Stress

Introduction

Mechanics of materials is a branch of mechanics that studies the internal effects of stress and strain in a solid body that is subjected to an external loading. Stress is associated with the strength of the material from which the body is made, while strain is a measure of the deformation of the body. In addition to this, mechanics of materials includes the study of the body's stability when a body such as a column is subjected to compressive loading.

Equilibrium of a Deformable Body

We will review some of the main principles of statics that will be used throughout the text.

External Loads. A body is subjected to only two types of external loads; namely, surface forces and body forces, Fig. (1). Surface Forces.

Surface Forces. *Surface forces* are caused by the direct contact of one body with the surface of another. In all cases these forces are distributed over the *area* of contact between the bodies. If this area is small in comparison with the total surface area of the body, then the surface force can be *idealized* as a single ***concentrated force,*** which is applied to a point on the body. For example, the force of the ground on the wheels of a bicycle can be considered as a concentrated force. If the surface loading is applied along a narrow strip of area, the loading can be *idealized* as a ***linear distributed load,*** w (s). Here the loading is measured as having an intensity of force/length along the strip and is represented graphically by a series of arrows along the lines. ***The resultant force* FR *of w(s) is equivalent to the area under the distributed loading curve, and this resultant acts through the centroid C or geometric center of this area.***

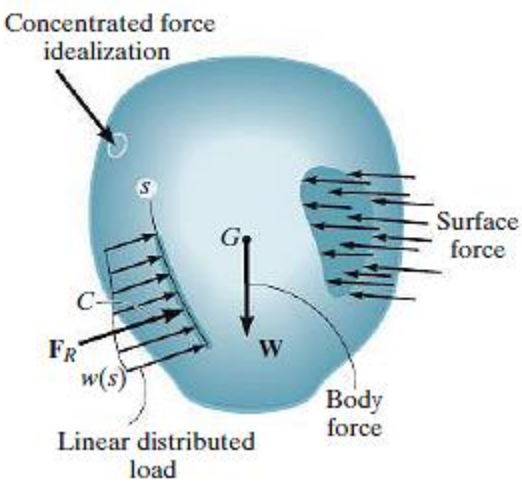
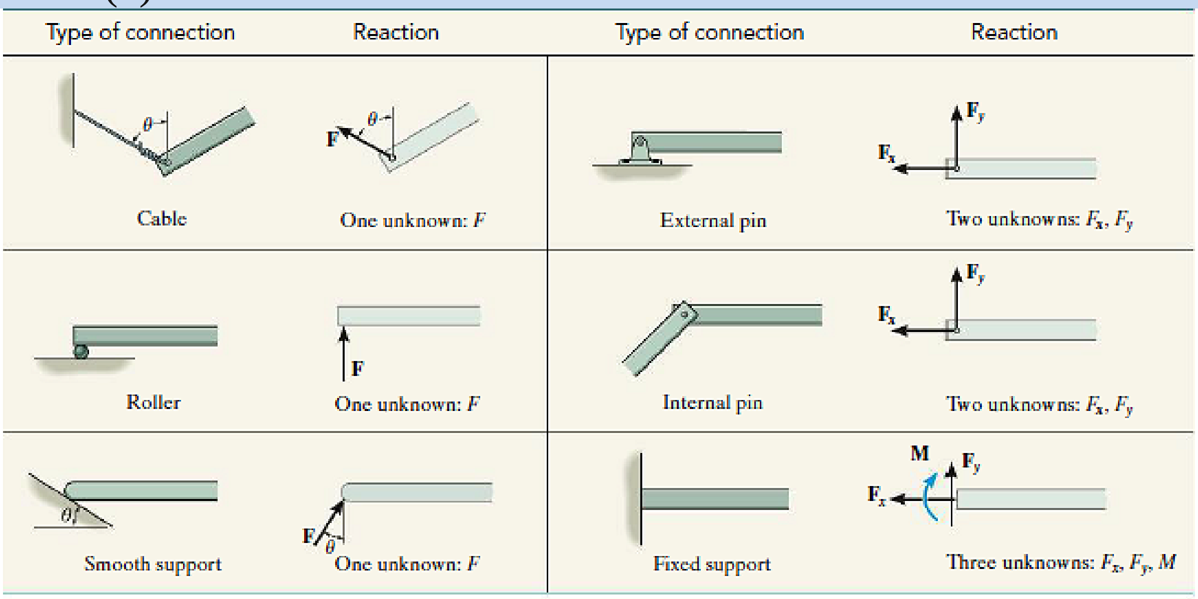


Fig (1)

Body Forces. A ***body force*** is developed when one body exerts a force on another body without direct physical contact between the bodies. Examples include the effects caused by the earth's gravitation or its electromagnetic field.

