1- QUADRATURE PHASE SHIFT KEYING (QPSK)

As a matter of fact, in communication systems, we have two main resources. These are the transmission power and the channel bandwidth. The channel bandwidth depends upon the bit rate or signaling rate f¿. In digital band pass transmission, we use a carrier for transmission. This carrier is transmitted over a channel. If two or more bits are combined in some symbols, then the signaling rate will be reduced. Thus, the frequency of the carrier needed is also reduced. This reduces the transmission channel bandwidth. Hence, because of grouping of bits in symbols, the transmission channel bandwidth can be reduced. In quadrature phase shift keying (QPSK), two successive bits in the data sequence are grouped together. This reduces the bits rate or signaling rate (i.e., f¿) and thus reduces the bandwidth of the channel.*

In case of BPSK, we know that when symbol changes the level, the phase of the carrier is changed by 180°. Because, there were only two symbols in BPSK, the phase shift occurs in two levels only. However, in QPSK, two successive bits are combined. In fact, this combination of two bits forms four distinct symbols. When the symbol is changed to next symbol, then the phase of the carrier is changed by 45° (π/4 radians). Table 1.1 shows these symbols and their phase shifts.

Table 1.1. Symbol and corresponding phase shifts in QPSK

S. No.	Input successive bits		Symbol	Phase shift in carrier
1	1(1v)	0(-1v)	S_1	π/4
2	0(-1v)	0(-1v)	S_2	3π/4
3	0(-1v)	1(1v)	S_3	5π/4
4	1(1v)	1(1v)	S_4	7π/4

Hence as shown in Table 1.1, there are four symbols and the phase is shifted by π/4 for each symbol.

1.2 GENERATION OF QPSK

Figure 1.1 shows the block diagram of QPSK transmitter. Here, the input binary sequence is first converted to a bipolar NRZ type of signal. This signal is denoted by b(t). It represents binary ‘1' by + 1 V and binary '0' by - 1 V. This signal has been shown in figure 1.2(a). The demultiplexer divides b(t) into two separate bit streams of the odd numbered and even numbered bits. Here, b_e (t) represents even numbered sequence and b_0 (t) represents odd numbered sequence. The symbol duration of both of these odd and even numbered sequences is $2T_b$. Hence, each symbol consists of two bits. Figure 1.2(b) and (c) illustrate the waveform of $b_e(t)$ and $b_0(t)$ It may be observed that the first even bit occurs after the first odd bit. Hence, even numbered bit sequence $b_e(t)$ starts with the delay of one bit period due to first odd bit. Thus, first symbol of $b_e(t)$ is delayed by one bit period 'T_b' with respect to first symbol of $b_0(t)$. This delay of T_b is known as offset. This shows that the change in levels of $b_e(t)$ and $b_0(t)$ cannot occur at the same time due to offset or staggering.

