5.1Spread Spectrum

Although bandwidth is a valuable commodity in wireless systems, increasing the transmit signal bandwidth can sometimes improve performance. Spread spectrum is a technique that increases signal bandwidth beyond the minimum necessary for data communication. There are many reasons for doing this. Spread-spectrum techniques can hide a signal below the noise floor, making it difficult to detect. Spread spectrum also mitigates the performance degradation due to inter symbol and narrowband interference. In conjunction with a RAKE receiver, spread spectrum can provide coherent combining of different multipath components. Spread spectrum also allows multiple users to share the same signal bandwidth, since spread signals can be superimposed on top of each other and demodulated with minimal interference between them. Finally, the wide bandwidth of spread-spectrum signals is useful for location and timing acquisition.

Spread spectrum first achieved widespread use in military applications because of its inherent property of hiding the spread signal below the noise floor during transmission, its resistance to narrowband jamming and interference, and its low probability of detection and interception. For commercial applications, the narrowband interference resistance has made spread spectrum common in cordless phones. The ISI rejection and bandwidth- sharing capabilities of spread spectrum are very desirable in cellular systems and wireless LANs. As a result, spread spectrum is the basis for both second- and third-generation cellular systems as well as second-generation wireless LANs.

5.2 Spread-Spectrum Principles

Spread spectrum is a modulation method applied to digitally modulated signals that increases the transmit signal bandwidth to a value much larger than is needed to transmit the underlying information bits. There are many signaling techniques that increase the transmit bandwidth above the minimum required for data transmission - for example, coding and frequency modulation. However, these techniques do not fall in the category of spread spectrum. The following three properties are needed for a signal to be spread-spectrum modulated [1].

1. The signal occupies a bandwidth much larger than is needed for the information signal.

2. The spread-spectrum modulation is done using a spreading code, which is independent of the data in the signal.

3. Dispreading at the receiver is done by correlating the received signal with a synchronized copy of the spreading code.

In order to make these notions precise, we return to the signal space representation of Section 5.1 to investigate embedding an information signal of bandwidth B within a much larger transmit signal bandwidth B, than is needed. From (5.3), a set of linearly independent signals $s_i(t)$, i = 1, ., M, of bandwidth B and time duration T can be written using a basis function representation as

$$s_i(t) = \sum_{j=1}^{N} s_{ij}\phi_j(t), \qquad 0 \le t < T,$$

where the basis functions ϕ_j (t) are orthonormal and span an N-dimensional space. One of these signals is trans- mitted every T seconds to convey $\log_2 M/T$ bits per second. the minimum number of basis functions needed to represent these signals is approximately 2BT. Since the $\{s_i(t)\}_{i=1}^{M}$ are linearly inde- pendent, this implies M≈ 2BT. To embed these signals into a higher-dimensional space, we chose N>> M. The receiver uses an M-branch structure where the ith branch correlates the received signal with s_i (t). The receiver outputs the signal corresponding to the branch with the maximum correlator output.

Suppose we generate the signals s_i (t) using random sequences, so that the sequence of coefficients s1j are chosen based on a random sequence generation, where each coefficient has mean zero and variance E_s/N. Thus, the signals s_i (t) will have their energies uniformly distributed over the signal space of dimension N. Consider an interference or jamming signal within this signal space. The interfering signal can be represented as

$$I(t) = \sum_{j=1}^{N} I_j\phi_j(t),$$

with total energy over $[0,\ T]$ given by

$$\int_0^T I^2(t)\,dt = \sum_{j=1}^{N} I_j^2 = E_J.$$

Suppose the signal s(t) is transmitted. Neglecting noise, the received signal is the sum of the transmitted signal plus interference:

$$x(t) = s_i(t) + I(t)$$

The output of the correlator in the it branch of the receiver is then

$$x_i = \int_0^T x(t)s_i(t)\,dt = \sum_{j=1}^{N}(s_{ij}^2 + I_j s_{ij}),$$

where the first term in this expression represents the signal and the second term represents the interference. It can be shown [1] that the signal-to-interference power ratio (SIR) of this signal is

