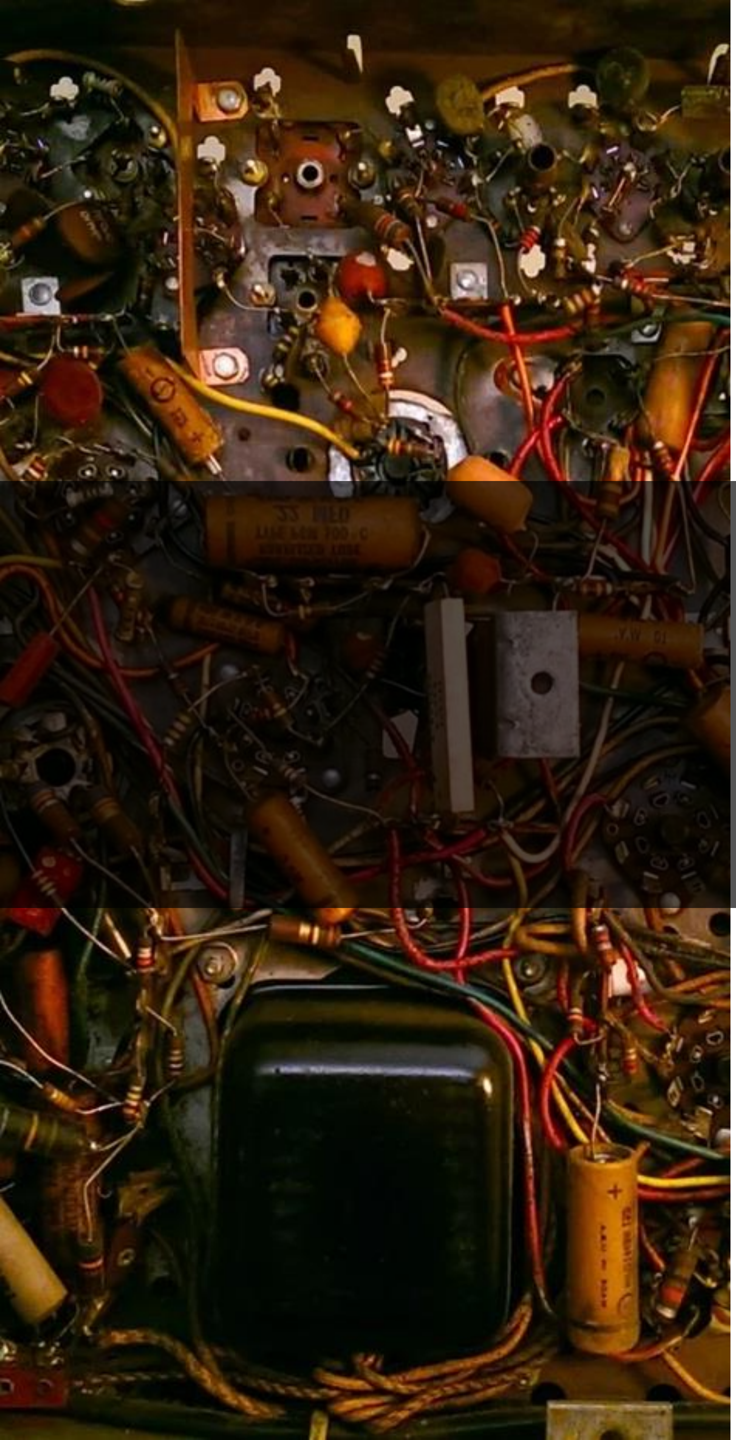


University of Al Maarif

Department of Medical Instruments Techniques Engineering

Class: 2nd

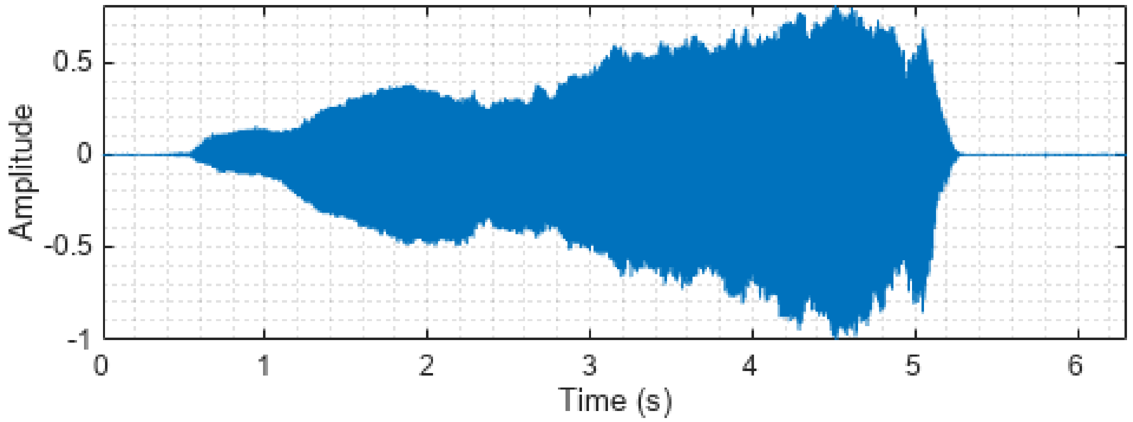
Digital Electronics

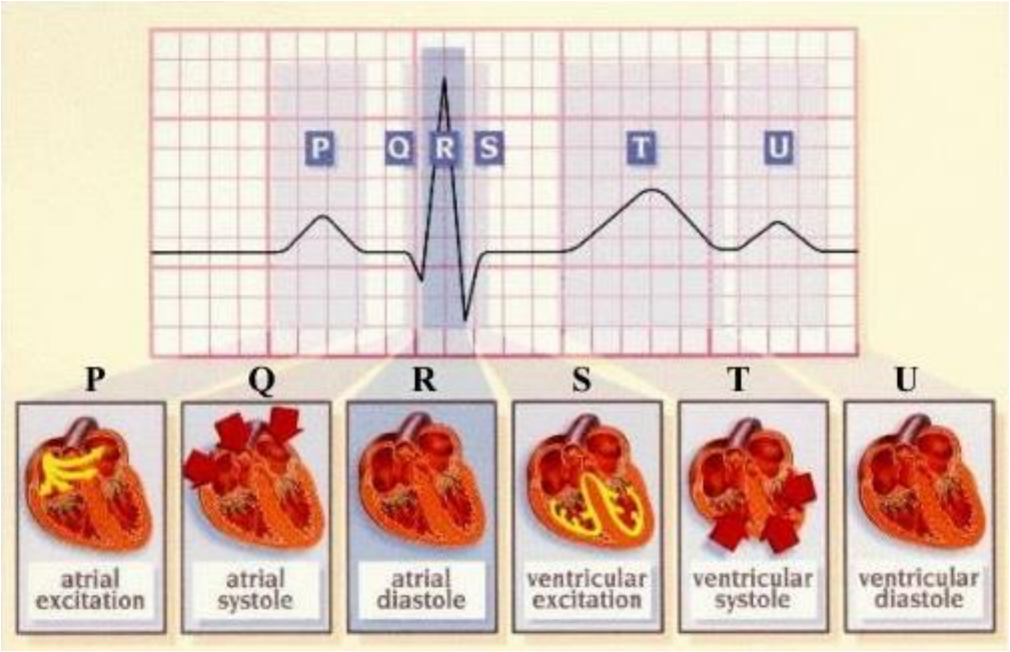
MIET2203

Lecture 1: Number system

Lecturer: Mohammed Jabal

Objective

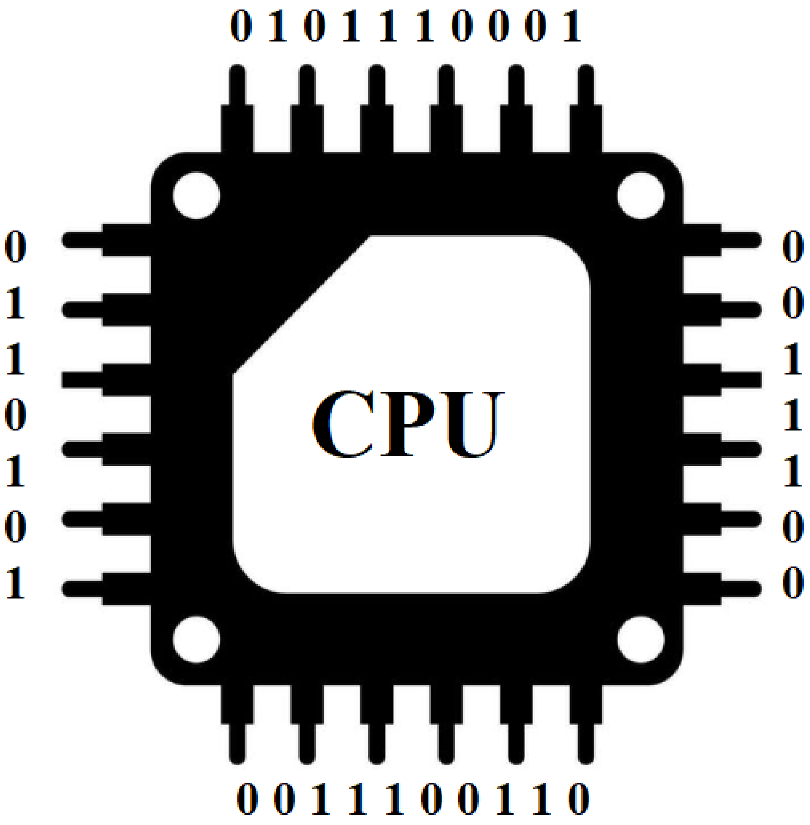


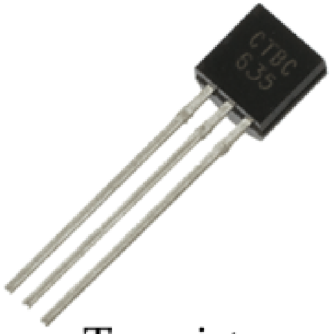


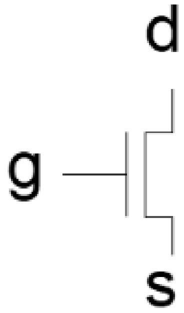
5 + 4 = 9
3 x 2 = 6
10/2 = 5

[0, 1, 2, 3, 4, 5, 6, 7, 8, 9]

