Average Shear Stress

Shear stress has been defined as the stress component that acts *in the plane* of the sectioned area. To show how this stress can develop, consider the effect of applying a force **F** to the bar in Fig. (1–a). If the supports are considered rigid, and **F** is large enough, it will cause the material of the bar to deform and fail along the planes identified by *AB* and *CD*. A free-body diagram of the unsupported center segment of the bar, Fig. (1–b), indicates that the shear force $V = F/2$ must be applied at each section to hold the segment in equilibrium. The ***average shear stress*** distributed over each sectioned area that develops this shear force is defined by

$$\tau_{avg} = \frac{V}{A} \qquad (1)$$

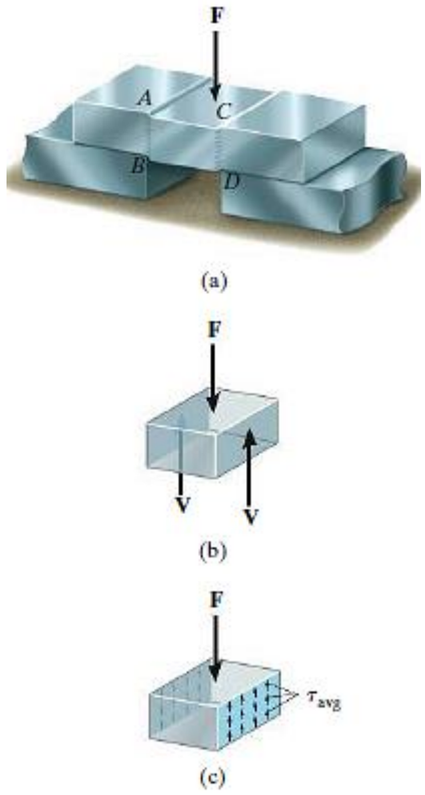


Fig (1)

The distribution of average shear stress acting over the sections is shown in Fig. (1–c). Notice that τ_{avg} is in the *same direction* as **V,** since the shear stress must create associated forces all of which contribute to the internal resultant force **V** at the section.

The loading case discussed here is an example of ***simple or direct shear,*** since the shear is caused by the *direct action* of the applied load **F.** This type of shear often occurs in various types of simple connections that use bolts, pins, welding material, etc. In all these cases, however, application of Eq. (1) is *only approximate.* A more precise investigation of the shear-stress distribution over the section often reveals that much larger shear stresses occur in the material than those predicted by this equation. Although this may be the case, application of Eq. (1) is generally acceptable for many problems in engineering design and analysis. For example, engineering codes allow its use when considering design sizes for fasteners such as bolts and for obtaining the bonding strength of glued joints subjected to shear loadings.

Shear Stress Equilibrium. Fig. (2–a) shows a volume element of material taken at a point located on the surface of a sectioned area which is subjected to a shear stress τ_{zy}. Force and moment equilibrium requires the shear stress acting on this face of the element to be accompanied by shear stress acting on three other faces. To show this we will first consider force equilibrium in the *y* direction. Then

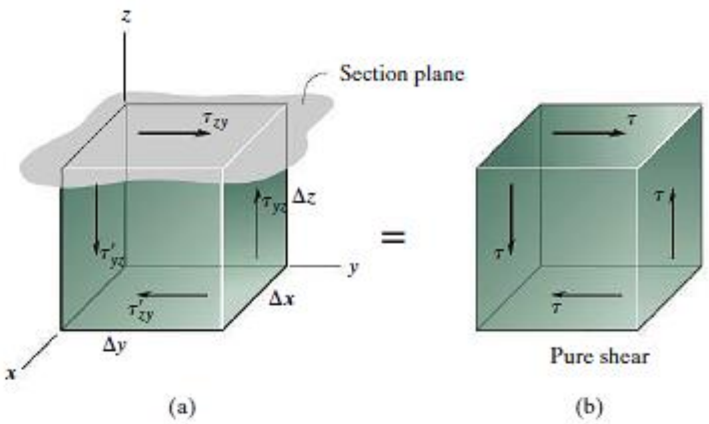


Fig (2)

$$\Sigma F_y = 0; \qquad \overbrace{\overbrace{\tau_{zy}}^{\text{stress}}\overbrace{(\Delta x\, \Delta y)}^{\text{area}}}^{\text{force}} - \tau'_{zy}\, \Delta x\, \Delta y = 0$$

