AL MAAREF UNIVERSITY COLLEGE
Civil Engineering Department
Mechanic of Materials

Second stage
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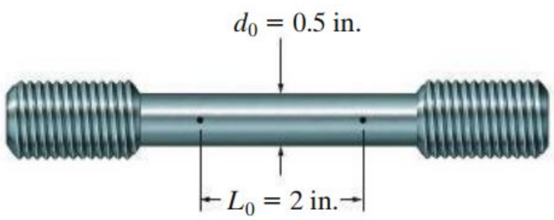
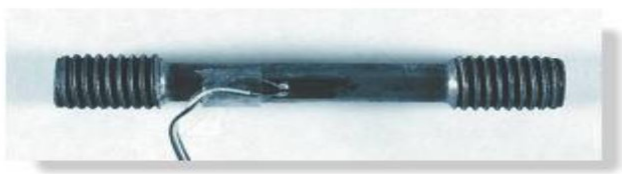
Mechanical Properties of Materials

1. The Tension and Compression Test

The strength of a material depends on its ability to sustain a load without undue deformation or failure. This property is inherent in the material itself and must be determined by experiment. One of the most important tests to perform in this regard is the tension or compression test. Although several important mechanical properties of a material can be determined from this test, it is used primarily to determine the relationship between the average normal stress and average normal strain in many engineering materials such as metals, ceramics, polymers, and composites.

To perform a tension or compression test a specimen of the material is made into a "standard" shape and size. It has a constant circular cross section with enlarged ends, so that failure will not occur at the grips. Before testing, two small punch marks are placed along the specimen's uniform length. Measurements are taken of both the specimen's initial cross-sectional area, A0, and the gauge-length distance L_0 between the punch marks. For example, when a metal specimen is used in a tension test it generally has an initial diameter of $d_0 = 0.5$ in. (13 mm) and a gauge length of $L_0 = 2$ in. (51 mm), Fig. 3–1 . In order to apply an axial load with no bending of the specimen, the ends are usually seated into ball-and-socket joints. A testing machine like the one shown in Fig. 3–2 is then used to stretch the specimen at a very slow, constant rate until it fails. The machine is designed to read the load required to maintain thisuniform stretching. At frequent intervals during the test, data is recorded of the applied load $\boldsymbol{P}$, as read on the dial of the machine or taken from a digital readout. Also, the elongation $\delta = L - L_0$ between the punch marks on the specimen may be measured using either a caliper or a mechanical or optical device called an extensometer. This value of δ (delta) is then used to calculate the average normal strain in the specimen. Sometimes, however, this measurement is not taken, since it is also possible to read the strain directly by using an electrical-resistance strain gauge , which looks like the one shown in Fig. 3–3 . The operation of this gauge is based on the change in electrical resistance of a very thin wire or piece of metal foil under strain. Essentially the gauge is cemented to the specimen along its length. If the cement is very strong in comparison to the gauge, then the gauge is in effect an integral part of the specimen, so that when the specimen is strained in the direction of the gauge, the wire and specimen will experience the same strain. By measuring the electrical resistance of the wire, the gauge may be calibrated to read values of normal strain directly.





2. The Stress–Strain Diagram

It is not feasible to prepare a test specimen to match the size, A_0 and L_0, of each structural member. Rather, the test results must be reported so they apply to a member of any size. To achieve this, the load and corresponding deformation data are used to calculate various values of the stress and corresponding strain in the specimen. A plot of the results produces a curve called the ***stress–strain diagram.*** There are two ways in which it is normally described.

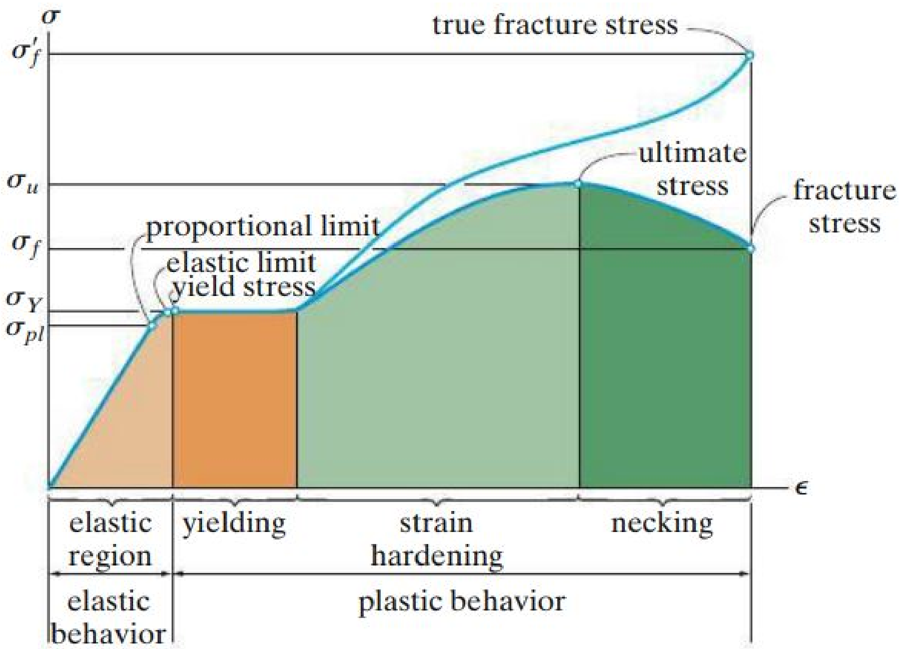
Conventional Stress–Strain Diagram. We can determine the ***nominal*** or ***engineering stress*** by dividing the applied load P by the specimen's *original* cross-sectional area A_0 . This calculation assumes that the stress is *constant* over the cross section and throughout the gauge length. We have

$$\boxed{\sigma = \frac{P}{A_0}} \qquad (3\text{–}1)$$

Likewise, the ***nominal*** or ***engineering strain*** is found directly from the strain gauge reading, or by dividing the change in the specimen's gauge length, δ , by the specimen's original gauge length L_0 . Here the strain is assumed to be constant throughout the region between the gauge points. Thus,

$$\boxed{\epsilon = \frac{\delta}{L_0}} \qquad (3\text{–}2)$$

If the corresponding values of σ and ϵ are plotted so that the vertical axis is the stress and the horizontal axis is the strain, the resulting curve is called a ***conventional stress–strain diagram*** . Realize, however, that two stress–strain diagrams for a particular material will be quite similar, but will never be exactly the same. This is because the results actually depend on variables such as the material's composition, microscopic imperfections, the way it is manufactured, the rate of loading, and the temperature during the time of the test. We will now discuss the characteristics of the conventional stress–strain curve as it pertains to *steel*, a commonly used material for fabricating both structural members and mechanical elements. Using the method described above, the characteristic stress–strain diagram for a steel specimen is shown in Fig. 3–4. From this curve we can identify four different ways in which the material behaves, depending on the amount of strain induced in the material.