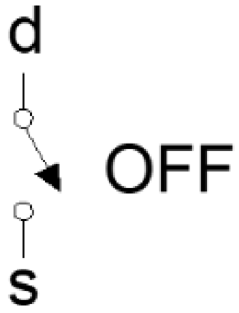
Objective



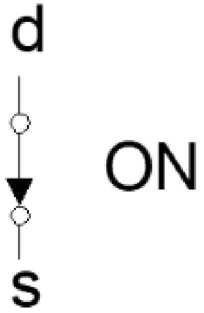

Transistor



g = 0.0V



g = 5.0V



g: gate
d: drain
s: source

Number Syms

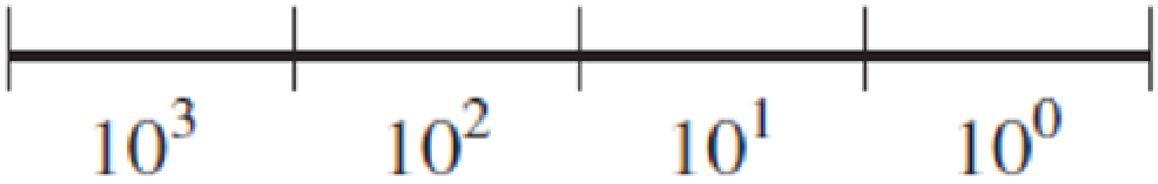
Numbers are used to express quantities. There are many numeration systems used in the field of digital electronics, one of the most important being the binary system of numeration on which is based the computer science. Each of the various numeration systems and codes has its advantages and disadvantages.

