Probability is the chance of something happening. It is a number between 0 and 1, where 0 means it is **impossible** and 1 means it is **certain**.
For **example,** the probability of flipping a **coin** and getting heads is 1/2, because there are two equally likely outcomes (heads or tails).
Here is another **example:**
The probability of rolling a 6 on a **die** is 1/6, because there are 6 equally likely outcomes (1, 2, 3, 4, 5, or 6).

$$\text{Probability(Event)} = \frac{\text{Favorable Outcomes}}{\text{Total Outcomes}}$$

Numerically the probability value always lies between 0 and 1.

$$0 \leq P(E) \leq 1$$

It is expressed in percentage, decimal, or fraction.

If the probability of the happening $= p$
and the probability of not happening $= q$

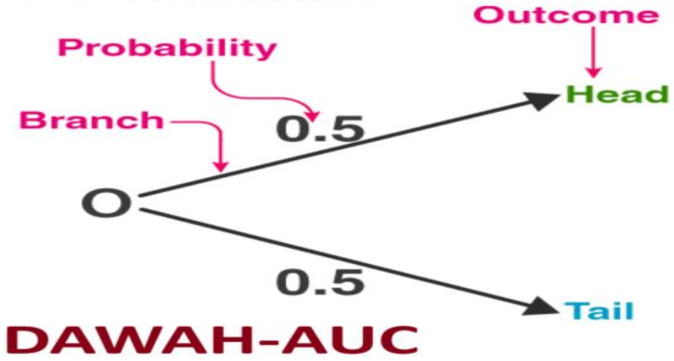
then $$p+q = \frac{m}{m+n} + \frac{n}{m+n} = \frac{m+n}{m+n} = 1 \quad \text{or} \quad p+q = 1.$$

Examples

- Probability of getting a head, on tossing a two-faced coin.

$$P(\text{Head}) = \frac{\text{Number of Heads}}{\text{Total Outcomes}} = \frac{1}{2}$$

تحليلات هندسية [[[[DAWAH]]]] **Probability**



The probability of getting an odd number, on rolling a six-faced dice.

$$P(\text{Odd Number}) = \frac{\text{Number of odd numbers}}{\text{Total Numbers}} = \frac{3}{6} = \frac{1}{2}$$

Different Probability Formulas

1-Probability formula with the complementary rule: occur when there are just two outcomes, and one event is exactly opposite to another event. For an event with probability P(A) its compliment is P(A') if A is an event, then **P(not A) = 1 - P(A)** **The same** **P(A') = 1 - P(A).**
P(A) + P(A') = 1

- **In an examination, the event of success and the event of failure are complementary events.**
  - **P(Success)+P($\overline{(Failure)}$)=1**

Example A football team plays 120 matches and wins 80 matches. What is the probability of the team winning the next match?

Solution

Total number of matches played = 120

Number of matches won by the team = 80

$$\text{Probability of winning} = \frac{\text{Number of matches won}}{\text{Total football team plays}}$$

$$= \frac{80}{120} = \frac{2}{3}$$

$\therefore$ The probability is $\frac{2}{3}$

Example ALI takes two coins and flips them both at once. What is the probability of getting heads on both the coins?

Solution

Sample space on flipping two coins = {(H, H), (H, T), (T, H), (T, T)}
Total number of outcomes = 4
Favourable outcome of two heads = 1

$\therefore$ The probability is $\frac{1}{4}$

Example Sare had a jar containing 8 red balls, 5 blue balls, and 7 green balls. If pick a ball from the jar at random. What is the probability that the ball which is picked is either a red or a blue ball?

Solution Let us take a count of the number of balls in the jar.
Number of red balls = 8
Number of blue balls = 5
Number of green balls = 7
Total number of balls = 20

$$P(\text{red or a blue ball}) = \frac{\text{No of red balls} + \text{No of blue balls}}{\text{Total number of balls}}$$

$$= \frac{8+5}{20} = \frac{13}{20}$$

$\therefore$ The probability is $\frac{13}{20}$

(∩)Intersection ……. union (∪)

Example The probability of Kareem being selected in a Football team is 0.3 and the probability of Mahdy being selected is 0.5. The probability of both of them being selected in the team is 0.2. What is the probability that either one of them is selected for the team?

Solution

Probability of Kareem's selection =P(S)=0.3
Probability of Mahdy's selection = P(M)=0.5
Probability of both of them being selected = **P(S∩M)=0.2**
Probability of either of them being selected = P(S∪M)=?

$$P(S \cup M) = P(S) + P(M) - P(S \cap M)$$
$$= 0.3 + 0.5 - 0.2 = 0.8 - 0.2 = 0.6$$