$$\tau_{zy} = \tau'_{zy}$$

In a similar manner, force equilibrium in the z direction yields $\tau_{yz} = \tau'_{yz}$. Finally, taking moments about the x axis,

$$\Sigma M_x = 0; \qquad \overbrace{\overbrace{-\overbrace{\tau_{zy}}^{\text{stress}}\overbrace{(\Delta x\, \Delta y)}^{\text{area}}}^{\text{force}}\ \overbrace{\Delta z}^{\text{arm}}}^{\text{moment}} + \tau_{yz}(\Delta x\, \Delta z)\, \Delta y = 0$$

$$\tau_{zy} = \tau_{yz}$$

so that

$$\tau_{zy} = \tau'_{zy} = \tau_{yz} = \tau'_{yz} = \tau$$

In other words, ***all four shear stresses must have equal magnitude and be directed either toward or away from each other at opposite edges of the element*** , Fig. (2–b). This is referred to as the *complementary property of shear*, and under the conditions shown in Fig. (2), the material is subjected to *pure shear*.

Important Points

- If two parts are *thin or small* when joined together, the applied loads may cause shearing of the material with negligible bending. If this is the case, it is generally assumed that an *average shear stress* acts over the cross-sectional area.
- When shear stress τ acts on a plane, then equilibrium of a volume element of material at a point on the plane requires associated shear stress of the same magnitude act on three adjacent sides of the element.

Procedure for Analysis

The equation $\tau_{avg} = V/A$ is used to determine the *average shear stress* in the material. Application requires the following steps.

Internal Shear.

- Section the member at the point where the average shear stress is to be determined.
- Draw the necessary free-body diagram, and calculate the internal shear force **V** acting at the section that is necessary to hold the part in equilibrium.

Average Shear Stress.

- Determine the sectioned area A, and determine the average shear stress $\tau_{avg} = V/A$.
- It is suggested that τ_{avg} be shown on a small volume element of material located at a point on the section where it is determined. To do this, first draw τ_{avg} on the face of the element, coincident with the sectioned area A. This stress acts in the *same direction* as **V**. The shear stresses acting on the three adjacent planes can then be drawn in their appropriate directions following the scheme shown in Fig. 1–20.

EXAMPLE (1) Determine the average shear stress in the 20-mm-diameter pin at A and the 30-mm-diameter pin at B that support the beam in Fig. (3–a).

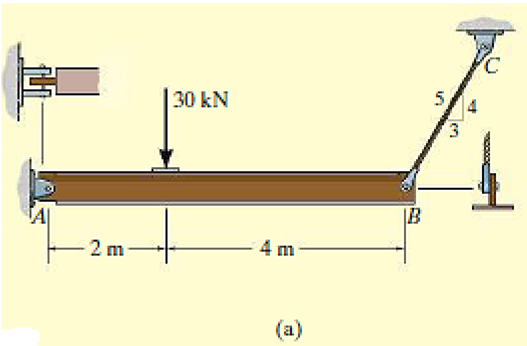
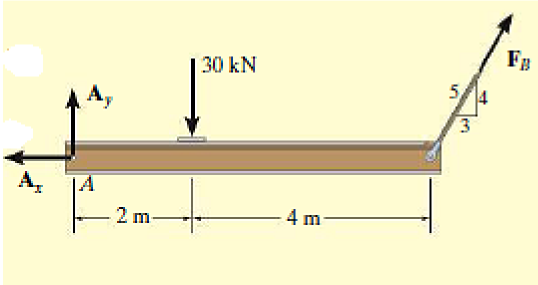


Fig (3)

SOLUTION

Internal Loadings. The forces on the pins can be obtained by considering the equilibrium of the beam, Fig. (3–b).



(b)

Fig (3)

$\circlearrowleft + \Sigma M_A = 0;$

$$F_B\left(\frac{4}{5}\right)(6\text{ m}) - 30\text{ kN}(2\text{ m}) = 0 \qquad F_B = 12.5\text{ kN}$$

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \quad (12.5\text{ kN})\left(\frac{3}{5}\right) - A_x = 0 \qquad A_x = 7.50\text{ kN}$$

$$+\uparrow\Sigma F_y = 0; \quad A_y + (12.5\text{ kN})\left(\frac{4}{5}\right) - 30\text{ kN} = 0 \quad A_y = 20\text{ kN}$$

Thus, the resultant force acting on pin *A* is

$$F_A = \sqrt{A_x^2 + A_y^2} = \sqrt{(7.50\text{ kN})^2 + (20\text{ kN})^2} = 21.36\text{ kN}$$