$$SIR = \frac{E_s}{E_j} \cdot \frac{N}{M}.$$

This result is independent of the distribution of the interferer's energy over the N-dimensional signal space. In other words, by spreading the interference power over a larger dimension N than the required signaling dimension M, the SIR is increased by G = N/M, where G is called the processing gain or spreading factor. In practice, spread-spectrum systems have processing gains on the order of 10-1000. Since $N \approx 2B_s T$ and $M \approx 2BT$, we have $G \approx B_s / B$, the ratio of the spread signal bandwidth to the information signal bandwidth. Processing gain is often defined as this bandwidth ratio or something similar, but its underlying meaning is generally related to the performance improvement of a spread-spectrum system relative to a non-spread system in the presence of interference Note that block and convolution coding are also techniques that improve performance in the presence of noise or interference by increasing signal bandwidth. An interesting trade-off arises as to whether, given a specific spreading bandwidth, it is more beneficial to use coding or spread spectrum. The answer depends on the specifics of the system design

Spread spectrum is typically implemented in one of two forms: direct sequence (DS) or frequency hopping (FH). In direct-sequence spread-spectrum (DSSS) modulation, the modulated data signal s(t) is multiplied by a wideband spreading signal or code $s_c(t)$, where $s_c(t)$ is constant over a time duration T_c and has amplitude equal to 1 or -1. The spreading code bits are usually referred to as chips, T_c is called the chip time, and $1/T_c$ is called the chip rate. The bandwidth $B_c \approx 1/T_c$ of $s_c(t)$ is roughly $B_c/B \approx T_s/T_c$, times larger than the bandwidth B of the modulated signal s(t), and the number of chips per bit, T_s/T_c, is an integer approximately equal to G, the processing gain of the system. Multiplying the modulated signal by the spreading signal results in the convolution of these two signals in the frequency domain. Thus, the transmitted signal $s(t)\, s_c(t)$ has frequency response $S(f) * S_c(f)$, which has a bandwidth of roughly $B_c + B$. The multiplication of a spreading signal $s_c(t)$ with a modulated data signal s(t) over one symbol time T_s , is illustrated in Figure 5.1.

For an AWGN (additive white Gaussian noise) channel, the received spread signal is $s(t)s_c(t) + n(t)$ for n(t) the channel noise. If the receiver multiplies this signal by a synchronized replica of the spreading signal, the result is $s(t)\, s_c^2(t) + n(t)s_c(t)$. Since

$s_c(t) = \pm1$, we have $s_c^2(t) = 1$. Moreover, $n'(t) = n(t)s_c(t)$ has approximately the same white Gaussian statistics as $n(t)$ if $s_c(t)$ is zero mean and sufficiently wideband (i.e., if its autocorrelation approximates a delta function). Thus, the received signal is $s(t)\,s_c^2(t) + n(t)s_c(t) = s(t) + n'(t)$, which indicates that spreading and dispreading have no impact on signals transmitted over AWGN channels. However, spreading and dispreading have tremendous benefits when the channel introduces narrowband interference or ISI.

We now illustrate the narrowband interference and multipath rejection properties of direct-sequence spread spectrum (DSSS) in the frequency domain: more details will be given in later sections. We first consider narrow- band interference rejection, as shown in Figure 5.2. Neglecting noise, we see that the receiver input consists of the spread modulated signal $S(f) * S_c(f)$ and the narrowband interference $I(f)$. The dispreading in the receiver recovers the data signal $S(f)$. However, the interference signal $I(t)$ is multiplied by the spreading signal $s_c(t)$, resulting in their convolution $I(f)\,S_c(f)$ in the frequency domain. Thus, receiver dispreading has the effect of distributing the interference power over the bandwidth of the spreading code. The demodulation of the modulated signal $s(t)$ effectively acts as a low pass filter, removing most of the energy of the spread interference, which reduces its power by the processing gain $G \approx B_c/B$.