A. Decimal Number System (Base 10).
B. Binary Number System (Base 2).
C. Octal Number System (Base 8).
D. Hexadecimal Number System (Base 16).

Note: Base = Radix

A. Decimal Number System

In the decimal numbering system, each position contains 10 different possible digits. These digits are [0, 1, 2, 3, 4, 5, 6, 7, 8, 9]. Each position in a multidigit number will have a weighting factor based on a power of 10.



- The least significant position has a weighting factor of 10^0.
- The most significant position (leftmost) has a weighting factor of 10^3.

Example 1.1:

To evaluate the decimal number 4623, the digit in each position Is multiplied by the appropriate weighting factor:

4 6 2 3

$$3 \times 10^0 = 3$$
$$2 \times 10^1 = 20$$
$$6 \times 10^2 = 600$$
$$4 \times 10^3 = +\underline{4000}$$
$$4623$$

Example 1.2:

in the case of the decimal number 3586.265, the integer part (i.e. 3586) can be expressed as:

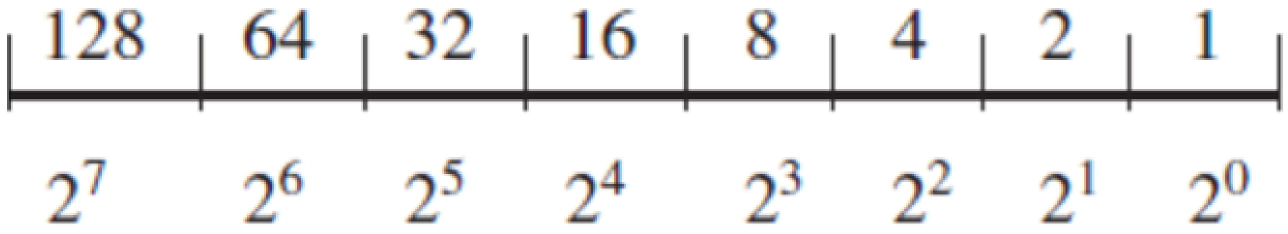
$$3586 = 6\times10^{0} + 8\times10^{1} + 5\times10^{2} + 3\times10^{3} = 6+80+500+3000 = 3586$$

and the fractional part can be expressed as:

$$265 = 2\times10^{-1} + 6\times10^{-2} + 5\times10^{-3} = 0.2+0.06+0.005 = 0.265$$

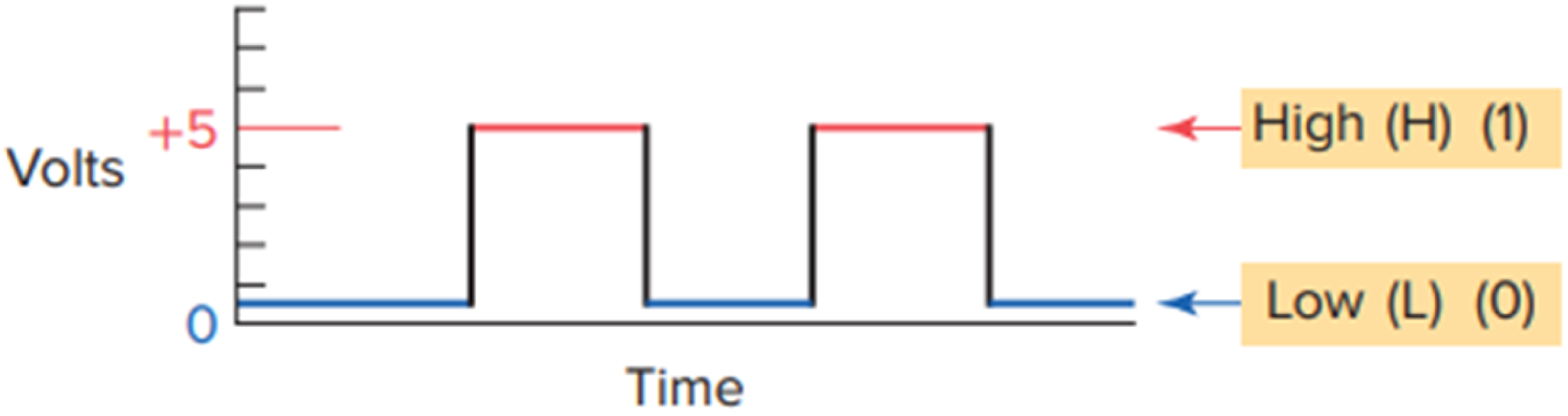
B. Binary Number System

The binary number system uses only two digits instead of ten as the decimal number system. Those two digits are [0, 1]. In binary system, digits are called bits (Binary Digits). Digits are arranged right to left in doubling values of weight as shown in the figure (instead of multiplying the weight by 10 as in the case of decimal system).



