2. MINIMUM SHIFT KEYING (MSK)

We have discussed QPSK technique in last article. The bandwidth requirement of QPSK is high. Filters or other methods can overcome these problems, but they have other side effects. For example filters alter the amplitude of the waveform.

MSK overcomes these problems. In MSK, the output waveform is continuous in phase hence there are no abrupt changes in amplitude. The side lobes of MSK are very small hence bandpass filtering is not required to avoid interchannel interference. Figure 2.1 shows the waveform of MSK. The binary bit sequence is shown at the top. Figure 2.1(a) shows the corresponding NRZ

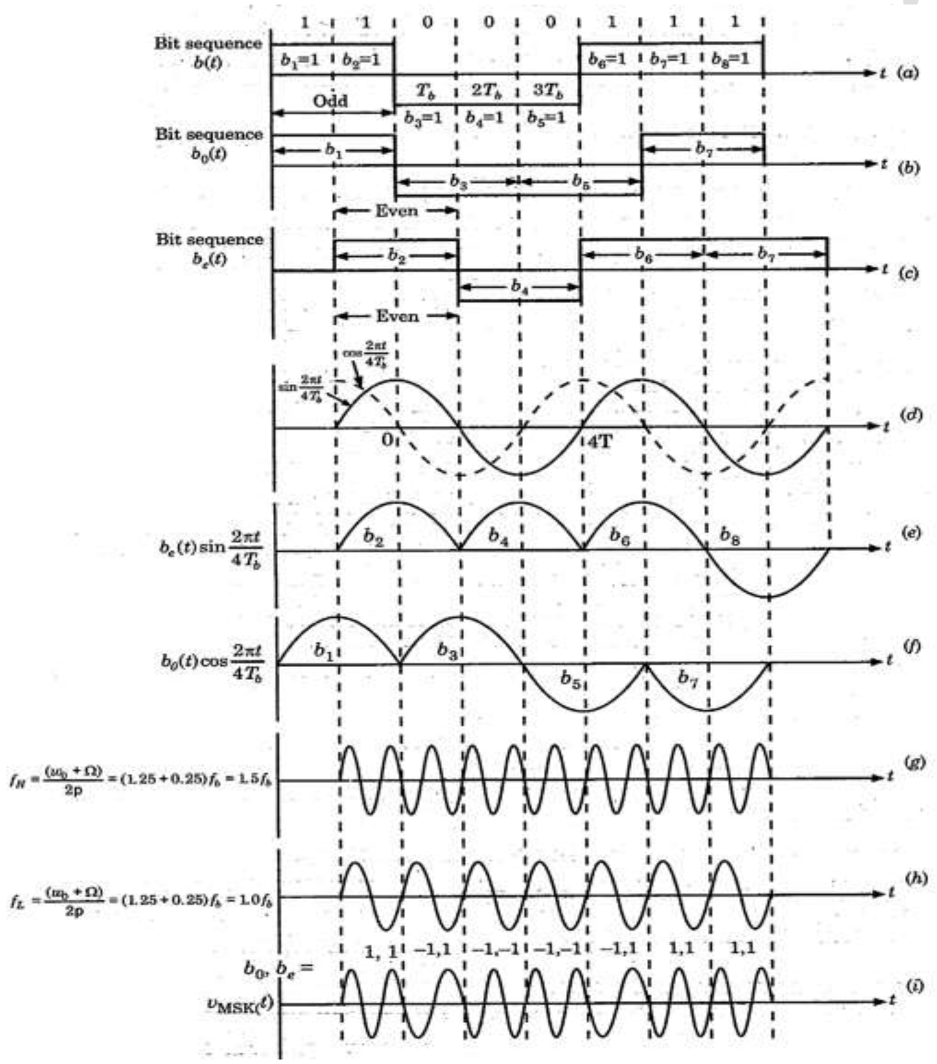

Fig.2.1(a) Bipolar NRZ waveform representing bit sequence (b) Odd bit sequence waveforms b_0 (t) (c) Even bit sequence waveform b_e (t) (d) Waveforms of frequency $f_b/4$ used for smoothing of b_e (t) and b_0 (t) (e) Modulating waveform of even sequence (f) Modulating waveform of odd sequence (g) Waveform of frequency f_H (h) Waveform of frequency f_L (i) MSK waveform. waveform b(t). From b(t), two waveforms are generated for odd and even bits. b_0 (t) represents

odd bits and b_e (t) represents even bits. Figure 2.1(b) and (c) shows the waveform of b_0 (t) and b_e (t). As shown in those waveforms b_1, b_2, b_5 etc. are represented by odd waveform i.e., b_0 (t).

