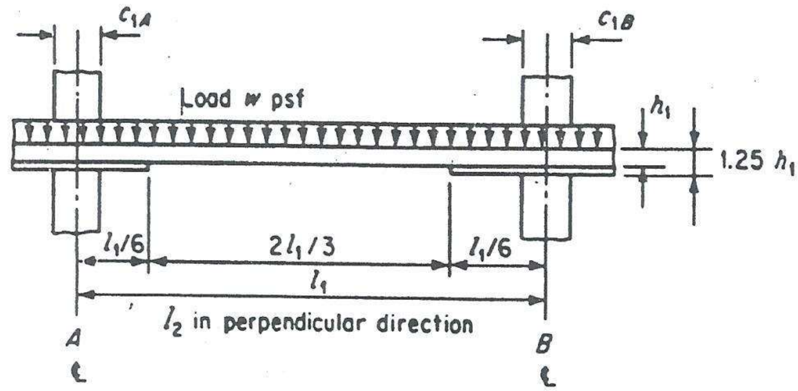
**Table A.13b Coefficients for slabs with variable moment of inertia†**

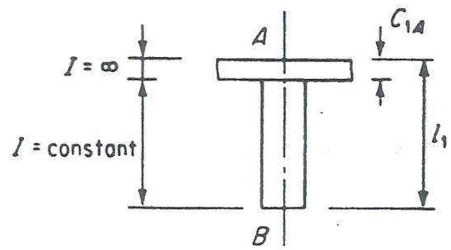



| Column dimension | | Uniform load FEM = coeff. $(wl_2 l_1^2)$ | | Stiffness factor‡ | | Carryover factor | |
|---|---|---|---|---|---|---|---|
| $\frac{c_{1A}}{l_1}$ | $\frac{c_{1B}}{l_1}$ | $M_{AB}$ | $M_{BA}$ | $k_{AB}$ | $k_{BA}$ | $COF_{AB}$ | $COF_{BA}$ |
| 0.00 | 0.00 | 0.088 | 0.088 | 4.78 | 4.78 | 0.541 | 0.541 |
| | 0.05 | 0.087 | 0.089 | 4.80 | 4.82 | 0.545 | 0.541 |
| | 0.10 | 0.087 | 0.090 | 4.83 | 4.94 | 0.553 | 0.541 |
| | 0.15 | 0.085 | 0.093 | 4.87 | 5.12 | 0.567 | 0.540 |
| | 0.20 | 0.084 | 0.096 | 4.93 | 5.36 | 0.585 | 0.537 |
| | 0.25 | 0.082 | 0.100 | 5.00 | 5.68 | 0.606 | 0.534 |
| 0.05 | 0.05 | 0.088 | 0.088 | 4.84 | 4.84 | 0.545 | 0.545 |
| | 0.10 | 0.087 | 0.090 | 4.87 | 4.95 | 0.553 | 0.544 |
| | 0.15 | 0.085 | 0.093 | 4.91 | 5.13 | 0.567 | 0.543 |
| | 0.20 | 0.084 | 0.096 | 4.97 | 5.38 | 0.584 | 0.541 |
| | 0.25 | 0.082 | 0.100 | 5.05 | 5.70 | 0.606 | 0.537 |
| 0.10 | 0.10 | 0.089 | 0.089 | 4.98 | 4.98 | 0.553 | 0.553 |
| | 0.15 | 0.088 | 0.092 | 5.03 | 5.16 | 0.566 | 0.551 |
| | 0.20 | 0.086 | 0.094 | 5.09 | 5.42 | 0.584 | 0.549 |
| | 0.25 | 0.084 | 0.099 | 5.17 | 5.74 | 0.606 | 0.546 |
| 0.15 | 0.15 | 0.090 | 0.090 | 5.22 | 5.22 | 0.565 | 0.565 |
| | 0.20 | 0.089 | 0.094 | 5.28 | 5.47 | 0.583 | 0.563 |
| | 0.25 | 0.87 | 0.097 | 5.37 | 5.80 | 0.604 | 0.559 |
| 0.20 | 0.20 | 0.092 | 0.092 | 5.55 | 5.55 | 0.580 | 0.580 |
| | 0.25 | 0.090 | 0.096 | 5.64 | 5.88 | 0.602 | 0.577 |
| 0.25 | 0.25 | 0.094 | 0.094 | 5.98 | 5.98 | 0.598 | 0.598 |