Figure 5.3 illustrates ISI rejection, which is based on a similar premise. Suppose the spread signal $s(t)s_c(t)$

is transmitted through a two-ray channel with impulse response $h(t) = \alpha\delta(t) + \beta\delta(t-\tau)$. Then $H(f) = \alpha + \beta e^{-j2\pi f\tau}$, resulting in a receiver input in the absence of noise equal to $H(f)[S(f) * S_c(f)]$ in the frequency domain or $[s(t)s_c(t)] * h(t) = \alpha s(t)s_c(t) + \beta s(t-\tau)s_c(t-\tau)$ in the time domain. Suppose that the receiver despreading process multiplies this signal by a copy of $s_c(t)$ synchronized to the first path of this two-ray model. This results in the time-domain signal $\alpha s(t)s_c^2(t) + \beta s(t-\tau)s_c(t-\tau)s_c(t)$. Since the second multipath component $\beta s'(t) = \beta s(t-\tau)s_c(t-\tau)s_c(t)$ includes the product of asynchronized copies of $s_c(t)$, it remains spread out over the spreading code bandwidth, and the demodulation process will remove most of its energy. More precisely, as described in Section 13.2, the demodulation process effectively attenuates the multipath component by the autocorrelation $\rho_c(\tau)$ of the spreading code at delay τ. This autocorrelation can be quite small when $\tau > T_c$, on the order of $1/G \approx T_c/T_s$, resulting in significant mitigation of the ISI when the modulated signal is spread over a wide bandwidth. Since the spreading code autocorrelation determines the ISI rejection of the spread-spectrum system, it is important to use spreading codes with good autocorrelation properties, as discussed in the next section.

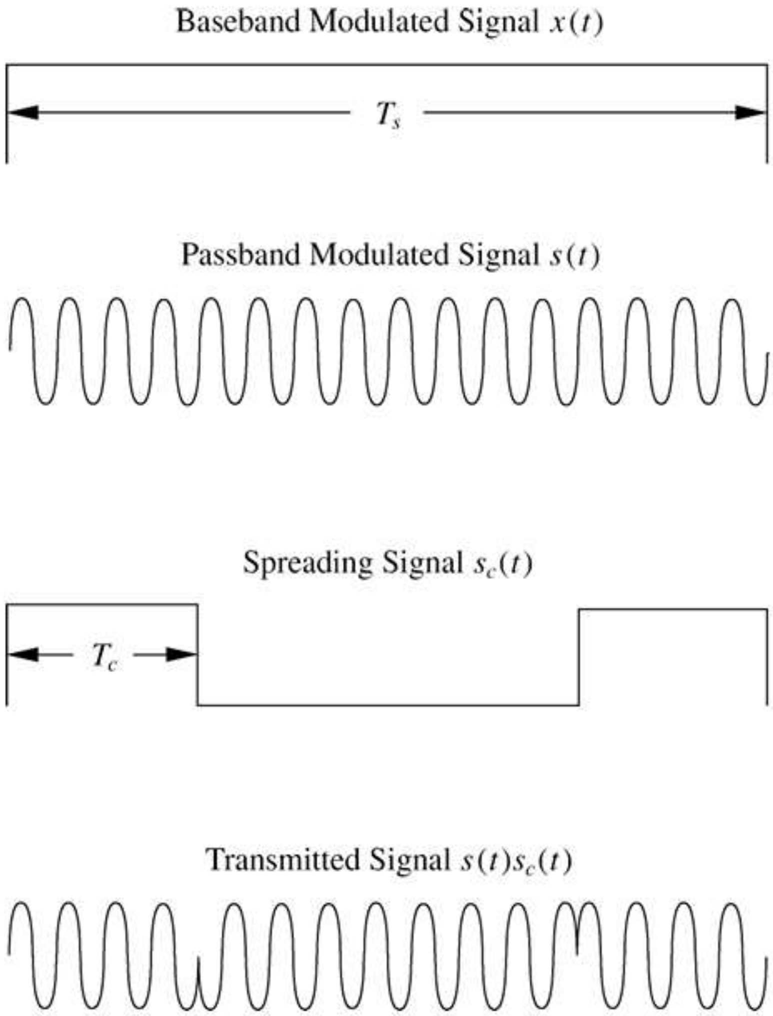

Figure 5.1: Spreading signal multiplication.

