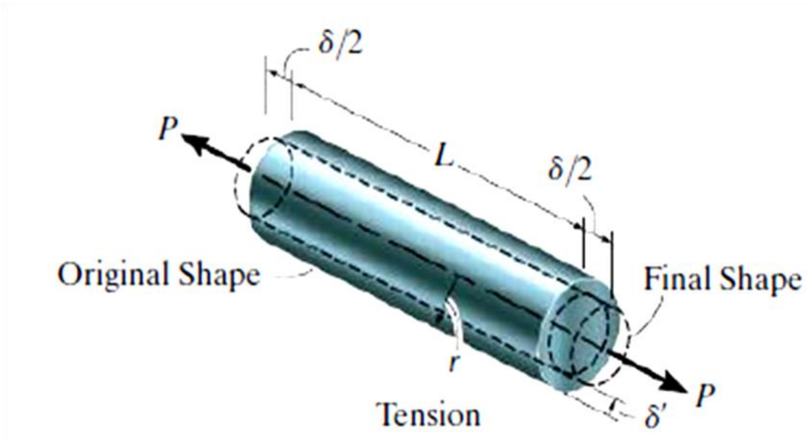
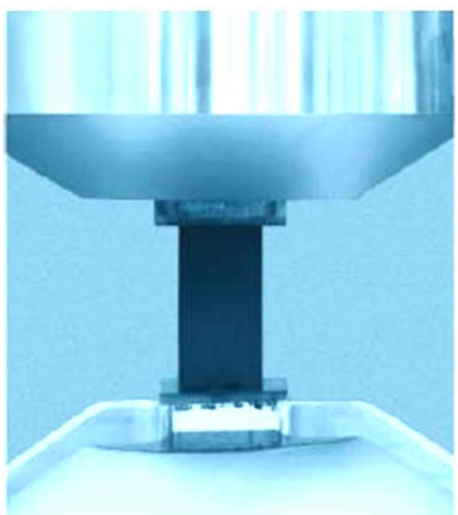
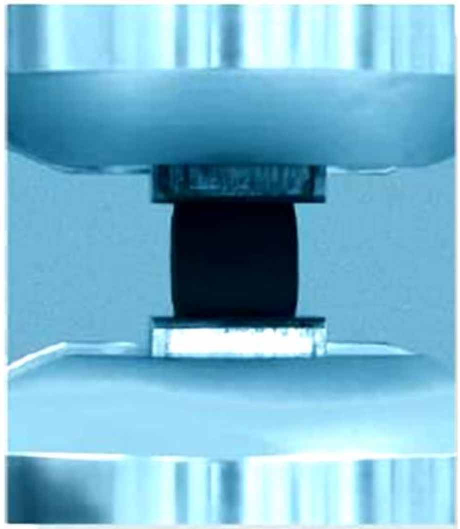
1- Poisson's Ratio

When a deformable body is subjected to an axial tensile force, not only does it elongate but it also contracts laterally.
Consider a bar having an original radius r and length L and subjected to the tensile force P in Figure.





(a) The bar subjected to longitudinal elongation (positive strain).



(b) The bar subjected to the lateral contraction (negative strain).

This force elongates the bar by an amount δ, and its radius contracts by an amount δ'.

Strains in the longitudinal or axial direction is,

$$\epsilon_{\text{long}} = \frac{\delta}{L}$$

And the lateral or radial direction is,

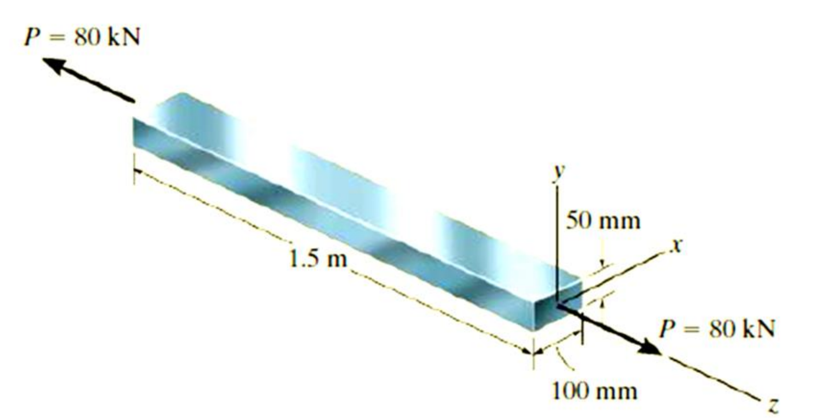
$$\epsilon_{\text{lat}} = \frac{\delta'}{r}$$