† Applicable when $c_1/l_1 = c_2/l_2$. For other relationships between these ratios, the constants will be slightly in error.

‡ Stiffness is $K_{AB} = k_{AB}E(l_2 h_1^3/12l_1)$ and $K_{BA} = k_{BA}E(l_2 h_1^3/12l_1)$.

**Table A.13c Coefficients for columns with variable moment of inertia**




| Slab depth $c_{1A}/l_1$ | Uniform load FEM = coeff. $(wl_2 l_1^2)$ | | Stiffness factors | | Carryover factors | |
|---|---|---|---|---|---|---|
| | $M_{AB}$ | $M_{BA}$ | $k_{AB}$ | $k_{BA}$ | $COF_{AB}$ | $COF_{BA}$ |
| 0.00 | 0.083 | 0.083 | 4.00 | 4.00 | 0.500 | 0.500 |
| 0.05 | 0.100 | 0.075 | 4.91 | 4.21 | 0.496 | 0.579 |
| 0.10 | 0.118 | 0.068 | 6.09 | 4.44 | 0.486 | 0.667 |
| 0.15 | 0.135 | 0.060 | 7.64 | 4.71 | 0.471 | 0.765 |
| 0.20 | 0.153 | 0.053 | 9.69 | 5.00 | 0.452 | 0.875 |
| 0.25 | 0.172 | 0.047 | 12.44 | 5.33 | 0.429 | 1.000 |

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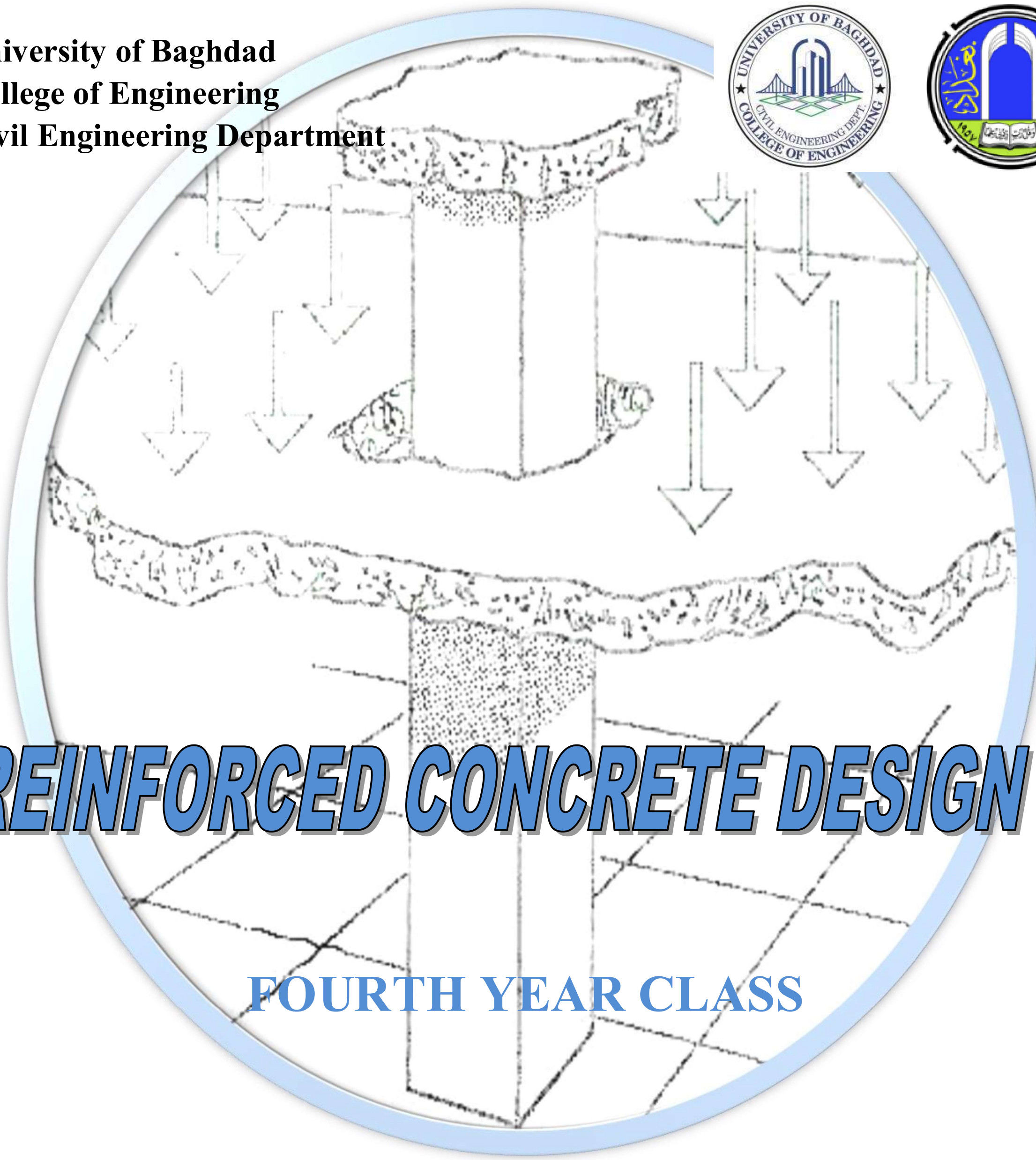