∴ The probability of either of them being selected is 0.6

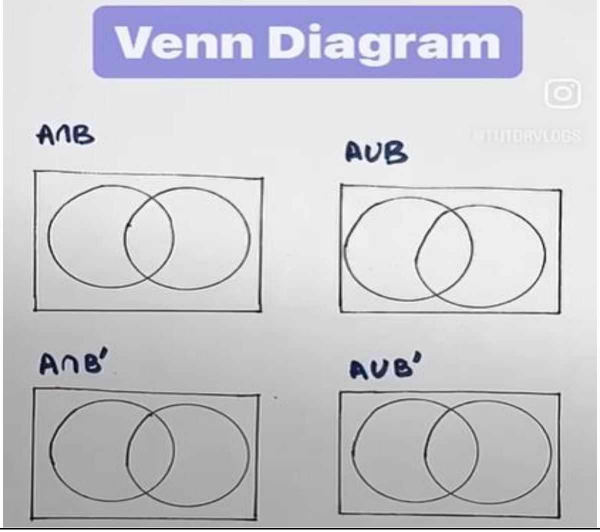
2- Probability formula with addition rule: When calculating the probability of **either one of two events** from occurring, it is as simple as adding the probability of each event and then subtracting the probability of both of the events occurring:

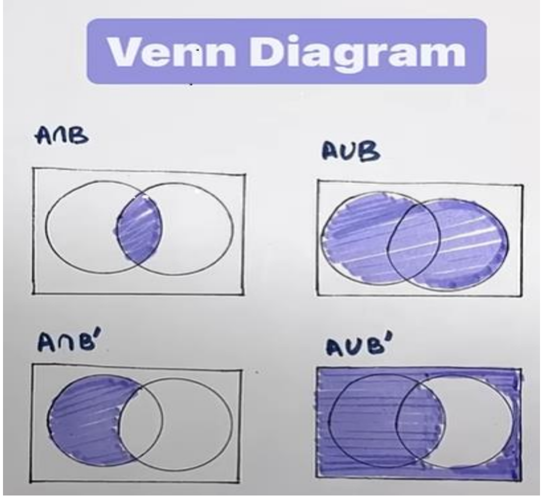
P(A or B) = P(A) + P(B) - P(A and B)

We must subtract P(A and B) to avoid double counting!

P(A or B) = P(A) **+** P(B) **-** P(A∩B)

P(A ∪ B) = P(A) **+** P(B) **-** P(A∩B)





EXAMPLE **At a local language school, 40% of the students are learning Spanish, 20% of the students are learning German, and 8% of the students are learning**

both Spanish and German. What is the probability that a randomly selected student is learning Spanish or German?

Solution: **P(Spanish or German)=P(Spanish)+P(German)−P(Spanish andGerman)=0.4+0.2−0.08=0.52**

EXAMPLE There are 50 students enrolled in the third year of a CET. During this semester, the students have to take some elective courses. **18 students** decide to take an elective in Electronics, **27 students** decide to take an elective in communication, and **10 students** decide to take an elective in both. Electronics and communication What is the probability that a student takes an elective in Electronics and communication?

Solution:

P(Electronics and communication)=P(Electronics)+P(communication)−P(Electronics and communication)=18/50+27/50−10/50=0.7

EXAMPLE Suppose a bag contains **20 balls**. **10 of the balls are white**, **7 of the balls are red,** and **3 of the balls are blue**. Suppose **one ball** is selected at random from the bag.

1. Are the events "selecting a white ball" and "selecting a red ball" mutually exclusive? Why?
2. What is the probability of selecting a white or red ball?

Solution:

1. The events "selecting a white ball" and "selecting a red ball" are mutually exclusive because the events cannot happen at the same time. **It is not possible for the selected ball to be both white and red.**
2. P(white or red)=P(white)+P(red)=10/20+7/20=0.85

3-Probability formula with the conditional rule: When event **A** is already known to have occurred, the probability of event **B** is known as conditional probability and is given by: $P(B|A) = \dfrac{P(A\cap B)}{P(A)}$

Example: You have a fair six-sided **die**. You want to determine the probability of rolling an **even number**, given that the number rolled is **greater than four. >4**

The possible outcomes (sample space) for a six-sided die are six.

Event A: Rolling an even number. Event A would mean rolling {2,4,6}.
Event B: Rolling a number >4. Event B would mean rolling {5,6}.

$$P(A)=\frac{\text{number of favorable outcomes}}{\text{total number of outcomes}}=\frac{3}{6}=\frac{1}{2}$$

P(B) is the probability of rolling a number >4. Two numbers are(>4)they are{5,6} out of the six possible outcomes. Thus, P(B) = 2/6 = 1/3.

To find **intersection** of events A and B …… **So P(A∩B) = 1/6.**

The **intersection of events A and B**. In this case, means rolling a number that is **even** and also **>4**. The only outcome that does both is rolling a **six.**

the conditional probability: P(B|A)

The formula for conditional probability is as follows:

$$P(B|A)=\frac{P(A\cap B)}{P(A)}=\frac{\frac{1}{6}}{\frac{1}{2}}=\frac{1}{3}$$

Example: My neighbor has **2 children**. I learn that he has a **son**, ALI. What is the probability that ALI's sibling is a boy?

Solution: Let the **boy** child be **B** and the girl child is G.