Conventional and true stress-strain diagrams
for ductile material (steel) (not to scale)

Fig. 3–4

Elastic Behavior. Elastic behavior of the material occurs when the strains in the specimen are within the light orange region shown in Fig. 3–4 . Here the curve is actually a *straight line* throughout most of this region, so that the stress is *proportional* to the strain. The material in this region is said to be *linear elastic* . The upper stress limit to this linear relationship is called the ***proportional limit*** , σ_{pl} . If the stress slightly exceeds the proportional limit, the curve tends to bend and flatten out as shown. This continues until the stress reaches the ***elastic limit*** . Upon reaching this point, if the load is removed the specimen will still return back to its original shape. Normally for steel, however, the elastic limit is seldom determined, since it is very close to the proportional limit and therefore rather difficult to detect.

Yielding. A slight increase in stress above the elastic limit will result in a breakdown of the material and cause it to *deform permanently*. This behavior is called ***yielding,*** and it is indicated by the rectangular dark orange region of the curve. The stress that causes yielding is called the ***yield stress*** or ***yield point,*** σ_Y , and the deformation that occurs is called ***plastic deformation*** . Although not shown in Fig. 3–4, for low-carbon steels or those that are hot rolled, the yield point is often distinguished by two values. The ***upper yield point*** occurs first, followed by a sudden decrease in load-carrying capacity to a ***lower yield point.*** Notice that once the yield point is reached, then as shown in Fig. 3–4, the specimen will continue to elongate (strain) *without any increase in load*. When the material is in this state, it is often referred to as being ***perfectly plastic.***

Strain Hardening. When yielding has ended, an increase in load can be supported by the specimen, resulting in a curve that rises continuously but becomes flatter until it reaches a maximum stress referred to as the ***ultimate stress,*** $\boldsymbol{\sigma_u}$. The rise in the curve in this manner is called ***strain hardening,*** and it is identified in Fig. 3–4 as the region in light green.

Necking. Up to the ultimate stress, as the specimen elongates, its cross-sectional area will decrease. This decrease is fairly *uniform* over the specimen's entire gauge length; however, just after, at the ultimate stress, the cross-sectional area will begin to decrease in a *localized* region of the specimen. As a result, a constriction or "neck" tends to form in this region as the specimen elongates further, Fig. 3–5 *a* . This region of the curve due to necking is indicated in dark green in Fig. 3–4 . Here the stress–strain diagram tends to curve downward until the specimen breaksat the ***fracture stress,*** $\boldsymbol{\sigma_f}$, Fig. 3–5 *b* .

Typical necking pattern which has occurred on this steel specimen just before fracture.

Necking

(a)

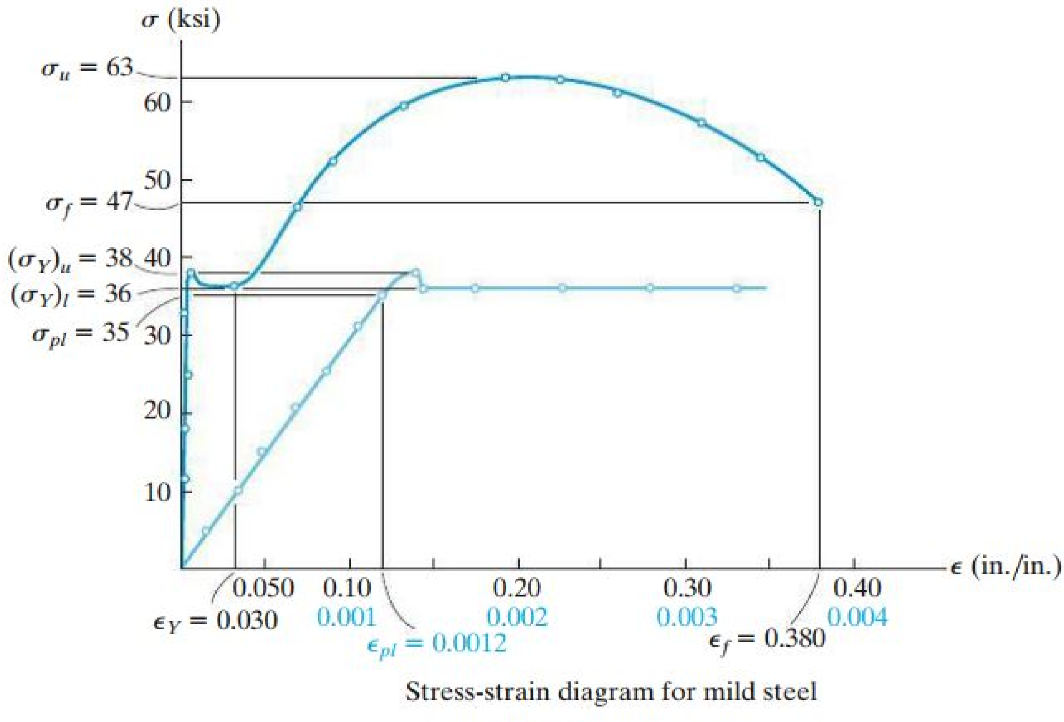
Failure of a ductile material

(b)

True Stress–Strain Diagram. Instead of always using the *original* cross-sectional area and specimen length to calculate the (engineering) stress and strain, we could have used the *actual* cross-sectional area and specimen length at the *instant* the load is measured. The values of stress and strain found from these measurements are called *true stress* and *true strain* , and a plot of their values is called the ***true stress–strain diagram*** . When this diagram is plotted it has a form shown by the light-blue curve in Fig. 3–4 . Note that the conventional and true σ - ϵ diagrams are practically coincident when the strain is small. The differences between the diagrams begin to appear in the strain-hardening range, where the magnitude of strain becomes more significant. In particular, there is a large divergence within the necking region.

