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Ground Water Flow

Ground water represents that portion of water under the surface of the earth. It results from infiltration of rain, surface runoff and snow after melting into the soil. Subsequently, this water transports through the soil into the ground water level, where it eventually moves back to surface streams, lakes, rivers or oceans.

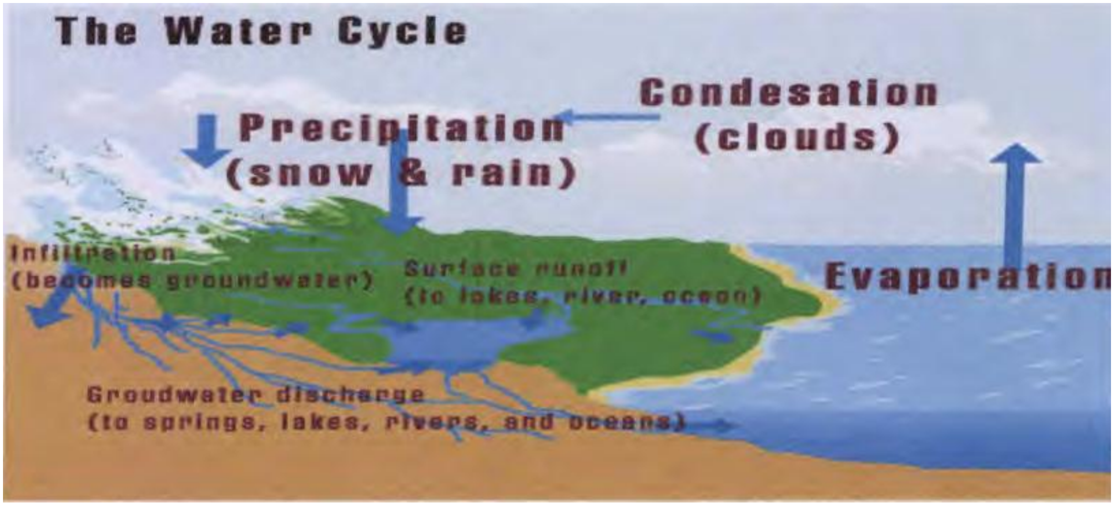

Figure 1: Water cycle in nature

Permeability:
It describes fluid ability to pass through the soil pores and voids. Mostly, permeability depends on the soil and fluid type. For most civil engineering applications, water is the applicable fluid. Therefore, it is important to describe the soil type. The course soil such as sand has high permeability, while the fine soil such as clay soil has low permeability.

Importance of Permeability:

Knowledge of the permeability properties of soil is necessary to:

1. Estimating the quantity of underground seepage.
2. Solving problems involving pumping seepage water from construction excavation
3. Stability analyses of earth structures and earth retaining walls subjected to seepage forces.

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Permeability of porous medium:- depends on

1- The characteristics of the porous medium.
2- The characteristics of the flowing fluid.

The permeability of a medium is measured in terms of **hydraulic conductivity** (also known as **the coefficient of permeability).**

The coefficient of permeability is equal to the volume of water which flow in unit time through unit a cross-sectional area of the medium under a unit hydraulic gradient at the prevailing temperature.
The hydraulic conductivity therefore has the dimensions of (L/T).

Flow of water through Porous media:-

Ground water flows whenever there is an existing of a difference in head between two points. This flow can either be laminar or turbulent. Most often, ground water flows with such a small velocity that the resulting flow is laminar.

The rate of flow (discharge) is measured by using the **Darcy's Law:**

The fundamental premise for Darcy's law to work is:

1- The flow is laminar, no turbulent flows
2- Fully saturated
3- The flow is in steady state, no temporal variation.

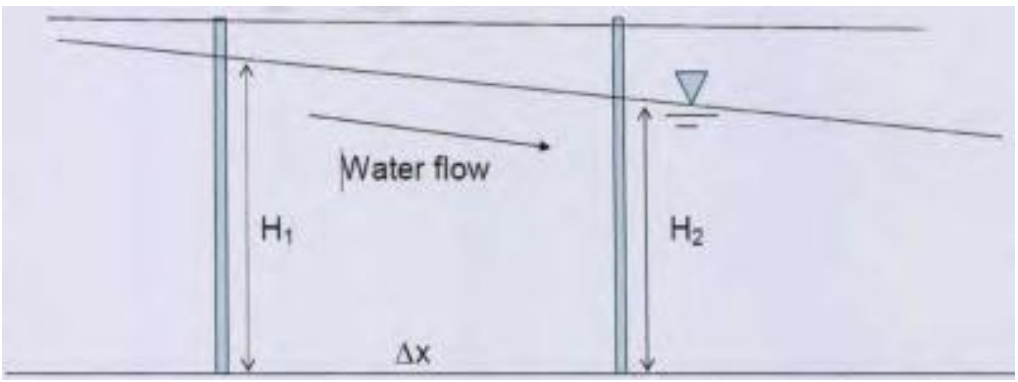

Figure 2: Water flow and hydraulic gradient between two wells

$Q = K.A.(\Delta h/L)$ *(1)*

Where:-

Q: is the rate of flow.

K:is the coefficient of permeability.

A: is the cross-sectional area.

Δh: is the difference in head between two points.

L:is the length between these two points (Δx).

Darcy's Law can be written also as:

$V = k.(\Delta h/L)$ *(2)*

Darcy's law states that how fast the groundwater flow in the aquifer which depends on two parameters:

1- How large is the hydraulic gradient of the water head *(i=dH/dx);* and
2- The parameter describing how permeable the aquifer porous medium.