The pin at *A* is supported by two fixed "leaves" and so the free-body diagram of the center segment of the pin shown in Fig. (3–c) has *two* shearing surfaces between the beam and each leaf. The force of the beam (21.36 kN) acting on the pin is therefore supported by shear force on each of these surfaces. This case is called *double shear* . Thus,

$$V_A = \frac{F_A}{2} = \frac{21.36\text{ kN}}{2} = 10.68\text{ kN}$$

In Fig. (3–a), note that pin *B* is subjected to single shear, which occurs on the section between the cable and beam, Fig. (3–d). For this pin segment,

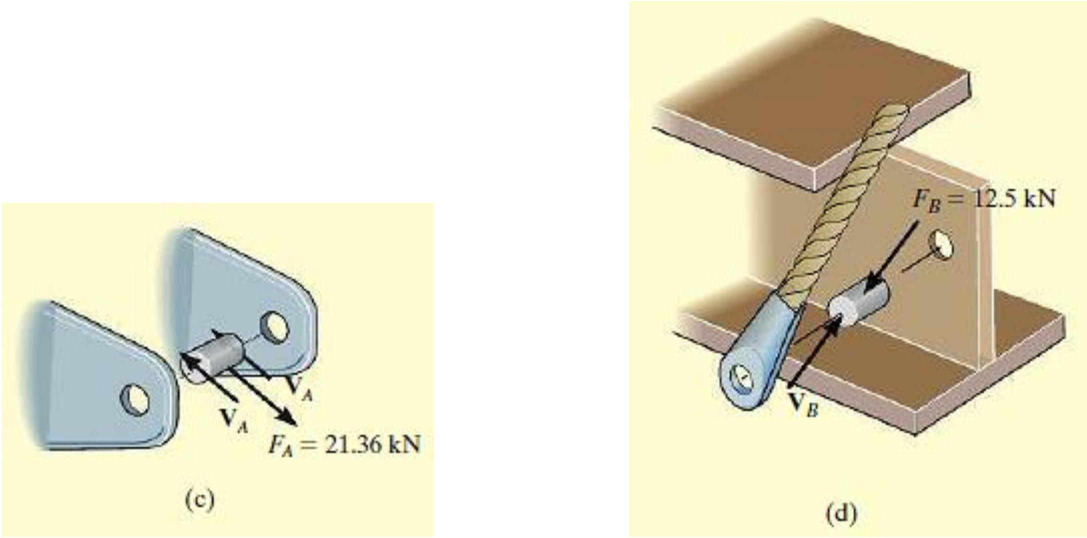


Fig (3)

$$V_B = F_B = 12.5 \text{ kN}$$

Average Shear Stress.

$$(\tau_A)_{avg} = \frac{V_A}{A_A} = \frac{10.68(10^3)\text{ N}}{\frac{\pi}{4}(0.02\text{ m})^2} = 34.0\text{ MPa} \qquad \textit{Ans.}$$

$$(\tau_B)_{avg} = \frac{V_B}{A_B} = \frac{12.5(10^3)\text{ N}}{\frac{\pi}{4}(0.03\text{ m})^2} = 17.7\text{ MPa} \qquad \textit{Ans.}$$

EXAMPLE (2) If the wood joint in Fig. (4–a) has a width of 150 mm, determine the average shear stress developed along shear planes *a – a* and *b – b*. For each plane, represent the state of stress on an element of the material.

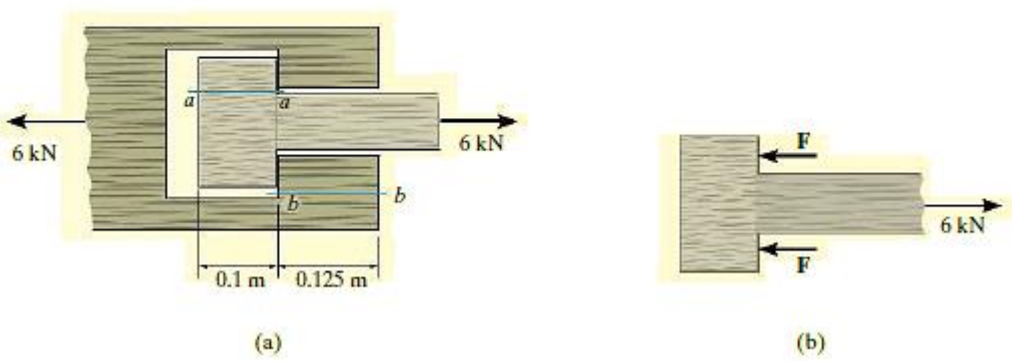


Fig (4)