Here it can be seen from the conventional σ - ϵ diagram that the specimen *actually* supports a *decreasing load* , since A_0 is constant when calculating engineering stress, $\sigma = P/A_0$. However, from the true σ - ϵ diagram, the actual area A within the necking region is always decreasing until fracture, σ_f, and so the material actually sustains *increasing stress* , since $\sigma = P/A$.

Although the true and conventional stress–strain diagrams are different, most engineering design is done so that the material supports a stress within the elastic range. This is because the deformation of the material is generally not severe and the material will restore itself when the load is removed. The true strain up to the elastic limit will remain small enough so that the error in using the engineering values of σ and ϵ is very small (about 0.1%) compared with their true values. This is one of the primary reasons for using conventional stress–strain diagrams. The above concepts can be summarized with reference to Fig. 3–6 , which shows an actual conventional stress–strain diagram for a mild steel specimen. In order to enhance the details, the elastic region of the curve has been shown in light blue color using an exaggerated strain scale, also shown in light blue. Tracing the behavior, the proportional limit is reached at σ_{pl} = 35 ksi (241 MPa), where ϵ_{pl} = 0.0012 in./in. This is followed by an upper yield point of 1s*Y*2*u* = 38 ksi (262 MPa), then suddenly a lower yield point of $(\sigma_Y)_\mu$ = 36 ksi (248 MPa). The end of yielding occurs at a strain of ϵ_Y = 0.030 in./in. , which is 25 times greater than the strain at the proportional limit! Continuing, the specimen undergoes strain hardening until it reaches the ultimate stress of σu = 63 ksi (434 MPa), then it begins to neck down until a fracture occurs, σ_f = 47 ksi (324 MPa). By comparison, the strain at failure, ϵ_f= 0.380 in./in. , is 317 times greater than ϵ_{pl}!.



Stress-strain diagram for mild steel

Fig. 3–6

3. Stress–Strain Behavior of Ductile and Brittle Materials

Materials can be classified as either being ductile or brittle, depending on their stress–strain characteristics.

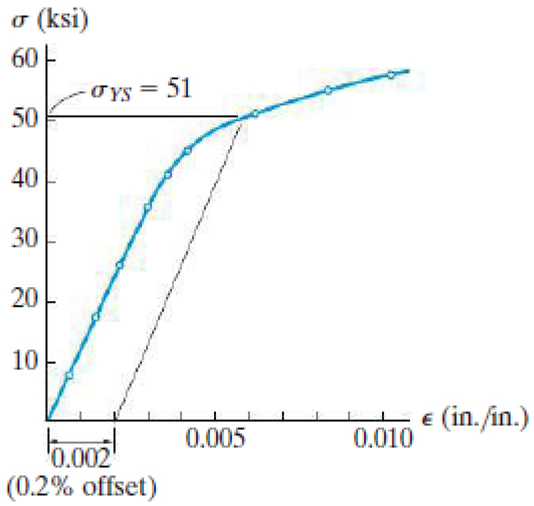
Ductile Materials. Any material that can be subjected to large strains before it fractures is called a ***ductile material.*** Mild steel, as discussed previously, is a typical example. Engineers often choose ductile materials for design because these materials are capable of absorbing shock or energy, and if they become overloaded, they will usually exhibit large deformation before failing. One way to specify the ductility of a material is to report its percent elongation or percent reduction in area at the time of fracture. The ***percent elongation*** is the specimen's fracture strain expressed as a percent. Thus, if the specimen's original gauge length is L_0 and its length at fracture is L_f , then

$$\text{Percent elongation} = \frac{L_f - L_0}{L_0}(100\%) \qquad (3\text{–}3)$$

As seen in Fig. 3–6 , since $\epsilon_f = 0.380$, this value would be 38% for a mild steel specimen. The ***percent reduction in area*** is another way to specify ductility. It is defined within the region of necking as follows:

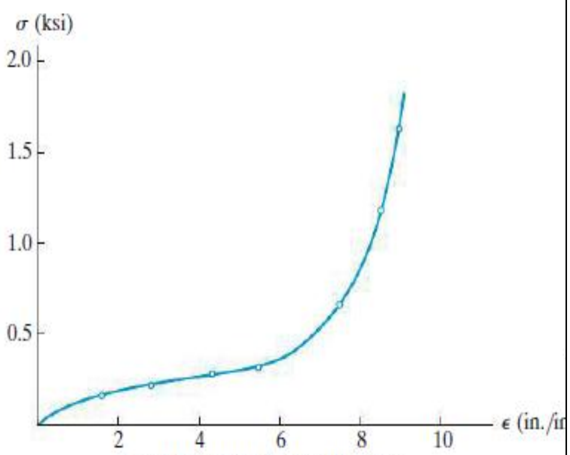
$$\text{Percent reduction of area} = \frac{A_0 - A_f}{A_0}(100\%) \qquad (3\text{–}4)$$

Here A_0 is the specimen's original cross-sectional area and A_f is the area of the neck at fracture. Mild steel has a typical value of 60%. Besides steel, other metals such as brass, molybdenum, and zinc may also exhibit ductile stress–strain characteristics similar to steel, whereby they undergo elastic stress–strain behavior, yielding at constant stress, strain hardening, and finally necking until fracture. In most metals, however, constant yielding will *not occur* beyond the elastic range. One metal for which this is the case is aluminum. Actually, this metal often does not have a well-defined *yield point* , and consequently it is standard practice to define a ***yield strength*** using a graphical procedure called the ***offset method*** . Normally for structural design a 0.2% strain (0.002 in./in.) is chosen, and from this point on the P axis, a line parallel to the initial straight-line portion of the stress–strain diagram is drawn. The point where this line intersects the curve defines the yield strength. An example of the construction for determining the yield strength for an aluminum alloy is shown in Fig. 3–7 . From the graph, the yield strength is σ_{YS} = 51 ksi (352 MPa). Apart from metals, 0.2 % strain is used as the offset to determine the yield strength of many plastics.