<u>How to compute the hydraulic conductivity:-</u>

1- In the Lab.
   a. <u>Constant Head permeameter:</u> **the coefficient of permeability** of relatively more permeable soils can be determined in laboratory by the constant — head permeability test. The test is conducted in an instrument known as Constant Head permeameter. Darcy's Law for flow of water through porous Medium (Soil) is applied for computing **the coefficient of permeability,** this method is used to compute the **coefficient of permeability** for granular soils (Sandy Soils , Gravel)

   Where:
   $Q = K.A.(\Delta h/L)$

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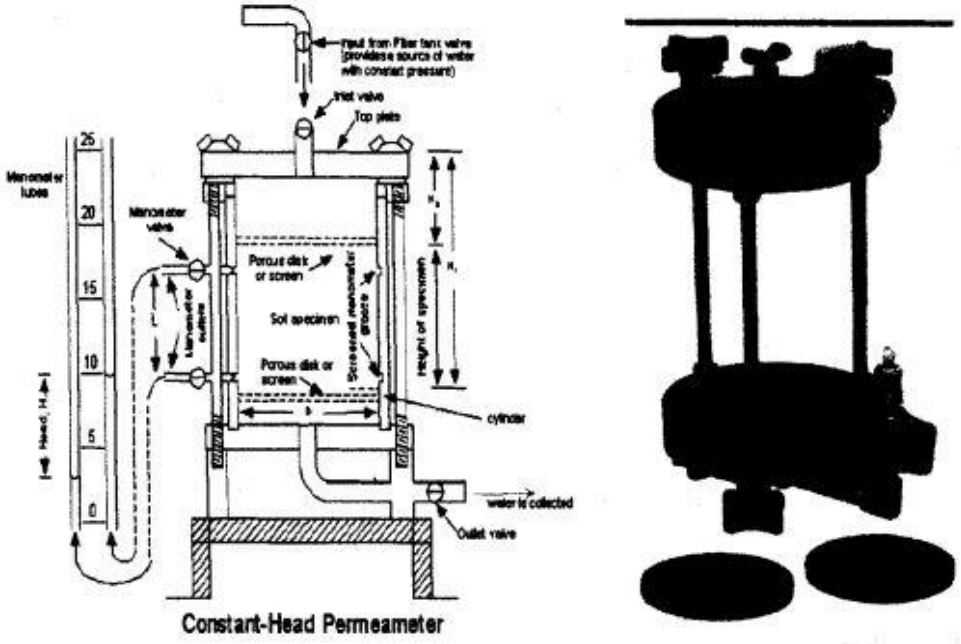

Figure 3: Constant- head permeameter test

b. Variable Head Permeameter: this method is used to compute the **coefficient of permeability** for Fine Texture- Compacted Soils and Low Permeability Soils (silty clay, clay soils).

The device consist of a cylinder attached to a vertical glass tube of small diameter, the cylinder pressed into the soil to a known depth then the whole apparatus filled with water, as the water percolate through the soil in the cylinder the water drop in the tube.

$$adh = -qdt$$

Where:
a: is the cross-sectional area of the tube

$$a\,dh = -(A.\,K.\,i)\,dt$$

Where:
A:is the cross-sectional area of the cylinder.
i=h/L: hydraulic gradient.
L: length of soil Specimen.

$$\boldsymbol{a\,dh = -A.\,K\,(h/L)dt}$$

$$\boldsymbol{(A.\,k.\,dt)/\,(a\,L) = (dh/h)}$$

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Integrating: $(A.k/\ a\ L)\ \int dt = -\,dh/h$

$$(A.\ K\ /a\ L)\ (t_2 - t_1) = \log\ (h_1/h_2)$$

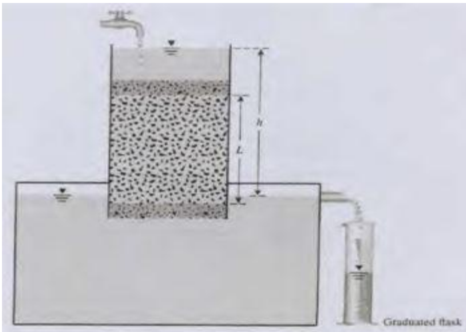
$$K\ =\ (a\,.L\ /A.t)\ \log\ (h1/h2)$$

2- in situ

a. By using an auger to make a hole in the soil with determined diameter and height if the water table with the ground level, computing the time to fill the hole with water and then calculating the coefficient of permeability.

b. if the water table below the ground level (under the end of the hole filling the hole with water, and computing the time to percolate the water through the soil.

Example 1:



Referring to the shown figure, calculate the hydraulic conductivity (K) in m/s. The tap valve was switched on to allow water pass through the medium in the cylinder. Then, the passed water moved to the bottom tank. The collected water in the graduated cylinder was found to be365 cm^3every three min. The water level difference between the top cylinder and the bottom tank was 72 cm. the length of the permeable layer is 46 cm and the area of the cross sectional is 23 cm^2.