The duration of each bit in b_0 (t) or b_e (t) is $2T_b$ whereas it is T_b in b(t) i.e.,

$$T_s = 2T_b$$

The waveforms b_0 (t) and b_e (t) have an offset of T_b. This offset is essential in MSK. Two waveforms sin $2л(t/4T_b)$ and cos $2\pi(t/4T_b)$ are generated as shown in figure 7.31(d). The waveform of sin $2\pi(t, 4T_b)$ passes through zero at the end of symbol time in b_0 (t). Hence, one symbol duration of b_0 (t) consists of complete half cycle of cos $2л(t/4T_b)$. This means that similarly, one symbol duration of b_e (t) contains half cycle of sin $2\pi(t/T_b)$. Thus there is a phase shift of 'T_b' in sine and cosine waveforms. b_e (t) is multiplied by sin $2л(t/4T_b)$ and b_0 (t) is multiplied by cos $2л(t/4T_b)$. These product waveforms are shown in figure 7.31 (e) and (f). The transmitted MSK signal is represented as under:

$$s(t) = \sqrt{2P_s}\,[b_e(t)\sin(2\pi t/4T_b)]\cos(2\pi f_c t) + \sqrt{2P_s}\,[b_0(t)\cos(2\pi t/4T_b)]\sin(2\pi f_c t)$$

This means that the product signal b_e (t) sin $(2\pi t/4T_b)$ and b_0 (t) cos $(2\pi t/4T_b)$ modulate the quadrature carriers of frequency f_c .We can write last equation as,

$$s(t) = \sqrt{2P_s}\left[\frac{b_o(t)+b_e(t)}{2}\right]\sin 2\pi\left(f_c+\frac{1}{4T_b}\right)t+\sqrt{2P_s}\left[\frac{b_o(t)-b_e(t)}{2}\right]\sin 2\pi\left(f_c-\frac{1}{4T_b}\right)t$$

We know that $f_b = \frac{1}{T_b}$, then the last equation (7.74) becomes,

$$s(t) = \sqrt{2P_s}\left[\frac{b_0(t)+b_e(t)}{2}\right]\sin 2\pi\left(f_c+\frac{f_b}{4}\right)t+\sqrt{2P_s}\left[\frac{b_0(t)-b_e(t)}{2}\right]\sin 2\pi\left(f_c-\frac{f_b}{4}\right)t$$

Let $$C_H(t) = \frac{b_0(t)+b_e(t)}{2}$$

and $$C_L(t) = \frac{b_0(t)+b_e(t)}{2}$$

and let $$f_H = f_c+\frac{f_b}{4}$$

and $$f_L = f_c-\frac{f_b}{4}$$

with these substitutions, equation (7.75) becomes,

$$s(t) = \sqrt{2P_s}\; C_H(t)\sin(2\pi f_H t)+\sqrt{2P_s}\; C_L(t)\sin(2\pi f_L t)$$

If $b_0(t) = b_e(t)$ then $C_L(t) = 0$ and $C_H(t) = \pm 1$, then last equation becomes,

$$s(t) = \sqrt{2P_s}\; C_H(t)\sin(2\pi f_H t)$$

Hence, the transmitted frequency is f_H.