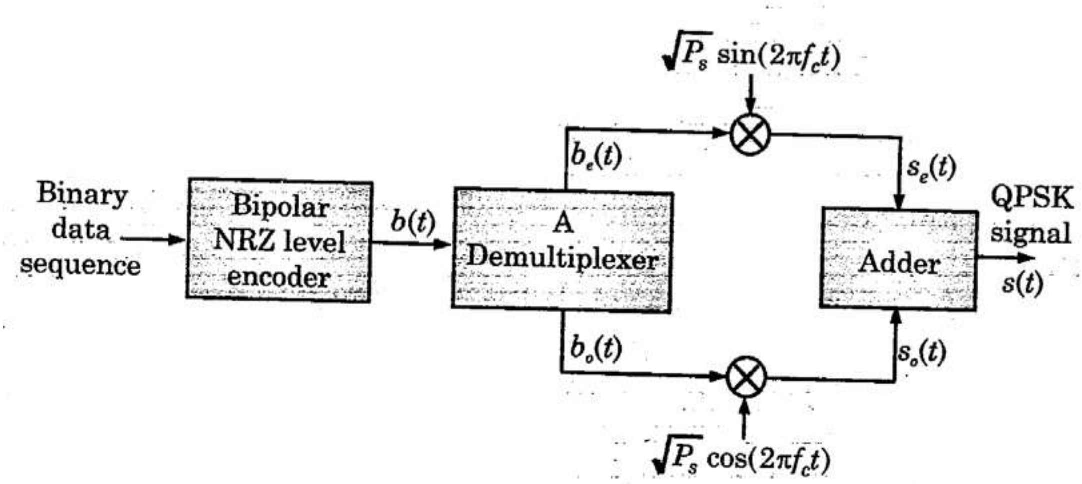

Fig. 1.1 Generation of QPSK.

Also , the bit steam $b_e(t)$ modulates carrier $\sqrt{P_s}\cos(2\pi f_c t)$ and $b_0(t)$ modulates $\sqrt{P_s}\sin(2\pi f_c t)$. These modulates are the balanced modulators. The two carriers $\sqrt{P_s}\cos(2\pi f_c t)$ and $\sqrt{P_s}\sin(2\pi f_c t)$ have been shown in figure 1.2(d) and (e) there carriers are also known as **quadrature carriers** .

The two modulated signals can be written as ,

$$s_e(t) = b_e(t)\sqrt{P_s}\,\sin(2\pi f_c t)$$

And

$$s_0(t) = b_0(t)\sqrt{P_s}\,\cos(2\pi f_c t)$$

Hence, s_e (t) and $s_0(t)$ are basically BPSK signals. The only difference is that $T = 2T_b$ here. The value of $b_e(t)$ and $b_0(t)$ would be + 1Vor - 1V. Figure 1.2 (f) and (g) shows the waveforms of 1.2 here. $S_e(t)$ and $s_0(t)$. The adder in **figure 1.1 adds these two signals $b_e(t)$ and $b_0(t)$.**