# REINFORCED CONCRETE DESIGN II

## FOURTH YEAR CLASS

# 4

## 2018 - 2019

MHND

**Shear in slabs**

One-way shear or beam-action shear: involves an inclined crack extending across the entire width of the panel.
Two-way shear or punching shear: involves a truncated cone or pyramid-shaped surface around the column

For each applicable factored load combination, design strength shall satisfy:
- $\phi V_n \geq V_u$ at all sections in each direction for one-way shear.
- $\phi v_n \geq v_u$ at the critical sections for two-way shear.

Interaction between load effects shall be considered.

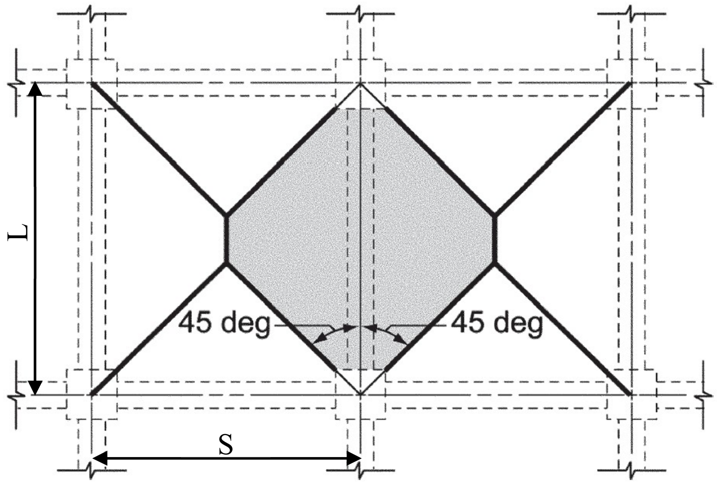
$V_n = V_c + V_s$
$v_n = v_c$ (nominal shear strength for two-way members without shear reinforcement).
$v_n = v_c + v_s$ (nominal shear strength for two-way members with shear reinforcement other than shearheads).