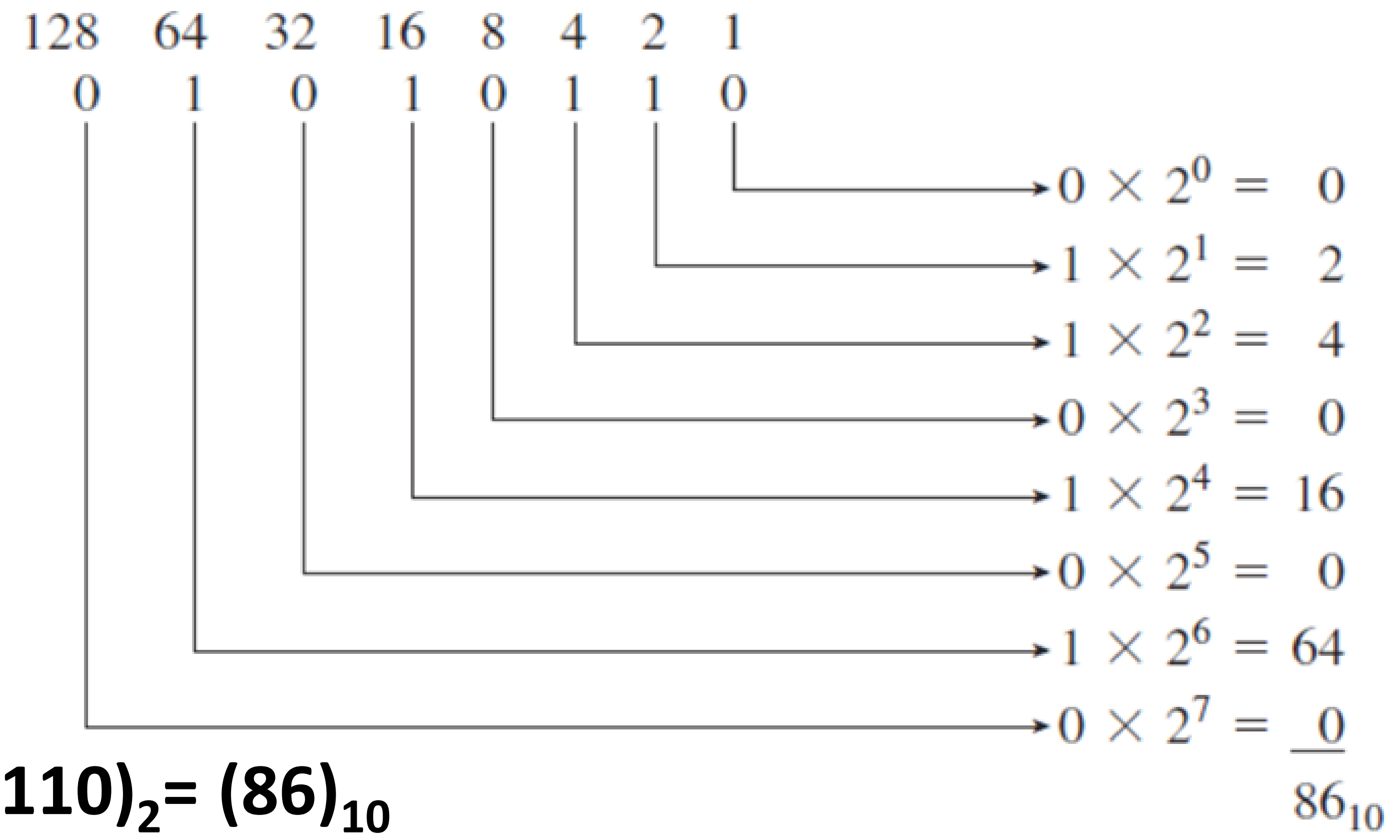
B. Binary Number System

With digital circuits it is easy to distinguish between two voltage levels (i.e., +5 V and 0 V), which can be related to the binary digits 1 and 0. Therefore, the binary system can be applied quite easily to computer systems.



Example 1.3:

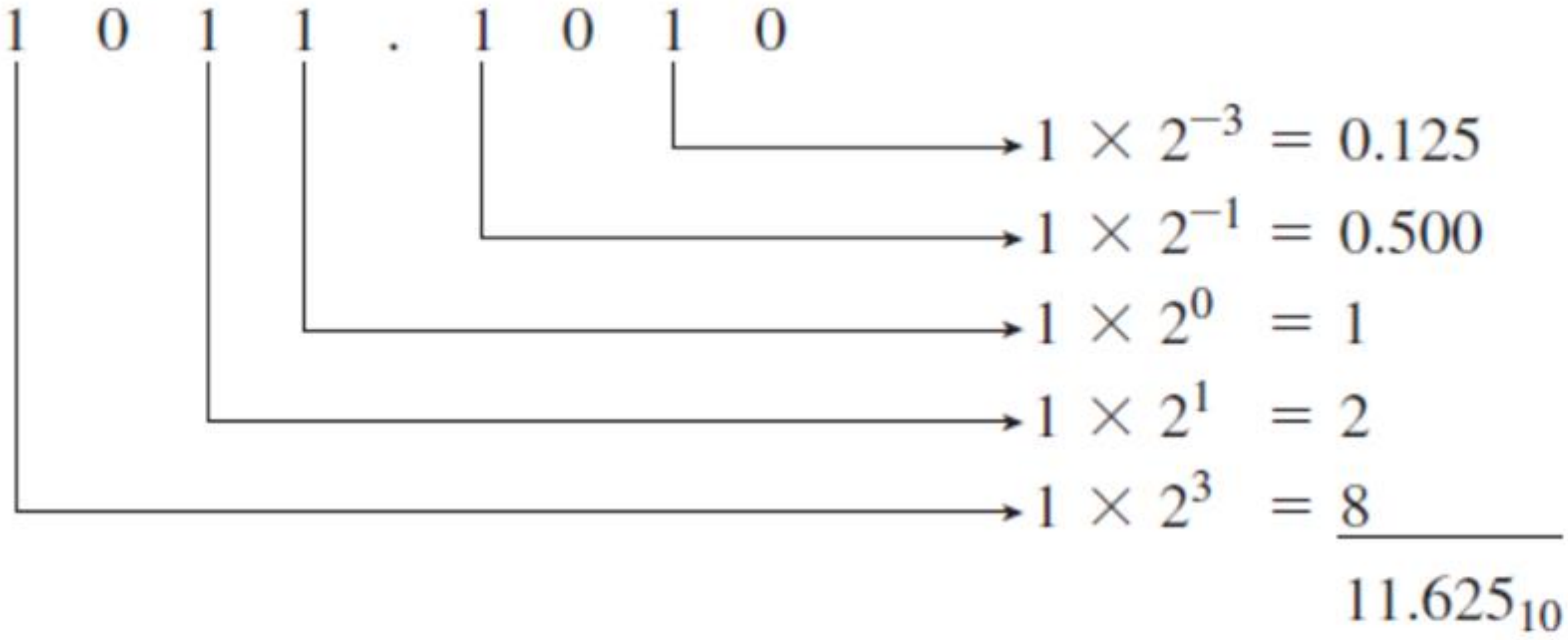
Find the Decimal Equivalent of the binary number $(01010110)_2$?



