation 2 represents the Laplace Transform of equation 1.

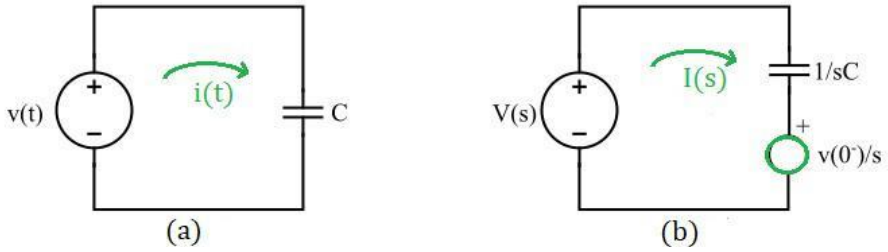
If the initial current flowing through the conductor is 0 then equation 2 will be:

$V(s) = LsI(s)$ ------ (2

Hence, the above analysis shows that the the inductor L in time-domain is converted to 'sL' in the frequency domain. Also, figure 2(b) shows the Laplace Transformed Circuit

3. Pure Capacitive Circuit

The below given image represents a pure capacitive circuit.



Applying KVL in figure 3(a):

$$i(t) = C\frac{dv(t)}{dt} \quad ------ (1)$$

Taking the Laplace transform of equation 1

$I(s)=C(sV(s)-v(0-))$

$I(s)=sCV(s)-Cv(0-)$

$I(s)=sCV(s)-q(0-)$

$$V(s) = \frac{I(s)}{sC} + \frac{v(0^-)}{s} \quad --------- (2)$$

Here, v(0−)is the initial voltage across the capacitor. The equation 2 represents the Laplace Transform of equation 1.

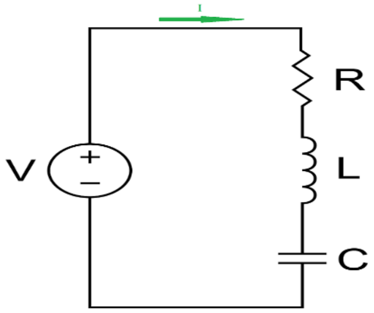
If the voltage across the capacitor is 0, i.e., capacitor is discharge then equation 2 will be:

$$V(s) = \frac{I(s)}{sC}$$

Important Note for Laplace Transform in Circuit Analysis

Reactive Element Name	Reactive Element Symbol	Laplace Transform (if initial conditions are 0)
Resistor	R	R
Capacitor	C	$\frac{1}{sC}$
Inductor	L	sL

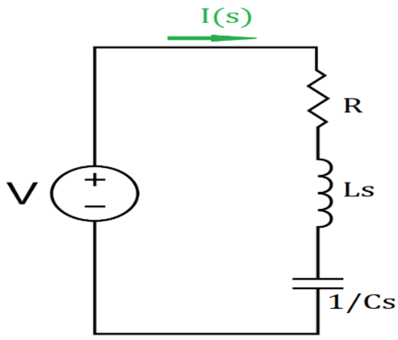
Example: The given figure represents the RLC circuit. Find the output voltage of the given circuit in s-domain.



Solution:

Converting the circuit in the Laplace domain. It will help us to treat the capacitor and inductor as resistor.

The Laplace transformed circuit is given below:



Applying KVL in the above circuit:

$$V(s) = I(s)[R + sL + \frac{1}{sC}]$$

Assuming the initial conditions are 0. The output voltage is:

$$V(s) = I(s)(\frac{sRC+s^2LC+1}{sc})$$

EXAMPLE. A RESISTANCE R IN SERIES WITH INDUCTANCE L IS CONNECTED WITH E.M.F. E (t). THE CURRENT i IS GIVEN BY

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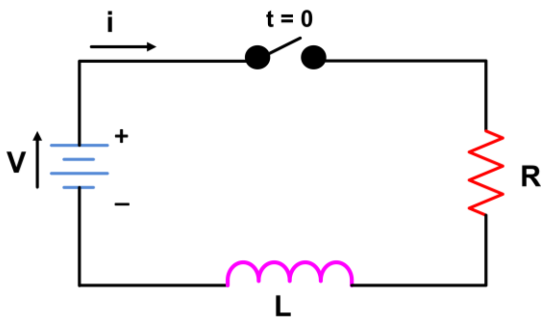

$$L\frac{di}{dt} + Ri = E(t).$$

If the switch is connected at t = 0 and disconnected at t = a, find the current i in terms of t.