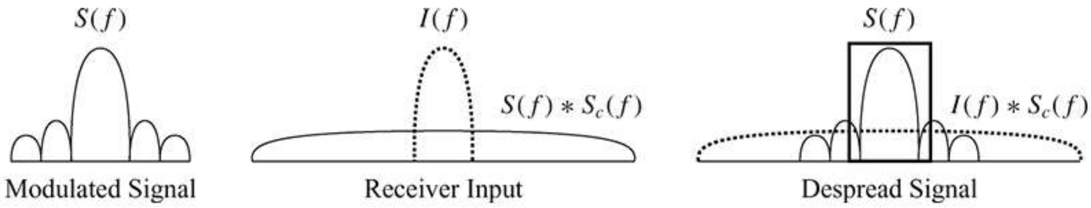

Figure 5.2: Narrowband interference rejection in DSSS.

The basic premise of frequency-hopping spread spectrum (FHSS) is to hop the modulated data signal over a wide bandwidth by changing its carrier frequency according to the value of a spreading code $s_c(t)$. This process is illustrated in Figure 5.1. The chip time T_c of $s_c(t)$ dictates the time between hops - that is, the time duration over which the modulated data signal is centered at a given carrier frequency f_i before hopping to a new carrier frequency. The hop time can exceed a symbol time, $T_c = KT_s$ for some integer k, which is called slow frequency hopping (SFH); or the carrier can be changed multiple times per symbol, $T_c = T_s/k$ for some integer k, which is called fast frequency hopping (FFH). In FFH there is frequency diversity on every symbol, which protects each symbol against narrowband interference and spectral nulls due to frequency-selective fading. The bandwidth of the FH system is approximately equal to NB, where N is the number of carrier frequencies available for hopping and B is the bandwidth of the data signal. The signal is generated using a frequency synthesizer that determines the modulating carrier frequency from the chip sequence, typically using a form of FM modulation such as CPFSK. In the receiver, the signal is demodulated using a similar frequency synthesizer, synchronized to the chip sequence 'The concept of frequency hopping was invented during World War II by film star Hendy Lamar and composer George Antheil. Their patent for a "secret communications system" used a chip sequence generated by a player piano roll to hop between 88 frequencies. The design was intended to make radio-guided torpedoes hard to detect jam.

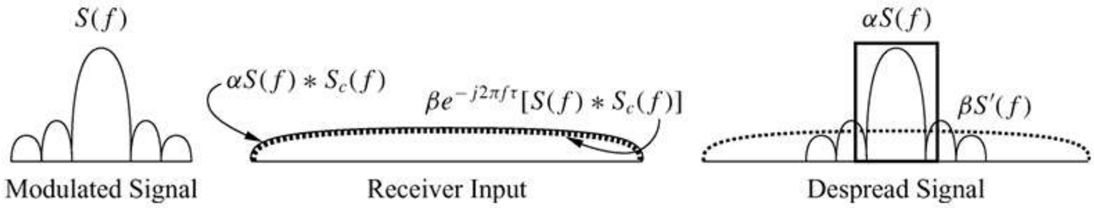

Figure 5.3: ISI rejection in DSSS.