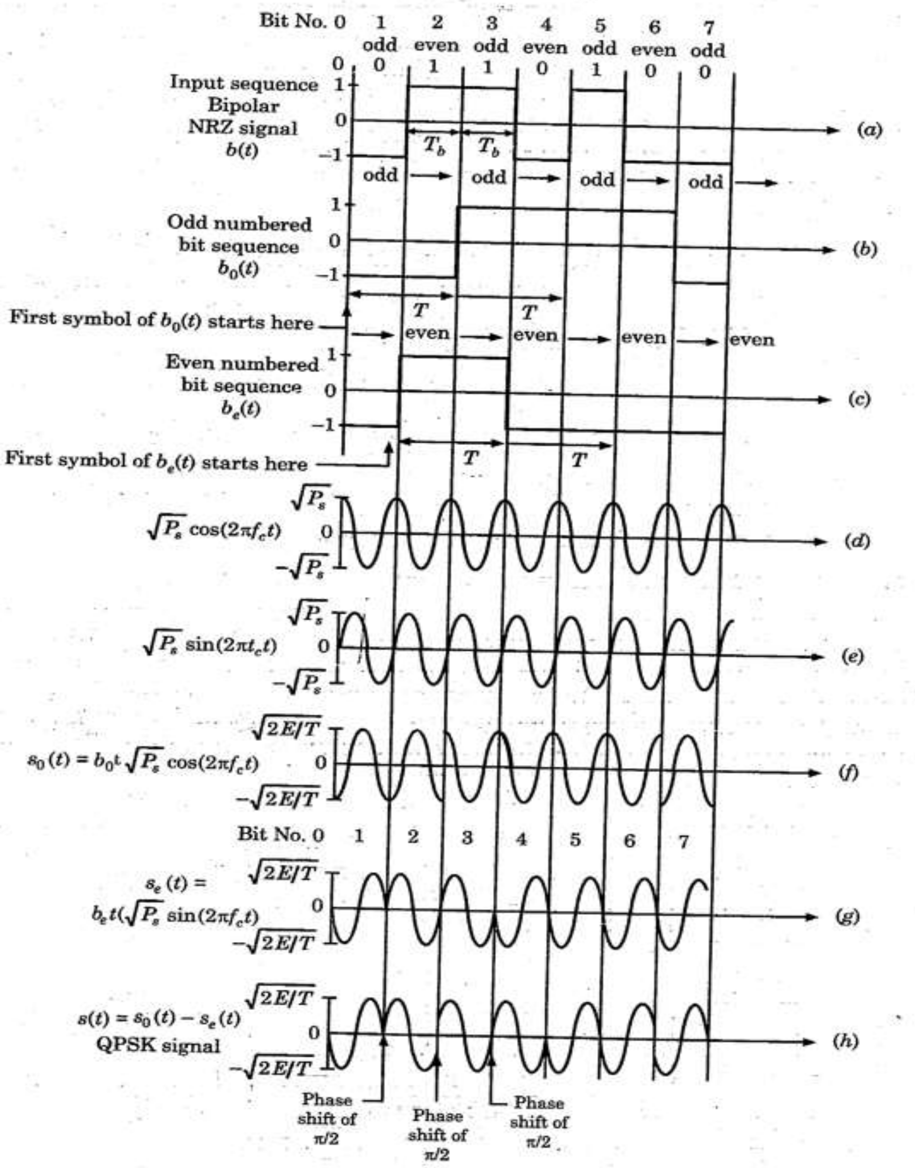

Fig. 1.2 QPSK waveforms, (a) Input sequence and its corresponding NRZ waveform, (b) Odd numbered bit sequence and its corresponding waveform (c) Even numbered bit sequence and its NRZ waveform (d) Basis function $f_1(t)$ (e) Basis function $f_2(t)$(f) Binary PSK waveform for odd numbered channel (g) Binary PSK waveform for even numbered channel (h) Final QPSK waveform.

The output of the adder is QPSK signal and it is given by,

$$s(t) = s_0(t) + s_e(t)$$

Or

$$s(t) = b_0(t)\sqrt{P_s}\,cos(2\pi f_c t) + b_e(t)\sqrt{P_s}\,sin(2\pi f_c t)$$

Figure 1.2(h) shows the QPSK signal represented by following equation In QPSK signal in figure 1.2(h), if there is any phase change, it occurs at minimum duration of T_b This is because the two signals $s_e(t)$ and $s_0(t)$ have an offset of 'T_b'. Due to this offset, the phase shift in QPSK signal is $\frac{\pi}{2}$.

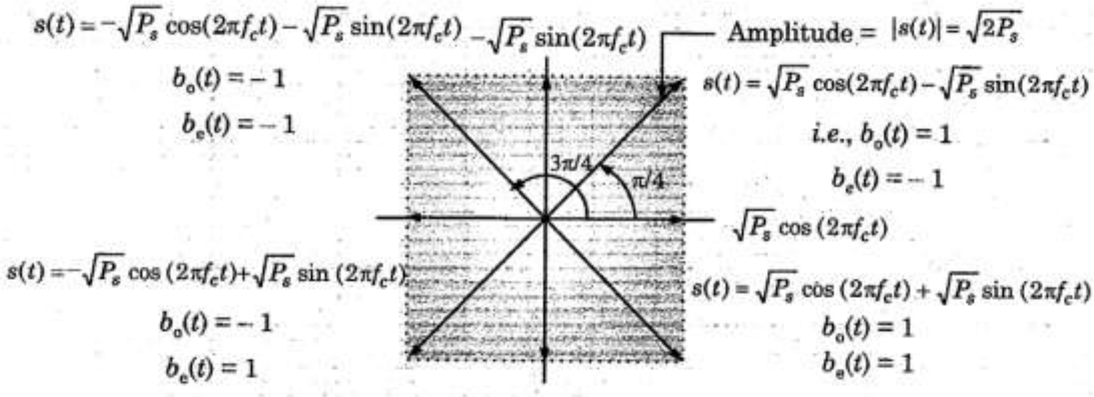

Fig. 1.3 The Phasor diagram of QPSK signal.