In the early 1800s, the French scientist S. D. Poisson realized that within the *elastic range* the *ratio* of these strains is a *constant*, since the deformations δ and δ' are proportional. This constant is referred to as ***Poisson's ratio,*** ϑ (nu), and it has a numerical value that is unique for a particular material that is both ***homogeneous and isotropic***. Stated mathematically it is,

$$\nu = -\frac{\epsilon_{\text{lat}}}{\epsilon_{\text{long}}} \qquad (1)$$

The **negative sign** is included here since ***longitudinal elongation*** (positive strain) causes ***lateral contraction*** (negative strain), and vice versa.
Poisson's ratio is a ***dimensionless*** quantity, and for most nonporous solids it has a value that is generally between ¼ and 1/3.
For an "ideal material" having no lateral deformation when it is stretched or compressed, Poisson's ratio will be 0. The *maximum* possible value for Poisson's ratio is 0.5.
Therefore **$0 <= \vartheta <= 0.5$**.

Example:
A bar made of A-36 steel has the dimensions shown in Fig. If an axial force of P = 80 KN is applied to the bar, determine the change in its length and the change in the dimensions of its cross section after applying the load. The material behaves elastically. Where $\nu_{st} = 0.32$, E_{st} = 200 GPa.



SOLUTION

The normal stress in the bar is

$$\sigma_z = \frac{P}{A} = \frac{80(10^3)\ \text{N}}{(0.1\ \text{m})(0.05\ \text{m})} = 16.0(10^6)\ \text{Pa}$$

From the table for A-36 steel E_{st} = 200 GPa, and so the strain in the z direction is,

$$\epsilon_z = \frac{\sigma_z}{E_{st}} = \frac{16.0(10^6)\ \text{Pa}}{200(10^9)\ \text{Pa}} = 80(10^{-6})\ \text{mm/mm}$$

The axial elongation of the bar is therefore,

$$\delta_z = \epsilon_z L_z = [80(10^{-6})](1.5\text{ m}) = 120\,\mu\text{m} \qquad \textit{Ans.}$$

Using Eq. 1, where $\vartheta_{st} = 0.32$ as found from the table, the lateral contraction strains in *both* the *x* and *y* directions are,

$$\epsilon_x = \epsilon_y = -\nu_{st}\,\epsilon_z = -0.32[80(10^{-6})] = -25.6\,\mu\text{m/m}$$

Thus the changes in the dimensions of the cross section are,

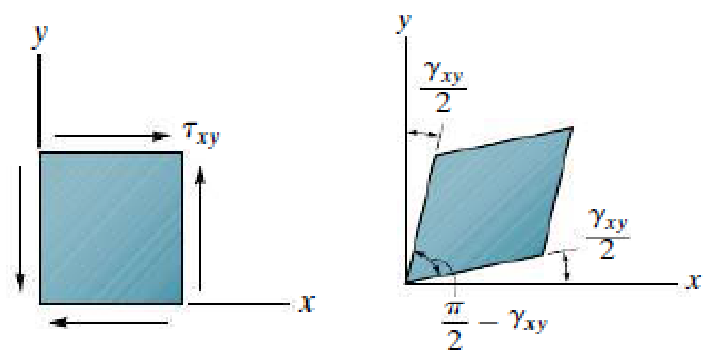
$$\delta_x = \epsilon_x L_x = -[25.6(10^{-6})](0.1\text{ m}) = -2.56\,\mu\text{m} \qquad \textit{Ans.}$$

And,

$$\delta_y = \epsilon_y L_y = -[25.6(10^{-6})](0.05\text{ m}) = -1.28\,\mu\text{m} \qquad \textit{Ans.}$$

2- The Shear Stress–Strain Diagram

When a small element of material is subjected to *pure shear*, equilibrium requires that equal shear stresses must be developed on four faces of the element. These stresses $\boldsymbol{\tau_{xy}}$ must be directed toward or away from diagonally opposite corners of the element, as shown in Fig. *a*. Furthermore, if the material is *homogeneous* and *isotropic*, then this shear stress will distort the element uniformly, Fig. *b*.