SOLUTION

Internal Loadings. Referring to the free-body diagram of the member, Fig. (4–b),

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 6\text{ kN} - F - F = 0 \qquad F = 3\text{ kN}$$

Now consider the equilibrium of segments cut across shear planes *a* – *a* and *b* – *b*, shown in Figs. (4–c) and (4–d).

$$\xrightarrow{+} \Sigma F_x = 0; \qquad V_a - 3\text{ kN} = 0 \qquad V_a = 3\text{ kN}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 3\text{ kN} - V_b = 0 \qquad V_b = 3\text{ kN}$$

Average Shear Stress.

$$(\tau_a)_{\text{avg}} = \frac{V_a}{A_a} = \frac{3(10^3)\text{ N}}{(0.1\text{ m})(0.15\text{ m})} = 200\text{ kPa} \qquad \textit{Ans.}$$

$$(\tau_b)_{\text{avg}} = \frac{V_b}{A_b} = \frac{3(10^3)\text{ N}}{(0.125\text{ m})(0.15\text{ m})} = 160\text{ kPa} \qquad \textit{Ans.}$$

The state of stress on elements located on sections *a* – *a* and *b* – *b* is shown in Figs. (4–c) and (4–d), respectively.

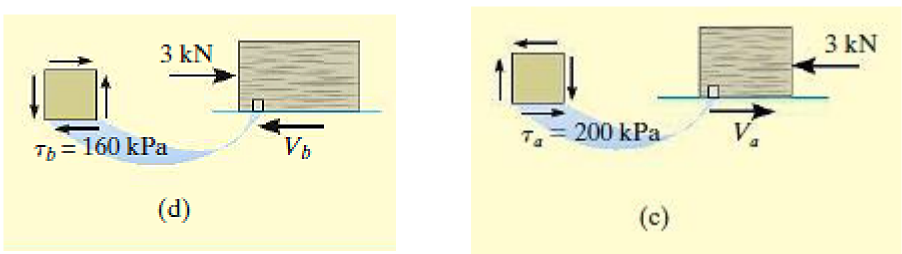


Fig (4)

EXAMPLE (3) the inclined member in Fig. (5–a) is subjected to a compressive force of 600 lb. Determine the average compressive stress along the smooth areas of contact defined by *AB* and *BC* , and the average shear stress along the horizontal plane defined by *DB* .

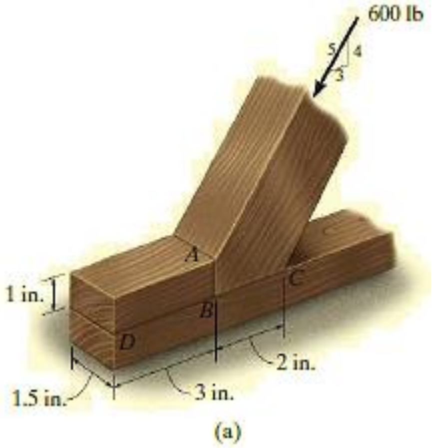


Fig (5)

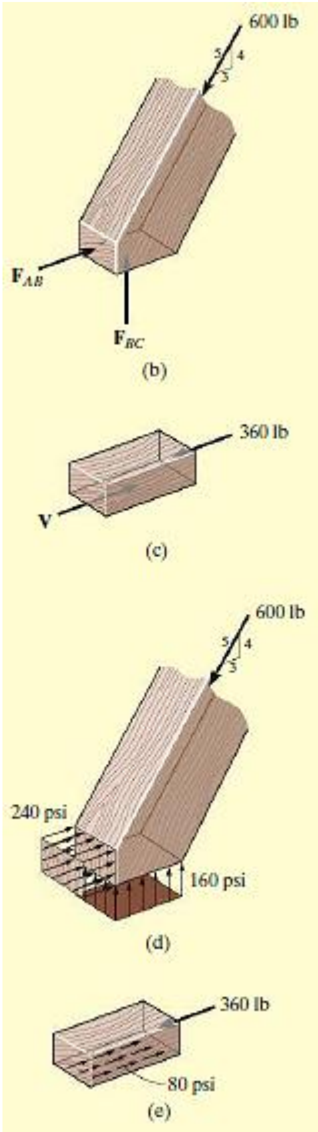


Fig (5)