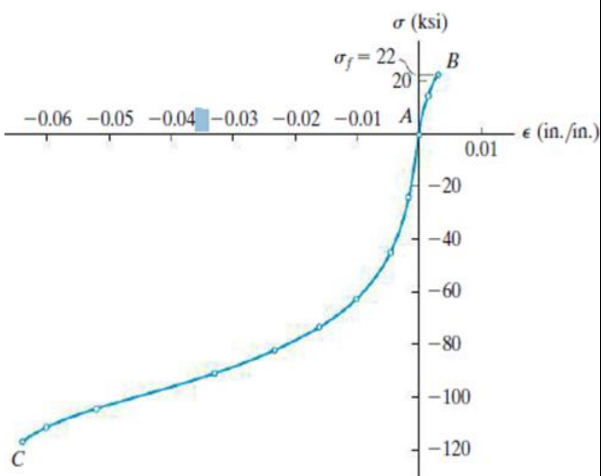
Yield strength for an aluminum alloy

Fig. 3–7



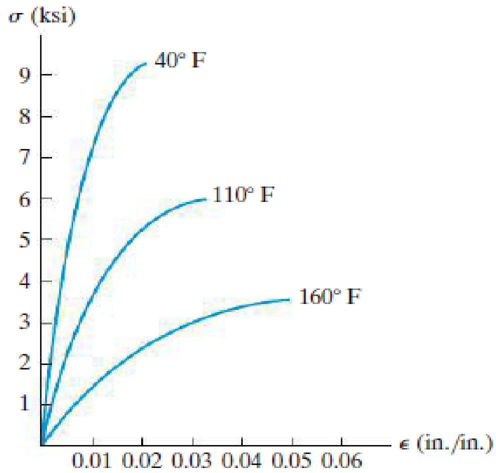
$\sigma-\epsilon$ diagram for natural rubber

Fig. 3–8



$\sigma-\epsilon$ diagram for gray cast iron

Fig. 3–9

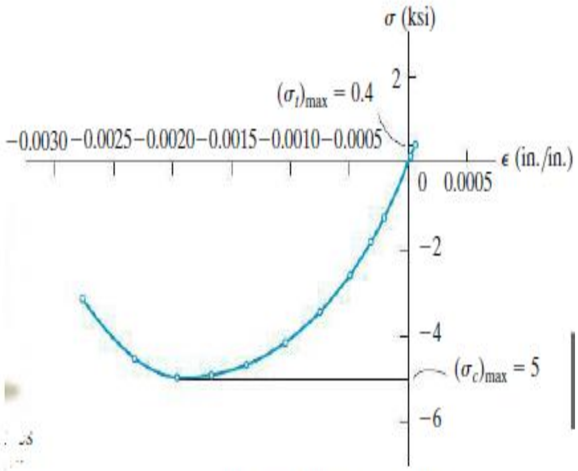


$\sigma-\epsilon$ diagrams for a methacrylate plastic

Fig. 3–12

Realize that the yield strength is not a physical property of the material, since it is a stress that causes a *specified* permanent strain in the material. In this text, however, we will assume that the yield strength, yield point, elastic limit, and proportional limit all *coincide* unless otherwise stated. An exception would be natural rubber, which in fact does not even have a proportional limit, since stress and strain are *not* linearly related. Instead, as shown in Fig. 3–8, this material, which is known as a polymer, exhibits *nonlinear elastic behavior*. Wood is a material that is often moderately ductile, and as a result it is usually designed to respond only to elastic loadings. The strength characteristics of wood vary greatly from one species to another, and for each species they depend on the moisture content, age, and the size and arrangement of knots in the wood. Since wood is a fibrous material, its tensile or compressive characteristics will differ greatly when it is loaded either parallel or perpendicular to its grain. Specifically, wood splits easily when it is loaded in tension perpendicular to its grain, and consequently tensile loads are almost always intended to be applied parallel to the grain of wood members.

Brittle Materials. Materials that exhibit little or no yielding before failure are referred to as ***brittle materials.*** Gray cast iron is an example, having a stress–strain diagram in tension as shown by portion *AB* of the curve in Fig. 3–9. Here fracture at σ_f = 22 ksi (152 MPa) took place initially at an imperfection or microscopic crack and then spread rapidly across the specimen, causing complete fracture. Since the appearance of initial cracks in a specimen is quite random, brittle materials do not have a well-defined tensile fracture stress. Instead the *average* fracture stress from a set of observed tests is generally reported. A typical failed specimen is shown in Fig. 3–10 *a* . Compared with their behavior in tension, brittle materials, such as gray cast iron, exhibit a much higher resistance to axial compression, as evidenced by portion *AC* of the curve in Fig. 3–9. For this case any cracks or imperfections in the specimen tend to close up, and as the load increases the material will generally bulge or become barrel shaped as the strains become larger, Fig. 3–10 *b*. Like gray cast iron, concrete is classified as a brittle material, and it also has a low strength capacity in tension. The characteristics of its stress–strain diagram depend primarily on the mix of concrete (water, sand, gravel, and cement) and the time and temperature of curing. A typical example of a "complete" stress–strain diagram for concrete is given in Fig. 3–11. By inspection, its maximum compressive strength is about 12.5 times greater than its tensile strength, $(\sigma_c)_{max}$ = 5 ksi (34.5 MPa) versus $(\sigma_t)_{max}$ = 0.40 ksi (2.76 MPa). For this reason, concrete is almost always reinforced with steel bars or rods whenever it is designed to support tensile loads. It can generally be stated that most materials exhibit both ductile and brittle behavior. For example, steel has brittle behavior when it contains a high carbon content, and it is ductile when the carbon content is reduced. Also, at low temperatures materials become harder and more brittle, whereas when the temperature rises they become softer and more ductile. This effect is shown in Fig. 3–12 for a methacrylate plastic.