Support Reactions. The surface forces that develop at the supports or points of contact between bodies are called reactions. As a general rule, if the support prevents translation in a given direction, then a force must be developed on the member in that direction. Likewise, if rotation is prevented, a couple moment must be exerted on the member.

Table (1)



Type of connection	Reaction	Type of connection	Reaction
Cable	One unknown: F	External pin	Two unknowns: F_x, F_y
Roller	One unknown: F	Internal pin	Two unknowns: F_x, F_y
Smooth support	One unknown: F	Fixed support	Three unknowns: F_x, F_y, M

Equations of Equilibrium. Equilibrium of a body requires both a ***balance of forces,*** to prevent the body from translating or having accelerated motion along a straight or curved path, and a ***balance of moments,*** to prevent the body from rotating. These conditions can be expressed mathematically by two vector equations

$$\boxed{\begin{aligned} \Sigma \mathbf{F} &= \mathbf{0} \\ \Sigma \mathbf{M}_O &= \mathbf{0} \end{aligned}} \qquad (1)$$

The above two equations can be written in scalar form as six equations, namely,

$$\boxed{\begin{aligned} \Sigma F_x &= 0 & \Sigma F_y &= 0 & \Sigma F_z &= 0 \\ \Sigma M_x &= 0 & \Sigma M_y &= 0 & \Sigma M_z &= 0 \end{aligned}} \qquad (2)$$

Often in engineering practice the loading on a body can be represented as a system of *coplanar forces*. If this is the case, and the forces lie in the $x - y$ plane, then the conditions for equilibrium of the body can be specified with only three scalar equilibrium equations; that is,

$$\boxed{\begin{aligned} \Sigma F_x &= 0 \\ \Sigma F_y &= 0 \\ \Sigma M_O &= 0 \end{aligned}} \qquad (3)$$

Internal Resultant Loadings. In mechanics of materials, statics is primarily used to determine the resultant loadings that act within a body. For example, consider the body shown in Fig. (2) *a*, which is held in equilibrium by the four external forces. In order to obtain the internal loadings acting on a specific region within the body, it is necessary to pass an imaginary section or "cut" through the region where the internal loadings are to be determined. The two parts of the body are then separated, and a free-body diagram of one of the parts is drawn, Fig. (2) *b*. Although the exact distribution of this internal loading may be *unknown*, we can use the equations of equilibrium to relate the external forces on the bottom part of the body to the distribution's *resultant force and moment*, $\mathbf{F}_R$ and $\mathbf{M}_{R_O}$, *at any specific point O* on the sectioned area, Fig. (2) *c* .