Solution:

Given:
Q = Vol./t= 365 cm^3/ (3min*60 sec) = 2.02 cm^3 /s.
i = dh/L

= 72/46 = 1.56

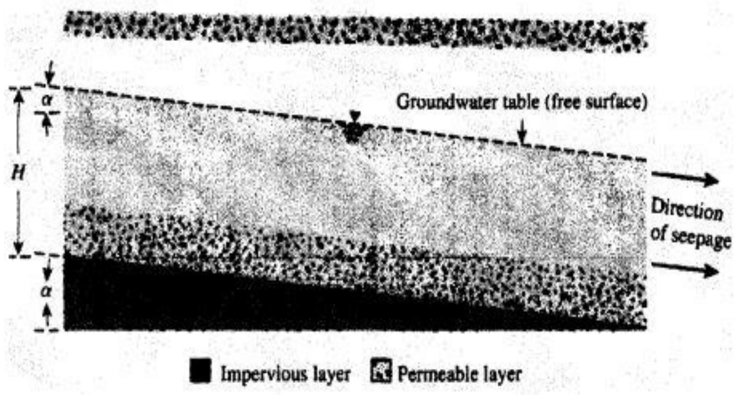
Q = KiA

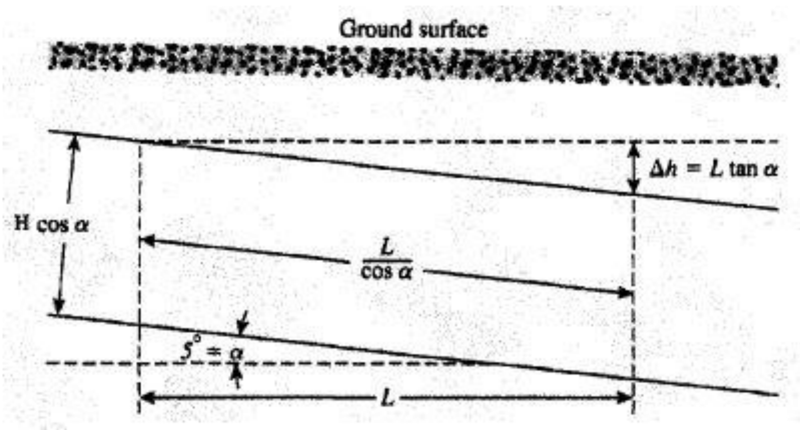
2.02 = K *1.56* 23

= 0.056 cm/s.

Example 2:

A permeable soil layer is underlain by an impervious layer as shown in the figure with K= $4.8*10^{-3}$ cm/s for the permeable layer. Calculate the rate of seepage through this layer cm^3/s/cm width if H is 3 m and α=5^0





Solution:

i= dh/L

= L tan α/ (L/cos α)

= sin α= sin 5

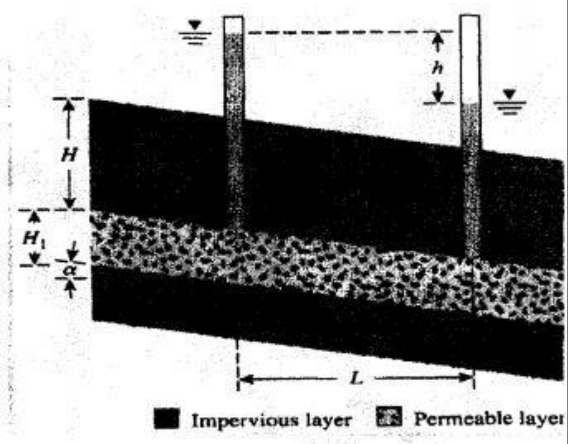
Q = KiA

= $4.8*10^{-3}$ * sin 5 * (3m*100 cm)*COS 5*(lcm)

= 0.125 cm^3/s/cm

Example 3:

Find the flow rate in m^3/s/m length (at right angles to the cross section shown in the figure through the permeable soil layer. Notably, the depth of the impervious layer is 8 m and the permeable layer is 3 m, the water level difference between the two wells is 4 m, the distance between the two wells is 50 m, α=8° and K= $8*10^{-4}$ m/s.



Solution:

i = dh/L

= h/ (L/cos α)

= 4 m/(50 m/cos 8^0) = 0.081 m/s

A of 1 unit length = H1 * cos α

= 3 m * cos 8^0

= 2.97 m^2/m

Q = KiA

= $8*10^{-4}$ *0.081 *2.97

= 1.93 * 10^{-4} m^3/s/m

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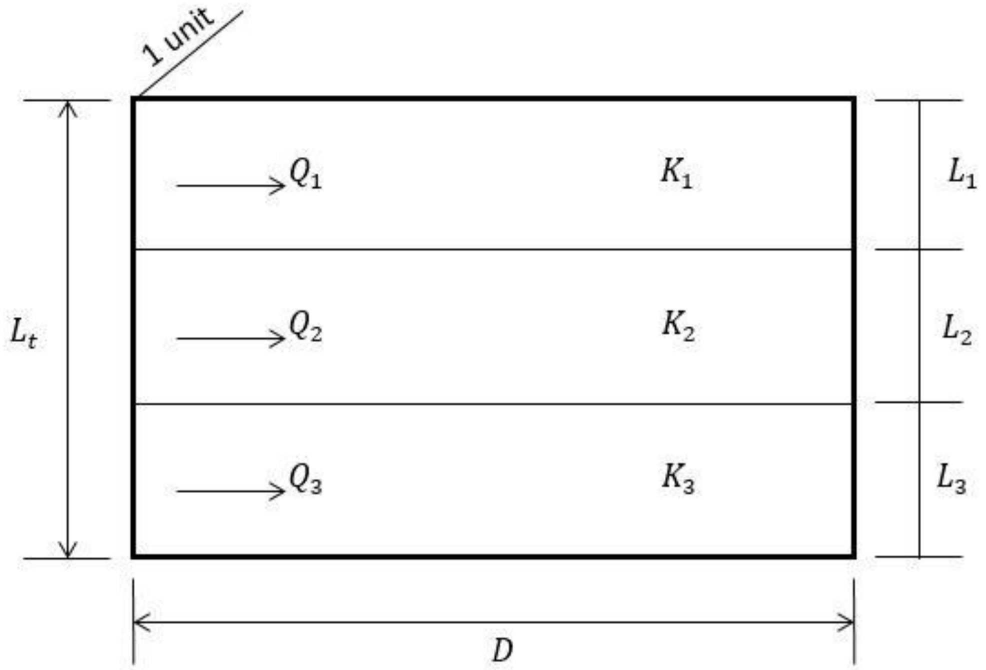
The hydraulic conductivity of multiple layers: -

