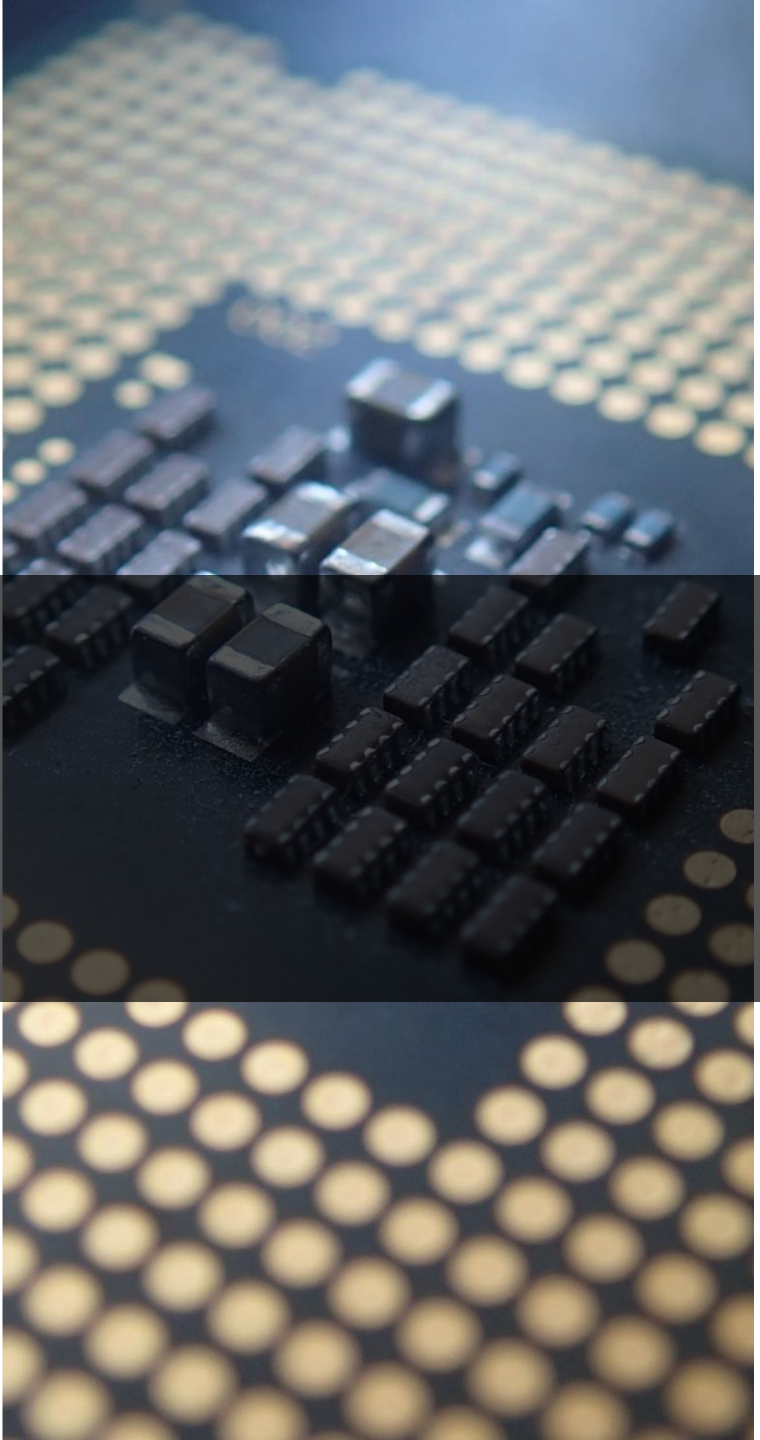

University of Al Maarif

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Class: 2nd

Digital Electronics

MIET2203

Lecture 6: Karnaugh map

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Karnaugh map

- A **Karnaugh** map provides a **pictorial** method of grouping together **expressions** with **common** factors and therefore **eliminating** unwanted **variables**. The **Karnaugh** map can also be described as a **special arrangement** of a **truth table**. The diagram below illustrates the correspondence between the Karnaugh map and the truth table for the general case of a two-variable problem.