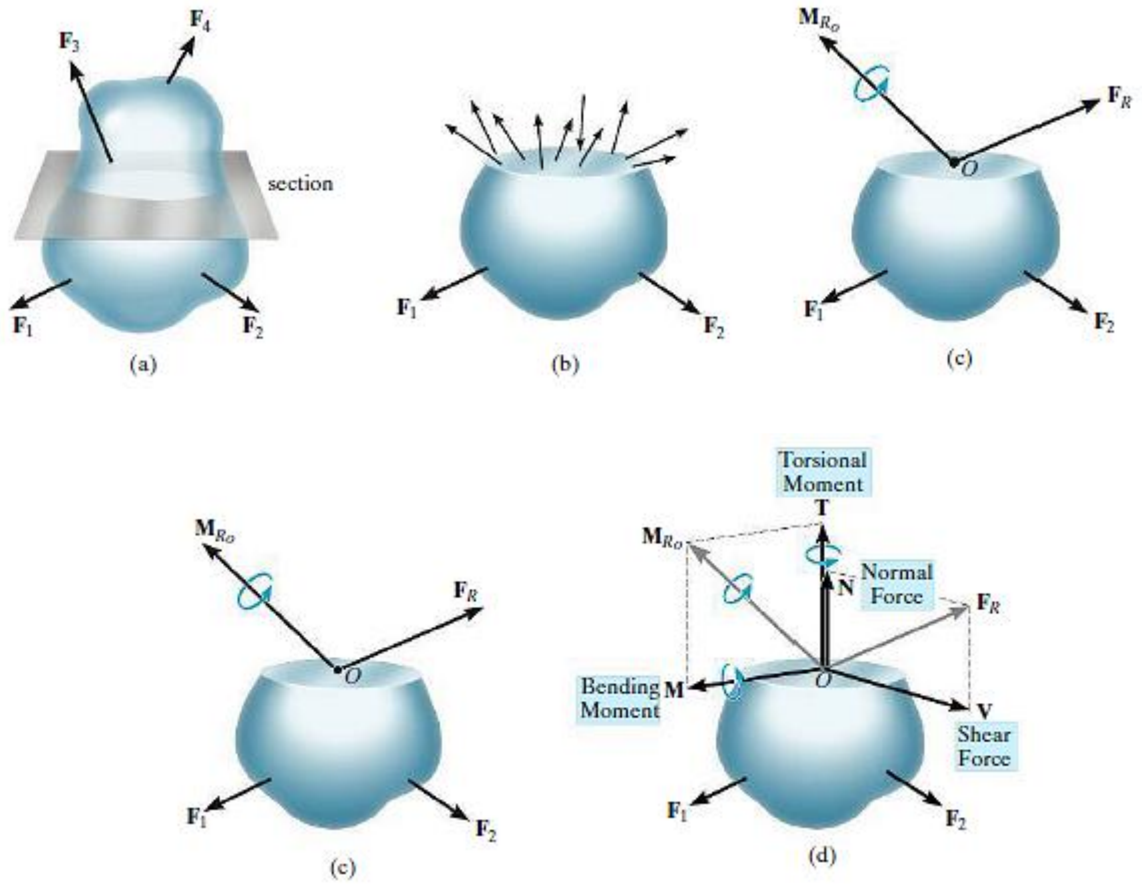


Fig (2)

Three Dimensions. Later in this text we will show how to relate the resultant loadings, $\mathbf{F}_R$ and $\mathbf{M}_{R_O}$, to the *distribution of force* on the sectioned area, and thereby develop equations that can be used for analysis and design. To do this, however, the components of $\mathbf{F}_R$ and $\mathbf{M}_{R_O}$ acting both normal and perpendicular to the sectioned area must be considered, Fig. (2) *d*. Four different types of resultant loadings can then be defined as follows:

Normal force, N. This force acts perpendicular to the area. It is developed whenever the external loads tend to push or pull on the two segments of the body.

Shear force, V. The shear force lies in the plane of the area, and it is developed when the external loads tend to cause the two segments of the body to slide over one another.

Torsional moment or torque, T. This effect is developed when the external loads tend to twist one segment of the body with respect to the other about an axis perpendicular to the area.

Bending moment, M. The bending moment is caused by the external loads that tend to bend the body about an axis lying within the plane of the area.

Coplanar Loadings. If the body is subjected to a *coplanar system of forces*, Fig. (3) *a*, then only normal-force, shear-force, and bending-moment components will exist at the section, Fig. (3) *b* . If we use the *x*, *y*, *z* coordinate axes, as shown on the left segment, then **N** can be obtained by applying $\sum Fx = 0$, and **V** can be obtained from $\sum Fy = 0$. Finally, the bending moment $\mathbf{M}_O$ can be determined by summing moments about point *O* (the *z* axis), $\sum M_O = 0$, in order to eliminate the moments caused by the unknowns **N** and **V** .