1- multiple horizontal layers:



$Q = KiA$

$Q_1 = K_1 i_1 A_1$

$Q_2 = K_2 i_2 A_2$

$Q_3 = K_3 i_3 A_3$

$Q_t = Q_1 + Q_2 + Q_3$

$A = (L) \times (1)$

$A_t = (L_t) \times (1)$

$A_1 = (L_1) \times (1)$

$A_2 = (L_2) \times (1)$

$A_3 = (L_3) \times (1)$

$L_t = L_1 + L_2 + L_3$

$A_t = A_1 + A_2 + A_3$

$i_t = i_1 = i_2 = i_3$

$\Delta h_t/D = \Delta h_1/D = \Delta h_2/D = \Delta h_3/D$

$Q_t = K_h . A_t . (\Delta h_t/D)$

$Q_1 = K_1 . A_1 . (\Delta h_1/D)$

$Q_2 = K_2 . A_2 . (\Delta h_2/D)$

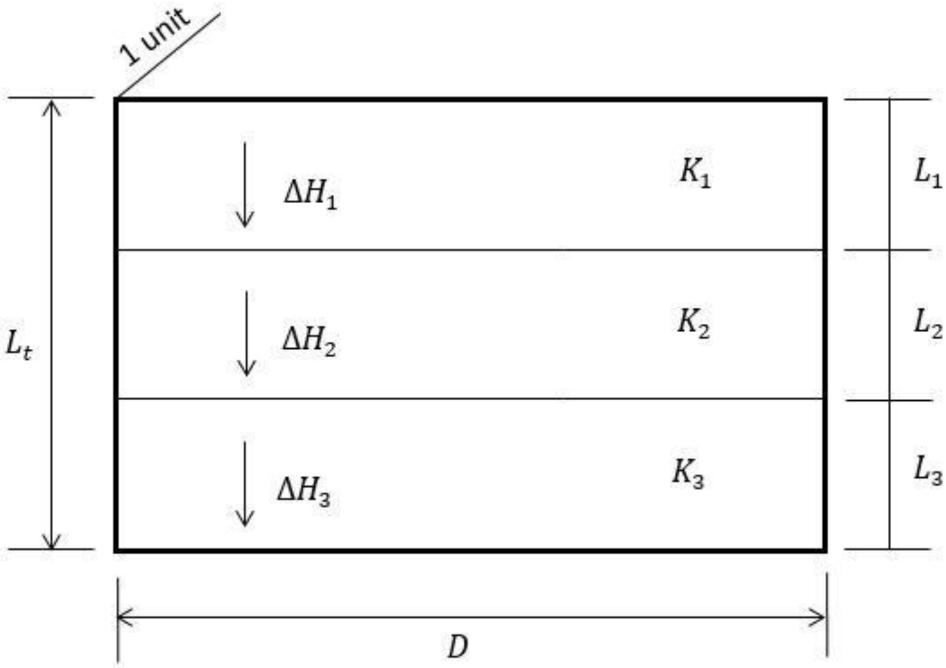
$Q_h = K_3 . A_3 . (\Delta h_3/D)$

$$K_h . A_t . (\Delta h_t/D) = K_1 . A_1 . (\Delta h_1/D) + K_2 . A_2 . (\Delta h_2/D) + K_3 . A_3 . (\Delta h_3/D)$$

$K_h . A_t . = K_1 . A_1 + K_2 . A_2 + K_3 . A_3$

$$K_h = \frac{K_1 . L_1 . + K_2 . L_2 + K_3 . L_3}{L_1 + L_2 + L_3} = \frac{\sum KL}{L}$$

2- Multiple vertical layers:



$\Delta h_t = \Delta h_1 + \Delta h_2 + \Delta h_3$

$Q_t = Q_1 = Q_2 = Q_3$

$Q_t = K_v . A_t . (\Delta h_t/L_t)$,

$Q_1 = K_1 . A_1 . (\Delta h_1/L_1)$,

$Q_2 = K_2 . A_2 . (\Delta h_2/L_2)$,

$Q_3 = K_3 . A_3 .(\Delta h_3/L_3)$,

$A_t = A_1 = A_2 = A_3$

$L_t = L_1 + L_2 + L_3$

$$Q_t . \frac{L_t}{K_v . A_t .} = Q_1 . \frac{L_1}{K_1 . A_1 .} + Q_2 . \frac{L_2}{K_2 . A_2 .} + Q_3 . \frac{L_3}{K_3 . A_3 .}$$

$$\frac{L_t}{K_v} = \frac{L_1}{K_1} + \frac{L_2}{K_2} + \frac{L_3}{K_3}$$

$$K_v = \frac{L_1 + L_2 + L_3}{\frac{L_1}{K_1} + \frac{L_2}{K_2} + \frac{L_3}{K_3}}$$

Example 1

Three layered soil of (200 cm) thickness each, with total horizontal permeability of (0.35 cm/hr), the permeability of second layer is equal to twice the permeability of the first layer, and the permeability of the third layer is equal to the half of the permeability of the first layer. If the three layers are confined by two impervious layers, find the permeability of each layer and the total flow assuming that the hydraulic gradient is equal to one unit?