Solution. Conditions under which current i flows are $i = 0$ at $t = 0$,

$$E(t) = \begin{cases} E, & 0 < t < a \\ 0, & t > a \end{cases}$$

Given equation is $$L\frac{di}{dt} + Ri = E(t) \quad ....(1)$$

Taking Laplace transform of (1), we get

Note: Instead of

$\bar{i}$ **we can use I(s)**

$$L\,[s\bar{i} - i\,(0)] + R\bar{i} = \int_0^\infty e^{-st} E\,(t)\,dt$$

$$L\,\overline{si} + R\bar{i} = \int_0^\infty e^{-st} E\,(t)\,dt$$

$$[i\,(0) = 0]$$

$$(L\,s + R)\,\bar{i} = \int_0^\infty e^{-st}\,.\,E\,dt = \int_0^a e^{-st} E\,dt + \int_a^\infty e^{-st} E\,dt$$

$$= E\left[\frac{e^{-st}}{-s}\right]_0^a + 0 = \frac{E}{s}\,[1 - e^{-as}] = \frac{E}{s} - \frac{E}{s}\,e^{-as}$$

$$\bar{i} = \frac{E}{s\,(Ls + R)} - \frac{Ee^{-as}}{s\,(Ls + R)}$$

$$i = L^{-1}\left[\frac{E}{s\,(Ls + R)}\right] - L^{-1}\left[\frac{E\,e^{-as}}{s\,(Ls + R)}\right] \qquad ...(2)$$

$$L^{-1}\left[\frac{E}{s\,(Ls+R)}\right] = \frac{E}{L}L^{-1}\left[\frac{1}{s\left(s+\frac{R}{L}\right)}\right]$$ (Resolving into partial fractions)

Now we have to find the value of $L^{-1}\left[\frac{E}{s\,(Ls+R)}\right]$

$$\mathrm{L}^{-1}\left[\frac{E\,e^{-as}}{s\,(Ls+R)}\right] = \frac{E}{R}\left[1-e^{-\frac{R}{L}(t-a)}\right]u\,(t-a)$$

[By the second shifting theorem] On substituting the values of the inverse transforms in (2) we get

$$i = \frac{E}{R}\left[1-e^{-\frac{R}{L}t}\right]-\frac{E}{R}\left[1-e^{-\frac{R}{L}(t-a)}\right]u\,(t-a)$$

$$i = \frac{E}{R}\left[1-e^{-\frac{R}{L}t}\right] \quad \text{for } 0 < t < a, \quad [u\,(t-a) = 0]$$

$$i = \frac{E}{R}\left[1-e^{-\frac{R}{L}t}\right]-\frac{E}{R}\left\{1-e^{-\frac{R}{L}(t-a)}\right\} \quad \text{for } t > a \quad [u\,(t-a)=1]$$

$$= \frac{E}{R}\left[e^{-\frac{R}{L}(t-a)}-e^{-\frac{R}{L}t}\right] = \frac{E}{R}e^{-\frac{R}{L}t}\left[e^{\frac{Ra}{L}}-1\right] \qquad \textbf{Ans.}$$

Example: There is no initial energy stored in this circuit. Find i1(t) and i2(t) for t > 0.

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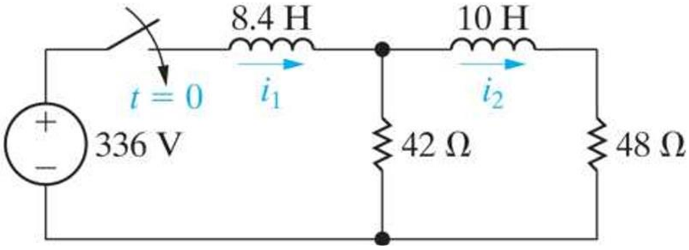
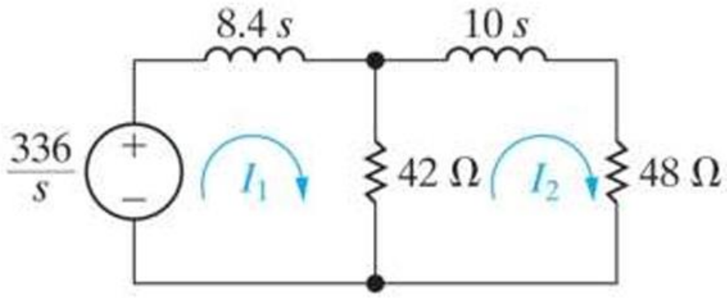