Answer: $(01010110)_2 = (86)_{10}$

Example 1.4:

Convert the fractional binary number $(1011.1010)_2$ to decimal?



Answer: $(1011.1010)_2 = (11.625)_{10}$

Decimal-to-Binary Conversion

- For the integer part, the binary equivalent can be found by successively dividing the integer part of the decimal number by 2 and recording the remainders until the quotient becomes ‘0’. The remainders written in reverse order constitute the binary equivalent.

- For the fractional part, it is found by successively multiplying the fractional part of the decimal number by 2 and recording the carry until the result of multiplication is ‘0’. The carry sequence written in forward order constitutes the binary equivalent.

Example 1.5:

We will find the binary equivalent of $(13.375)_{10}$

- The integer part = 13

Divisor	Dividend	Remainder
2	13	—
2	6	1
2	3	0
2	1	1
—	0	1

- **The binary equivalent of $(13)_{10}$ is therefore $(1101)_2$**

Example 1.5:

- The fractional part = 0.375

- 0.375 × 2 = 0.75 with a carry of 0
- 0.75 × 2 = 0.5 with a carry of 1
- 0.5 × 2 = 0 with a carry of 1

- The binary equivalent of $(0.375)_{10} = (.011)_2$

- Therefore, the binary equivalent of $(13.375)_{10} = (1101.011)_2$

C. Octal Number System

The octal number system has a radix of 8 and therefore has eight distinct digits. All higher-order numbers are expressed as a combination of these on the same pattern as the one followed in the case of the binary and decimal number systems.

[0, 1, 2, 3, 4, 5, 6, 7]

The place values for the different digits in the octal number system are 8^0, 8^1, 8^2 and so on (for the integer part) and 8^{-1}, 8^{-2}, 8^{-3} and so on (for the fractional part).

Decimal representation of Octal number

The decimal equivalent of the octal number $(137.21)_8$ is determined as follows:

- The integer part = 137
- The decimal equivalent = $7 \times 8^0 + 3 \times 8^1 + 1 \times 8^2 = 7 + 24 + 64 = 95$

- The fractional part = .21
- The decimal equivalent = $2 \times 8^{-1} + 1 \times 8^{-2} = 0.265$

- Therefore, the decimal equivalent of $(137.21)_8 = (95.265)_{10}$

Decimal-to-Octal Conversion

The process of decimal-to-octal conversion is similar to that of decimal-to-binary conversion. The progressive division in the case of the integer part and the progressive multiplication while working on the fractional part here are by '8' which is the radix of the octal number system. Again, the integer and fractional parts of the decimal number are treated separately. The process can be best illustrated with the help of an example.

Example1.6:

We will find the octal equivalent of $(73.75)_{10}$.

- The integer part = 73

Divisor	Dividend	Remainder
8	73	—
8	9	1
8	1	1
—	0	1

- **The octal equivalent of $(73)_{10} = (111)_8$**

Example1.6:

- The fractional part = 0.75

- $0.75 \times 8 = 0$ with a carry of 6
- The octal equivalent of $(0.75)_{10} = (.6)_8$

- Therefore, the octal equivalent of $(73.75)_{10} = (111.6)_8$

D. Hexadecimal Number System

The hexadecimal number system is a radix-16 number system and its 16 basic digits.

[0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F]

The place values or weights of different digits in a mixed hexadecimal number are 16^0, 16^1, 16^2 and so on (for the integer part) and 16^{-1}, 16^{-2}, 16^{-3} and so on (for the fractional part). The decimal equivalent of A, B, C, D, E and F are 10, 11, 12, 13, 14 and 15 respectively, for obvious reasons.

Decimal representation of hexadecimal number

The decimal equivalent of the hexadecimal number $(1E0.2A)_{16}$ is determined as follows:

- **The integer part = 1E0**
- **The decimal equivalent = $0 \times 16^0 + 14 \times 16^1 + 1 \times 16^2 = 0 + 224 + 256 = 480$**

- **The fractional part = .2A**
- **The decimal equivalent = $2 \times 16^{-1} + 10 \times 16^{-2} = 0.164$**

- **Therefore, the decimal equivalent of $(1E0.2A)_{16} = (480.164)_{10}$**

Decimal-to-Hexadecimal Conversion

The process of decimal-to-hexadecimal conversion is also similar. Since the hexadecimal number system has a base of 16, the progressive division and multiplication factor in this case is 16. The process is illustrated further with the help of an example.