(a) (b)

An example of such a diagram for a ductile material is shown in Fig. 3. Like the tension test, this material when subjected to shear will exhibit linear-elastic behavior and it will have a defined *proportional limit* τ_{pl}. Also, strain hardening will occur until an *ultimate shear stress* τ_u is reached. And finally, the material will begin to lose its shear strength until it reaches a point where it *fractures*, τ_f.

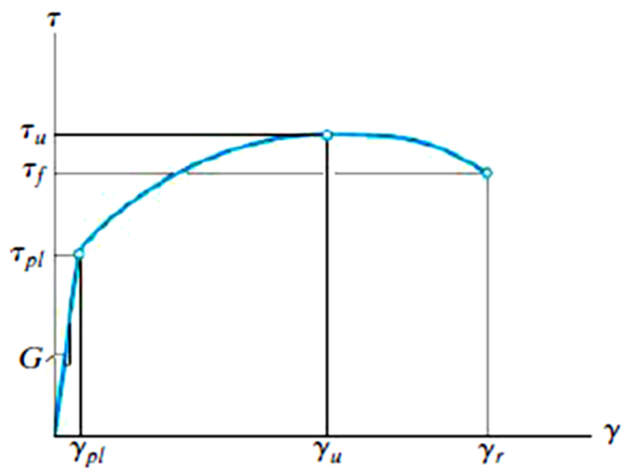


Fig. (3)

For most engineering materials, like the one just described, the elastic behavior is ***linear***, and so Hooke's law for shear can be written as,

$$\tau = G\gamma$$

Here G is called the ***shear modulus of elasticity*** or the ***modulus of rigidity***. Its value represents the slope of the line on the $\tau - \gamma$ diagram, that is, $G = \tau_{pl} / \gamma_{pl}$.

Notice that the units of measurement for G will be the *same* as those for τ (Pa or psi), since g is measured in radians, a dimensionless quantity.

It will be given that the three material constants, E, ν, and G are actually *related* by the equation,

$$G = \frac{E}{2(1 + \nu)}$$

Provided E and G are known, the value of ν can then be determined from this equation rather than through experimental measurement. For example, in the case of A-36 steel, $E_{st} = 29(10^3)$ ksi and $G_{st} = 11(10^3)$ ksi.

So that, $\nu_{st} = 0.32$.

3-Failure of Materials Due to Creep and Fatigue

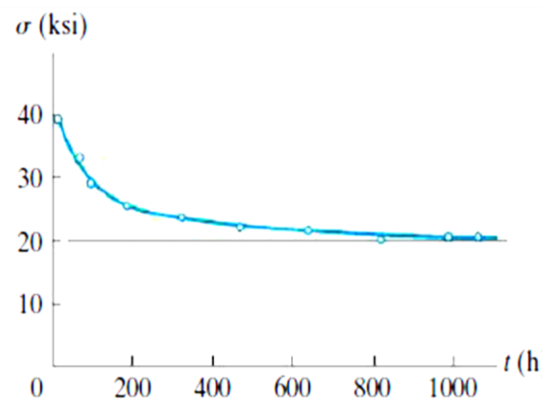
The mechanical properties of a material have up to this point been discussed only for a static or slowly applied load at constant temperature. In some cases, however, a member may have to be used in an environment for which loadings must be sustained over long periods of time at elevated temperatures, or in other cases, the loading may be repeated or cycled. We will not consider these effects in this book, although we will briefly mention how one determines a material's strength for these conditions, since they are given special treatment in design.

Creep:

When a material has to support a load for a very long period of time, it may continue to deform until a sudden fracture occurs or its usefulness is impaired. This time-dependent permanent deformation is known as creep. In the general sense, therefore, both stress and/or temperature play a significant role in the rate of creep.

