To find the inverse transforms of $\mathcal{Z}$-transformation, another approach is to be used rather than partial fractions this approach can not be applied for Laplace transforms.

(1) Linear Factors

$$\frac{H(z)}{(az+b)(cz+d)} = \frac{Az}{az+b} + \frac{Bz}{cz+d}$$

Ex: $$\frac{z+4}{(z-3)(z-2)} = \frac{Az}{(z-3)} + \frac{Bz}{(z-2)}$$

(2) Repeated Factors

$$\frac{H(z)}{(az+b)^n} = \frac{A_n \cdot z}{(az+b)^n} + \frac{A_{n-1} \cdot z}{(az+b)^{n-1}} + \dots \frac{A_1 \cdot z}{(az+b)}$$

Ex: $$\frac{z+4}{(z+3)^3} = \frac{Az}{(z+3)^3} + \frac{Bz}{(z+3)^2} + \frac{Cz}{(z+3)}$$

(3) Quadratic Factors

$$\frac{H(z)}{az^2+bz+c} = \frac{Az^2+Bz}{az^2+bz+c}$$

EX: $\dfrac{z-1}{(z-4)(z^2+1)} = \dfrac{Az}{(z-4)} + \dfrac{Bz^2+Cz}{(z^2+1)}$

if az^2+bz+c has to be repeated.

$$\frac{H(z)}{(az^2+bz+c)^n} = \frac{A_n z^2+B_n z}{(az^2+bz+c)^n} + \dots + \frac{A_1 z^2+b_1 z}{(az^2+bz+c)}$$

1. If $X(z) = \dfrac{2z^2+3z}{z^2-2z-8}$, find $\mathcal{Z}^{-1}\{X(z)\}$

$$\frac{2z^2+3z}{(z+2)(z-4)} = \frac{Az}{(z+2)} + \frac{Bz}{(z-4)}$$

$$2z^2+3z = (Az^2-4Az) + (Bz^2+2Bz)$$

z^2: $A+B=2 \Rightarrow B=2-A \quad (1)$

z^1: $3=-4A+2B$; From (1)

$3=-4A+2(2-A) \Rightarrow A=1/6$

$\therefore B = 2-\dfrac{1}{6} = \dfrac{11}{6}$

$$\mathcal{Z}^{-1}\left\{\frac{2z^2+3z}{z^2-2z+8}\right\} = \frac{1}{6}\,\mathcal{Z}^{-1}\left\{\frac{z}{z+2}\right\} + \frac{11}{6}\,\mathcal{Z}^{-1}\left\{\frac{z}{z+4}\right\}$$

$$\therefore \{x_k\} = \frac{(-2)^k}{6} + \frac{11}{6}(4)^k$$

Method(2)

$$2z^2+3z = Az(z-4) + Bz(z+2)$$

set : $(z+2)=0 \Rightarrow z=-2$

$$2(-2)^2+3(-2) = A(-2)(-2-4) + B(-2)\underbrace{\cancel{(-2+2)}}_{0}$$

$$8-6 = 12A \Rightarrow A = \frac{1}{6}$$

Set : $(z-4)=0 \Rightarrow z=4$

$$2(4)^2+3(4) = A(4)\underbrace{\cancel{(4-4)}}_{0} + B(4)(4+2)$$

$$32+12 = 24B$$

$$B = \frac{24}{44} = \frac{6}{11}$$

2. Find $\mathcal{Z}^{-1}\left\{\dfrac{z^2-4z}{(z-2)^2}\right\}$

$$\left\{\frac{z^2-4z}{(z-2)^2} = \frac{Az}{(z-2)^2} + \frac{Bz}{(z-2)}\right\} \times (z-2)^2$$