A	B	F
0	0	0
0	1	1
1	0	1
1	1	1

Truth Table.

B \ A	0	1
0	0	1
1	1	1

F.

Karnaugh map

• The values around the edge of the map can be thought of as coordinates. right-hand as an example, the square on the top right-hand corner of the map in the above diagram has coordinates A=1 and B=0. This square corresponds to the row in the truth table where A=1 and B=0 and F=1. Note that the value in the F column represents a particular function to which the Karnaugh map corresponds.

• The number of cells in a Karnaugh map, as well as the number of rows in a Truth table, is equal to the (total number of possible input variable combinations). For three variables, the number of cells is $2^3 = 8$. For four variables, the number of cells is $2^4 = 16$.

Karnaugh map

Example:

A	B	F
0	0	0
0	1	1
1	0	1
1	1	1

B \ A	0	1
0	0	1
1	1	1

F =

Karnaugh Map Simplification of SOP Expressions

The process that results in an expression containing the fewest possible terms with the fewest possible variables is called **minimization**. After an SOP expression has been mapped, a minimum SOP expression is obtained by grouping the 1s and determining the minimum SOP expression from the map.

Grouping the 1s

You can group 1s on the Karnaugh map according to the following rules by enclosing those adjacent cells containing 1s. The goal is to maximize the size of the groups and to minimize the number of groups.

1. A group must contain either 1, 2, 4, 8, or 16 cells, which are all powers of two. In the case of a 3-variable map, $2^3 = 8$ cells is the maximum group.
2. Each cell in a group must be adjacent to one or more cells in that same group, but all cells in the group do not have to be adjacent to each other.
3. Always include the largest possible number of 1s in a group in accordance with rule 1.
4. Each 1 on the map must be included in at least one group. The 1s already in a group can be included in another group as long as the overlapping groups include noncommon 1s.

Karnaugh Map Simplification of SOP Expressions

EXAMPLE 4–27

Group the 1s in each of the Karnaugh maps in Figure 4–33.

AB \ C	0	1
00	1	
01		1
11	1	1
10		

(a)

AB \ C	0	1
00	1	1
01	1	
11		1
10	1	1

(b)

AB \ CD	00	01	11	10
00	1	1		
01	1	1	1	1
11				
10		1	1	

(c)

AB \ CD	00	01	11	10
00	1			1
01	1	1		1
11	1	1		1
10	1		1	1

(d)

FIGURE 4–33

Karnaugh Map Simplification of SOP Expressions

Solution

The groupings are shown in Figure 4–34. In some cases, there may be more than one way to group the 1s to form maximum groupings.