For practical purposes, when creep becomes important, a member is usually designed to resist a specified creep strain for a given period of time. An important mechanical property that is used in this regard is called the creep strength.

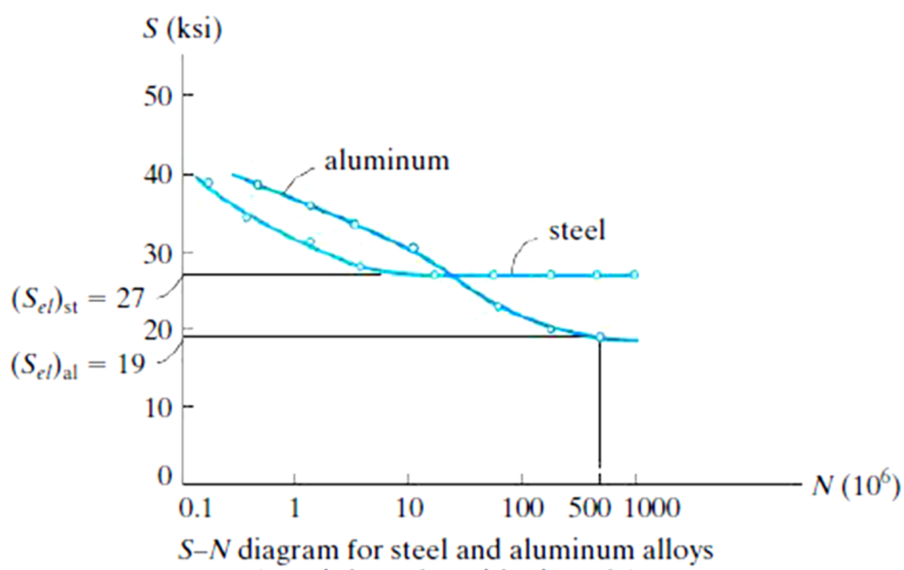
An example of the results for stainless steel at a temperature of 1200°F and prescribed creep strain of 1% is shown in Fig. below. As noted, this material has a yield strength of 40 ksi (276 MPa) at room temperature (0.2% offset) and the creep strength at 1000 h is found to be approximately σ_c = 20 ksi (138 MPa).



σ–t diagram for stainless steel at 1200°F and creep strain at 1%

Fatigue:

When a metal is subjected to repeat cycles of stress or strain, it causes its structure to break down, ultimately leading to fracture. This behavior is called fatigue, In order to specify a safe strength for a metallic material under repeated loading, it is necessary to determine a limit below which no evidence of failure can be detected after applying a load for a specified number of cycles. This limiting stress is called the endurance or fatigue limit. Using a testing machine for this purpose, a series of specimens are each subjected to a specified stress and cycled to failure. The results are plotted as a graph representing the stress S (or σ) on the vertical axis and the number of cycles-to-failure N on the horizontal axis. This graph is called an S–N diagram or stress–cycle diagram,



S–N diagram for steel and aluminum alloys
(*N* axis has a logarithmic scale)

Important Points

- *Poisson's ratio*, ν, is a ratio of the lateral strain of a homogeneous and isotropic material to its longitudinal strain. Generally these strains are of opposite signs, that is, if one is an elongation, the other will be a contraction.

- The *shear stress–strain diagram* is a plot of the shear stress versus the shear strain. If the material is homogeneous and isotropic, and is also linear elastic, the slope of the straight line within the elastic region is called the modulus of rigidity or the shear modulus, G.

- There is a mathematical relationship between G, E, and ν.

- *Creep* is the time-dependent deformation of a material for which stress and/or temperature play an important role. Members are designed to resist the effects of creep based on their material creep strength, which is the largest initial stress a material can withstand during a specified time without exceeding a specified creep strain.

- *Fatigue* occurs in metals when the stress or strain is cycled. This phenomenon causes brittle fracture of the material. Members are designed to resist fatigue by ensuring that the stress in the member does not exceed its *endurance* or *fatigue limit*. This value is determined from an S–N diagram as the maximum stress the material can resist when subjected to a specified number of cycles of loading.