The sample space is **S = {BB,BG,GB,GG}**

Assume that boys and girls are equally likely to be born, the 4 elements of S are equally likely. *The event,* **X,** *that the neighbor has a son is the set*

X = {BB,BG,GB} **Hence, P(X) = 3/4**

The event, **Y**, that the neighbor has **two sons** is the set **Y = {BB}**

Then, **P(Y ∩ X) = 1/4**

Now, using the conditional probability formula

$$P(Y\mid X)=\frac{P(Y\cap X)}{P(X)}=\frac{\frac{1}{4}}{\frac{3}{4}}=\frac{1}{3}$$

Example: A fair die is rolled. What is the probability of A given B where A is the event of getting an even number and B is ≤ 3

Solution: To find: P(A | B) using the given information.

When a die is rolled, the sample space = {1, 2, 3, 4, 5, 6}.

A is the event of getting an **even** number. Thus**, A = {2, 4, 6}.**

B is ≤ 3. Thus, **B = {1, 2,3}.**

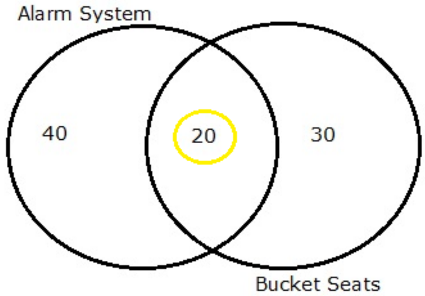
Then, A ∩ B = {2}.

Now, using the conditional probability formula,

P(A | B) = P(A∩B) / P(B)

P(A | B) = (1/6)/(3/6) = 1/3

Example / In a group of 100 sports car buyers, 40 bought alarm systems, 30 purchased bucket seats, and 20 purchased an alarm system and bucket seats. If a car buyer chosen at random bought an alarm system, what is the probability they also bought bucket seats?
P(A). It's given in the question as 40%, or 0.4.
P(A∩B). This is the intersection of A and B: both happening together. It's given in the question 20 out of 100 buyers, or 0.2.
P(B|A) = P(A∩B) / P(A) = 0.2 / 0.4 = 0.5.
The probability that a buyer bought bucket seats, given that they purchased an alarm system, is 50%.



Venn diagram for 90 buyers, showing that 20 alarm buyers also purchased bucket seats.

Example: This question uses the following contingency table:

	Have pets	Do not have pets	Total
Male	0.41	0.08	0.49
Female	0.45	0.06	0.51
Total	0.86	0.14	1

What is the probability a **randomly selected person is male**, given that they own a **pet**?
Step 1: say M is for male and **PO stands for pet owner**, so the formula becomes:
P(M|PO) = P(M∩PO) / P(PO)
Step 2: Figure out P(M∩PO) from the table. The intersection of **male/pets** (the intersection on the table of these two factors) **is 0.41**.

	Have pets	Do not have pets	Total
Male	0.41	0.08	0.49
Female	0.45	0.06	0.51
Total	0.86	0.14	1

Step 3: Figure out P(PO) from the table. From the total column, 86% (0.86) of respondents had a pet.

	Have pets	Do not have pets	Total
Male	0.41	0.08	0.49
Female	0.45	0.06	0.51
Total	0.86	0.14	1

Step 4: Insert your values into the formula:
P(M|PO) = P(M∩PO) / P(M) = 0.41 / 0.86 = 0.477, or 47.7%.

4-Probability formula with multiplication rule: Whenever an event is the intersection of two other events, events A and B need to occur simultaneously. Then

- $P(A \cap B) = P(A) \cdot P(B)$ (in case of independent events)
- $P(A \cap B) = P(A) \cdot P(B|A)$ (in case of **dependent events**)

- **Example: Find the probability of getting a number <5 when a dice is rolled by using the probability formula.**

Solution ///To find: Probability of getting a number **<5**
Given: Sample space, S = {1,2,3,4,5,6} ,,Therefore, n(S) = 6

- Let A be the event of getting a number less than 5. Then A = {1,2,3,4}
  So, n(A) = 4
- Using the probability equation,
  P(A) = (n(A))/(n(s))
  p(A) = 4/6= 2/3 **Answer:** The probability of getting a number less than 5 is 2/3.
- **Example:** What is the probability of getting a sum of 9 when two dice are thrown?

- **Solution:** There is a **total of 36 possibilities** when we throw **two dice**. To get the desired outcome., **9**, we can have the following favorable outcomes. **(4,5),(5,4),(6,3)(3,6).** There are 4 favorable outcomes.
  Probability of an event P(E) = (Number of favorable outcomes) ÷ (Total outcomes in a sample space)
  Probability of getting number 9 = 4 ÷ 36 = 1/9

Tossing **Three Coins**

The number of total outcomes on Tossing three coins simultaneously is equal to $2^3 = 8$. For these outcomes, we can find the probability of getting one head, two heads, three heads, and no head. A similar probability can also be calculated for the number of tails. Total number of outcomes = 2^3 = 8 Sample Space = **{(H, H, H), (H, H, T), (H, T, H), (T, H, H), (T, T, H), (T, H, T), (H, T, T), (T, T, T)}**

$P(0H) = P(3T) = \text{Number of outcomes with no heads}/\text{Total Outcomes} = 1/8$

