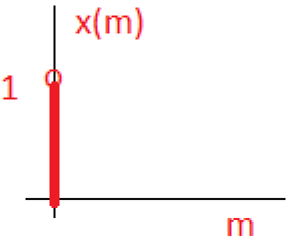
Z- Transform

Example 1

Determine the Z-transform, the region of convergence (ROC), and the Fourier transform of the following signal.

$$x(m) = \delta(m) = \begin{cases} 1 & m = 0 \\ 0 & m \neq 0 \end{cases}$$

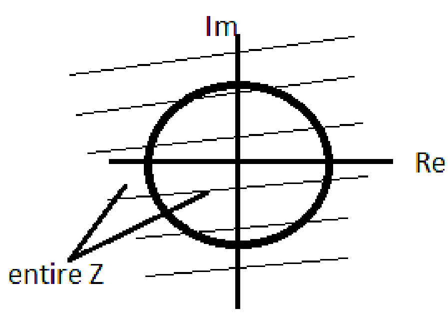


Sol.

$$X(z) = \sum_{m=-\infty}^{\infty} x(m) z^{-m} = \sum_{m=-\infty}^{\infty} \delta(m) z^{-m} = \delta(0)\, z^{0} = 1$$

So that the F. T. is δ (m)

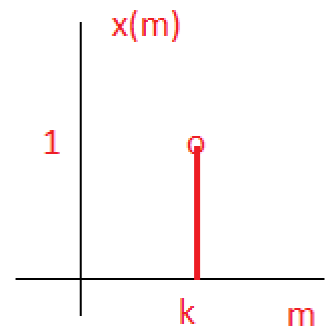
And ROC of X (z) is the set of all the values of z for which X (z) attains a finite computable value. i.e., are all values of z.



Example 2

Determine the Z-transform, the region of convergence (ROC), and the Fourier transform of the following signal.

$$x(m) = \delta(m-k) = \begin{cases} 1 & m = k \\ 0 & m \neq k \end{cases}$$



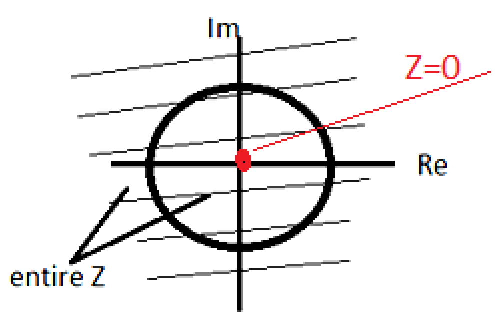
Sol.

$$X(z) = \sum_{m=-\infty}^{\infty} x(m-k)z^{-m} = \sum_{m=-\infty}^{\infty} \delta(m-k)z^{-m} = z^{-k}$$

The ROC of X (z) is the entire Z-Plane except the point Z=0.

The F. T. is obtaining by Z= $e^{j\omega}$:

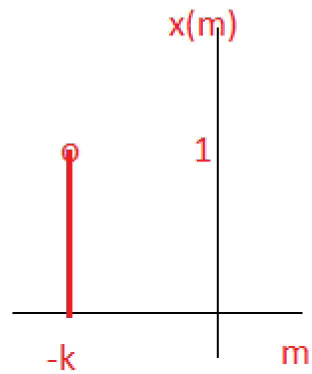
i.e., $X(e^{j\omega}) = e^{-j\omega k}$



Example 3

Determine the Z-transform, the region of convergence (ROC), and the Fourier transform of the following signal.

$$x(m) = \delta(m+k) = \begin{cases} 1 & m = -k \\ 0 & m \neq -k \end{cases}$$



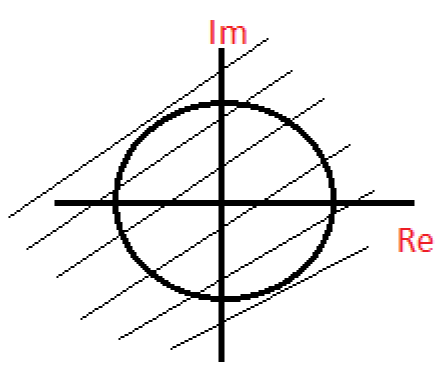
Sol.

$$X(z) = \sum_{m=-\infty}^{\infty} x(m+k) z^{-m} = \sum_{m=-\infty}^{\infty} \delta(m+k) z^{-m} = z^{k}$$

The ROC of X (z) is the entire Z-Plane except the point Z = ∞.