Solution

$$K_h = \frac{K_1 . L_1 . + K_2 . L_2 + K_3 . L_3}{L_1 + L_2 + L_3} = 0.35\ cm/hr$$

$K_2 = 2\ K_1$

$K_3 = 1/2\ K_1$

$L_1 = L_2 = L_3 = 200\ cm$

$$0.35 = \frac{\mathrm{K_1.L_1.} + 2\,\mathrm{K_2.L_2} + 1/2\,\mathrm{K_3.L_3}}{\mathrm{L_1 + L_2 + L_3}}$$

$\mathrm{K_1} = 0.3\ cm/hr$, $\mathrm{K_2} = 0.6$, $\mathrm{K_3} = 0.15$

$\mathrm{Q_t = K_h.A_t.(\Delta h_t}/L_t)$

$Q = 0.35.(3 \times 200 \times 100).(1) = 21000\, cm^3/hr$

Example

Find the vertical permeability of the three layered soil (100 cm) thickness each, the permeability of the first layer is (1.27 cm/hr), the second layer is (0.127 cm/hr), and the third one is (12.70 cm/hr) respectively. If the water pounded on the surface is (350 cm), find the flow and the head losses between the layers?

Solution

$$\mathrm{K_v} = \frac{(\mathrm{L_1 + L_2 + L_3})}{\left(\frac{L_1}{\mathrm{K_1}} + \frac{L_2}{\mathrm{K_2}} + \frac{L_3}{\mathrm{K_3}}\right)}$$

$$\mathrm{K_v} = (3 \times 100)/\left(\frac{100}{1.27} + \frac{100}{0.127} + \frac{100}{12.7}\right)$$

$\mathrm{Q_t = K_v.A_t.(\Delta h_t}/L_t)$

$$\mathrm{Q_t} = (0.343)(100 \times 100)(\frac{350}{300})$$

$\mathrm{Q_t} = 4004.0\ cm^3/hr$

$\mathrm{Q_1 = K_1.A_1.(\Delta h_1}/L_1)$

$4004.0 = 1.27(100 \times 100)\ (\Delta \mathrm{h_1}/100)$

$\Delta \mathrm{h_1} = 31.53\, cm$

$\mathrm{Q_2 = K_2.A_2.(\Delta h_2}/L_2)$

$4004.0 = 0.127(100 \times 100)\ (\Delta \mathrm{h_2}/100)$

$\Delta \mathrm{h_2} = 315.30\, cm$

$\mathrm{Q_3 = K_3.A_3.(\Delta h_3}/L_3)$

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$$4004.0 = 12.7(100 \times 100)\ (\Delta \mathrm{h}_3/100)$$

$$\Delta \mathrm{h}_3 = 3.15\ cm$$

Some Definition:-

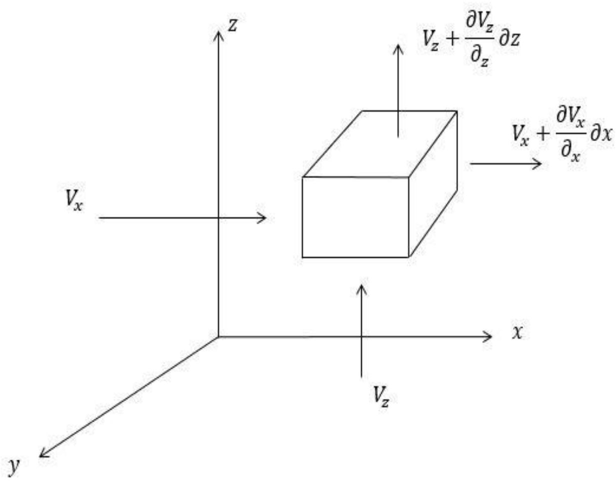
1- Aquifer: A saturated permeable geological unit that can transmit significant quantity of water under ordinary hydraulic head. Types of Aquifers:-

a- Unconfined aquifer: the aquifer that occurs near the ground surface and bounded from the bottom with impervious layer, using (Dupuit-Forchheimer Eq. to compute discharge and the head losses in any point throw the aquifer).

b- Confined aquifer: the aquifer that occurs between two impervious layers and in higher depth, using (Darcy's law to compute discharge and head losses at any point throw the aquifer).

Steady State Saturated flow (Lap lace's Equation):-

Consider an element of soil of size dx, dy. Where $(dx = dy)$ through which the flow is taking place (the third dimension along z-axis is large, it is taken as unity)



Let the velocity at the inlet faces equalV_x, and V_y respectively, and the outlet faces equal $V_x, +(\partial V_x/\partial x).dx$, and $V_y + (\partial V_y/\partial y).dy$

As the flow is steady and the soil is incompressible the discharge entering the

element is equal to that
leaving element.

Thus $V_x\, dy + V_y\, dx = (V_x + (\partial V_x/\partial x).dx).dy + (V_y + (\partial V_y/\partial y).dy).dx$

Or $\left((\partial V_x/\partial x) + \left(\partial V_y/\partial y\right)\right).dx.dy = 0$

Or $(\partial V_x/\partial x) + \left(\partial V_y/\partial y\right) = 0 \ldots\ldots\ldots\ldots$ ① (the continuity equation for two-dimensional flow)

Let h be the total head at any point, the horizontal and vertical components of the hydraulic gradient are respectively:

$i_x = -(\partial \mathrm{h}/\partial x) \qquad i_y = -(\partial \mathrm{h}/\partial y)$

The minus indicates that the head decreases in the direction of flow.