$\phi = 0.75$
$V_u$ is the factored shear force at the slab section considered.
$V_n$ is the nominal shear strength.
$V_c$ is the nominal shear strength provided by concrete.
$V_s$ is the nominal shear strength provided by shear reinforcement.
$v_n$ is the equivalent concrete stress corresponding to nominal two-way shear strength of slab.
$v_u$ is the maximum factored two-way shear stress calculated around the perimeter of a given critical section.
$v_{ug}$ is the factored shear stress on the slab critical section for two-way action due to gravity loads without moment transfer.

**shear cap**: a projection below the slab used to increase the slab shear strength. It shall project below the slab soffit and extend horizontally from the face of the column a distance at least equal to the thickness of the projection below the slab soffit.

**Shear in slab with beams**

Shear shall be checked at a distance **d** from the face of the support (beam).



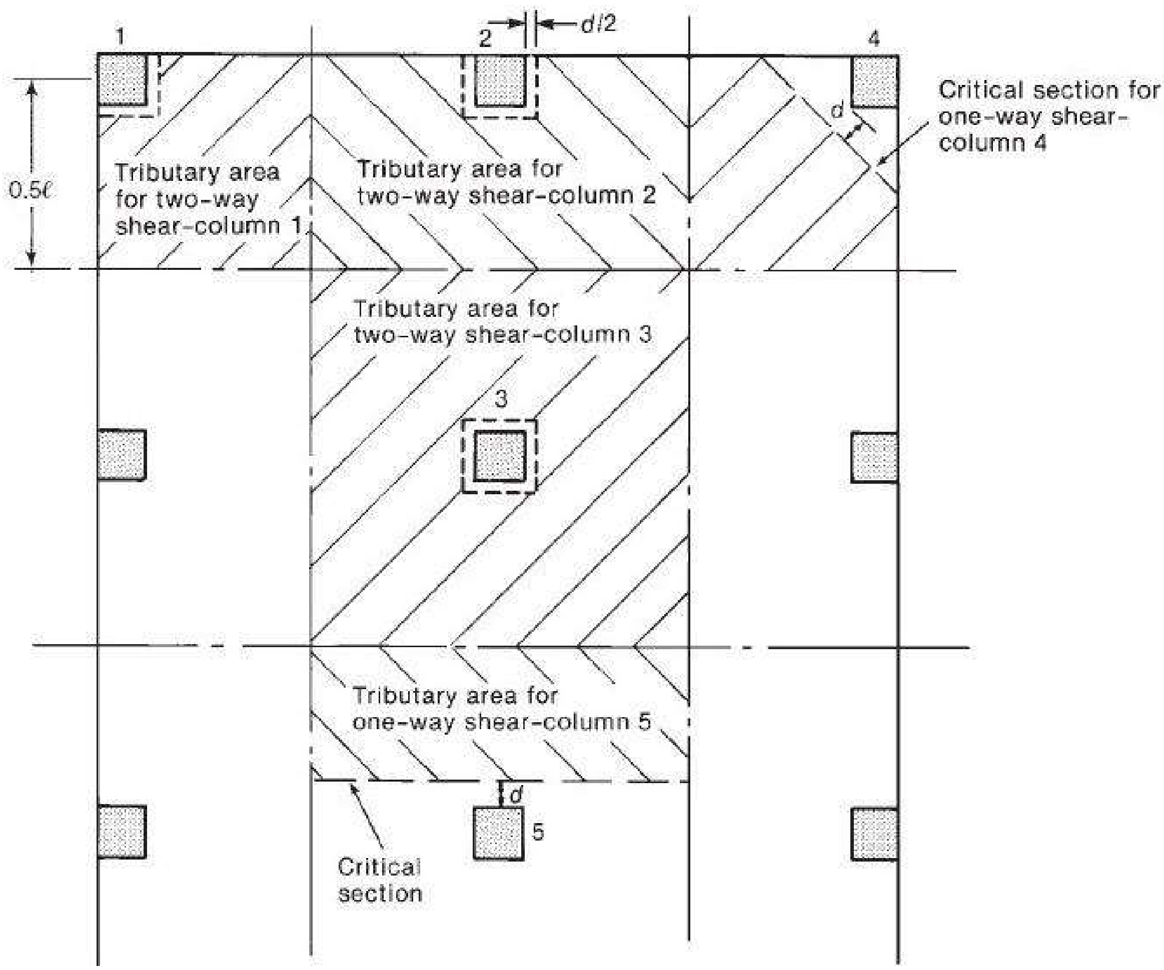

*Tributary area for shear on an interior beam.*

**Shear in flat plate and flat slab**

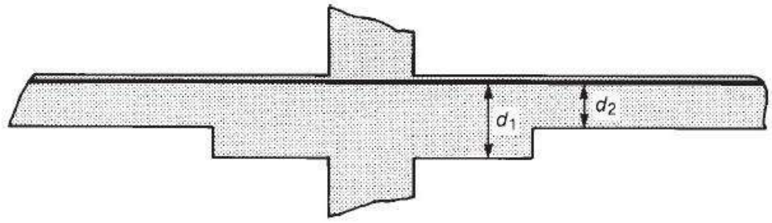
Types:-

1- Beam action (one – way shear action)

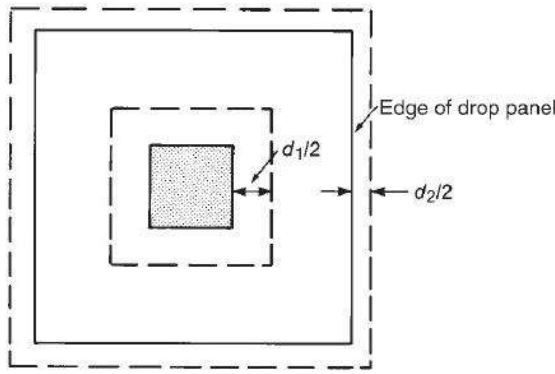
2- Punching shear (two – way shear action)




Critical sections and tributary areas for shear in flat plate.

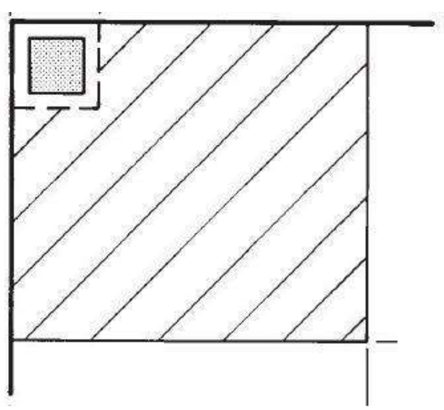



(a) Section through drop panel.

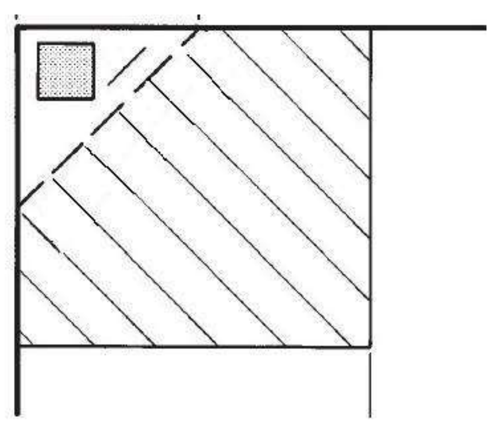



(b) Critical sections.

Critical sections in a slab with drop panels.



(a) Two-way shear.



(b) One-way shear.

Critical shear perimeters and tributary areas for corner column.