1.3 Reception of QPSK (i.e. Detection of QPSK)

Figure 1.4 shows the QPSK receiver. This is synchronous reception. Hence, the coherent carrier is to be recovered from the received signal s(t). The received signal s(t) is first raised to its 4th power, i.e., $s^4(t)$. After that, it is allowed to pass through a band pass filter (BPF) which is centered around $4f_c$. The output of the band pass filter is a coherent carrier of frequency $4f_c$ This is divided by 4 and it provides two coherent quadrature carriers, i.e., cos ($2\pi f_c$ t) and sin ($2\pi f_c$ t). These coherent carriers are applied to two synchronous demodulators. These synchronous demodulators consist of multiplier and an integrator.*

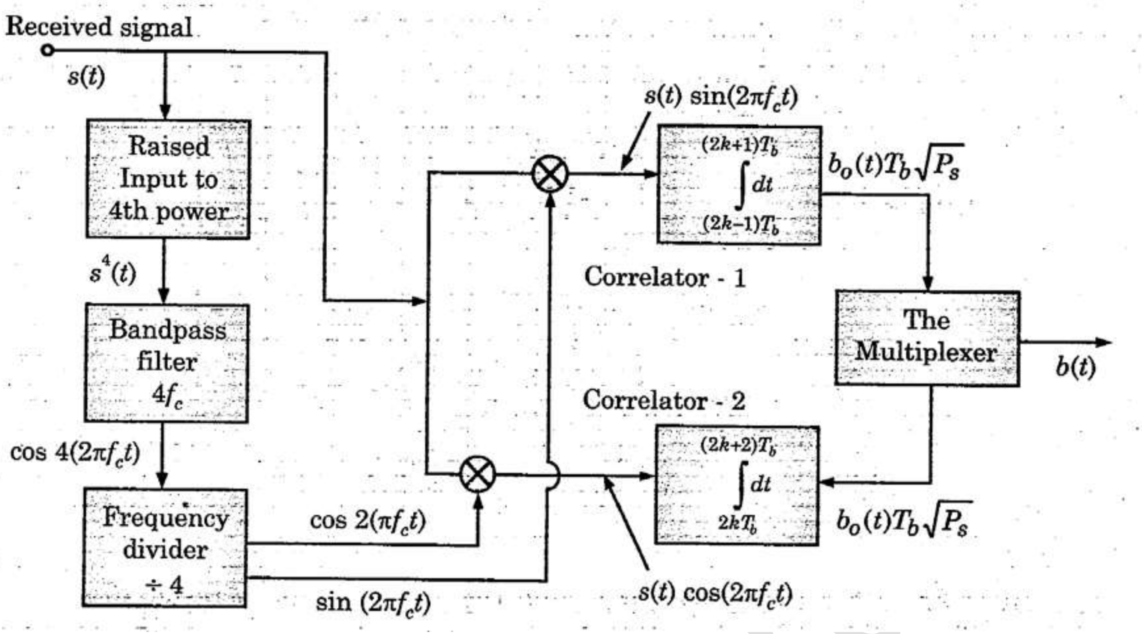

Fig. 1.4 Reception of QPSK.

The incoming signal is applied to both the multipliers. Here, the integrator integrates the product signal over two bit interval (i.e., $T_s = 2T_b$). At the end of this period, the output of integrator is sampled. The outputs of the two integrators are sampled at the offset of one bit period, T_b . Hence, the output of the multiplexer is the signal b(t). This means that the odd and even sequences are combined by multiplexer.