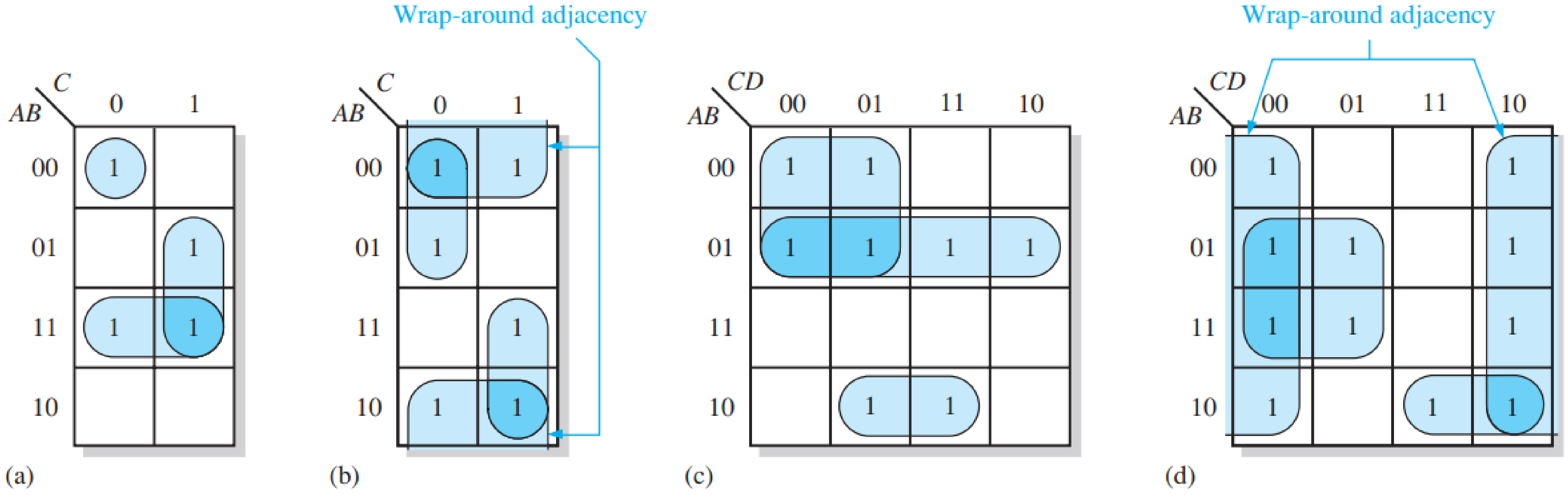
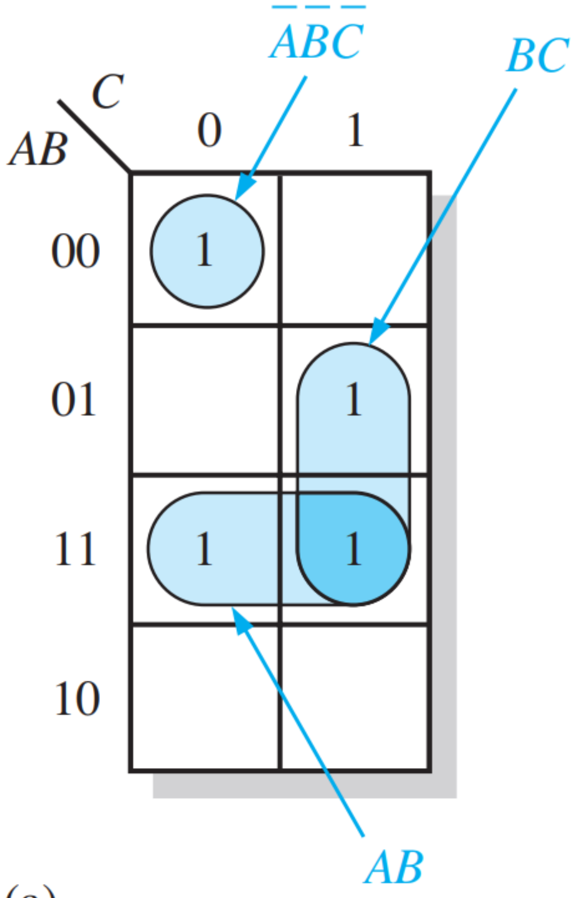


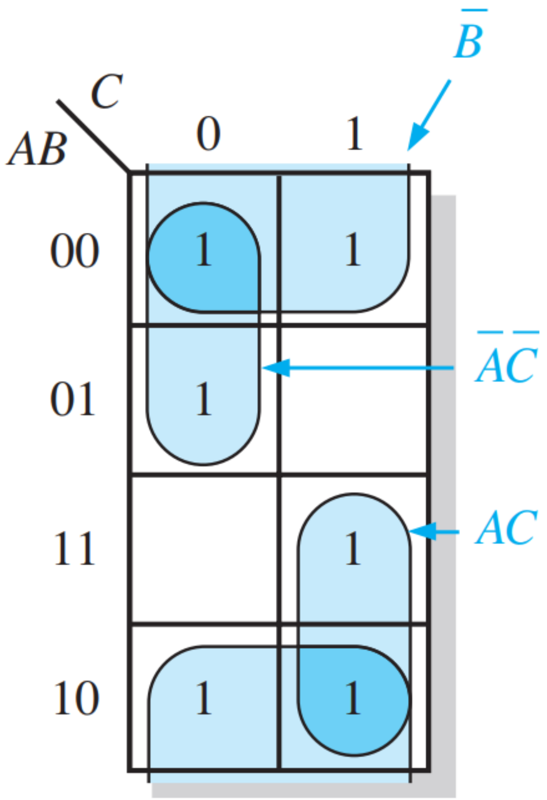
FIGURE 4–34

Karnaugh Map Simplification of SOP Expressions



(a)