The F. T. is obtaining by Z= $e^{j\omega}$:

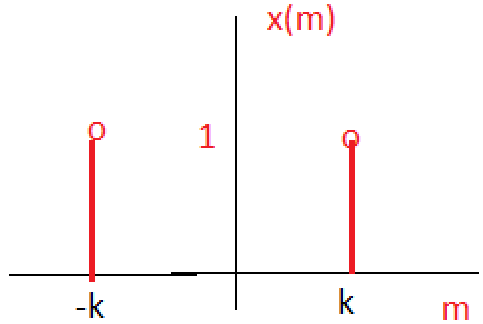
i.e., $X(e^{j\omega}) = e^{j\omega k}$



Example 4

Determine the Z-transform, the region of convergence (ROC), and the Fourier transform of the following signal.

$$x(m) = \delta(m+k) + \delta(m-k) = \begin{cases} 1 & m = \pm k \\ 0 & m \neq \pm k \end{cases}$$



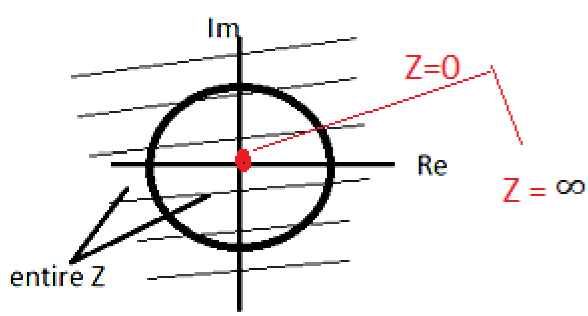
Sol.

$$X(z) = \sum_{m=-\infty}^{\infty} x(m) z^{-m} = \sum_{m=-\infty}^{0} x(m) z^{-m} + \sum_{m=0}^{\infty} x(m) z^{-m} = z^{-k} + z^{+k}$$

The ROC of X (z) is the entire Z-Plane except the points Z = 0 and Z = ∞.

The F. T. is obtaining by Z= $e^{j\omega}$:

i.e., $X(e^{j\omega}) = e^{-j\omega k} + e^{+j\omega k} = 2\cos(\omega k)$



Example 5

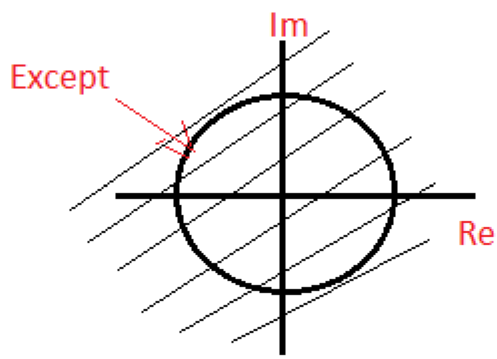
Determine the Z-transform, the region of convergence (ROC), of the signal u (m).

Sol:

$$X(z) = \sum_{m=-\infty}^{\infty} x(m)z^{-m} = \sum_{m=0}^{\infty} x(m)z^{-m} = \sum_{m=0}^{\infty} u(m)z^{-m} = 1 + z^{-1} + z^{-2} + \cdots$$

$$= \frac{z}{z-1}$$

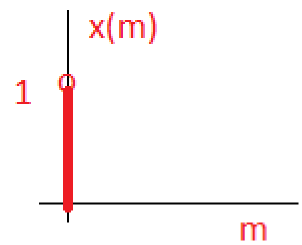
i.e., ROC = all values / Z =1



Example 6

Determine the Z-transform, the region of convergence (ROC), of the following signal.

$$x(m) = \begin{cases} 0 & m \geq 0 \\ \alpha^m & m < 0 \end{cases}$$



Sol:

$$X(z) = \sum_{m=-\infty}^{\infty} x(m)z^{-m} = \sum_{m=-\infty}^{0} x(m)z^{-m} + \sum_{m=0}^{\infty} x(m)z^{-m} = \sum_{-\infty}^{0} \alpha^m z^{-m} + 0$$

$$= \sum_{-\infty}^{0} \alpha^m z^{-m} = \sum_{0}^{\infty} (\alpha^{-1} z)^m$$

i.e. $|\propto^{-1} z| < 1 \quad \rightarrow \quad ROC \; is \; at \; |z| < |\propto|$

$$x(z) = 1 + \left(\frac{\propto}{z}\right)^{-1} + \left(\frac{\propto}{z}\right)^{-2} + \left(\frac{\propto}{z}\right)^{-3} + \cdots = \frac{\propto}{\propto - z}$$

$$X(j\omega) = \frac{\propto}{\propto - e^{j\omega}}$$