- $P(1H) = P(2T) = \text{Number of Outcomes with one head}/\text{Total Outcomes} = 3/8$
- $P(2H) = P(1T) = \text{Number of outcomes with two heads} /\text{Total Outcomes} = 3/8$
- $P(3H) = P(0T) = \text{Number of outcomes with three heads}/\text{Total Outcomes} = 1/8$

Probability Theorems

- The following theorems of probability are helpful to understand the applications of probability and calculations probability.
- Theorem 1: The sum of the probability of happening of an event and not happening of an event is equal to 1. P(A) + P(A') = 1.
- Theorem 2: The probability of an impossible event or the probability of an event not happening is always equal to 0. P(ϕ) = 0.
- Theorem 3: The probability of a sure event is always equal to 1.
- P(A) = 1

- Theorem 4: The probability of happening of any event always lies between 0 and 1. $0 \leq P(A) \leq 1$
- Theorem 5: If there are two events A and B, we can apply the formula of the union (U) of two sets and we can derive the formula for the probability of happening of event A or event B as follows.
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- Also for two mutually exclusive events A and B, we have $P(A \cup B) = P(A) + P(B)$
- **Bayes' theorem describes the probability of an event based on the condition of occurrence of other events. It is also called conditional probability. It helps in calculating the probability of happening of one event based on the condition of happening of another event.**
  - **Probability of B given A……P(B/A)**
  - **Probability of A…..P(A)**
  - **Probability of B…….P(B)**

The formula for Bayes' theorem is $$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

where, $P(A|B)$ denotes how often event A happens on a condition that B happens.
where, $P(B|A)$ denotes how often event B happens on a condition that A happens.

- **$P(A)$ the likelihood of occurrence of event A.**
- **$P(B)$ the likelihood of occurrence of event**

Example: Out of 50 football players, 40 play club football, 30 play football for the national team and 20 players play for both club and national team. If a player is selected at random and he plays for the club team what is the probability that he plays for the national team too?

Solution: In the given question

P(A ∩ B) = 20/50 = 0.4

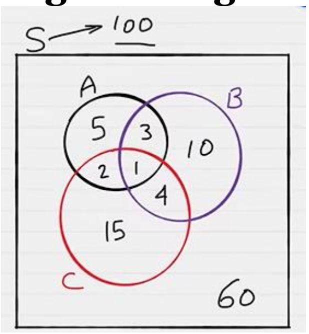
P(B) =40/50 =0.8

So, P(A|B) = P(A ∩ B) / P(B)

P(A|B) = 0.4 / 0.8

P(A|B) = 0.5

EXAMPLE/from venn find using drawing and formula



$P(A)=\frac{\text{Number of outcom in } A}{\text{Total outcom in } S}$ ......S-Sample space

$P(A)=\frac{5+3+2+1}{100}=\frac{11}{100}=0.11$

$P(B)=\frac{\text{Number of outcom in } B}{\text{Total outcom in } S}=\frac{10+4+3+1}{100}=\frac{18}{100}=0.18$

FROM VENN

$P(A \cap B)=\frac{1+3}{100}=\frac{4}{100}=0.04$

BY FORMULA

P(A ∩ B) = P(A) * P(B|A)..... { P(B|A).....B given A}{ A is Sample space}

$P(A \cap B)=\frac{11}{100}*\frac{4}{11}=0.04$

....

EXAMPLE/From the same venn find P(A ∩ B')

--from venn $P(A \cap B')=\frac{7}{100}=0.07$

By formula P(A ∩ B') = P(A ∩(S- B)) = P(A ∩ S) - P(A ∩ B)

$= P(A) - P(A \cap B)=\frac{11}{100}-\frac{4}{100}=0.07$

....

EXAMPLE/From the same venn find P(A' ∩ B')

By formula P(A' ∩ B') = P((S-A) ∩(S- B))

= P(S ∩ S) - P(S ∩ B) - P(A ∩ S) + P(A ∩ B)

= P(S) - P(B)- P(A) + P(A ∩ B) $=\frac{100}{100}-\frac{18}{100}-\frac{11}{100}+\frac{4}{100}=0.75$

Random Variable

is a variable whose value is an outcome of a random phenomenon, for example, this can be viewed as the outcome of throwing a die where the process is fixed but the outcome is not.

outcome is a possible result of a probability experiment. For example, a portfolio earning 8% returns is an outcome.

An Event

An event is a single outcome or set of outcomes to which we assign a probability.

An experiment is an activity that has observable results.

An outcome is the result of the experiment.

A sample space of an experiment is the set of all possible outcomes of the experiment.

Each repetition of an experiment is called a trial.

Mutually Exclusive Events

are events that can not happen at the same time. Examples include: right and left hand turns, even and odd numbers on a die, winning and losing a game, or running and walking.

The basic probability(P) of an event happening (forgetting mutual exclusivity for a moment) is: $P = \frac{\text{Number of ways the event can happen}}{\text{total number of outcomes}}$

Example:

A poll finds that 72% of Fallujah consider themselves football fans. If you randomly pick two people from the population, what is the probability the first person is a football fan and the second is as well? That the first one is and the second one isn't?