Now, let us consider the product signal at the output of upper multiplier, i.e.,

$$s(t)\,\sin\,(2\pi f_c\ t)\ =\ b_0(t)\,\sqrt{P_s}\,\cos(2\pi f_c t)\,\sin(2\pi f_c t) + b_e(t)\,\sqrt{P_s}\,\sin^2(2\pi f_c t)$$

This signal is integrated by the upper integrator in figure 1.4. Therefore, we have

$$\int_{(2k-1)T_b}^{(2k+1)\,T_b} s(t)\sin(2\pi f_c t)\,dt\ =\ b_0(t)\,\sqrt{P_s}\int_{(2k-1)\,T_b}^{(2k+1)\,T_b}\cos(2\pi f_c t)\,\sin(2\pi f_c t)\,dt$$

$$+\,b_e(t)\,\sqrt{P_s}\int_{(2k-1)\,T_b}^{(2k+1)\,T_b}\sin^2(2\pi f_c t)\,dt$$

Now, since $\frac{1}{2}\sin\,(2x)\ = \sin x\cdot\cos x$

and $\sin^2\,(x) = \frac{1}{2}[1-\cos\,(2x)]$

Therefore, using these two trigonometric identities in equation ,we get

$$\int_{(2k-1)T_b}^{(2k+1)T_b} s(t)\sin(2\pi f_c t)\,dt = \frac{b_o(t)\sqrt{P_s}}{2}\int_{(2k-1)T_b}^{(2k+1)T_b} \sin 4\pi f_c t\,dt + \frac{b_e(t)\sqrt{P_s}}{2}\int_{(2k-1)T_b}^{(2k+1)T_b} 1\cdot dt$$

$$-\frac{b_e(t)\sqrt{P_s}}{2}\int_{(2k-1)T_b}^{(2k+1)T_b} \cos 4\pi f_c t\,dt$$

In this equation, the first and third integration terms involve integration of sinusoidal carriers over two bit period. They have full (integral number of) cycles over two bit periods and thus integration will be zero, i.e.,

$$\int_{(2k-1)T_b}^{(2k+1)T_b} s(t)\sin(2\pi f_c t)\,dt = \frac{b_e(t)\sqrt{P_s}}{2}[t]_{(2k-1)T_b}^{(2k+1)T_b} = \frac{b_e(t)\sqrt{P_s}}{2}\times 2T_b = b_e(t)\sqrt{P_s}T_b$$

Hence, the upper integrator responds to even sequence only. Similarly, we can obtain the output of lower integrator as $b_0(t)\sqrt{P_s}T_b$.

1.4. Concept of Carrier Synchronization in QPSK

Both the carriers are to be synchronized properly in coherent detection in QPSK. Figure 1.5 shows the PLL system for carrier synchronization in QPSK.

The fourth power of the input signal consists of discrete frequency component at $4f_c$ We know that,

$$cos4(2\pi f_c t) = \cos(8\pi f_c t + 2\pi N)$$

where 'N' is the number of cycles over the bit period. It is always an integer value.

When the frequency division by four takes place, the RHS of this equation becomes $\left[2\pi f_c t + \frac{N\pi}{2}\right]$

indicates that the output has a fixed phase error of $\frac{N\pi}{2}$ Differential encoding can be used to nullify the phase error events. The PLL remains locked with the phase of '$4f_c$' and then output of PLL is divided by 4. This provides a coherent carrier. A 90° phase shift is added to this carrier to produce a quadrature carrier.