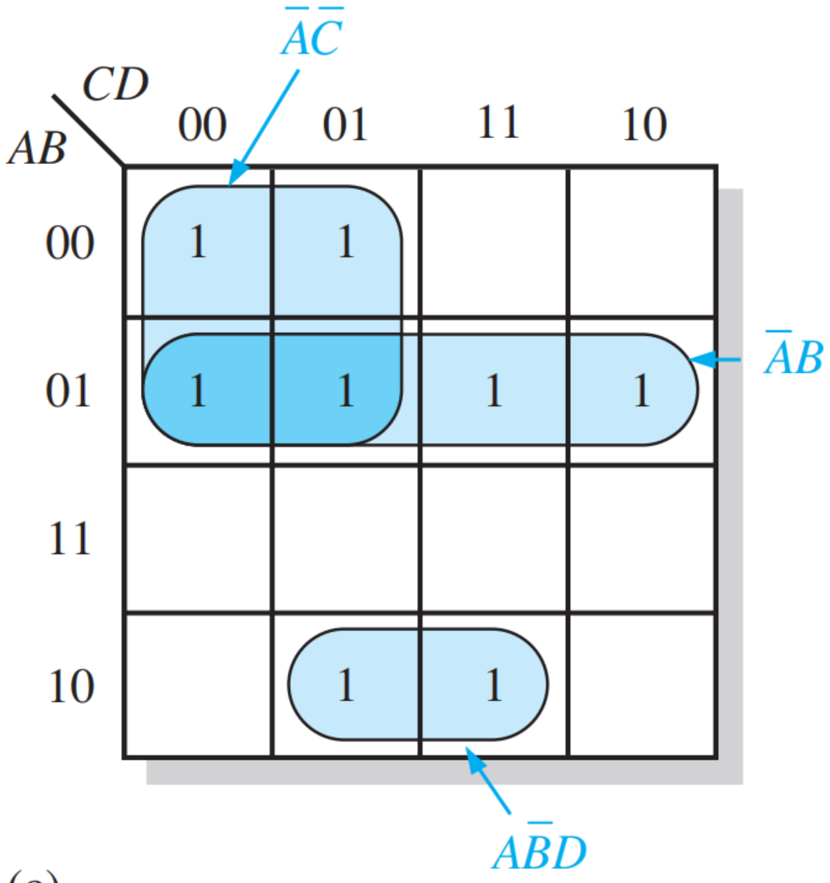
$$AB + BC + \overline{A}\,\overline{B}\,\overline{C}$$



(b)