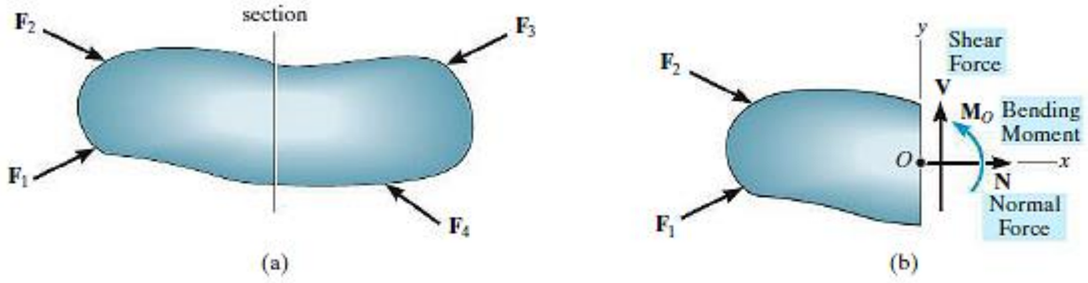


Fig (3)

Important Points

- *Mechanics of materials* is a study of the relationship between the external loads applied to a body and the stress and strain caused by the internal loads within the body.
- External forces can be applied to a body as *distributed* or *concentrated surface loadings*, or as *body forces* that act throughout the volume of the body.
- Linear distributed loadings produce a *resultant force* having a *magnitude* equal to the *area* under the load diagram, and having a *location* that passes through the *centroid* of this area.
- A support produces a *force* in a particular direction on its attached member if it *prevents translation* of the member in that direction, and it produces a *couple moment* on the member if it *prevents rotation*.
- The equations of equilibrium $\Sigma \mathbf{F} = \mathbf{0}$ and $\Sigma \mathbf{M} = \mathbf{0}$ must be satisfied in order to prevent a body from translating with accelerated motion and from rotating.
- When applying the equations of equilibrium, it is important to first draw the free-body diagram for the body in order to account for all the terms in the equations.
- The method of sections is used to determine the internal resultant loadings acting on the surface of the sectioned body. In general, these resultants consist of a normal force, shear force, torsional moment, and bending moment.

Procedure for Analysis

The resultant *internal* loadings at a point located on the section of a body can be obtained using the method of sections. This requires the following steps.

Support Reactions.

- First decide which segment of the body is to be considered. If the segment has a support or connection to another body, then *before* the body is sectioned, it will be necessary to determine the reactions acting on the chosen segment. To do this draw the free-body diagram of the *entire body* and then apply the necessary equations of equilibrium to obtain these reactions.

Free-Body Diagram.

- Keep all external distributed loadings, couple moments, torques, and forces in their *exact locations*, before passing an imaginary section through the body at the point where the resultant internal loadings are to be determined.
- Draw a free-body diagram of one of the "cut" segments and indicate the unknown resultants **N**, **V**, **M**, and **T** at the section. These resultants are normally placed at the point representing the geometric center or *centroid* of the sectioned area.
- If the member is subjected to a *coplanar* system of forces, only **N**, **V**, and **M** act at the centroid.
- Establish the x, y, z coordinate axes with origin at the centroid and show the resultant internal loadings acting along the axes.

Equations of Equilibrium.

- Moments should be summed at the section, about each of the coordinate axes where the resultants act. Doing this eliminates the unknown forces **N** and **V** and allows a direct solution for **M** (and **T**).
- If the solution of the equilibrium equations yields a negative value for a resultant, the *directional sense* of the resultant is *opposite* to that shown on the free-body diagram.

Example (1) Determine the resultant internal loadings acting on the cross section at *C* of the cantilevered beam shown in Fig. (4) *a* .

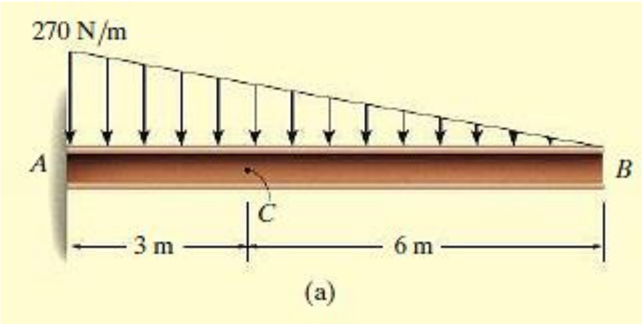


Fig (4)

SOLUTION

Support Reactions. The support reactions at *A* do not have to be determined if segment *CB* is considered.