σ−ϵ diagram for typical concrete mix

Fig. 3-11

3.4 Hooke's Law

As noted in the previous section, the stress–strain diagrams for most engineering materials exhibit a *linear relationship* between stress and strain within the elastic region. Consequently, an increase in stress causes a proportionate increase in strain. This fact was discovered by Robert Hooke in 1676 using springs and is known as *Hooke's law* . It may be expressed mathematically as

$$\boxed{\sigma = E\epsilon} \tag{3–5}$$

Here E represents the constant of proportionality, which is called the ***modulus of elasticity*** or ***Young's modulus,*** named after Thomas Young, who published an account of it in 1807. Equation 3–5 actually represents the equation of the *initial straight-lined portion* of the stress–strain diagram up to the proportional limit. Furthermore, the modulus of elasticity represents the *slope* of this line. Since strain is dimensionless, from Eq. 3–5 , $\boldsymbol{E}$ will have the same units as stress, such as psi, ksi, or pascals. As an example of its calculation, consider the stress–strain diagram for steel shown in Fig. 3–6 . Here σ_{pl} = 35 ksi and ϵ_{pl} = 0.0012 in./in., so that

$$E = \frac{\sigma_{pl}}{\epsilon_{pl}} = \frac{35 \text{ ksi}}{0.0012 \text{ in./in.}} = 29(10^3) \text{ ksi}$$

As shown in Fig. 3–13 , the proportional limit for a particular type of steel alloy depends on its carbon content; however, most grades of steel, from the softest rolled steel to the hardest tool steel, have about the same modulus of elasticity, generally accepted to be E_{st} = 2911032 ksi or 200 GPa. Values of E for other commonly used engineering materials are often tabulated in engineering codes and reference books. Representative values are also listed on the inside back cover of this book. It should be noted that the modulus of elasticity is a mechanical property that indicates the *stiffness* of a material. Materials that are very stiff, such as steel, have large values of E [E_{st} = 2911032 ksi or 200 GPa], whereas spongy materials such as vulcanized rubber may have low values [E_r = 0.10 ksi or 0.69 MPa]. The modulus of elasticity is one of the most important mechanical properties used in the development of equations presented in this text. It must always be remembered, though, that E can be used only if a material has *linear elastic behavior* . Also, if the stress in the material is *greater* than the proportional limit, the stress–strain diagram ceases to be a straight line and so Eq. 3–5 is no longer valid.

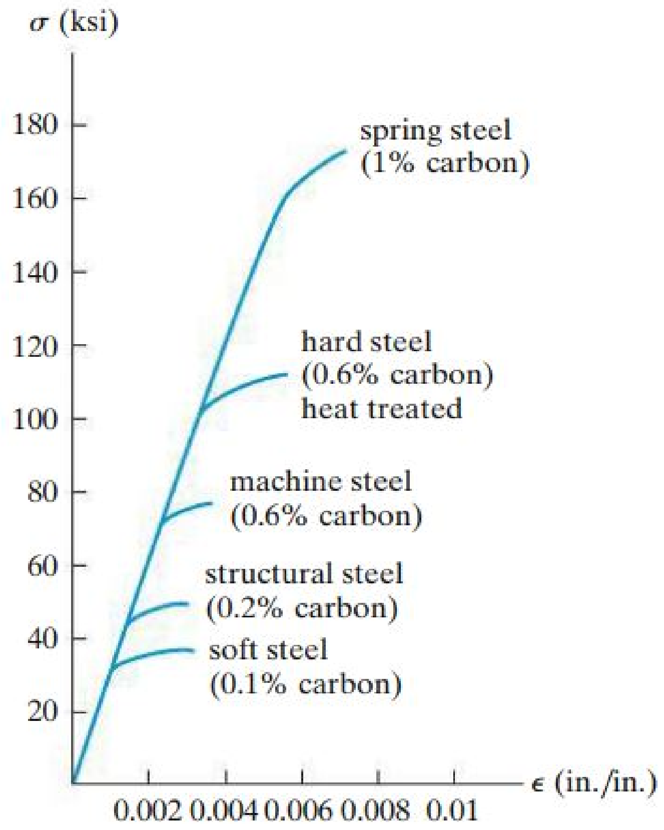


Fig. 3–13

3.5 Strain Energy

As a material is deformed by an external load, the load will do external work, which in turn will be stored in the material as internal energy. This energy is related to the strains in the material, and so it is referred to as ***strain energy.*** To obtain this strain energy let us consider a volume element of material from a tension test specimen Fig. 3–15. It is subjected to the uniaxial stress s. This stress develops a force $\Delta F = \sigma\, \Delta A = \sigma\, (\Delta x\, \Delta y)$ on the top and bottom faces of the element *after* the element of length Δ z undergoes a vertical displacement $\epsilon\, \Delta z$. By definition, *work* of ΔF is determined by the product of a force and the displacement in the direction of the force. Since the force is increased uniformly from zero to its final magnitude ΔF when the displacement $\epsilon\ \Delta z$ occurs, the work done on the element by the force is then equal to the *average* force magnitude ($\Delta F/2$) times the displacement $\epsilon\ \Delta z$. The conservation of energy requires this "external work" on the element to be equivalent to the "internal work" or strain energy stored in the element—assuming that no energy is lost in the form of heat. Consequently, the strain energy ΔU is $\Delta U = (1/2\ \Delta F)\ \epsilon\ \Delta z = (1/2\ \sigma\ \Delta x\ \Delta y)\ \epsilon\ \Delta z$. Since the volume of the element is $\Delta V = \Delta x\ \Delta y\ \Delta z$, then $\Delta U = 1/\ 2\ \sigma\epsilon\Delta V$. For applications, it is often convenient to specify the strain energy per unit volume of material. This is called the ***strain-energy density,*** and it can be expressed as

$$u = \frac{\Delta U}{\Delta V} = \frac{1}{2}\sigma\epsilon \qquad (3\text{–}6)$$