If $b_0(t) = -\,b_e(t)$, then $C_H(t) = 0$ and $C_L(t) = \pm 1$. Then equation (7.79) becomes,

$$s(t) = \sqrt{2P_s}\; C_H(t)\sin(2\pi f_L t)$$

Hence, the transmitted frequency is f_L

The frequencies f_H and f_L are chosen such that cos($2\pi\ f_H$ t) and sin ($2\pi\ f_L$ t) are orthogonal over the interval T_bu. For orthogonality following relation must be satisfied i.e.,

$$\int_0^{T_b}\sin(2\pi f_H t)\sin(2\pi f_L t)\,dt = 0$$

This relation will be satisfied if we have integers 'm' and 'n' such that,

$$2\pi\,(f_H-f_L)\,T_b = n\pi$$

and $$2\pi(f_H+f_L)\,T_b = m\pi$$

Let us substitute values of f and f1 from equations in above relations. we get

$$2\pi\left(f_c+\frac{f_b}{4}-f_c+\frac{f_b}{4}\right)T_b = n\pi$$

or $$f_b\, T_{b-} = n$$

or $$f_b \times \frac{1}{f_b} = n \quad \Rightarrow \quad n = 1$$

$$2\pi\left(f_c+\frac{f_b}{4}+f_c-\frac{f_b}{4}\right)T_b = m\,\pi$$

or $$4f_c\, T_b = m$$

or $$4f_c \times \frac{1}{f_b} = m \quad \Rightarrow \quad f_c = \frac{m}{4} f_b$$

with $n = 1$ **We get**

$$2\pi(f_H - f_L)\, T_b = 1 \times \pi$$

or $$(f_H - f_L) = \frac{1}{2T_b} = \frac{f_b}{2}$$

Here, n = 1 means the difference between f_H and f_L is minimum and at the same time, (MSK) they are orthogonal. Therefore, this technique is called minimum shift keying (MSK). This minimum difference is given by equation above we know that $f_c \frac{m}{4} f_b$ This shows that carrier frequency 'f' is integer multiple of $\frac{f_b}{4}$ we get

$$f_H = f_c + \frac{f_b}{4} = m\frac{f_b}{4} + \frac{f_b}{4}$$

or $$f_H = (m+1)\frac{f_b}{4}$$

Similarly, substituting $f_b = m\frac{f_b}{4}$ We get

$$f_L = (m-1)\frac{f_b}{4}$$

Figure 2.1(g) and (h) shows the waveforms of sin(2π f_H t) and sin (2 π f_L t). For these waveforms m = 5. f_H and f_L are calculated with m = 5. Figure 2.1(i) shows the final MSK , we know that if b_0 (t) = b_e (t), then transmitted waveform is of frequency f_H. And if b_0 (t) = - b_e (t) then the transmitted waveform , which has frequency of f_L. This shows that MSK is basically FSK with reduced bandwidth and continuous phase.

2.1 Signal Space Representation of MSK and Distance between the Signal Points (i.e., Geometrical Representation of MSK)

$$s(t) = C_H(t)\sqrt{P_sT_s}\cdot\sqrt{\frac{2}{T_s}}\sin(2\pi f_H t) + C_L(t)\sqrt{P_sT_s}\cdot\sqrt{\frac{2}{T_s}}\sin(2\pi f_L t)$$

Here let, $\phi_H(t) = \sqrt{2/T_s}\sin(2\pi f_H t)$

$$\phi_L(t) = \sqrt{2/T_s}\sin(2\pi f_L t)$$

The carriers ϕ_H (t) and ϕ_L (t) are in quadrature. They are in quadrature because their frequencies are in quadrature. In QPSK the carriers are in quadrature be- cause of phase shift. Depending on the values of $C_H(t)$ and $C_L(t)$, there will be four signal points in ϕ_H ϕ_L plane. This has been illustrated in figure 2.2. The distance of each signal point from the origin is $\sqrt{P_sT_s}$