Example1.7:

Let us determine the hexadecimal equivalent of $(82.25)_{10}$.

- The integer part = 82

Divisor	Dividend	Remainder
16	82	—
16	5	2
—	0	5

- **The hexadecimal equivalent of $(82)_{10} = (52)_{16}$**

Example1.7:

- **The fractional part = 0.25**

- **$0.25 \times 16 = 0$ with a carry of 4**

- **Therefore, the hexadecimal equivalent of $(82.25)_{10} = (52.4)_{16}$**

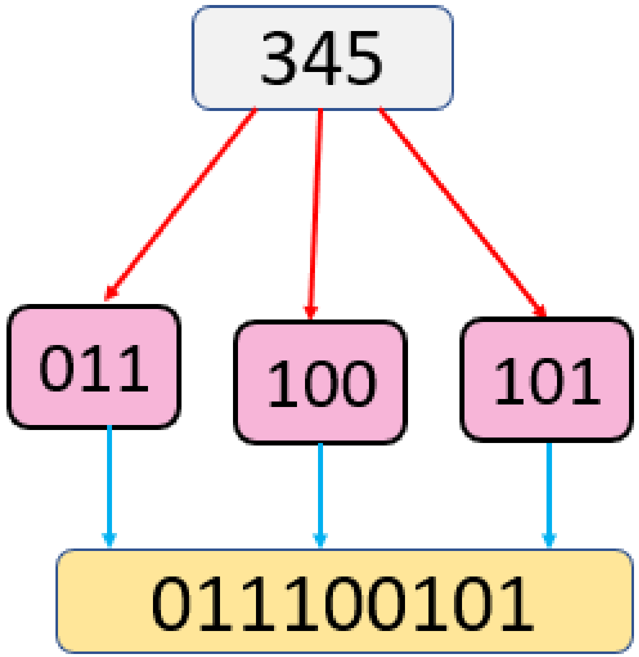
Binary–Octal and Octal–Binary Conversions

An octal number can be converted into its binary equivalent by replacing each octal digit with its three-bit binary equivalent. We take the three-bit equivalent because the base of the octal number system is 8 and it is the third power of the base of the binary number system, i.e. 2. All we have then to remember is the three-bit binary equivalents of the basic digits of the octal number system. A binary number can be converted into an equivalent octal number by splitting the integer and fractional parts into groups of three bits, starting from the binary point on both sides. The 0s can be added to complete the outside groups if needed.

Binary–Octal and Octal–Binary Conversions

Octal (Base 8)	Binary (Base 2)
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

Octal–Binary Conversions



$(345)_8 = (011100101)_2$

Example1.8:

Let us find the binary equivalent of $(374.26)_8$.