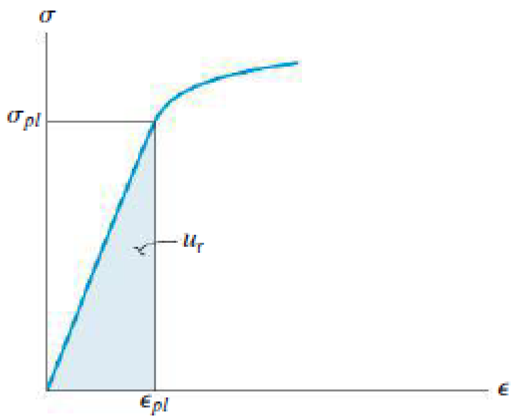
Finally, if the material behavior is *linear elastic*, then Hooke's law applies, $\sigma = E\epsilon$, and therefore we can express the *elastic strain-energy density* in terms of the uniaxial stress s as

$$u = \frac{1}{2}\frac{\sigma^2}{E} \qquad (3\text{–}7)$$

Modulus of Resilience. In particular, when the stress s reaches the proportional limit, the strain-energy density, as calculated by Eq. 3–6 or 3–7, is referred to as the ***modulus of resilience,*** i.e.,

$$\boxed{u_r = \frac{1}{2}\sigma_{pl}\,\epsilon_{pl} = \frac{1}{2}\frac{\sigma_{pl}^2}{E}} \qquad (3\text{–}8)$$

From the elastic region of the stress–strain diagram, Fig. 3–16 *a*, notice that u_r is equivalent to the shaded *triangular area* under the diagram. Physically the modulus of resilience represents the largest amount of internal strain energy per unit volume the material can absorb without causing any permanent damage to the material. Certainly this becomes important when designing bumpers or shock absorbers

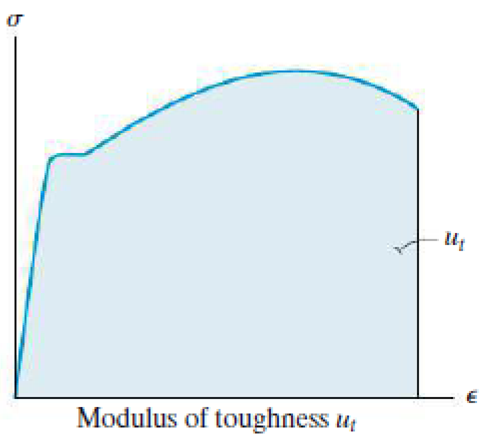


Modulus of resilience u_r

(a)

Fig. 3–16

Modulus of Toughness. Another important property of a material is the ***modulus of toughness*** , *ut*. This quantity represents the *entire area* under the stress–strain diagram, Fig. 3–16 *b* , and therefore it indicates the maximum amount of strain-energy the material can absorb just before it fractures. This property becomes important when designing members that may be accidentally overloaded. Note that alloying metals can also change their resilience and toughness. For example, by changing the percentage of carbon in steel, the resulting stress–strain diagrams in Fig. 3–17 show how the degrees of resilience (Fig. 3–16 *a*) and toughness (Fig. 3–16 *b*) can be changed.



(b)

Fig. 3–16 (cont.)

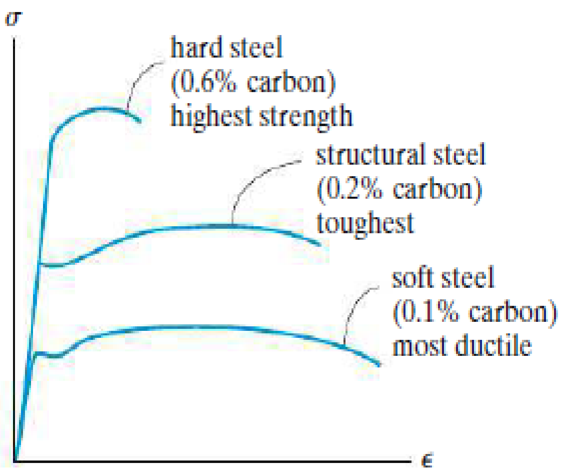


Fig. 3–17

EXAMPLE 3.1

A tension test for a steel alloy results in the stress–strain diagram shown in Fig. 3–18. Calculate the modulus of elasticity and the yield strength based on a 0.2% offset. Identify on the graph the ultimate stress and the fracture stress.

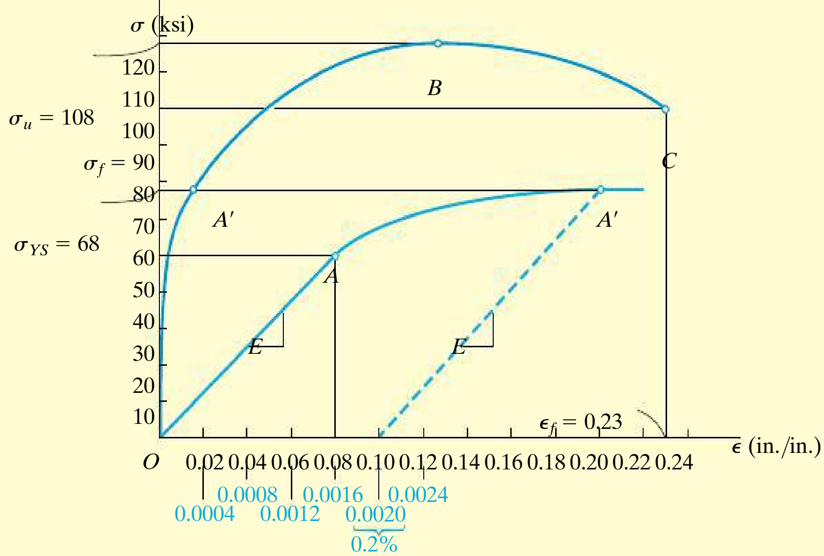


Fig. 3–18

SOLUTION

Modulus of Elasticity. We must calculate the *slope* of the initial straight-line portion of the graph. Using the magnified curve and scale shown in blue, this line extends from point O to an estimated point A, which has coordinates of approximately (0.0016 in./in., 50 ksi). Therefore,

$$E = \frac{50\text{ ksi}}{0.0016\text{ in./in.}} = 31.2(10^3)\text{ ksi} \qquad \textit{Ans.}$$

Note that the equation of line OA is thus $\sigma = 31.2(10^3)\epsilon$.