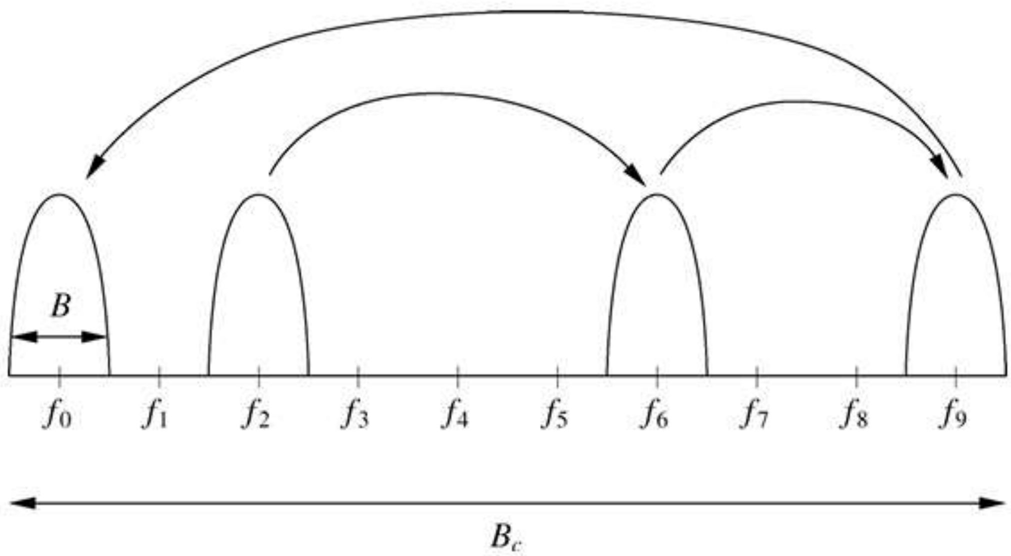

Figure 5.4: Frequency hopping.

$S_c(t)$, that generates the sequence of carrier frequencies from this chip sequence for downconversion. As with DS, FH has no impact on performance in an AWGN channel. However, it does mitigate the effects of narrowband interference and multipath.

Consider a narrowband interferer of bandwidth B at a carrier frequency f_i corresponding to one of the carriers used by the FH system. The interferer and FH signal occupy the same bandwidth only when carrier f_i is generated by the hop sequence. If the hop sequence spends an equal amount of time at each of the carrier frequencies, then interference occurs a fraction 1/N of the time, and thus the interference power is reduced by roughly 1/N. However, the nature of the interference reduction is different in FH versus DS systems. In particular, DS results in a reduced-power interference all the time, whereas FH has a full power interferer a fraction of the time. In FFH systems the interference affects only a fraction of a symbol time, so coding may not be required to compensate for this interference. In SFH systems the interference affects many symbols, so typically coding with interleaving is needed to avoid many simultaneous errors in a single code word. Frequency hopping is commonly used in military systems, where the interferers are assumed to be malicious jammers attempting to disrupt communications.

We now investigate the impact of multipath on an FH system. For simplicity, we consider a two-ray channel that introduces a multipath component with delay T. Suppose the receiver synchronizes to the hop sequence associated with the line-of-sight signal path. Then the LOS path is demodulated at the desired carrier frequency. However, the multipath component arrives at the receiver with a delay T. If $T > T_c$ then the receiver will have hopped to a new carrier frequency $f_j \neq f_i$ for down conversion when the multipath component, centered at carrier frequency f_i, arrives at the receiver. Since the multipath occupies a different frequency band than the LOS signal component being demodulated, it causes negligible interference to the demodulated signal. Thus, the demodulated signal does not exhibit either flat or frequency-selective fading for $T > T_c$. If $T < T_c$ then the impact of multipath depends on the bandwidth B of the modulated data signal as well as on the hop rate. First consider an FFH system where $T_c \ll T_s$. Since we also assume $T < T_c$, it follows that $T < T_c \ll T_s$. Since all the multipath arrives within a symbol time, the multipath introduces a complex amplitude gain and the signal experiences flat fading. Now consider an SFH system where $T_c \gg T_s$. Since we also assume $T < T_c$, all the multipath will arrive while the signal is at the same carrier frequency, so the impact of multipath is the same as if there were no frequency hopping: for $B < 1/T$ the signal experiences flat fading, and for $B > 1/T$ the signal experiences frequency-selective fading. The fading channel also varies slowly over time, since the equivalent low pass channel is a function of the carrier frequency and thus changes whenever the carrier hops to a new frequency. In summary, frequency hopping removes the impact of multipath on demodulation of the LOS component whenever $T > T_C$- For $T < T_c$, an FFH system will exhibit flat fading, whereas an SFH system will exhibit slowly varying flat fading for $B < 1/T$ and slowly varying frequency-selective fading for $B > 1/T$. The performance analysis under time- varying flat or frequency-selective fading is the same as for systems without hopping , respectively.