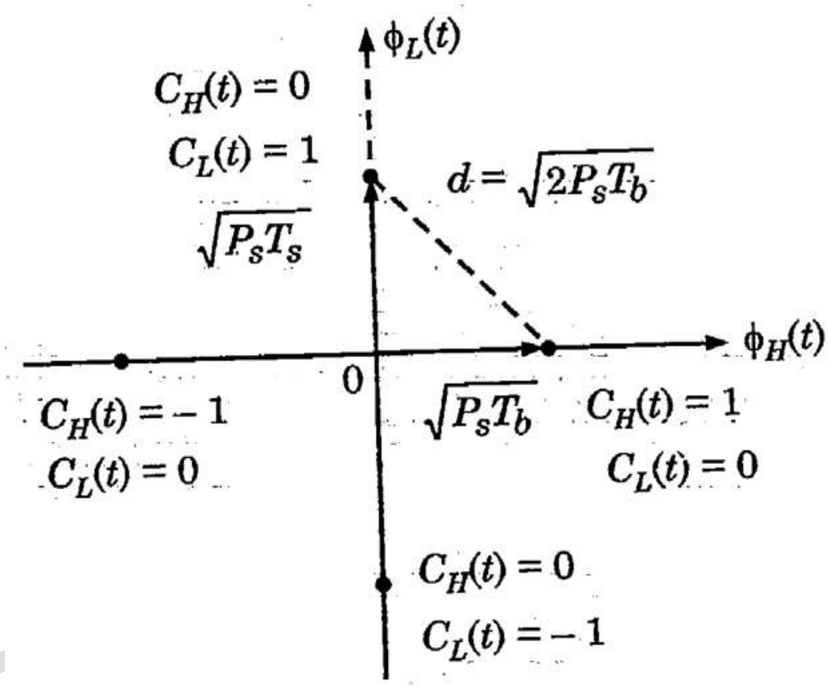

Fig. 2.2 Geometrical (Signal Space) representation of MSK signals.

Distance Between Signal Points

Since the points are symmetric, the distance between any two nearest points is same, i.e.,

EX: Find the minimum distance between the two MSK signal points.

Assume that the **bit energy** is:

$$E_b = 5\ Joules$$

Solution

The minimum Euclidean distance for MSK is given by:

$$d_{min} = \sqrt{2E_b}$$

Substitute:

$$d_{min} = \sqrt{2 \times 5}$$

$$d_{min} = \sqrt{10}$$

$$d_{min} \approx 3.162$$

$$d^2 = \left(\sqrt{P_s T_s}\right)^2 + \left(\sqrt{P_s T_s}\right)^2$$

or $$d = \sqrt{2P_s T_s}$$

or $$d = \sqrt{2E_s} \quad (\text{since } P_s T_s = E_s)$$

or $$d = \sqrt{4E_b} \quad (\text{since } E_s = 2E_b) = 2\sqrt{E_b}$$

These relations give distance between signal points in MSK. This distance is same as in QPSK.

2.3. Power Spectral Density (psd) and Bandwidth of MSK

The waveform which modulates $\sin(2\pi f_c t)$ is,

$$p(t) = \sqrt{2P_s}[b_0(t)\cos(2\pi t / 4T_b)]$$

or $$p(t) = \sqrt{2P_s}\ b_0(t)\cos(\pi f_b t / 2)$$

The power spectral density (psd) of above waveform is expressed as,

$$S_p(f) = \frac{32E_b}{\pi^2}\left[\frac{\cos(2\pi f T_b)}{1-(4fT_b)^2}\right]^2$$

when this signal modulates the carrier ‘f_c’ then the total power spectral density (psd) of baseband signal is divided by '4' and is placed at $\pm f_c$ i.e.,

.

$$S(f) = \frac{8E_b}{\pi^2}\left\{\frac{\cos 2\pi(f-f_c)T_b}{1-[4(f-f_c)T_b]}\right\}^2 + \frac{8E_b}{\pi^2}\left\{\frac{\cos 2\pi(f+f_c)T_b}{1-[4(f+f_c)T_b]^2}\right\}^2$$

The above equation gives power spectral density (psd) of MSK signal. Figure **2.3 shows the normalized spectral densities of MSK and QPSK**. Normalization means maximum amplitudes of signals are scaled with respect to '1',