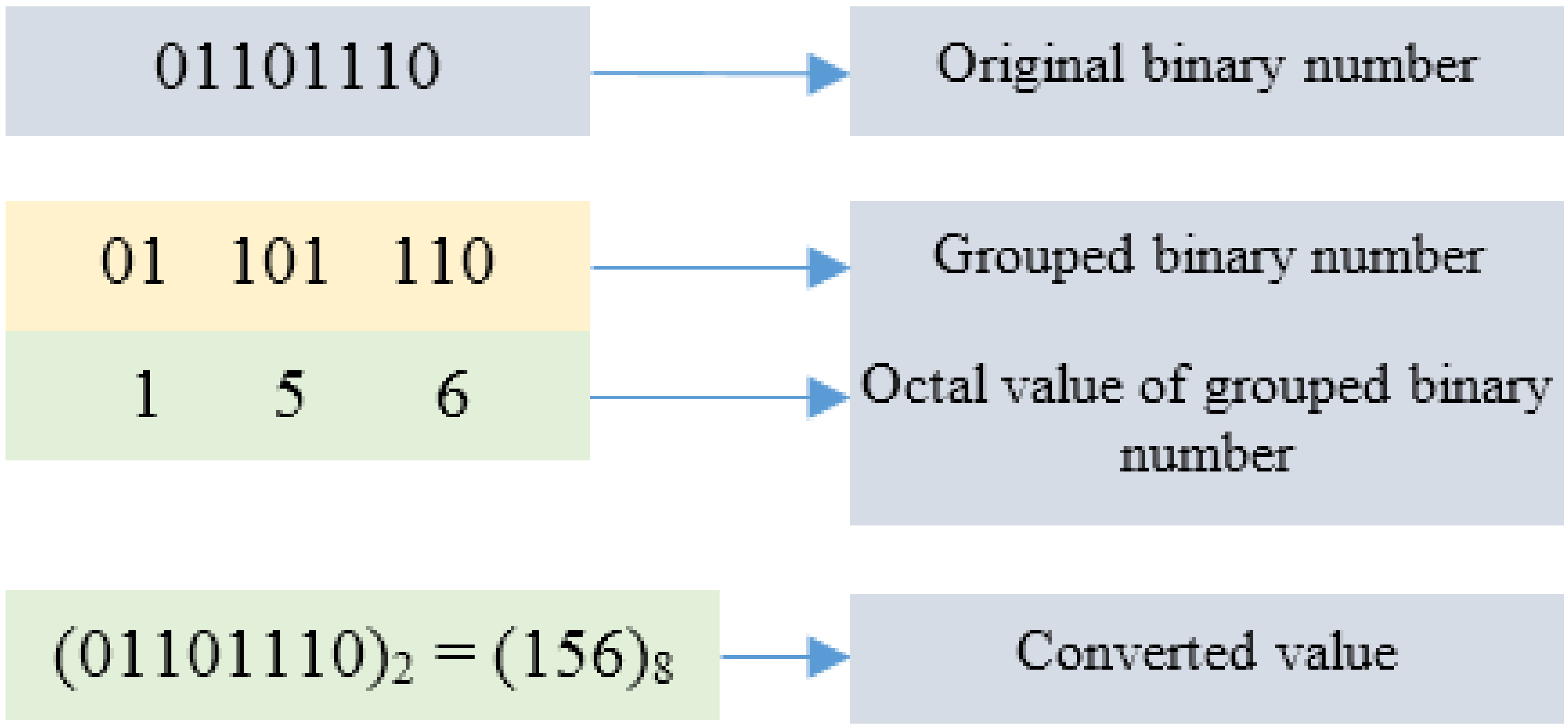
- **The binary equivalent =**

$$([011]\ [111]\ [100].[010]\ [110])_2 = (011111100.010110)_2$$

- **Any 0s on the extreme left of the integer part and extreme right of the fractional part of the equivalent binary number should be omitted. Therefore,**

$$(011111100.010110)_2 = (11111100.01011)_2$$

Binary–Octal Conversions



Example1.9:

Let us find the octal equivalent of $(1110100.0100111)_2$.

$$(1110100.0100111)_2 = ([1]\ [110]\ [100].[010]\ [011]\ [1])_2$$

$$= ([001]\ [110]\ [100].[010]\ [011]\ [100])_2 = (164.234)_8.$$

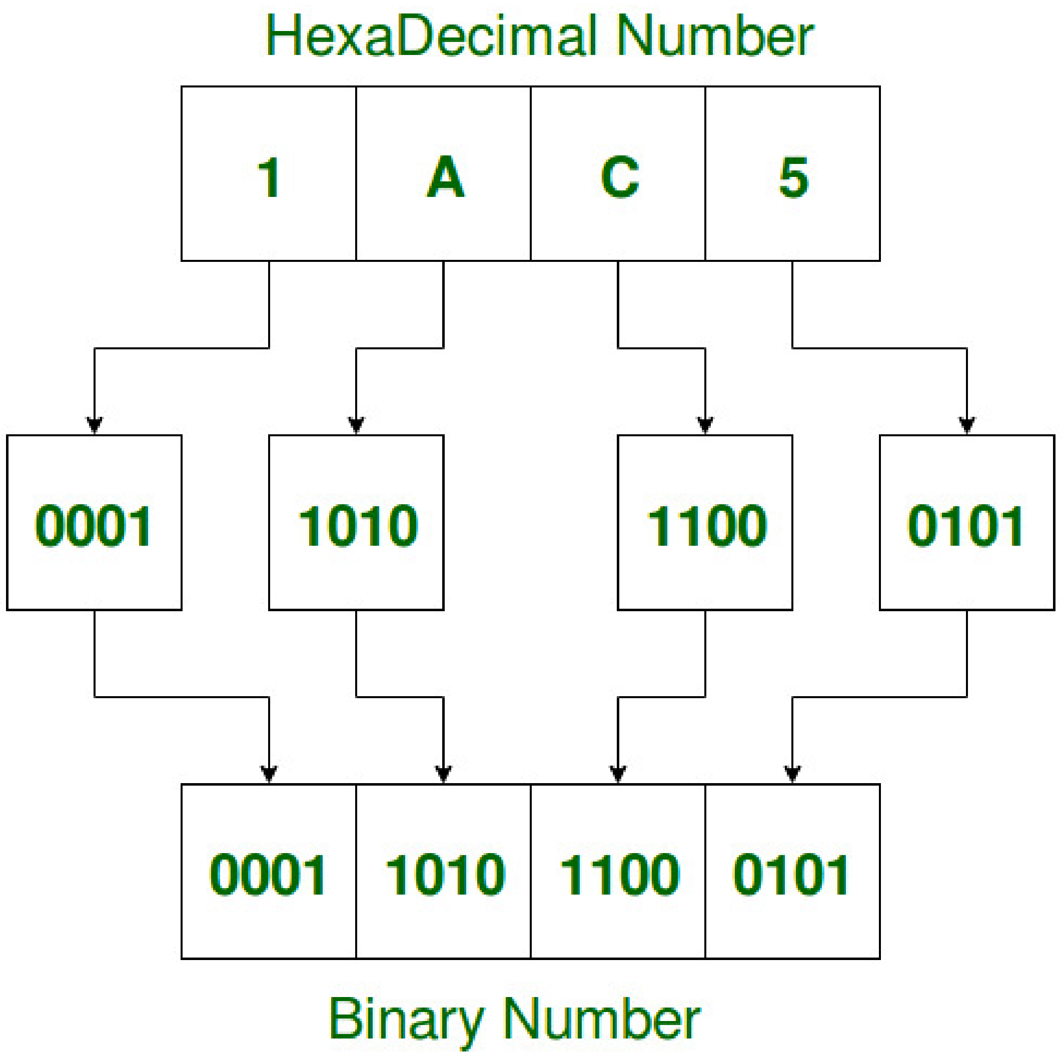
Hex–Binary and Binary–Hex Conversions

A hexadecimal number can be converted into its binary equivalent by replacing each hex digit with its four-bit binary equivalent. We take the four-bit equivalent because the base of the hexadecimal number system is 16 and it is the fourth power of the base of the binary number system. All we have then to remember is the four-bit binary equivalents of the basic digits of the hexadecimal number system. A given binary number can be converted into an equivalent hexadecimal number by splitting the integer and fractional parts into groups of four bits, starting from the binary point on both sides. The 0s can be added to complete the outside groups if needed.