Solution: one person being a football fan doesn't have an effect on whether the second randomly selected person is. Therefore, the events are independent and the probability can be found by multiplying the probabilities together:

First one and second are football fans:

P(A∩B) = P(A) · P(B) = 0.72 *0 .72 = 0.5184.
P(A∩B) = P(A) · P(B') =0 .72 * (1 – 0.72) = 0.202.
First one is a football fan, the second one isn't:
In the second part, I multiplied by the complement. As the probability of being a fan is 0.72, then the probability of not being a fan is (1 – .72), or .28.

Example:

A person has undertaken a mining job. The probabilities of completion of the job on time with and without rain are 0.42 and 0.90 respectively. If the probability that it will rain is 0.45, then determine the probability that the mining job will be completed on time.

Solution:

Let **A** be the event that the mining job will be completed on time and **B** be the event that it rains. We have

P(B) = 0.45,
P(no rain) = P(B′) = 1 - P(B) = 1 - 0.45 = 0.55
By multiplication law of probability,
P(A|B) = 0.42
P(A|B′) = 0.90
Since, events B and B′ form partitions of the sample space S, by total probability theorem, we have
P(A) = P(B) P(A|B) + P(B′) P(A|B′)
=0.45 × 0.42 + 0.55 × 0.9
= 0.189 + 0.495 = 0.684
So, the probability that the job will be completed on time is 0.684

Example : Let X and Y are two independent events such that P(X) = 0.3 and P(Y) = 0.7. Find P(X and Y), P(X or Y), P(Y not X), and P(neither X nor Y).

Solution: P(X) = 0.3 and P(Y) = 0.7 and events X and Y are independent of each other.

P(X and Y) = P(X ∩ Y) = P(X) P(Y) = 0.3 × 0.7 = 0.21

P(X or Y) = P(X ∪ Y) = P(X) + P(Y) – P(X ∩ Y) = 0.3 + 0.7 – 0.21 = 0.79

P(Y not X) = P(Y ∩ X′) = P(Y) – P(X ∩ Y) = 0.7 – 0.21 = 0.49

And P(neither X nor Y) = P(X′ ∩ Y′) = 1 – P(X ∪ Y) = 1 – 0.79 = 0.21

Example: The probability of rolling a **"5"** when you throw a **die** is **1/6** because there is one "5" on a die **and six possible outcomes**. If we call the probability of rolling a 5 "**Event A**", then the equation is:

$$P(A) = \frac{\text{Number of ways the event can happen}}{\text{total number of outcomes}}$$ ….. **P(A) = 1 / 6**

It's impossible to roll a 5 and a 6 together; the events are mutually exclusive

The events are written like this:

P(A and B) = 0

In English, all that means the probability of event A (rolling a 5) and event B (rolling a 6) happening together is 0.

However, when you roll a die, you *can* roll a 5 *OR* a 6 (the odds are 1 out of 6 for each event) and the sum of either event happening is the sum of both probabilities. In probability, it's written like this:

P(A or B) = P(A) + P(B)

P(rolling a 5 or rolling a 6) = P(rolling a 5) + P(rolling a 6)

P(rolling a 5 or rolling a 6) = 1/6 + 1/6 = 2/6 = 1/3.

It's impossible to roll a 1 and a 2 together.

- **1/The probability of an event E is between 1 and 0, i.e.,**
- **0 < P(E) < 1.**
- **2/The sum of probabilities of all mutually exclusive and exhaustive events is equal to 1.**

These two properties together define probability.

When we roll a die, the events 1, 2, 3, 4, 5, and 6 are mutually exclusive and exhaustive. The probability of any event occurring is between 0 and 1. The sum of probabilities of all these 6 events is equal to 1.

NOT MUTUALLY EXCLUSIVE EVENTS

Consider the case where two events A and B are not mutually exclusive. The probability of the event that either A or B or both occur is given as

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Example 4. *An urn contains 10 black and 10 white balls. Find the probability of drawing two balls of the same colour.*

Solution. Probability of drawing two black balls $= \dfrac{^{10}C_2}{^{20}C_2}$

$\therefore$ Probability of drawing two red balls $= \dfrac{^{10}C_2}{^{20}C_2}$

$\therefore$ Probability of drawing two balls of the same colour

$$= \frac{^{10}C_2}{^{20}C_2} + \frac{^{10}C_2}{^{20}C_2} = 2 \cdot \frac{^{10}C_2}{^{20}C_2} = 2 \cdot \frac{\dfrac{10 \times 9}{2 \times 1}}{\dfrac{20 \times 19}{2 \times 1}}$$

$$= \frac{9}{19}$$ **Ans.**

Mutually Exclusive Event Probability: Steps

Example . *A bag contains four white and two black balls and a second bag contains three of each colour. A bag is selected at random, and a ball is then drawn at random from the bag chosen. What is the probability that the ball drawn is white ?*

Solution. There are two mutually exclusive cases,

(*i*) when the first bag is chosen, (*ii*) when the second bag is chosen.