In addition to their interference and ISI rejection capabilities, both DSSS and FHSS provide a mechanism for multiple access, allowing many users to simultaneously share the spread bandwidth with minimal interference between users. In these multiuser systems, the interference between users is determined by the cross-correlation of their spreading codes. Spreading code designs typically have either good autocorrelation properties to mitigate ISI or good cross-correlation properties to mitigate multiuser interference. However, there is usually a trade-off between optimizing the two features. Thus, the best choice of code design depends on the number of users in the system and the severity of the multipath and interference. Trade-offs between frequency hopping and direct sequence in multiuser systems . Frequency hopping is also used in cellular systems to average out interference from other cells.

Example 5.1: Consider an SFH system with hop time T_c = 10 us and symbol time T_s = 1 μs. If the FH signal is transmitted over a multipath channel, for approximately what range of multipath delay spreads will the received dispread signal exhibit frequency-selective fading?

Solution: Based on the two-ray model analysis, the signal exhibits fading - flat or frequency-selective - only when the delay spread $T < T_c$ 10 μs. Moreover, for frequency-selective fading we require $B \approx 1/T_s\ 10^6 > 1/T$; that is, we require $T > 10^{-6} = 1$ us. So the dispread signal will exhibit frequency-selective fading for delay spreads ranging from approximately 1 μs to 10 μs.

5.2 Direct-Sequence Spread Spectrum (DSSS)

5.2.1 DSSS System Model

An end-to-end direct-sequence spread-spectrum system is illustrated in Figure 5.5. The multiplication by $s_c(t)$ and the carrier $\cos(2\pi f_c t)$ could be done in opposite order as well: down converting prior to dispreading allows the code synchronization and dispreading to be done digitally, but it complicates carrier phase tracking because this must be done relative to the wideband spread signal. For simplicity we illustrate the receiver only for in-phase signaling; a similar structure is used for the quadrature signal component. The data symbols s_I are first linearly A system that performs spreading and dispreading on the band pass modulated signal works as follows. The transmitter consists of a standard narrowband modulator that generates a passband modulated signal, followed by spreading. The receiver consists of dispreading, followed by a standard narrowband demodulator. This order of operations makes it straightforward to design a spread-spectrum system using existing narrowband modulators and demodulators, and operations such as carrier phase recovery are not affected by spreading. However, spread-spectrum systems today do as much of the signal processing as possible in the digital domain. Thus, spread-spectrum systems typically modulate the data symbols and multiply by the spreading code at baseband using digital signal processing, followed by A/D conversion and analog up conversion to the carrier frequency. In this case all functions prior to the carrier multiplication in Figure 5.5 are done digitally, with an A/D converter following the multiplication with $s_c(t)$. However, the carrier recovery loop for this system is more difficult to design because it operates on the spread signal. In particular, any nonlinear operation (e.g. squaring) that is used to remove either the data or the spreading sequence in carrier phase recovery can be seriously degraded by the noise associated with the spread signal.

modulated to form the baseband modulated signal $x(t) = \sum_I slg(t - lT_s)$ where g(t) is the modulator shaping pulse, T_S the symbol time, and s_I the symbol transmitted over the Ith symbol time. Linear modulation is used because DSSS is a form of phase modulation and therefore works best in conjunction with a linearly modulated data signal. The modulated signal is then multiplied by the spreading code $s_c(t)$ with chip time T_c, after which it is up converted through multiplication by the carrier $\cos(2\pi f_c t)$. The spread signal passes through the channel h(t), which also introduces narrowband interference I(t) and zero-mean AWGN n(t) with power spectral density No/2.