From Darcy's Law: $V_x = -K_x(\partial \mathrm{h}/\partial x) \qquad V_y = -K_y(\partial \mathrm{h}/\partial y)$

Substituting in Eq,①: $K_x(\partial^2 h/\partial x^2) + K_y(\partial^2 h/\partial y^2) = 0$

As the soil is isotropic $K_x = K_y$ therefore.

$(\partial^2 h/\partial x^2) + (\partial^2 h/\partial y^2) = 0 \ldots\ldots\ldots\ldots$ ② (Lap lace's Eq. in terms of head)

<u>Methods for Lap lace's Equation Solution:-</u>

a- Approximation method (Simplified) method by Dupuit — Forchheimer.
b- Electrical Analogy.
c- Numerical method (Relaxation & Iteration method).

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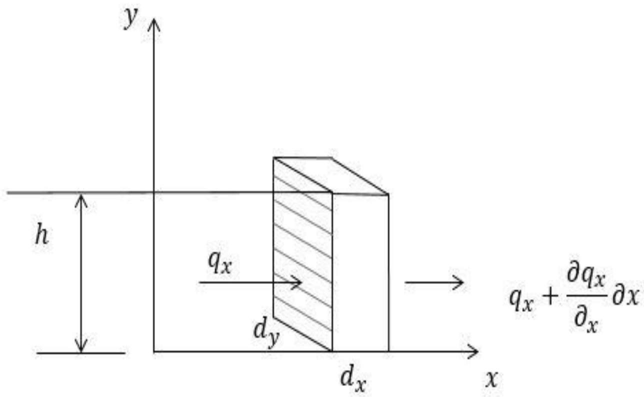
Dupuit-Forchheimer Equation (theory of free surface):-



Simplified Assumptions:

1- Neglect curvature of water surface (neglect the vertical flow component).

2- Stream lines are straight and parallel.

3- Hydraulic gradient equal to the slope of free surface.

4- Velocity is uniform across the section :

$V_x = -K_x(\partial \mathrm{h}/\partial x)$

$q_x =$ the discharge per unit width in the X-direction.

$q_x = V_x \,.A = -K_x(\partial \mathrm{h}/\partial x).(h) = -K_x(\partial \mathrm{h}^2/\partial x)$

Where $q_x =$ constant, then: $\partial q_x/\partial x = 0$

Or $-K_x(\partial^2 h^2/\partial^2 x) = 0$

Or $(\partial^2 h^2/\partial^2 x) = 0 \ldots\ldots\ldots\ldots$ 1

Integrating Eq. 1: $h^2 = Ax + B \ldots\ldots\ldots\ldots$ 2

Boundary conditions:

$At \quad x = 0 \quad h = y_1$

$At \quad x = L \quad h = y_2$

Substitution into Eq. 2:

$A = ({y_2}^2 - {y_1}^2)/L$, $B = {y_1}^2$, then:

$h^2 = ({y_2}^2 - {y_1}^2/L).x + {y_1}^2 \ldots\ldots\ldots\ldots$ 3

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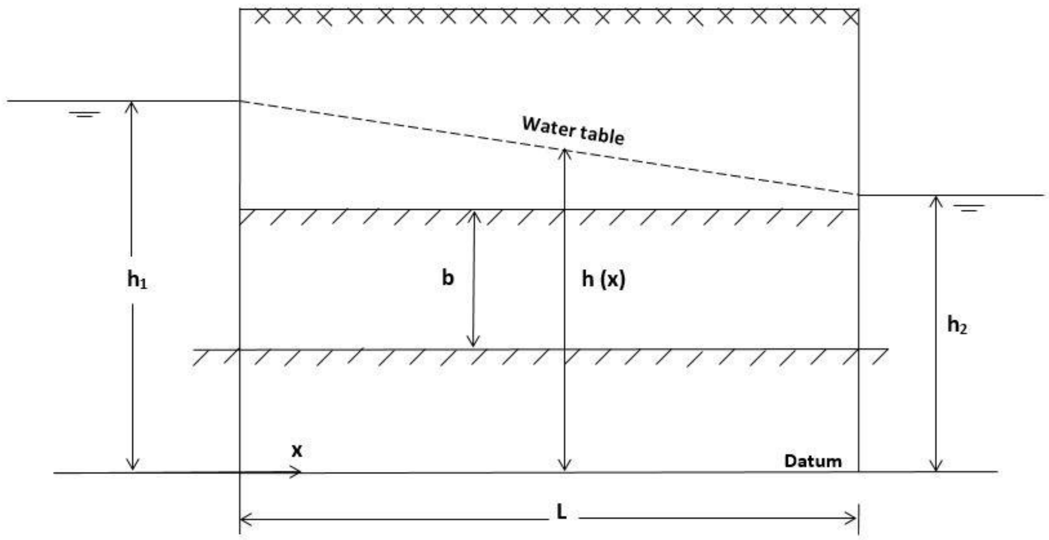
Flow between tow line sources in a confined aquifer of uniform thickness



$$\frac{d^2 h_{(x)}}{d\,x^2} = O$$

Subject to the condition

$h(O) = h_1 \qquad at \quad x = O$

$h(L) = h_2 \qquad at \quad x = L$

the hydraulic head in the flow system is given by solution the eq.