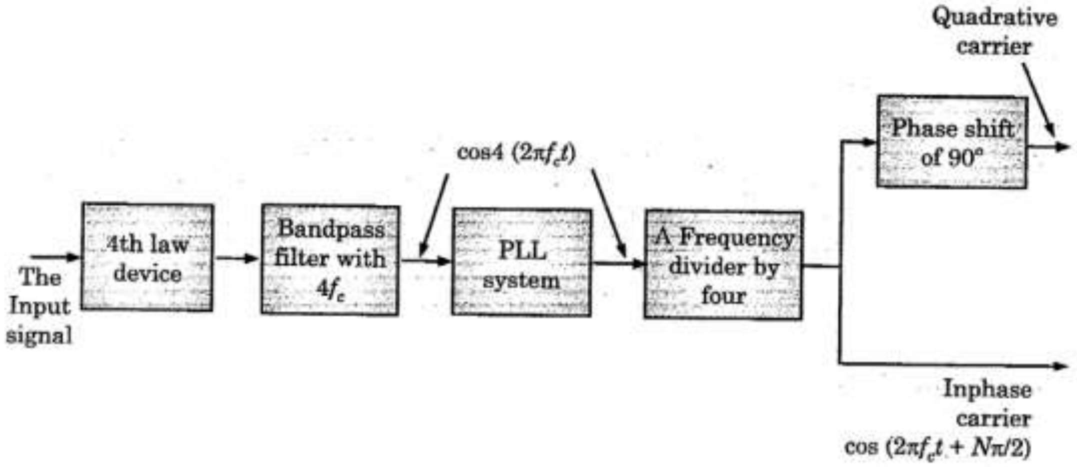

Fig. 1.5 PLL system used for carrier synchronization in QPSK.

1.5. Signal Space Representation in QPSK Signals

Figure 1.6 shows the phasor diagram of QPSK signal. Depending upon the combination of two successive bits, the phase shift occurs in carrier. This means that the QPSK signal can be written as,

$$s(t) = \sqrt{2P_s}\cos\left[2\pi f_c t + (2m+1)\frac{\pi}{4}\right] \qquad m = 0,\ 1,\ 2,\ 3$$

Here, above expression takes four values and they represent the phasors of figure 1.6. This equation can be expanded as under:

$$s(t) = \sqrt{2P_s}\cos(2\pi f_c t)\cos\left[(2m+1)\frac{\pi}{4}\right] - \sqrt{2P_s}\sin(2\pi f_c t)\sin\left[(2m+1)\frac{\pi}{4}\right]$$

Let us rearrange the above equation as under:

$$s(t) = \left\{\sqrt{P_s T_s}\cos\left[(2m+1)\frac{\pi}{4}\right]\right\}\sqrt{\frac{2}{T_s}}\cos(2\pi f_c t) - \left\{\sqrt{P_s T_s}\sin\left[(2m+1)\frac{\pi}{4}\right]\right\}\sqrt{\frac{2}{T_s}}\sin(2\pi f_c t)$$

Again, let $\qquad \phi_1(t) = \sqrt{\frac{2}{T_s}}\cos(2\pi f_c t)$

and $\phi_2(t) = \sqrt{\frac{2}{T_s}} \sin(2\pi f_c t)$

These two signals are known as orthogonal signals and they are used as carriers in QPSK modulator.

Let $b_0(t) = \sqrt{2} \cos\left[(2m+1)\frac{\pi}{4}\right]$

and $b_e(t) = -\sqrt{2} \sin\left[(2m+1)\frac{\pi}{4}\right]$

With the use of equations (7.57) to (7.60) we can write equation (7.56) as under:

$$s(t) = \sqrt{P_s T_s} \cdot \frac{1}{\sqrt{2}} b_0(t)\, \phi_1(t) + \sqrt{P_s T_s} \cdot \frac{1}{\sqrt{2}} b_e(t)\, \phi_2(t)$$

or $$s(t) = \sqrt{P_s \cdot \frac{T_s}{2}}\, b_0(t)\, \phi_1(t) + \sqrt{P_s \frac{T_s}{2}}\, b_e(t)\, \phi_2(t)$$

T_s = symbol duration and $T_s = 2T_b$

or $T_b = \frac{T_s}{2}$

Then the above equation becomes,

$$s(t) = \sqrt{P_s T_b}\, b_0(t)\, \phi_1(t) + \sqrt{P_s T_b} b_e(t)\, \phi_2(t)$$

Since bit energy $E_b = P_s T_b$

Therefore, $s(t) = \sqrt{E_b}\, b_0(t)\, \phi_1(t) + \sqrt{E_b} b_e(t)\, \phi_2(t)$

This equation gives signal space representation of QPSK signal. The two orthogonal signals ϕ_1 (t) and ϕ_2 (t) form the two axes of the signal space. Figure 1.6 shows the signal space representation. The possible 4 signal points have been shown by small circles on ϕ_1 ϕ_2 -plane.