Now the chance of choosing the first bag is $\frac{1}{2}$ and if this bag is chosen, the probability of drawing a white ball is 4/6. Hence the probability of drawing a white ball from first bag is

Example . *Three machines I, II and III manufacture respectively 0.4, 0.5 and 0.1 of the total production. The percentage of defective items produced by I, II and III is 2, 4 and 1 per cent respectively. For an item chosen at random, what is the probability it is defective ?*

Solution. The defective item produced by machine I $= \frac{0.4 \times 2}{100} = \frac{0.8}{100}$

The defective item produced by machine II $= \frac{0.5 \times 4}{100} = \frac{2}{100}$

The defective item produced by machine III $= \frac{0.1 \times 1}{100} = \frac{0.1}{100}$

The total defective items produced by machines I, II, III

$$= \frac{0.8}{100} + \frac{2}{100} + \frac{0.1}{100} = \frac{2.9}{100} = 0.029$$

The required probability $= \frac{0.029}{1} = 0.029$

Example: **"If P(A) = 0.20, P(B) = 0.35 and (P (A∪ B)) = 0.51, are A and B mutually exclusive?"**

Note: a union (∪) of two events occurring means that A or B occurs.

Step 1: ***Add up the probabilities of the separate events (A and B)*. In the above example:**

0.20 + 0.35 =0.55

Step 2: *Compare your answer to the given "union" statement (A ∪ B)*. If they are the same, the events are mutually exclusive. If they are different, they are not mutually exclusive. *Why*? **If**

For the experiment of throwing a die and observing the number of dots on the top face the sample space is the set S = {1,2,3,4,5,6} In the experiment of flipping a coin and observing whether it falls heads or tails, the sample space is S = {heads,tails} or simply S = {H,T}.

EXAMPLE Determining the Sample Space Two dice, identical except that one is green and the other is red, are tossed and the number of dots on the top face of each is observed. What is the sample space for this experiment?

Solution Each die can take on its six different values with the other die also taking on all of its six different values. We can express the outcomes as order pairs. For example, (2, 3) will mean 2 dots on the top face of the green die and 3 dots on the top face of the red die. The sample space S is below. A more colourful version is shown

S = {(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),
(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),
(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),
(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),
(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),
(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)}

they are mutually exclusive (they can't occur together), then the (∪)nion of the two events must be the sum of both, i.e. **0.20 + 0.35 = 0.55.**
In our example, 0.55 does not equal 0.51, so the events are ***not* mutually exclusive**.

EXAMPLE// If A and B are mutually exclusive events such that P(A) = 0.35 and P(B)=0.45, find
(i) P(A∪B)
(ii) P(A∩B)
(iii) P(A∩B')
(iv) P(A'∩B')

Solution

Since, it is given that, A and B are mutually exclusive events.

$\therefore P(A\cap B)=0[\because A\cap B=\phi] P(A)=0.35, P(B)=0.45$

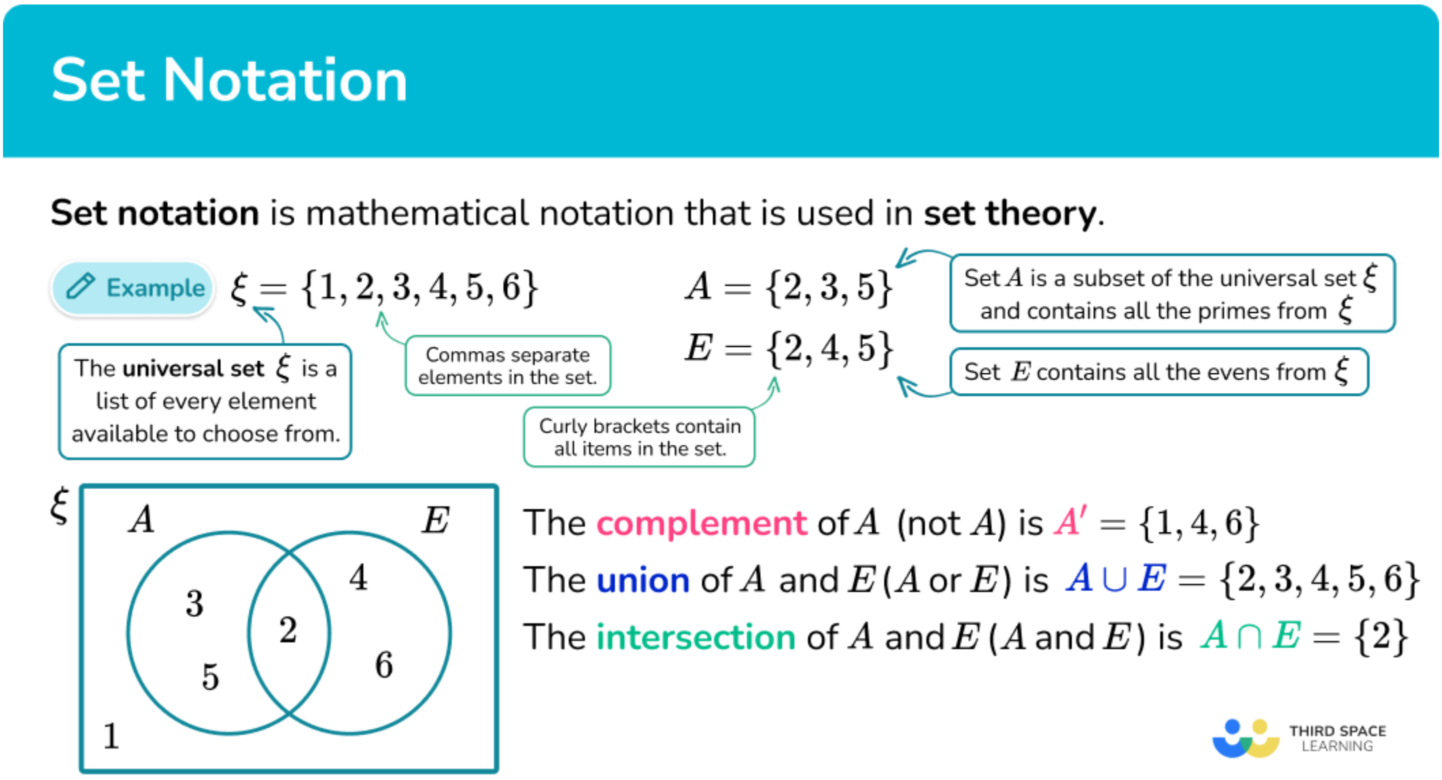
(i) $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