$$-\frac{336}{s}+(42+8.4s)I_1-42I_2=0$$

$$(10s+90)I_2-42I_1=0 \quad \Rightarrow \quad I_1=\frac{10s+90}{42}I_2$$



Substituting, $$-\frac{336}{s}+\left[\frac{(42+8.4s)(10s+90)}{42}-42\right]I_2=0$$

$$\Rightarrow \quad I_2(s)=\frac{336(42)}{s[(42+8.4s)(10s+90)-42^2}=\frac{168}{s^3+14s^2+24s}$$

$$I_1(s)=\frac{10s+90}{42}\left[\frac{168}{s^3+14s^2+24s}\right]=\frac{40s+360}{s^3+14s^2+24s}$$

$$=\frac{K_1}{s}+\frac{K_2}{s+2}+\frac{K_3}{s+12}$$

$$K_1 = \left. \frac{40s+360}{(s+2)(s+12)} \right|_{s=0} = 15;$$

$$K_2 = \left. \frac{40s+360}{s(s+12)} \right|_{s=-2} = -14; \qquad K_3 = \left. \frac{40s+360}{s(s+2)} \right|_{s=-12} = -1$$

$$I_1(s) = \frac{15}{s} + \frac{-14}{s+2} + \frac{-1}{s+12}$$

$$i_1(t) = \mathcal{L}^{-1}\left\{ \frac{15}{s} + \frac{-14}{s+2} + \frac{-1}{s+12} \right\}$$

$$= [15 - 14e^{-2t} - e^{-12t}]u(t) \text{ A}$$

The forced response is $15u(t)$ A;

The natural response is $[-14e^{-2t} - e^{-12t}]u(t)$ A.

$$I_2(s) = \frac{168}{s(s+2)(s+12)}$$

$$= \frac{K_1}{s} + \frac{K_2}{s+2} + \frac{K_3}{s+12}$$

$$K_1 = \frac{168}{(s+2)(s+12)}\bigg|_{s=0} = 7;$$

$$K_2 = \frac{168}{s(s+12)}\bigg|_{s=-2} = -8.4; \qquad K_3 = \frac{168}{s(s+2)}\bigg|_{s=-12} = 1.4$$

$$I_2(s) = \frac{7}{s} + \frac{-8.4}{s+2} + \frac{1.4}{s+12}$$

$$i_2(t) = \mathcal{L}^{-1}\left\{\frac{7}{s} + \frac{-8.4}{s+2} + \frac{1.4}{s+12}\right\}$$

$$= [7 - 8.4e^{-2t} + 1.4e^{-12t}]u(t) \text{ A}$$

The forced response is $7u(t)$ A;

The natural response is $[-8.4e^{-2t} - 1.4e^{-12t}]u(t)$ A.

$$i_1(t) = (15 - 14e^{-2t} - e^{-12t})\,u(t)\ A$$

$$i_2(t) = (7 - 8.4e^{-2t} + 1.4e^{-12t})\,u(t)\ A$$

At t = 0, the circuit has no initial stored energy, so $i_1(0) = 0$ and $i_2(0) = 0$. Now check the equations:

$$i_1(0) = (15 - 14 - 1)(1) = 0$$

$$i_2(0) = (7 - 8.4 + 1.4)(1) = 0$$

Example / A d.c. voltage of 3V is applied to an RC circuit with *R* = 2000_ and *C* = 0.001 F, where *q(0)* = 0. Find the voltage across the capacitor as a function of *t* .

Solution From Kirchhoff's voltage law, we get the differential equation

$$2000\frac{dq}{dt} + \frac{q}{0.001} = 3$$

$$\frac{dq}{dt} + 0.5q = 0.0015$$

Taking Laplace transforms of both sides of the equation where

$Q(s) = L\{q(t)\}$,

$$sQ(s) - q(0) + 0.5Q(s) = \frac{0.0015}{s}.$$

Here, we have used the derivative property of the Laplace transform,

q(t) = *sQ(s)* − *q*(0). Solve the algebraic equation

$$(s+0.5)Q = \frac{0.0015}{s} \quad \Leftrightarrow \quad Q = \frac{0.0015}{s(s+0.5)}.$$

We need to find the inverse Laplace transform of this function of *s* so that we expand using partial fractions, giving

$$Q = \frac{0.003}{s} - \frac{0.003}{s+0.5}$$

$$q = \mathcal{L}^{-1}\left\{\frac{0.003}{s} - \frac{0.003}{s+0.5}\right\} = 0.003 - 0.003e^{-0.5t}.$$

The voltage across the capacitor is

$$\frac{q(t)}{c} = \frac{0.003 - 0.003e^{-0.5t}}{0.001} = 3(1 - e^{-0.5t}).$$