$$h(x) = h_1 - \left(\frac{(h_1 - h_2)}{L}\right) \times x$$

$$Q = V_x . A \Rightarrow q_{(x)} = Q \times 1unit$$

$$q_{(x)} = V_x . b = -k . b \left(\frac{dh}{dx}\right) = k . b . \left(\frac{h_1 - h_2}{L}\right)$$

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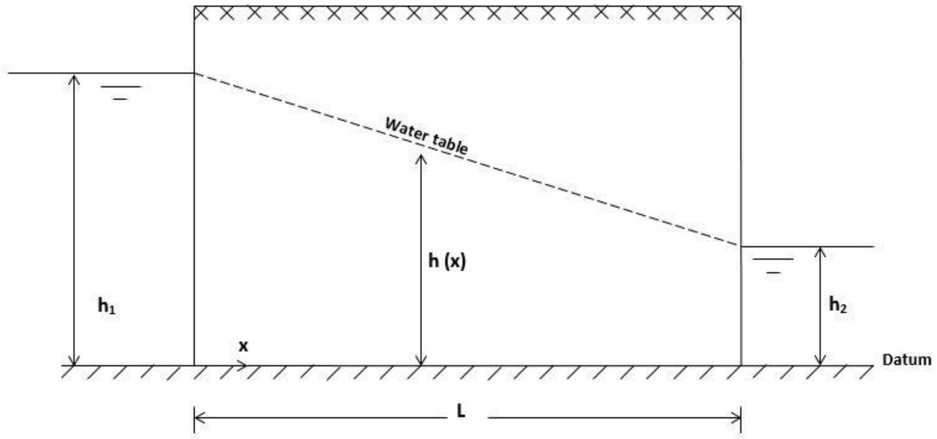
Flow between the line source in a horizontal unconfined aquifer



$$\frac{d^2 h^2{}_{(x)}}{d\,x^2} = O$$

$h(O) = h_1 \qquad at \quad x = O$

$h(L) = h_2 \qquad at \quad x = L$

$h^2(x) = {h_1}^2 + ({h_2}^2 - {h_1}^2)(x/L)$

$$V_{(x)} = -k.\frac{dh}{dx} = k.\frac{\left({h_1}^2 - {h_2}^2\right)/L}{2h_{(x)}} \Rightarrow$$

$q_{(x)} = V_{(x)}.h_{(x)} = k\left({h_1}^2 - {h_2}^2\right)/2L$

Example: Given $h_1 = 50\ m$, $h_2 = 20\ m$, $L = 100\ m$, $k = 5\ m/d$. Plot the variation of $V(x)$. dh/dx . q ? unconfined aquifer.

Solution:

$\boldsymbol{X}$	$\boldsymbol{h(x)}$	$\boldsymbol{V(x)}$	$\boldsymbol{\frac{dh}{dx}}$
20	45.6	1.15	-0.23
40	40.74	1.28	-0.257
60	35.2	1.49	-0.298
80	28.6	1.83	-0.367
100	20	2.625	-0.525

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$$h^2(x) = (50)^2 + \left[\frac{(20)^2 - (50)^2}{100}\right] X$$

$$h(x) = \sqrt{2500 - 21\,X}$$

$$V(x) = 5 \times \frac{((50)^2 - (20)^2)/100}{2\,h(x)} = \frac{52.5}{h(x)}$$

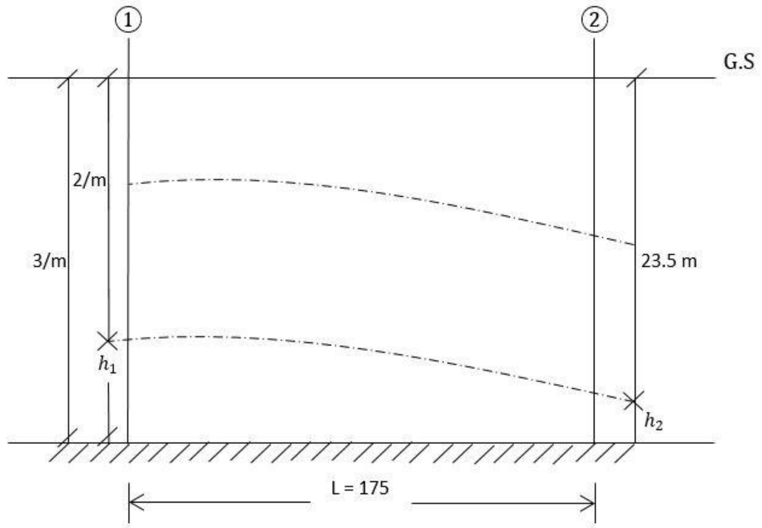
$$\frac{dh}{dx} = \frac{((20)^2 - (50)^2)/100}{2\,h(x)} = -\frac{10.5}{h(x)}$$

Example: water table flowing in sandy aquifer with hydraulic conductivity of (0.002 cm/s) and the aquifer thickness is (31 m), at well (1) water level (21 m) below ground surface, at well (2) water level (23.5 m) below ground surface. The distance between the two wells is (175 m).

Find:-

1- The discharge per unit width.
2- The hydraulic head at distance of (100 m).