Yield Strength. For a 0.2% offset, we begin at a strain of 0.2% or 0.0020 in./in. and graphically extend a (dashed) line parallel to OA until it intersects the σ–ϵ curve at A'. The yield strength is approximately

$$\sigma_{YS} = 68\text{ ksi} \qquad \textit{Ans.}$$

Ultimate Stress. This is defined by the peak of the σ–ϵ graph, point B in Fig. 3–18.

$$\sigma_u = 108\text{ ksi} \qquad \textit{Ans.}$$

Fracture Stress. When the specimen is strained to its maximum of $\epsilon_f = 0.23$ in./in., it fractures at point C. Thus,

$$\sigma_f = 90\text{ ksi} \qquad \textit{Ans.}$$

EXAMPLE 3.2

The stress–strain diagram for an aluminum alloy that is used for making aircraft parts is shown in Fig. 3–19. If a specimen of this material is stressed to 600 MPa, determine the permanent strain that remains in the specimen when the load is released. Also, find the modulus of resilience both before and after the load application.

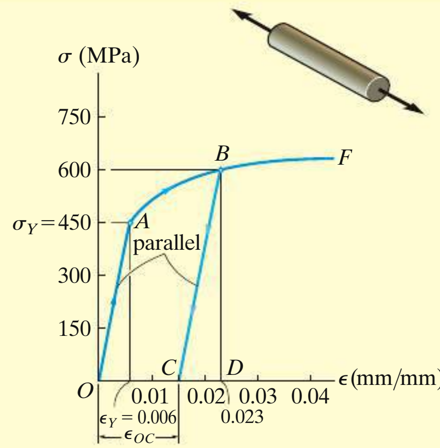


Fig. 3–19

SOLUTION

Permanent Strain. When the specimen is subjected to the load, it strain-hardens until point B is reached on the σ–ϵ diagram. The strain at this point is approximately 0.023 mm/mm. When the load is released, the material behaves by following the straight line BC, which is parallel to line OA. Since both lines have the same slope, the strain at point C can be determined analytically. The slope of line OA is the modulus of elasticity, i.e.,

$$E = \frac{450 \text{ MPa}}{0.006 \text{ mm/mm}} = 75.0 \text{ GPa}$$

From triangle CBD, we require

$$E = \frac{BD}{CD}; \qquad 75.0(10^9) \text{ Pa} = \frac{600(10^6) \text{ Pa}}{CD}$$

$$CD = 0.008 \text{ mm/mm}$$

This strain represents the amount of *recovered elastic strain*. The permanent strain, ϵ_{OC}, is thus

$$\epsilon_{OC} = 0.023 \text{ mm/mm} - 0.008 \text{ mm/mm}$$
$$= 0.0150 \text{ mm/mm} \qquad \textit{Ans.}$$

Note: If gauge marks on the specimen were originally 50 mm apart, then after the load is *released* these marks will be $50 \text{ mm} + (0.0150)(50 \text{ mm}) = 50.75$ mm apart.

Modulus of Resilience. Applying Eq. 3–8, we have*

$$(u_r)_{\text{initial}} = \frac{1}{2}\sigma_{pl}\,\epsilon_{pl} = \frac{1}{2}(450 \text{ MPa})(0.006 \text{ mm/mm})$$
$$= 1.35 \text{ MJ/m}^3 \qquad \textit{Ans.}$$

$$(u_r)_{\text{final}} = \frac{1}{2}\sigma_{pl}\,\epsilon_{pl} = \frac{1}{2}(600 \text{ MPa})(0.008 \text{ mm/mm})$$
$$= 2.40 \text{ MJ/m}^3 \qquad \textit{Ans.}$$

NOTE: By comparison, the effect of strain-hardening the material has caused an increase in the modulus of resilience; however, note that the modulus of toughness for the material has decreased since the area under the original curve, $OABF$, is larger than the area under curve CBF.

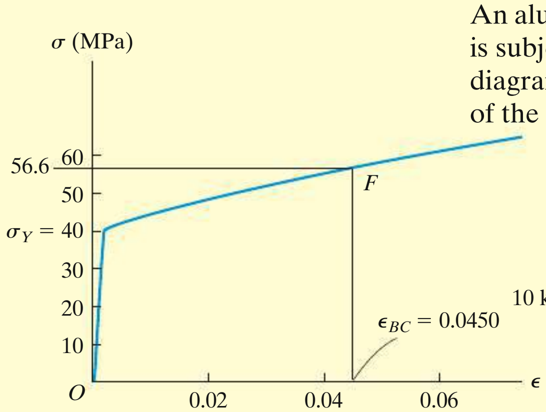
*Work in the SI system of units is measured in joules, where $1 \text{ J} = 1 \text{ N}\cdot\text{m}$.

3

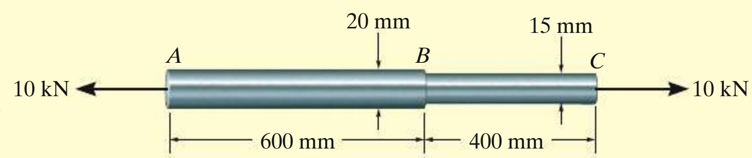
3

EXAMPLE 3.3

An aluminum rod, shown in Fig. 3–20*a*, has a circular cross section and is subjected to an axial load of 10 kN. If a portion of the stress–strain diagram is shown in Fig. 3–20*b*, determine the approximate elongation of the rod when the load is applied. Take $E_{al} = 70$ GPa.



(b)



(a)

Fig. 3–20

SOLUTION

For the analysis we will neglect the *localized deformations* at the point of load application and where the rod's cross-sectional area suddenly changes. (These effects will be discussed in Sections 4.1 and 4.7.) Throughout the midsection of each segment the normal stress and deformation are uniform.