$$z^2-4z = Az(1) + Bz(z-2)$$

$$z^2-4z = Az + Bz^2 - 2Bz$$

For $(z-2)=0 \Rightarrow z=2$

$$4-4(2) = A(2) + 0 \Rightarrow \boxed{A=-2}$$

z : $-4 = A-2B$, $-4 = -2 - 2B$, $\boxed{B=1}$ or

z^2 : $\boxed{1=B}$

$$\mathcal{Z}^{-1}\left\{\frac{z^2-4z}{(z-2)^2}\right\} = -2\,\mathcal{Z}^{-1}\left\{\frac{z}{(z-2)^2}\right\} + \mathcal{Z}^{-1}\left\{\frac{z}{z-2}\right\}$$

$$= -2k2^{k-1} + 2^k = 2^k - 2k2^{k-1}$$

$$\{x_k\} = (1-2^{-1}k)\,2^k \quad \text{Ans}$$

3. Find $\mathcal{Z}^{-1}\left\{\dfrac{z+2}{z^2-5z+6}\right\}$

$$\left\{\frac{z+2}{(z-3)(z-2)} = \frac{Az}{(z-3)} + \frac{Bz}{(z-2)}\right\} \times (z-3)(z-2)$$

$$z+2 = Az(z-2) + Bz(z-3),$$

$A=3$

$$3+2 = 3A(3-2) + 0 \Rightarrow A = 5/3$$

$B=2$

$$2+2 = 0 + 2B(2-3) \Rightarrow B=-2$$

$$\mathcal{Z}^{-1}\left\{\frac{z+2}{(z-3)(z-2)}\right\} = \frac{5}{3}\,\mathcal{Z}^{-1}\left\{\frac{z}{z-3}\right\} - 2\,\mathcal{Z}^{-1}\left\{\frac{z}{z-2}\right\}$$

$$\{x_k\} = \frac{5}{3}(3)^k - 2(2)^k \quad \underline{\text{Ans.}}$$

4. Find $\mathcal{Z}^{-1}\left\{\dfrac{z^2-3}{(z+2)(z^2+1)}\right\}$

$$\left\{\frac{z^2-3}{(z+2)(z^2+1)} = \frac{Az}{z+2} + \frac{Bz^2+Cz}{z^2+1}\right\} \times (z+2)(z^2+1)$$

$$z^2-3 = Az(z^2+1) + (Bz^2+Cz)(z+2)$$

$$= Az^3 + Az + Bz^3 + 2Bz^2 + Cz^2 + 2Cz$$

$z+2=0 \therefore z=-2$

$$(-2)^2-3 = A(-2)[(-2)^2+1] + 0$$

$$-1 = -10A \Rightarrow \boxed{A = \frac{-1}{10}}$$

z^3: $0 = A + B \Rightarrow \boxed{B = -A = \frac{1}{10}}$

z^2: $2B + C = 1 \quad C = 1 - \frac{2}{10} = \frac{8}{10}$

$$\mathcal{Z}^{-1}\left\{\frac{z^3-3}{(z+2)(z^2+1)}\right\} = -\frac{1}{10}\mathcal{Z}^{-1}\left\{\frac{z}{z+2}\right\} + \mathcal{Z}^{-1}\left\{\frac{\frac{z^2}{10}+\frac{8z}{10}}{z^2+1}\right\}$$

$$= -\frac{1}{10}\mathcal{Z}^{-1}\left\{\frac{z}{z+2}\right\} + \frac{1}{10}\mathcal{Z}^{-1}\left\{\frac{z^2}{z^2+1}\right\} + \frac{8}{10}\mathcal{Z}^{-1}\left\{\frac{z}{z^2+1}\right\}$$

we have;

$$\mathcal{Z}\{\cos n\theta\} = \frac{z^2 - \cos\theta}{z^2 - 2z\cos\theta + 1}, \text{ and if } \theta = \pi/2$$

$$\mathcal{Z}\left\{\cos n\frac{\pi}{2}\right\} = \frac{z^2}{z^2+1}, \text{ similarly}$$