SOLUTION

Internal Loadings. The free-body diagram of the inclined member is shown in Fig. (5–b). The compressive forces acting on the areas of contact are

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{AB} - 600\text{ lb}\left(\tfrac{3}{5}\right) = 0 \qquad F_{AB} = 360\text{ lb}$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{BC} - 600\text{ lb}\left(\tfrac{4}{5}\right) = 0 \qquad F_{BC} = 480\text{ lb}$$

Also, from the free-body diagram of the top segment *ABD* of the bottom member, Fig. (5–c), the shear force acting on the sectioned horizontal plane *DB* is

$$\xrightarrow{+} \Sigma F_x = 0; \qquad V - 360\text{ lb} = 0 \qquad V = 360\text{ lb}$$

Average Stress. The average compressive stresses along the horizontal and vertical planes of the inclined member are

$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{360 \text{ lb}}{(1 \text{ in.})(1.5 \text{ in.})} = 240 \text{ psi} \qquad \textit{Ans.}$$

$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{480 \text{ lb}}{(2 \text{ in.})(1.5 \text{ in.})} = 160 \text{ psi} \qquad \textit{Ans.}$$

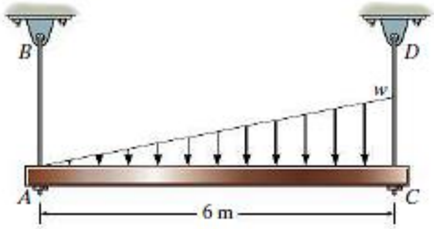
These stress distributions are shown in Fig. (5–d). The average shear stress acting on the horizontal plane defined by *DB* is

$$\tau_{\text{avg}} = \frac{360 \text{ lb}}{(3 \text{ in.})(1.5 \text{ in.})} = 80 \text{ psi} \qquad \textit{Ans.}$$

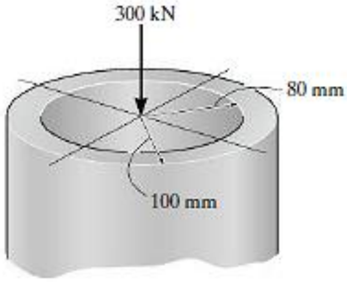
This stress is shown uniformly distributed over the sectioned area in Fig. (5–e) .

FUNDAMENTAL PROBLEMS

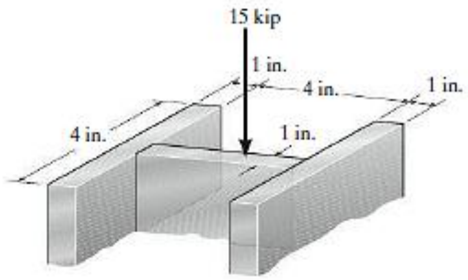
F-1 The uniform beam is supported by two rods *AB* and *CD* that have cross-sectional areas of 10 mm^2 and 15 mm^2, respectively. Determine the intensity *w* of the distributed load so that the average normal stress in each rod does not exceed 300 kPa.



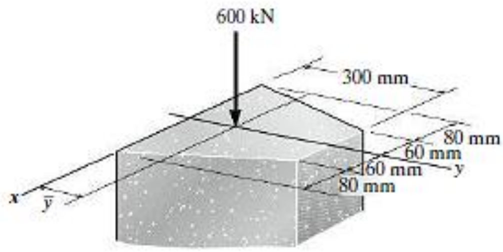
F-2 Determine the average normal stress developed on the cross section. Sketch the normal stress distribution over the cross section.



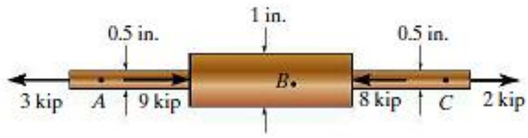
F-3 Determine the average normal stress developed on the cross section. Sketch the normal stress distribution over the cross section.



F-4 If the 600-kN force acts through the centroid of the cross section, determine the location $\bar{y}$ of the centroid and the average normal stress developed on the cross section. Also, sketch the normal stress distribution over the cross section.



F-5 Determine the average normal stress developed at points *A*, *B*, and *C*. The diameter of each segment is indicated in the figure.



F-6 Determine the average normal stress developed in rod *AB* if the load has a mass of 50 kg. The diameter of rod *AB* is 8 mm.