In order to find the elongation of the rod, we must first obtain the strain. This is done by calculating the stress, then using the stress–strain diagram. The normal stress within each segment is

$$\sigma_{AB} = \frac{P}{A} = \frac{10(10^3)\text{ N}}{\pi(0.01\text{ m})^2} = 31.83\text{ MPa}$$

$$\sigma_{BC} = \frac{P}{A} = \frac{10(10^3)\text{ N}}{\pi(0.0075\text{ m})^2} = 56.59\text{ MPa}$$

From the stress–strain diagram, the material in segment AB is strained *elastically* since $\sigma_{AB} < \sigma_Y = 40$ MPa. Using Hooke's law,

$$\epsilon_{AB} = \frac{\sigma_{AB}}{E_{al}} = \frac{31.83(10^6)\text{ Pa}}{70(10^9)\text{ Pa}} = 0.0004547\text{ mm/mm}$$

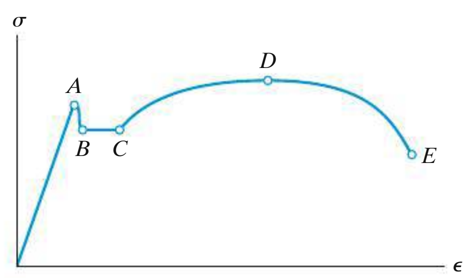
The material within segment BC is strained plastically, since $\sigma_{BC} > \sigma_Y = 40$ MPa. From the graph, for $\sigma_{BC} = 56.59$ MPa, $\epsilon_{BC} \approx 0.045$ mm/mm. The approximate elongation of the rod is therefore

$$\delta = \Sigma\epsilon L = 0.0004547(600\text{ mm}) + 0.0450(400\text{ mm})$$
$$= 18.3\text{ mm} \qquad \textit{Ans.}$$

FUNDAMENTAL PROBLEMS

F3–1. Define a homogeneous material.

F3–2. Indicate the points on the stress–strain diagram which represent the proportional limit and the ultimate stress.



F3–2

F3–3. Define the modulus of elasticity E.

F3–4. At room temperature, mild steel is a ductile material. True or false?

F3–5. Engineering stress and strain are calculated using the *actual* cross-sectional area and length of the specimen. True or false?

F3–6. As the temperature increases the modulus of elasticity will increase. True or false?

F3–7. A 100-mm long rod has a diameter of 15 mm. If an axial tensile load of 100 kN is applied, determine its change in length. Assume linear-elastic behavior with $E = 200$ GPa.

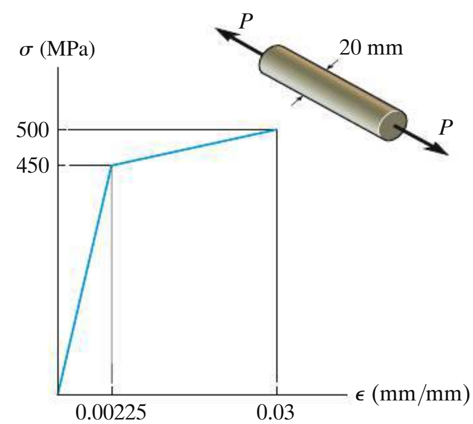
F3–8. A bar has a length of 8 in. and cross-sectional area of 12 in^2. Determine the modulus of elasticity of the material if it is subjected to an axial tensile load of 10 kip and stretches 0.003 in. The material has linear-elastic behavior.

F3–9. A 10-mm-diameter rod has a modulus of elasticity of $E = 100$ GPa. If it is 4 m long and subjected to an axial tensile load of 6 kN, determine its elongation. Assume linear-elastic behavior.

F3–10. The material for the 50-mm-long specimen has the stress–strain diagram shown. If $P = 100$ kN, determine the elongation of the specimen.

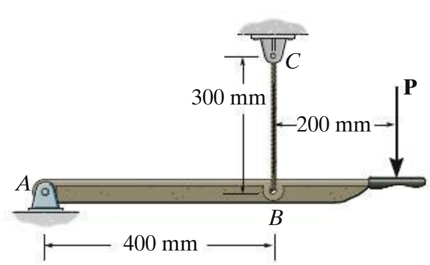
F3–11. The material for the 50-mm-long specimen has the stress–strain diagram shown. If $P = 150$ kN is applied and then released, determine the permanent elongation of the specimen.

3



F3–10/11

F3–12. If the elongation of wire BC is 0.2 mm after the force **P** is applied, determine the magnitude of **P**. The wire is A-36 steel and has a diameter of 3 mm.



F3–12

Important Points

- A *conventional stress–strain diagram* is important in engineering since it provides a means for obtaining data about a material's tensile or compressive strength without regard for the material's physical size or shape.
- *Engineering stress and strain* are calculated using the *original* cross-sectional area and gauge length of the specimen.
- A *ductile material*, such as mild steel, has four distinct behaviors as it is loaded. They are *elastic behavior, yielding, strain hardening*, and *necking*.
- A material is *linear elastic* if the stress is proportional to the strain within the elastic region. This behavior is described by *Hooke's law*, $\sigma = E\epsilon$, where the *modulus of elasticity E* is the slope of the line.
- Important points on the stress–strain diagram are the *proportional limit, elastic limit, yield stress, ultimate stress*, and *fracture stress*.
- The *ductility* of a material can be specified by the specimen's *percent elongation* or the *percent reduction in area*.
- If a material does not have a distinct yield point, a *yield strength* can be specified using a graphical procedure such as the *offset method*.
- *Brittle materials*, such as gray cast iron, have very little or no yielding and so they can fracture suddenly.
- *Strain hardening* is used to establish a higher yield point for a material. This is done by straining the material beyond the elastic limit, then releasing the load. The modulus of elasticity remains the same; however, the material's ductility *decreases*.
- *Strain energy* is energy stored in a material due to its deformation. This energy per unit volume is called *strain-energy density*. If it is measured up to the proportional limit, it is referred to as the *modulus of resilience*, and if it is measured up to the point of fracture, it is called the *modulus of toughness*. It can be determined from the area under the $\sigma-\epsilon$ diagram.