=0.35+0.45-0=0.80

(ii) P(A∩B)=0

(iii) P(A∩B')=P(A)−P(A∩B) **=0.35-0=0.35**

(iv) P(A'∩B')=P$(\overline{A \cup B})$ =1−P(A∪B) **=1-0.8=0.2**



EXAMPLE/

A box containing 4 bulbs, the probability of having one defected bulb is 0.5 and the probability to have zero defected bulb is 0.4. Calculate the probability of one defected bulb and zero defected bulb.

Solution: Probability of single bulb being defected is P(X) = 0.5

Probability of zero bulbs being defected is P(Y) = 0.4

As there can be either zero defected bulb or 1 defected bulb because these two events cannot occur simultaneously.

Hence, they are considered as mutually exclusive.

P(X or Y) = 0.5 + 0.4 = 0.9

EXAMPLE/ If P(X) = $\frac{1}{3}$ and P(Y) = $\frac{2}{3}$. Examine whether X and y are mutually exclusive X and Y are exhaustive

Solution: The events are said to be mutually exclusive if

$P(X \cap Y) = 0$

The events are said to be exhaustive if

$P(X \cup Y) = P(X) + P(Y) = \frac{1}{3} + \frac{2}{3} = 1$

$P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$

1=1-0

If P (X ∪ Y) is the sample space, then the above two conditions will **be satisfied. Hence, X and Y are mutually exclusive and exhaustive.**

Two Defining Properties of Probability

There are two important properties of probability.

SOLUTION

Since, it is given that, A and B are mutually exclusive events.

$\therefore \quad P(A \cap B) = 0 \quad [\because A \cap B = \phi]$

and $P(A) = 0.35$, $P(B)$, $P(B) = 0.45$

(i) P(A')=1-P(A)=1-0.35=0.65

(ii) P(B')= 1-P(B)=1-0.45=0.55

(iii) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.35+0.45-0 = 0.80$

(iv) $P(A \cap B) = 0$

(v) $P(A \cap B') = P(A) - P(A \cap B) = 0.35-0 = 035$

(vi) $P(A' \cap B') = P(A \cup B)' = 1 - P(A \cup B) = 1-0.8 = 0.2$

EXAMPLE/We are choosing 4 marbles from a bag containing 7 marbles. The total number of marble combinations there are is possible combinations

$${}_7C_4 = \frac{7!}{4!3!} = 35$$

a) The number of combinations that contain 3 green marbles +1 other:

$$_4C_3 = \frac{4!}{3!1!} = 4$$

First, the 3 greens. We are choosing 3 green marbles out of a total of
Now to choose 1 other – we're picking 1 marble out of the 3 non-green ones:

$$_3C_1 = \frac{3!}{2!1!} = 3$$

$$_4C_3 \times _3C_1 = 4 \times 3 = 12$$

The total number of combinations of 3 greens and 1 other

$$P(A) = \frac{12}{35}.$$

b) The number of combinations that contain 4 green balls:.

We are choosing 4 from 4 possible green balls: $_4C_4 = 1$ possible combination

So $P(B) = \frac{1}{35}$

$P(A \text{ or } B) = P(A) + P(B)$, so the probability of getting at least 3 green balls is $\frac{12}{35} + \frac{1}{35} = \frac{13}{35}$

Example

The probability that the price of oil will rise, P(B) = 0.5
The probability that the bus fare will increase if oil price rises, P(A|B) = 0.4
The probability that both oil prices and bus fares will rise,
P(AB) = 0.4*0.5 = 0.2
This may look complex but the logic is actually quite straight forward. There is a 50% chance that oil price will rise and if it rises there is a 40% chance that the bus fair will also rise. So, the joint probability of both oil price rise and bus fare rise is
50% of 40%, i.e., 0.5*0.4 = 0.20 or 20%.