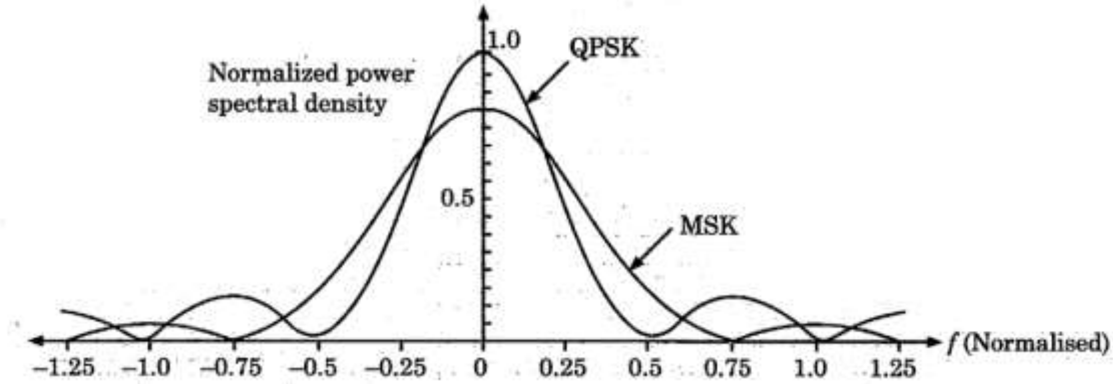

Fig. 2.3 Power spectral densities (psd) of MSK and QPSK.

The above plots show that the main lobe in MSK is wider than QPSK. The sides lobes in MSK are very small compared to QPSK.

Bandwidth Calculation of MSK

From figure 2.3, we observe that the width of main lobe in MSK is ± 0.75 i.e.,

$$fT_b = \pm 0.75$$

Or $$f = \pm 0.75\, f_b$$

Hence, bandwidth will be equal to width of the main lobe i.e.,

$$BW = 0.75\, f_b - (-0.75\, f_b) = 1.5\, f_b$$

Thus, the BW of MSK is higher than that of QPSK.

2.4. Generation of MSK

Figure 2.4 shows the block diagram of MSK transmitter. The two sinusoidal signals $\sin(2\pi f_c\, t)$ and $\cos(2\pi t/4T_b)$ are mixed (i.e., multiplied). The bandpass filters then pass only sum and

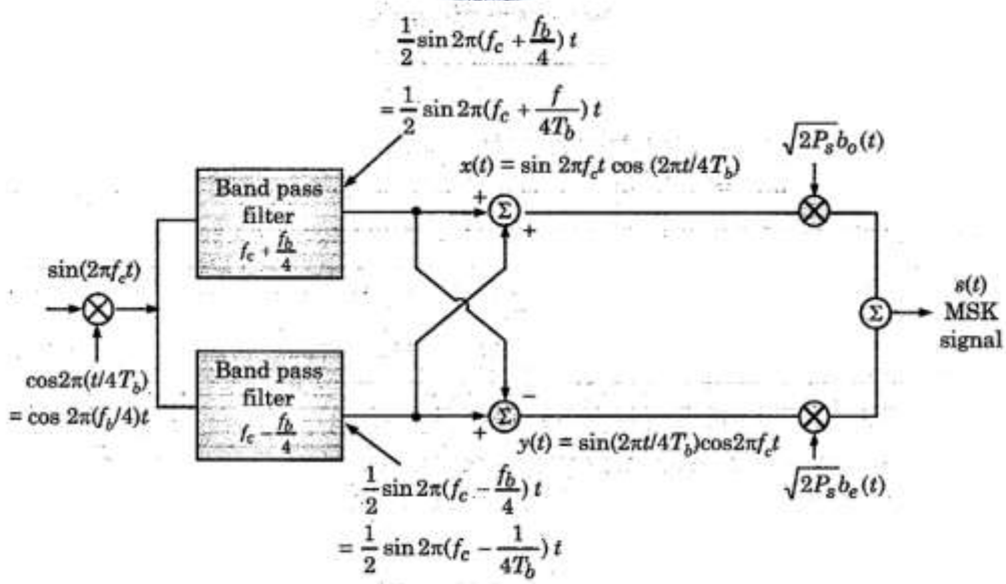

Fig. 2.4 MSK transmitter block diagram.

difference components and The outputs of bandpass filters (BPFs) are then added and subtracted such that two signals x(t) and y(t) are generated. Signal x(t) is multiplied by bo (t) and y(t) is multiplied by be(t). The outputs of the multipliers are then added to give final MSK signal.

2.5. Reception of MSK (i.e. Detection of MSK)

Figure 2.5 shows the block diagram of MSK receiver. MSK uses synchronous detection. The signals x(t) and y(t) are multiplied with the received MSK signal. Here x(t) and y(t) have same values as shown in transmitter block diagram of figure 2.5. The outputs of the multipliers are b_e (t) and b_0 (t). The integrators integrate over the period of $2T_b$. For the upper correlator, the sampling switch samples output of integrator at t = (2k + 1)T_b. Then the decision device decides whether b_0 (t) is + 1 or - 1. Similarly, lower correlator output is b_e(t). The outputs of two decision devices are staggered by T_b. The switch S_3 operates at t = kT_b and simply multiplexes the two correlator outputs.