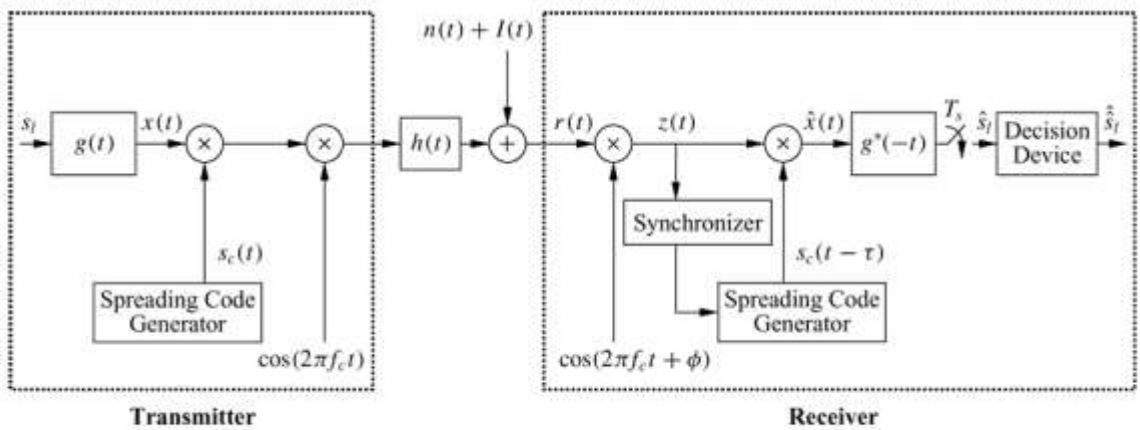

Figure 5.5: DSSS system model.

Assume the channel introduces several multipath components: h(t) = $\alpha 0 \delta$ (t - To) + $\alpha 0 \delta$ (t-T1)+. The received signal is first downconverted to baseband assuming perfect carrier recovery, so that the carrier cos(2π f_ct+ ∅) in the receiver has phase ∅ matched to the carrier phase of the incoming signal (e.g., if h(t) = δ(t) then ∅ = 0). The carrier recovery loop is typically assumed to lock to the carrier associated with the LOS (or minimum delay) multipath component. The synchronizer then uses the resulting baseband signal z(t) to align the delay T of the receiver's spreading code generator with one of the multipath component delays T_i . The spreading code generator then outputs the spreading code s_c(t - T), where $T = T_j$ if the synchronizer is perfectly aligned with the delay associated with the it multipath component. Ideally the synchronizer would lock to the multipath component with the largest amplitude. However, in practice this requires a complex search procedure, so instead the synchronizer typically locks to the first component it finds with an amplitude above a given threshold. This synchronization procedure can be quite complex, especially for channels with severe ISI or interference, and synchronization circuitry can make up a large part of any spread-spectrum receiver.

The multipath component at delay T is dispread via multiplication by the spreading code s_c(t −T). The other multipath components are not dispread, and most of their energy is removed, as we shortly show. After dispersing, the baseband signal $\hat{x}$(t) passes through a matched filter and decision device. Thus, there are three stages in the receiver demodulation for direct-sequence spread spectrum: down conversion, dispreading, and baseband demodulation. This demodulator is also called the single-user matched filter detector for DSSS. We now examine the three stages of this detector in more detail.

For simplicity, assume rectangular pulses are used in the modulation (g(t) = $\sqrt{2/T_s}\,, 0 \le t \le T_s$ matched filter g*(-t) then simply multiplies $\hat{x}$(t) by

$\sqrt{2/T_s}$The and integrates from zero to T_s to obtain the estimate of the transmitted symbol. Since perfect carrier recovery is assumed, the carrier phase offset ∅ in the receiver matches that of the incoming signal. We also assume perfect synchronization in the receiver.