Solution:



1) $q = k\ \left({h_1}^2 - {h_2}^2\right)/2L$

$k = 0.002\,cm/s$

$k = 0.002\,(3600 \times 24)/100 = 1.7\ m/day$

$L = 175\,m$

$h_1 = 31 - 21 = 10\ m\ , h_2 = 31 - 23.5 = 7.5\ m$
$q = 1.7\,((10)^2(7.5)^2)/(2 \times 175) = 0.212\ m^3/day$

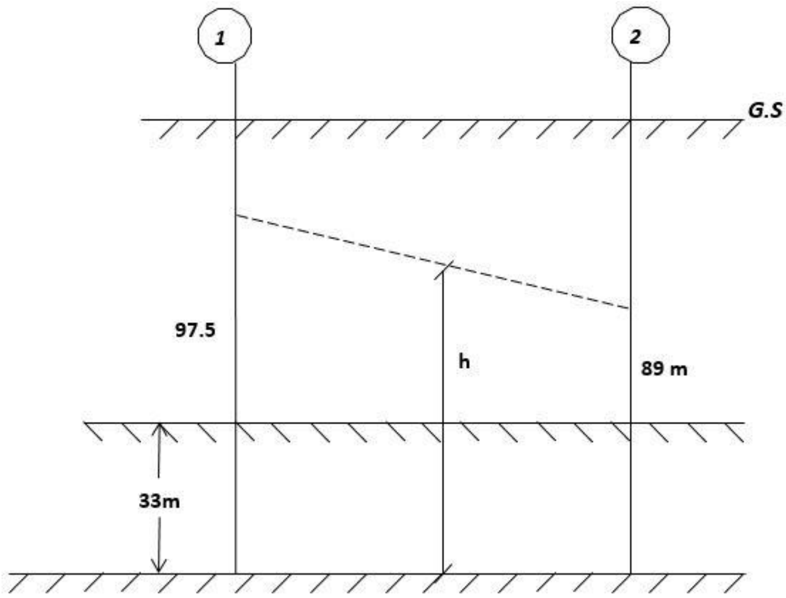
2) $h^2(x) = {h_1}^2 + ({h_2}^2 - {h_1}^2)(x/L)$
$x = 100\ m\ , L = 175\ m$
$h^2(x) = (10)^2 + ((7.5)^2(10)^2) \times \frac{100}{175} = 75\ m$

$h(x) = 8.66\ m$

Example: (33 m) thick confined aquifer, (7 km) wide, for two observation wells (1.2 km) apart head reading at well (1) was (97.5 m) and at well (2) was (89 m). if (k) equal to (1.2 m / day)

Find
1) The total daily flow.
2) The hydraulic head of an intermediate distance (x) between the wells.

Solution:



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1) $Q = K\ i\ A$

$$= (1.2) \times \frac{(97.5 - 891)}{1200} \times (33 \times 7000)$$

$$= 1963.5\ m^3/day \leftarrow$$

2) $I = \frac{Q}{K\ A}$

$$i = \frac{1963.5}{1.2\ (33 \times 7000)} = \frac{(97.5 - h)}{600}$$

$$h = 93.25\ m$$

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Electrical Analogy.

Analogy between the flow of ground water and electricity:-

(1) <u>For ground water:</u>

$Q = K.A.(\Delta h/L)$

$Q = K\,(BD).(\Delta h/L)$

Let $V = Q/(BD)$ and $\nabla h = (\Delta h/L)$

$V = K.\,h$ -------- (1)

Where:-

$\nabla = (\partial/\partial x) + (\partial/\partial y) + (\partial/\partial z)$

And $(\partial v_x/\partial x) + \left(\partial v_y/\partial y\right) + (\partial v_z/\partial z) =$ ------- (2) (continuity Eq.)

Sub. (1) in (2):-

$(\partial^2 h/\partial x^2) + (\partial^2 h/\partial y^2) + (\partial^2 h/\partial z^2) =$ $\nabla^2 h = 0$ (Lap Lace's Eq.)

(2) For Electricity:

$\Delta V = R.I$ --------- (1)

Where:-

R: Resistance (ohm)

I: Current (Ampere)

ΔV: Potential difference (volts)

$R = (1/\delta).(L/BD)$ -------- (2)

Where:-

δ: Specific Conductivity (1 / ohm .m)

Combining (1) and (2):

$I = \delta.(\Delta V/L).\ (BD)$

Let: $J = (I/BD)$ and $\nabla V = -(\Delta V/L)$

Where:-

J: Current density (Ampere / m^2)

$J = \nabla V$ --------- (3)

$(\partial J_x/\partial x) + \left(\partial J_y/\partial y\right) + (\partial J_z/\partial z) = 0$-------- (4) (Continuity Eq.)

Sub. (3) In (4):-

$(\partial^2 V/\partial x^2) + (\partial^2 V/\partial y^2) + (\partial^2 V/\partial z^2) = \nabla^2 V = 0$ (Lap Lace's Eq.)

Corresponding Elements between Ground flow and Electricity:-

Ground Water	**Electricity**
1- Hydraulic head differences $(\Delta \boldsymbol{h})$ (m)	1- potential difference (ΔV) (volt)
2- Hadronic conductivity $(\boldsymbol{K})$ (m /day)	2- Specific conductivity (δ) (1 / ohm.m)
3- flow rate $(\boldsymbol{Q})$ (m^3 / day)	3- Current (I) (Ampere)
4- Specific discharge $(\boldsymbol{V})$ (m/s)	4- Current density (J) (Ampere / m2)
5- Darcy's Law: $(\boldsymbol{v} = \boldsymbol{K}. \quad \boldsymbol{h})$	5- ohm's Law: $J = \delta. \quad V)$
6- Lap lace's Eq. $(\boldsymbol{\nabla^2 h = 0})$	6- Lap lace's Eq. $(\nabla^2 V = 0)$