From each signal point, we obtain two bits. For example, from point 'A', we obtain two bits as (1, 1) and from 'B' we obtain bits as (1, 1). The distance of any signal point from origin 'O', given as,

Distance $'OB' = \sqrt{P_s T_b + P_s T_b} = \sqrt{2 P_s T_b} = \sqrt{P_s T_s}$ $[\because 2T_b = T_s]$

or $'OB' = \sqrt{E_s}$ $[\because P_s T_s = E_s]$

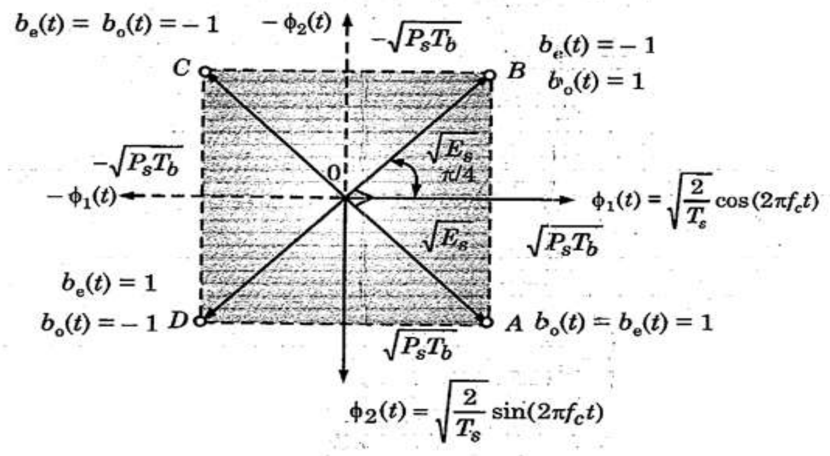

Fig. 1.6 The signal space representation for QPSK signals.

Hence, the length of each signal point from origin is $\sqrt{E_s}$. We know that b_e (t) and b_0 (t) represent two successive bits. There is an offset of 'T_b' between b_e (t) and b_0 (t). Therefore, b_e (t) and b_0 (t) both cannot change their levels simultaneously. Hence, either b_e (t) or b_0 (t) can change at a time. Let us say that b_e (t) = b_0 (t) = 1 is representing signal point 'A' in figure 7.29. In the next bit interval, if b_0 (t) = − 1, then signal point will be 'D'. Otherwise, if b_e (t) changes its level [i.e., b_e (t) = − 1], then next signal point will be 'B'. Hence, from signal point 'A', then next signal points will be either 'D' or 'B'.

1.6. Distance Between Signal Points

As a matter of fact, the ability to determine a bit without error is measured by the distance between two nearest possible signal points in the signal space. Such points differed in signal bit. For example, signal points 'A' and 'B' are two nearest points since they differ by a signal bit b_e (t). As 'A' and 'B' become closer to each other, the possibility of error increases. Therefore, this distance must be as large as possible. This distance is denoted by 'd'. In figure 1.6, the distance between signal points 'A' and 'B' can be given by,

$$d^2 = \left(\sqrt{E_s}\right)^2 + \left(\sqrt{E_s}\right)^2 = \sqrt{2E_s}$$

or

$$d = 2\sqrt{P_s T_b} = 2\sqrt{E_b}$$

1.7. Spectrum of QPSK Signal

The input sequence b(t) is of bit duration T_b . Also, it is a NRZ bipolar waveform. Recall, the power spectral density of such waveform can be given as,

$$S(f) = V_b^2\, T_b \left[\frac{\sin(\pi f T_b)}{\pi f T_b}\right]^2$$

Also, $V_b = \sqrt{P_s}$, then this equation becomes,

$$S(f) = P_s T_b \left[\frac{\sin(\pi f T_b)}{\pi f T_b}\right]^2$$

This equation gives power spectral density (psd) of signal b(t). This signal is divided into b_e (t) and b_0 (t) each of bit period $2T_b$. If we consider that symbols 1 and 0 are equally likely, then we can write power spectral densities (psds) of b_e (t) and b_0 (t) as,