Free-Body Diagram. The free-body diagram of segment *CB* is shown in Fig. (4) *b* . It is important to keep the distributed loading on the segment until *after* the section is made. Only then should this loading be replaced by a single resultant force. Notice that the intensity of the distributed loading at *C* is found by proportion, i.e., from Fig. (4) *a* , $w/6$ m = (270 N/m)/9 m, w = 180 N/m. The magnitude of the resultant of the distributed load is equal to the area under the loading curve (triangle) and acts through the centroid of this area. Thus, F = 1/2(180 N/m) (6 m) = 540 N, which acts 1/3 (6 m) = 2 m from *C* as shown in Fig. (4) *b*.

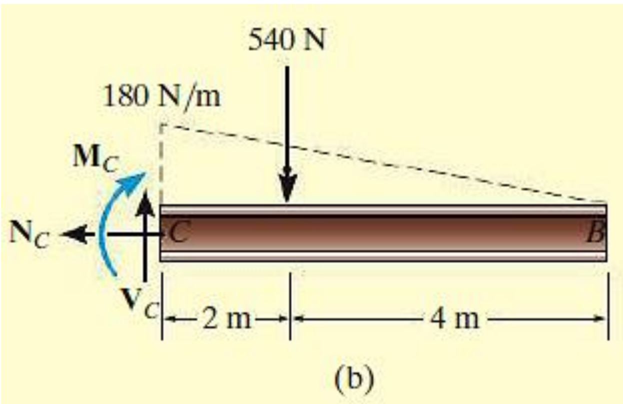


Fig (4)

Equations of Equilibrium. Applying the equations of equilibrium we have

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad -N_C = 0$

$N_C = 0$ *Ans.*

$+\uparrow \Sigma F_y = 0;$ $\quad V_C - 540\text{ N} = 0$

$V_C = 540\text{ N}$ *Ans.*

$\circlearrowleft + \Sigma M_C = 0;$ $\quad -M_C - 540\text{ N}(2\text{ m}) = 0$

$M_C = -1080\text{ N}\cdot\text{m}$ *Ans.*

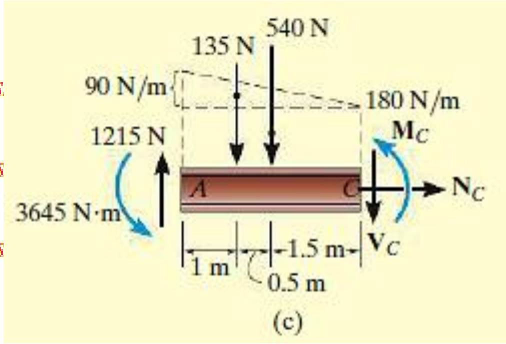


Fig (4)

NOTE: The negative sign indicates that $\mathbf{M}_C$ acts in the opposite direction to that shown on the free-body diagram. Try solving this problem using segment *AC*, by first obtaining the support reactions at *A*, which are given in Fig. (4) *c* .

Example (2) The 500-kg engine is suspended from the crane boom in Fig. (5) *a*. Determine the resultant internal loadings acting on the cross section of the boom at point *E*.

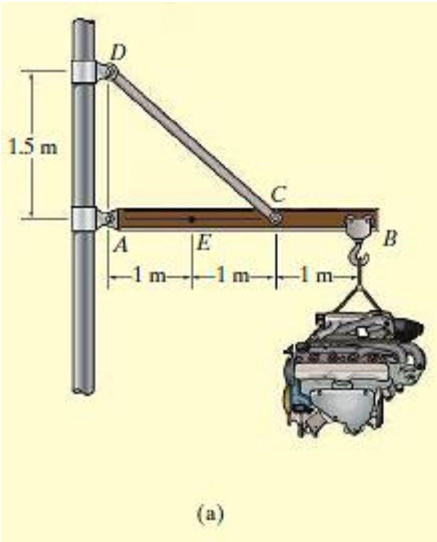


Fig (5)

SOLUTION

Support Reactions. We will consider segment *AE* of the boom, so we must first determine the pin reactions at *A*. Notice that member *CD* is a two-force member. The free-body diagram of the boom is shown in Fig. (5) *b*. applying the equations of equilibrium,