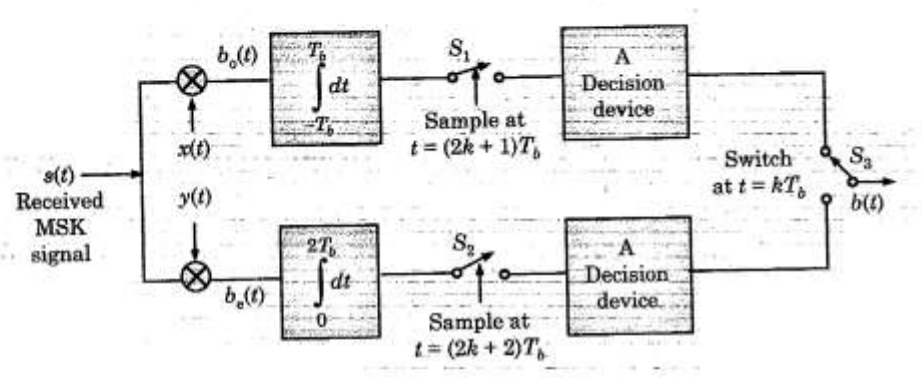

Fig. 2.5 MSK receiver block diagram.

2.6. Advantages and Drawbacks of MSK as Compared to QPSK

From the discussion of MSK, we can now compare the advantages of MSK over QPSK. Advantages:

1. The MSK baseband waveforms are smoother compared to QPSK.

2. MSK signal have continuous phase in all the cases, whereas QPSK has abrupt phase shift of $\frac{\pi}{2}$ or π

3. MSK waveform does not have amplitude variations, whereas QPSK signals have abrupt amplitude variations.

4. The main lobe of MSK is wider than that of QPSK. Main lobe of MSK contains around 99% of signal energy whereas QPSK main lobe contains around 90% signal energy.

5. Side lobes of MSK are smaller compared to that of QPSK. Hence, inter channel interference because of side lobes is significantly large in QPSK.

6. To avoid inter channel interference due to side lobes, QPSK needs bandpass filtering, whereas it is not required in MSK.

7. Bandpass filtering changes the amplitude waveform of QPSK because of abrupt changes in phase. This problem does not exist in MSK.

8. The distance between signal points is same in QPSK as well as in MSK. Hence, the probability of error is also same. However, there are some drawback of MSK.

(ii) Drawbacks

1. The bandwidth requirement of MSK is 1.5 f_b, whereas it is f_b in QPSK. Actually, this cannot be said serious drawback of MSK. Because power to bandwidth ratio of MSK is more. In fact, 99% of signal power can be transmitted within the bandwidth of 1.2 f_b in MSK. While QPSK needs around 8 f_b to transmit the same power.

2. The generation and detection of MSK is slightly complex. Because of incorrect synchronization, phase jitter can be present in MSK. This degrades the performance of MSK.

2.7 GAUSSIAN MINIMUM SHIFT KEYING (i.e., GMSK)

Like Minimum shift keying (MSK), Gaussian MSK (GMSK) yields a constant amplitude and continuous phase RF carrier signal. It only differs in use of a Gaussian baseband pulse shape in place of square pulse shape for MSK. Because, the Gaussian pulse rises and decays asymptotically with respect to a zero response level, it has a much more constrained bandwidth. A typical GMSK system has been shown in figure 7.36 along with an unfiltered MSK system. The unfiltered MSK is generated by direct FSK modulation of a carrier with a baseband signal which is scaled in amplitude to produce a modulation index of 0.5. This value of modulation index produces a difference of 180° phase shift for the