5.2.2 Advantages of Spread Spectrum

1. High Resistance to Interference & Jamming

Because the signal is spread over a wide bandwidth, narrowband interference affects only a small portion of the signal power.

2. Enhanced Security & Privacy

The spreading code makes the signal look like noise to unintended receivers. Only receivers with the correct code can dispread and decode it.

3. Reduced Multipath Effects

Spread spectrum helps reduce the impact of multipath fading, especially in CDMA systems.

4. Low Probability of Intercept (LPI)

The transmitted power is spread over a wide frequency range, making the signal harder to detect with standard receivers.

5. Multiple Access Capability

Using different codes allows multiple users to share the same frequency band simultaneously, like in **CDMA**.

6. Better Performance in Noisy Channels

Provides robustness in environments with high noise and interference (industrial or military environments).

5.2.3 Disadvantages of Spread Spectrum

1. Requires Larger Bandwidth

Spread spectrum uses bandwidth much wider than the minimum required, which can be inefficient in bandwidth-limited systems.

2. More Complex Transmitter & Receiver

Dispreading requires precise synchronization and code generation, increasing system complexity.

3. Power Consumption is Higher

More processing and wideband RF circuitry lead to higher energy use (important for mobile devices).

4. Difficult Code Management

Unique spreading codes must be assigned, managed, and synchronized across systems — challenging in large networks.

5. Synchronization Challenges

Receiver must know the exact spreading sequence timing. Achieving this can take time and require sophisticated algorithms.

6. Higher Cost

Hardware and processing requirements make spread-spectrum systems more expensive than narrowband systems.

5.3 Pseudo-Noise (PN) Sequence

A **Pseudo-Noise (PN) sequence** is a **deterministic binary sequence** that *appears random* but is generated by a mathematical rule, usually a **Linear Feedback Shift Register (LFSR)**.
It has noise-like statistical properties such as:

- nearly equal number of 1s and 0s
- random-looking patterns
- long periodicity
- very good autocorrelation (sharp peak at zero shift)

PN sequences are used in **spread spectrum communication**, **CDMA**, **GPS**, **synchronization**, and **scrambling**, because they allow a signal to be spread over a wide bandwidth and easily recovered through correlation.

5.3.1 Properties of PN Sequence

1. Periodicity

A PN sequence generated by an *n-stage* LFSR repeats after a fixed number of bits.
For a maximal-length sequence (m-sequence):

Period=2^n-1

2. Balance Property

Within one full period:

- Number of 1s = 2^{n-1}
- Number of 0s = 2^{n-1}-1

Meaning 1s and 0s are almost equal, giving a noise-like behavior.

3. Run Property

A run = consecutive identical bits.

PN sequences satisfy:

- Half of the runs are length 1
- One quarter are length 2
- One eighth are length 3
- ... and so on

This matches the behavior of a truly random noise sequence.

4. Autocorrelation Property (Most important)

Autocorrelation of an m-sequence (using ±1 mapping) is:

$$R(\tau) = \begin{cases} N, & \tau = 0 \\ -1, & \tau \neq 0 \end{cases}$$

5. Cross-Correlation Property

Between two different PN sequences of the same length, the cross-correlation takes only a small set of values.
For m-sequences:

$$R_{xy}(\tau) \in \{-1,\ -t,\ t-2\}$$

This makes PN sequences suitable for multi-user systems such as CDMA.

6. Spectral Flatness

The power spectrum of a PN sequence is almost flat, similar to white noise.
This ensures good spreading and resistance to interference.

7. Deterministic but Noise-Like

- Generated by a known rule (e.g., LFSR)
- Easy to reproduce at the receiver
- But statistically resembles real noise

This combination is ideal for spread-spectrum communication.