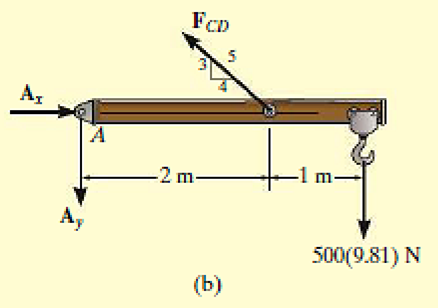


Fig (5)

$$\circlearrowleft +\Sigma M_A = 0; \qquad F_{CD}\left(\tfrac{3}{5}\right)(2\text{ m}) - [500(9.81)\text{ N}](3\text{ m}) = 0$$

$$F_{CD} = 12\,262.5\text{ N}$$

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad A_x - (12\,262.5\text{ N})\left(\tfrac{4}{5}\right) = 0$$

$$A_x = 9810\text{ N}$$

$$+\uparrow\Sigma F_y = 0; \qquad -A_y + (12\,262.5\text{ N})\left(\tfrac{3}{5}\right) - 500(9.81)\text{ N} = 0$$

$$A_y = 2452.5\text{ N}$$

Free-Body Diagram. The free-body diagram of segment *AE* is shown in Fig. (5) *c*.

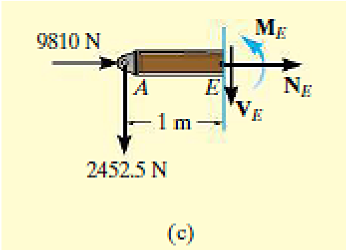


Fig (5)

Equations of Equilibrium.

$\xrightarrow{+}\Sigma F_x = 0;$ $\quad N_E + 9810 \text{ N} = 0$

$N_E = -9810 \text{ N} = -9.81 \text{ kN}$ *Ans.*

$+\uparrow\Sigma F_y = 0;$ $\quad -V_E - 2452.5 \text{ N} = 0$

$V_E = -2452.5 \text{ N} = -2.45 \text{ kN}$ *Ans.*

$\circlearrowleft +\Sigma M_E = 0;$ $\quad M_E + (2452.5 \text{N})(1 \text{ m}) = 0$

$M_E = -2452.5 \text{ N}\cdot\text{m} = -2.45 \text{ kN}\cdot\text{m}$ *Ans.*

Example (2) Determine the resultant internal loadings acting on the cross section at *G* of the beam shown in Fig. (6) *a*. Each joint is pin connected.

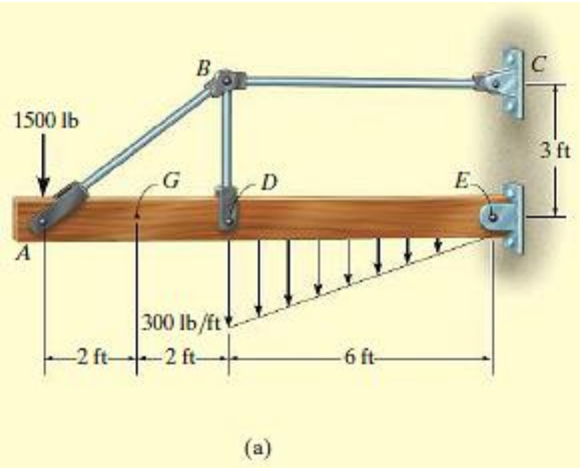


Fig (6)

SOLUTION

Support Reactions. Here we will consider segment *AG*. The free-body Diagram of the *entire* structure is shown in Fig. (6) *b*. Verify the calculated reactions at *E* and *C*. In particular, note that *BC* is a *two-force member* since only two forces act on it. For this reason the force at *C* must act along *BC*, which is horizontal as shown. Since *BA* and *BD* are also two-force members, the free-body diagram of joint *B* is shown in Fig. (6) *c*. Again, verify the magnitudes of forces **F***BA* and **F***BD*.