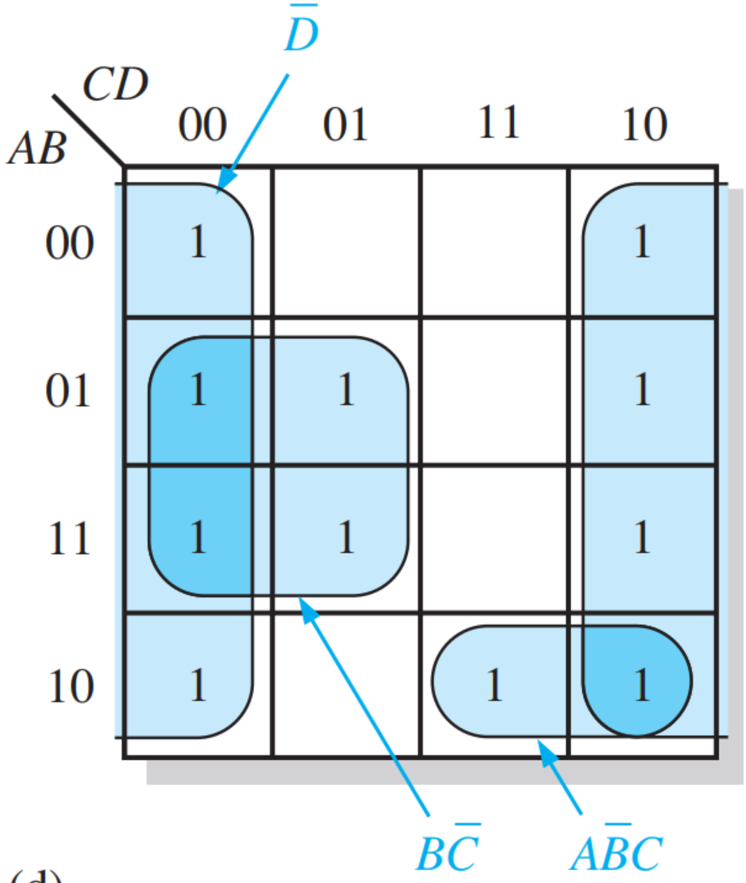
$$\overline{B} + \overline{A}\,\overline{C} + AC$$

Karnaugh Map Simplification of SOP Expressions



(c)

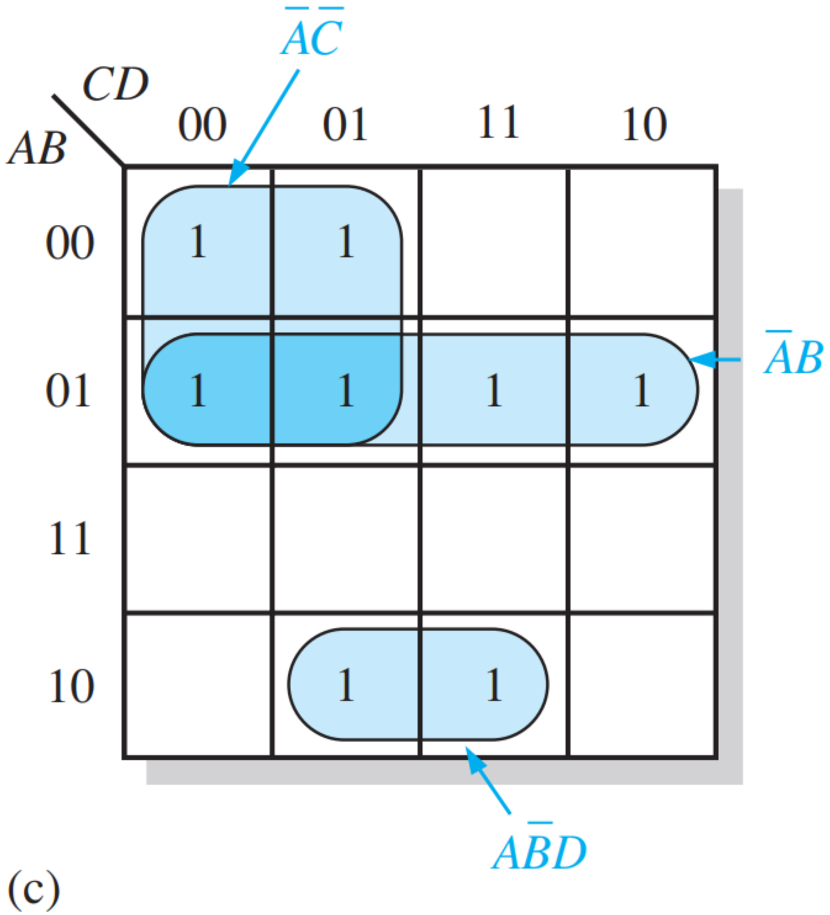
$$\overline{A}B + \overline{A}\,\overline{C} + A\overline{B}D$$