Hex–Binary and Binary–Hex Conversions

Hex (Base 16)	Binary (Base 2)
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
A	1010
B	1011
C	1100
D	1101
E	1110
F	1111

Hex–Binary Conversions



Example1.10:

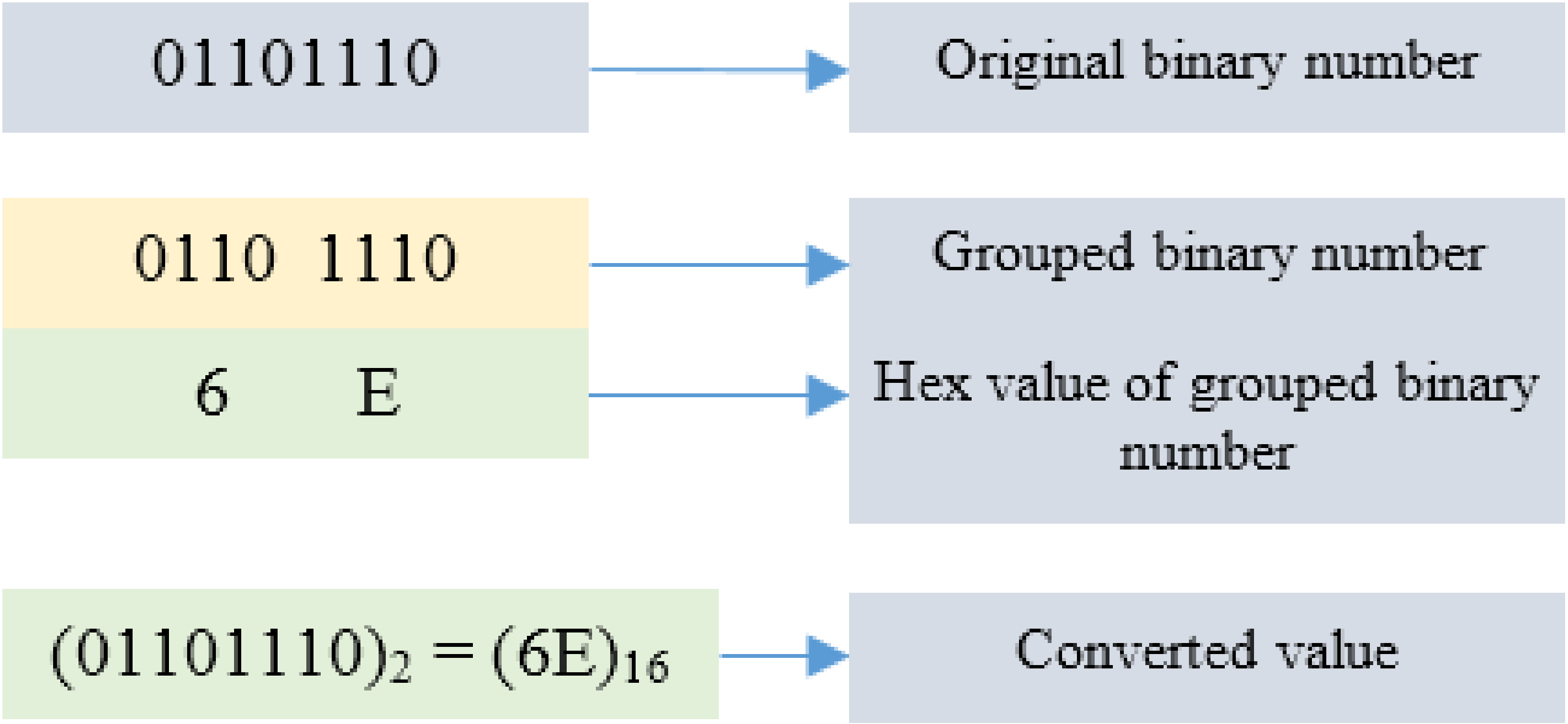
Let us find the binary equivalent of $(17E.F6)_{16}$.

- **The binary equivalent = ([0001] [0111] [1110].[1111] [0110])$_2$**

$= (000101111110.11110110)_2 = (101111110.1111011)_2$

- **The 0s on the extreme left of the integer part and on the extreme right of the fractional part have been omitted.**

Binary–Hex Conversions



Example1.11:

Let us find the hex equivalent of $(1011001110.011011101)_2$.

$$(1011001110.011011101)_2 = ([10]\ [1100]\ [1110].[0110]\ [1110]\ [1])_2$$

- **The hex equivalent = $([0010]\ [1100]\ [1110].[0110]\ [1110]\ [1000])_2 = (2CE.6E8)_{16}$**

Hex–Octal / Example1.12:

Let us find the octal equivalent of $(2F.C4)_{16}$.

- The binary equivalent = ([0010] [1111].[1100] [0100])$_2$

= $(00101111.11000100)_2$

= $(101111.110001)_2$ = ([101] [111].[110] [001])2 = $(57.61)_8$.

Octal–Hex / Example1.13:

Let us find the hex equivalent of $(762.013)_8$.

- **The octal number = $(762.013)_8$ = ([111] [110] [010].[000] [001] [011]$)_2$**

= $(111110010.000001011)_2$

= ([0001] [1111] [0010].[0000] [0101] [1000]$)_2$ = $(1F2.058)_{16}$.

References

[1] Digital fundamentals / Thomas L. Floyd. —Eleventh edition.

[2]