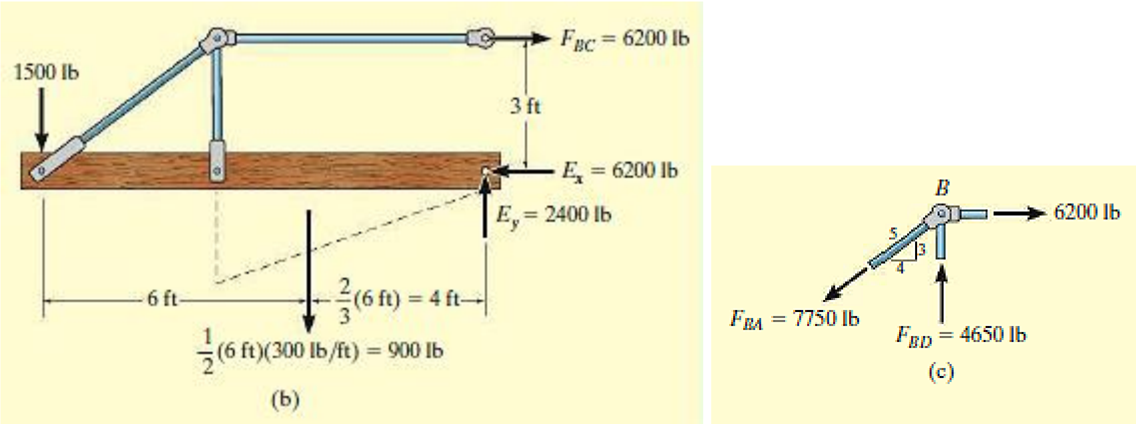


Fig (6)

Free-Body Diagram. Using the result for $\mathbf{F}_{BA}$, the free-body diagram of segment *AG* is shown in Fig. (6) *d*.

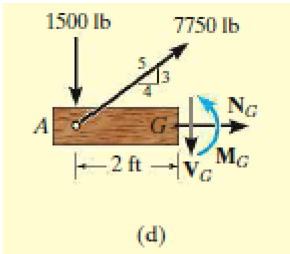


Fig (6)

Equations of Equilibrium.

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 7750 \text{ lb}\left(\tfrac{4}{5}\right) + N_G = 0 \qquad N_G = -6200 \text{ lb} \qquad \textit{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad -1500 \text{ lb} + 7750 \text{ lb}\left(\tfrac{3}{5}\right) - V_G = 0$$

$$V_G = 3150 \text{ lb} \qquad \textit{Ans.}$$

$$\circlearrowleft + \Sigma M_G = 0; \quad M_G - (7750 \text{ lb})\left(\tfrac{3}{5}\right)(2 \text{ ft}) + 1500 \text{ lb}(2 \text{ ft}) = 0$$

$$M_G = 6300 \text{ lb} \cdot \text{ft} \qquad \textit{Ans.}$$

Example (3) Determine the resultant internal loadings acting on the cross section at *B* of the pipe shown in Fig. (7) *a*. End *A* is subjected to a vertical force of 50 N, a horizontal force of 30 N, and a couple moment of 70 N / m. Neglect the pipe's mass.

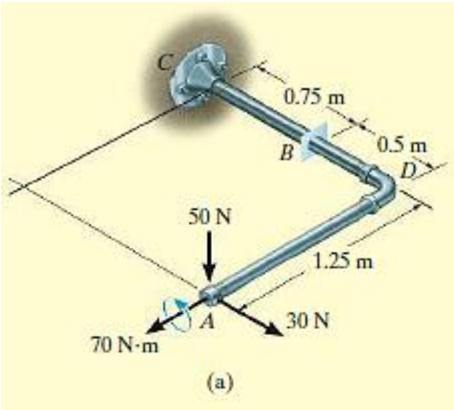


Fig (7)

SOLUTION

The problem can be solved by considering segment AB, so we do not need to calculate the support reactions at C.

Free-Body Diagram. The *x*, *y*, *z* axes are established at *B* and the free-body diagram of segment *AB* is shown in Fig. (7) *b*. The resultant force and moment components at the section are assumed to act in the *positive coordinate directions* and to pass through the *centroid* of the cross-sectional area at *B*.

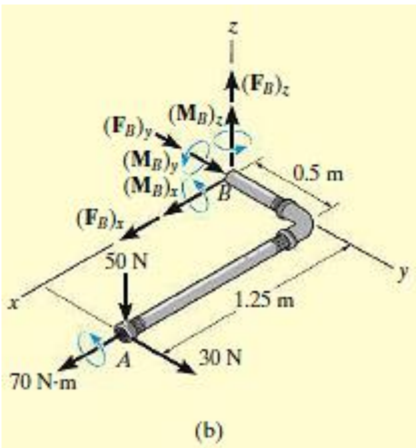


Fig (7)