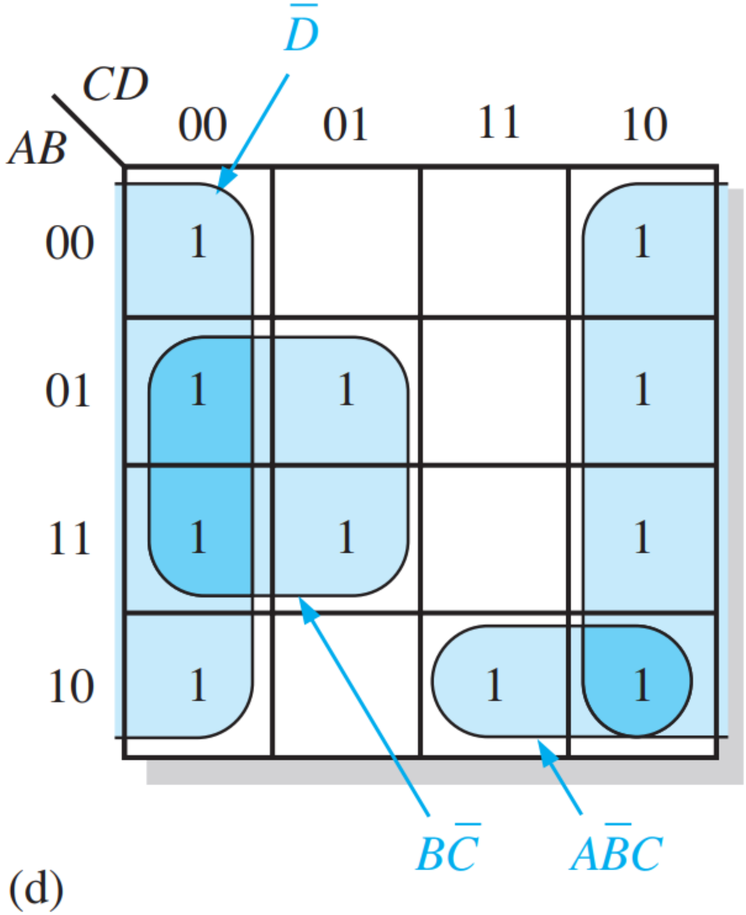
(d)

$$\overline{D} + A\overline{B}C + B\overline{C}$$

Karnaugh Map Simplification of SOP Expressions



$$\overline{A}B + \overline{A}\,\overline{C} + A\overline{B}D$$



$$\overline{D} + A\overline{B}C + B\overline{C}$$

Karnaugh Map Simplification of SOP Expressions

B	A	X
0	0	1
0	1	0
1	0	0
1	1	1

	$\bar{B}$	B
$\bar{A}$	1	0
A	0	1

$$X = \bar{A}\bar{B} + AB$$

Karnaugh Map Simplification of SOP Expressions

C	B	A	X
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

	$\bar{C}$	C
$\bar{A}\bar{B}$	1	0
$\bar{A}B$	1	1
AB	0	0
$A\bar{B}$	1	0

$$X = \bar{A}B + \bar{B}\bar{C}$$

Karnaugh Map Simplification of SOP Expressions

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	0	0	0
$\bar{A}B$	1	1	1	1
AB	1	1	1	1
$A\bar{B}$	0	0	0	0

$$X = B$$

			D	C	
		$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
		0	0	0	1
B		0	1	1	0
B	A	0	1	1	0
	A	0	0	1	0

$$X = BD + ACD + \bar{A}\bar{B}C\bar{D}$$

Don't care

- Some logic circuits can be designed so that there are certain input conditions for which there are no specified output levels, usually because these input conditions will never occur. In other words, there will be certain combinations of input levels where we "don't care" weather the output is HIGH or LOW. This is illustrated in the following truth table.

C	B	A	X
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	X
1	0	0	X
1	0	1	1
1	1	0	1
1	1	1	1

Don't care

	$\bar{C}$	C
$\bar{A}\bar{B}$	0	x
$\bar{A}B$	0	1
AB	X	1
$A\bar{B}$	0	1

$$X = C$$

References

[1] Digital fundamentals / Thomas L. Floyd. —Eleventh edition.

[2]