Probability of Atleast One of the Events Occuring

This refers to the addition rule.

The additional rule determines the probability of at least one of the events occurring.

P (A or B) = P (A) + P (B) - P (A and B)

Example 1:

At a certain high school, 2% of students play water polo. If 8 students are randomly selected, what is the probability that at least one of them plays water polo?

Step 1: Using this information, we say (water polo player)=0.02 and (not water polo player)=1−0.02=0.98.

Step 2: we are told that 8 students will be randomly selected. This means the number of trials we are conducting is =8.

Step 3: Calculate the probability of "at least one" using the formula

(at least one)=1− (failure)

(at least one water polo player)=1− (not water polo player)

$=1-(0.98)^8=1-0.8508=0.1492$

The probability of at least one student being a water polo player is 0.1492, or 14.92%.

Example . *A bag contains 10 white and 15 black balls. Two balls are drawn in succession. What is the probability that first is white and second is black ?*

Solution. Probability of drawing one white ball $= \frac{10}{25}$

Probability of drawing one black ball without replacement = 15/24

Required probability of drawing first white ball and second black ball $= \frac{10}{25} \times \frac{15}{24} = \frac{1}{4}$

Example 7. *An article manufactured by a company consists of two parts A and B. In the process of manufacture of part A, 9 out of 100 are likely to be defective. Similarly, 5 out of 100 are likely to be defective in the manufacture of part B. Calculate the probability that the assembled article will not be defective (assuming that the events of finding the part A non-defective and that of B are independent).*

Solution. Probability that part A will be defective $= \frac{9}{100}$

Probability that part A will not be defective $= \left(1 - \frac{9}{100}\right) = \frac{91}{100}$

Probability that part B will be defective $= \frac{5}{100}$

Probability that part B will not be defective $= \left(1 - \frac{5}{100}\right) = \frac{95}{100}$

Probability that the assembled article will not be defective = (Probability that part A will not be defective) × (Probability that part B will not be defective)

$$= \left(\frac{91}{100}\right) \times \left(\frac{95}{100}\right) = 0.8645 \qquad \textbf{Ans.}$$

Example . *A problem of statistics is given to three students A, B and C whose chances of solving it are* $\frac{1}{2}, \frac{3}{4}$ *and* $\frac{1}{4}$ *respectively. What is the probability that the problem will be solved ?* (A.M.I.E., Winter 2001)

Solution. The probability that A can solve the problem $= \frac{1}{2}$

The probability that A cannot solve the problem $= 1 - \frac{1}{2}$.

Similarly the probability that B and C cannot solve the problem are $\left(1-\frac{3}{4}\right)$ and $\left(1-\frac{1}{4}\right)$.

∴ The probability that A, B, C cannot solve the problem

$$= \left(1-\frac{1}{2}\right)\times\left(1-\frac{3}{4}\right)\times\left(1-\frac{1}{4}\right) = \frac{1}{2}\times\frac{1}{4}\times\frac{3}{4} = \frac{3}{32}.$$

Hence the probability that the problem can be solved

$$= 1-\frac{3}{32} = \frac{29}{32}$$

Example . *A student takes his examination in four subjects α, β, γ, δ. He estimates his chances of passing in α as $\frac{4}{5}$, in β as $\frac{3}{4}$, in γ as $\frac{5}{6}$ and in δ as $\frac{2}{3}$. To qualify, he must pass in α and at least two other subjects. What is the probability that he qualifies ?*

Solution. $P(\alpha) = \frac{4}{5},\ P(\beta) = \frac{3}{4},\ P(\gamma) = \frac{5}{6},\ P(\delta) = \frac{2}{3}$

There are four possibilities of passing at least two subjects

(*i*) Probability of passing β, γ and failing δ

$$= \frac{3}{4}\times\frac{5}{6}\times\left(1-\frac{2}{3}\right) = \frac{3}{4}\times\frac{5}{6}\times\frac{1}{3} = \frac{5}{24}$$

(*ii*) Probability of passing γ, δ and failing β

$$= \frac{5}{6}\times\frac{2}{3}\times\left(1-\frac{3}{4}\right) = \frac{5}{6}\times\frac{2}{3}\times\frac{1}{4} = \frac{5}{36}$$

(*iii*) Probability of passing δ, β and failing γ

(*iv*) Probability of passing β, γ, δ

$$= \frac{3}{4}\times\frac{5}{6}\times\frac{2}{3} = \frac{5}{12}.$$

Probability of passing at least two subjects

$$= \frac{5}{24}+\frac{5}{36}+\frac{1}{12}+\frac{5}{12} = \frac{61}{72}$$

Probability of passing α and at least two subjects

$$= \frac{4}{5}\times\frac{61}{72} = \frac{61}{90}$$

Department of Computer Engineering Techniques

College of Techniques Engineering

University of Al Maarif

K. DAWAH .ABBAS-2024-2025