Equations of Equilibrium. Applying the six scalar equations of equilibrium, we have

$$\Sigma F_x = 0; \qquad (F_B)_x = 0 \qquad \textit{Ans.}$$

$$\Sigma F_y = 0; \qquad (F_B)_y + 30\text{ N} = 0 \qquad (F_B)_y = -30\text{ N} \qquad \textit{Ans.}$$

$$\Sigma F_z = 0; \qquad (F_B)_z - 50\text{ N} = 0 \qquad (F_B)_z = 50\text{ N} \qquad \textit{Ans.}$$

$$\Sigma (M_B)_x = 0; \qquad (M_B)_x + 70\text{ N}\cdot\text{m} - 50\text{ N}\,(0.5\text{ m}) = 0$$

$$(M_B)_x = -45\text{ N}\cdot\text{m} \qquad \textit{Ans.}$$

$$\Sigma (M_B)_y = 0; \qquad (M_B)_y + 50\text{ N}\,(1.25\text{ m}) = 0$$

$$(M_B)_y = -62.5\text{ N}\cdot\text{m} \qquad \textit{Ans.}$$

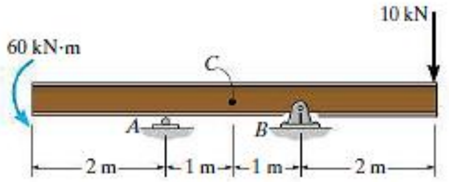
$$\Sigma (M_B)_z = 0; \qquad (M_B)_z + (30\text{ N})(1.25) = 0 \qquad \textit{Ans.}$$

$$(M_B)_z = -37.5\text{ N}\cdot\text{m}$$

NOTE: What do the negative signs for $(FB)_y$, $(MB)_x$, $(MB)_y$ and $(MB)_z$ indicate? The normal force $N_B = |\,(FB)_y\,| = 30$ N, whereas the shear force is $V_B = \sqrt{(0)^2 + (50)^2} = 50$ N. Also, the torsional moment is $T_B = |\,(MB)_y\,| = 62.5$ N.m and the bending moment is $M_B = \sqrt{(45)^2 + (37.5)^2} = 58.6$ N . m.

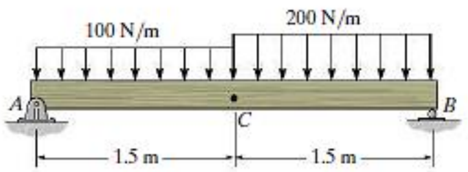
FUNDAMENTAL PROBLEMS

F1-1 Determine the internal normal force, shear force, and bending moment at point *C* in the beam.



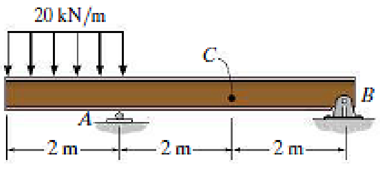
F1–1

F1-2 Determine the internal normal force, shear force, and bending moment at point *C* in the beam.



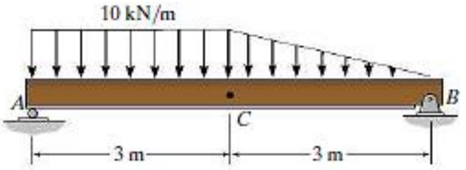
F1–2

F1-3 Determine the internal normal force, shear force, and bending moment at point *C* in the beam.



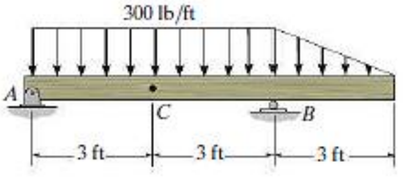
F1–3

F1-4 Determine the internal normal force, shear force, and bending moment at point *C* in the beam.



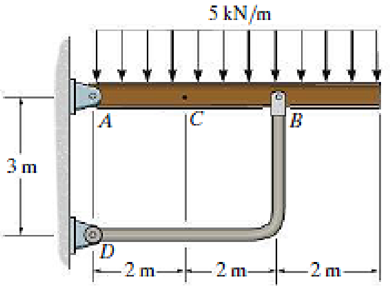
F1–4

F1-5 Determine the internal normal force, shear force, and bending moment at point *C* in the beam.



F1–5

F1-6 Determine the internal normal force, shear force, and bending moment at point *C* in the beam.



F1–6