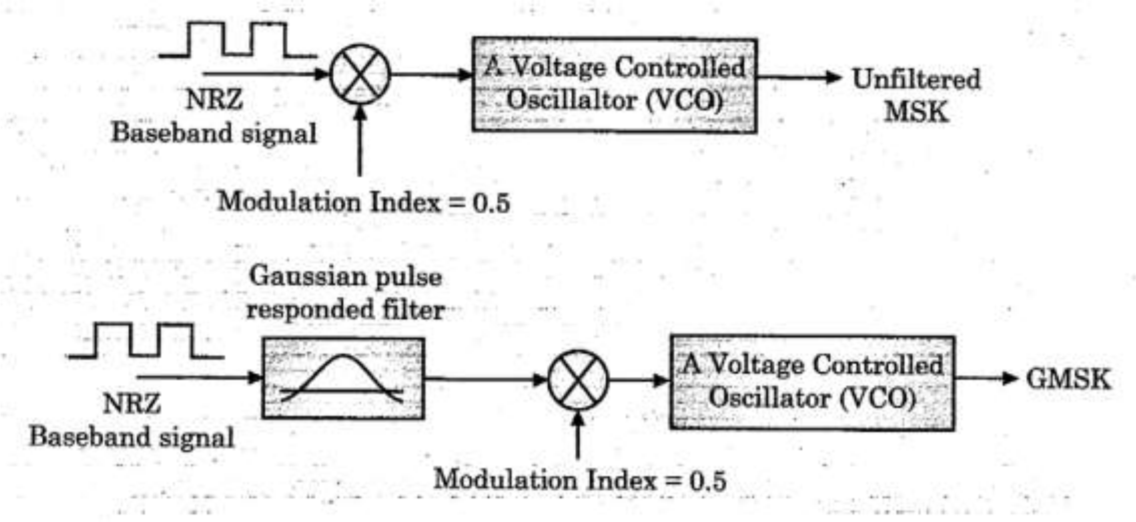

Fig. GMSK as unfiltered MSK.

two *2.6* data values. However, in GMSK, there is ISI (Inter symbol interference) which is a bandwidth limiting factor. GMSK is employed in GSM digital cellular radios and cellular digital packet data (CDPD) applications.

Gaussian Minimum Shift Keying (GMSK) is a modification of MSK. A filter used to reduce the bandwidth of a baseband pulse train prior to modulation is called a pre-modulation filter. The Gaussian pre-modulation filter smooths the phase trajectory of the MSK signal and hence limiting the instantaneous frequency variations. The result is an FM modulated signal with a much narrower bandwidth. This bandwidth reduction does not come for free since the pre-modulation filter smears the individual pulses in pulse train. As a consequence of this smearing in time, adjacent pulses interfere with each other generating what is commonly called inter-symbol interference or ISI. In the applications, where GMSK is used, the trade-off between power efficiency and bandwidth efficiency is well worth the cost.

Bit Error Rate (BER) for GMSK is given by

$$P_e = Q\left(\sqrt{\frac{2\alpha E_b}{N_0}}\right)$$

EX: find the Bit Error Rate (BER)

(Q= 4.8 * 10^{-4})

Given:

- Bit energy:

$$E_b = 2\text{ J}$$

- Noise spectral density:

$$N_0 = 0.5$$

- Modulation constant for GMSK (typical):

$$\alpha = 0.68$$

$$P_e = Q\left(\sqrt{\frac{2\alpha E_b}{N_0}}\right)$$

$$\frac{2\alpha E_b}{N_0} = \frac{2 \times 0.68 \times 2}{0.5}$$
$$= \frac{2.72 \times 2}{0.5}$$
$$= \frac{5.44}{0.5}$$
$$= 10.88$$

So:

$$\sqrt{10.88} \approx 3.299$$

$$P_e = 4.8 \times 10^{-4} \times 3.299$$

$$P_e = 1.58 \times 10^{-3}$$

where a is a constant related to BT_b.

Table. 2.1 GMSK Parameter a Related to BT_b.

S.NO	The value of BT_b	The value of α
1	0.25	0.68
2	∞	0.85

It may be noted that the case where BT→ ∞ corresponds to MSK (t.e. the filter is all pass for a fixed symbol interval T).

Recall that the probability of error for plain MSK is given by

$$P_e \cong Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

Here, we can conclude that PGMSK > PMSK. This arises from the trade off between power and bandwidth efficiency. GMSK achieves a better bandwidth efficiency than MSK at the expense of power efficiency.