$$S_e(f) = P_s T_s \left[\frac{\sin(\pi f T_s)}{\pi f T_s}\right]^2$$

and

$$S_0(f) = P_s T_s \left[\frac{\sin(\pi f T_s)}{\pi f T_s}\right]^2$$

In these two equations, we have just replaced T_b by T_s , and T_s is the period of bit in b_e (t) and b_0 (t). Because, in phase and quadrature components [b_e (t) and b_0 (t)] are statistically independent, the baseband power spectral density of QPSK signal equals the sum of the individual power

spectral densities of b_e (t) and b_0 (t) i.e.,

$$S_B(f) = S_e(f) + S_0(f)$$

or

$$S_B(f) = 2P_sT_s\left[\frac{\sin(\pi f\,T_s)}{\pi f T_s}\right]^2$$

This equation gives baseband power spectral density of QPSK signal. Upon modulation of carrier frequency f_c , the spectral density given by above equation is shifted at ±f. Thus plots of power spectral density of QPSK will be similar to that BPSK.

1.8. Bandwidth of QPSK Signal

We have observed that the bandwidth of BPSK signal is equal of $2f_b$. Here, $T_b = \frac{1}{f_b}$ is the one bit period. In QPSK, the two waveforms b_e (t) and b_0 (t) form the baseband signals. One bit period for both of these signals is equal to $2T_b$. Therefore, bandwidth of QPSK signal will be

$$\boldsymbol{BW = 2 \times \frac{1}{2T_b} = f_b}$$

Hence, the bandwidth of QPSK signal

is half of the bandwidth of BPSK signal. Earlier, we have observed that noise immunity of QPSK and BPSK is same. This shows that inspite of the reduction in bandwidth in QPSK, the noise immunity remains same as compared to BPSK. BW of QPSK can also be obtained from figure 1.7

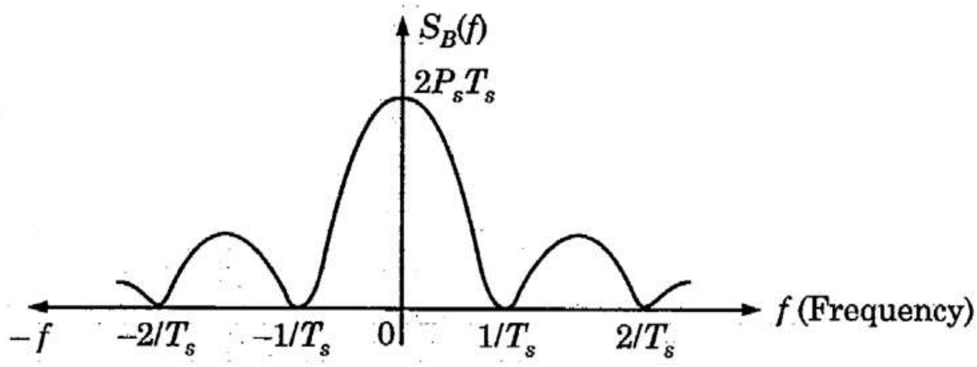

Fig. 1.7 Plot of power spectral density (psd) of QPSK signal.

$$BW = \text{Highest frequency} - \text{Lowest frequency in main lobe}$$

$$BW = \frac{1}{T_s} - \left(-\frac{1}{T_s}\right) \text{ since carrier frequency } f_c \text{ cancels our}$$

$$BW = \frac{2}{T_s}$$

We know that $T_s = 2T_b$

or $BW = \frac{2}{2T_b} = \frac{1}{T_b} = f_b$

1.9. Advantages of QPSK

QPSK has some certain advantages as compared to BPSK and DPSK as under:

(i) For the same bit error rate, the bandwidth required by QPSK is reduced to half as compared to BPSK.

(ii) Because of reduced bandwidth, the information transmission rate of QPSK is higher.