Factored one-way shear

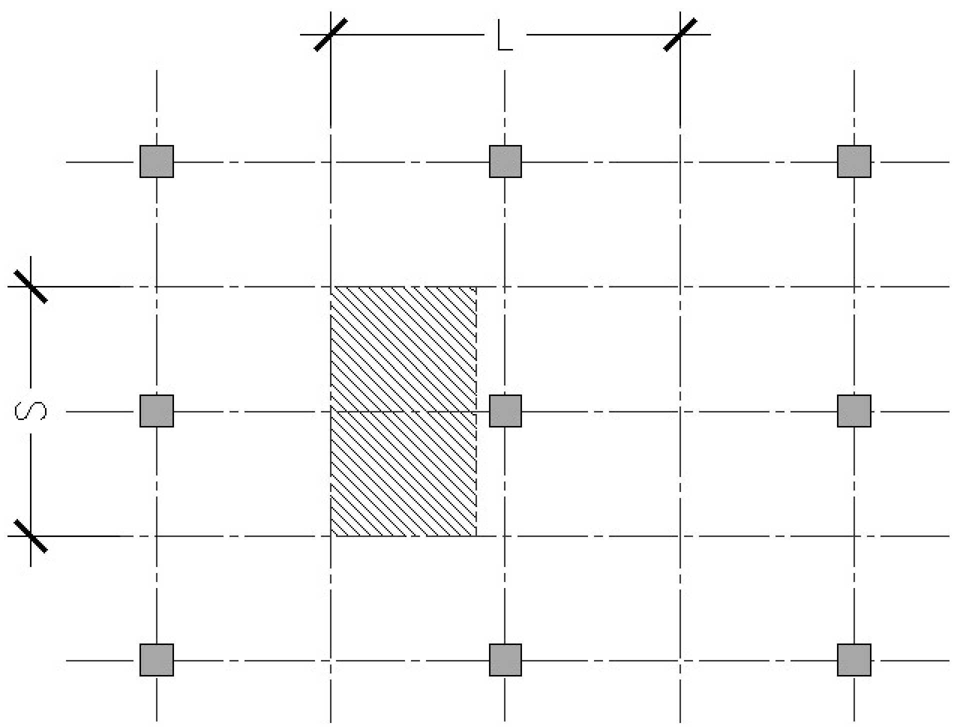
For slabs built integrally with supports, $V_u$ at the support shall be permitted to be calculated at the face of support.

Sections between the face of support and a critical section located a distance **d** from the face of support for nonprestressed slabs shall be permitted to be designed for $\mathbf{V_u}$ at that critical section if (a) through (c) are satisfied:

(a) Support reaction, in direction of applied shear, introduces compression into the end regions of the slab.

(b) Loads are applied at or near the top surface of the slab.

(c) No concentrated load occurs between the face of support and critical section.



**One-way shear strength**

Nominal one-way shear strength at a section (**$V_n$**) shall be calculated by:

$V_n = V_c + V_s$

Cross-sectional dimensions shall be selected to satisfy:

$$V_u \le \phi\left(V_c + 0.66\sqrt{f_c'}\, b_w d\right)$$

For nonprestressed members without axial force, $V_c$ shall be calculated by:

$$V_c = 0.17\sqrt{f_c'}\ b\ d$$

unless a more detailed calculation is made in accordance with Table 22.5.5.1.

Table 22.5.5.1 - Detailed method for calculating $V_c$

| $V_c$ | | |
|---|---|---|
| Least of (a), (b), and (c): | $\left(0.16\sqrt{f_c'} + 17\,\rho_w \frac{V_u d}{M_u}\right)\ b\ d$ | (a) |
| | $\left(0.16\sqrt{f_c'} + 17\,\rho_w\right)\ b\ d$ | (b) |
| | $0.29\sqrt{f_c'}\ b\ d$ | (c) |

$M_u$ occurs simultaneously with $V_u$ at the section considered.

Effect of any openings in members shall be considered in calculating $V_n$.

At each section where $V_u > \phi V_c$, transverse reinforcement shall be provided such that the equation

$$V_s \ge \frac{V_u}{\phi} - V_c$$

is satisfied.

The critical section extending across the entire width at a distance d from:-

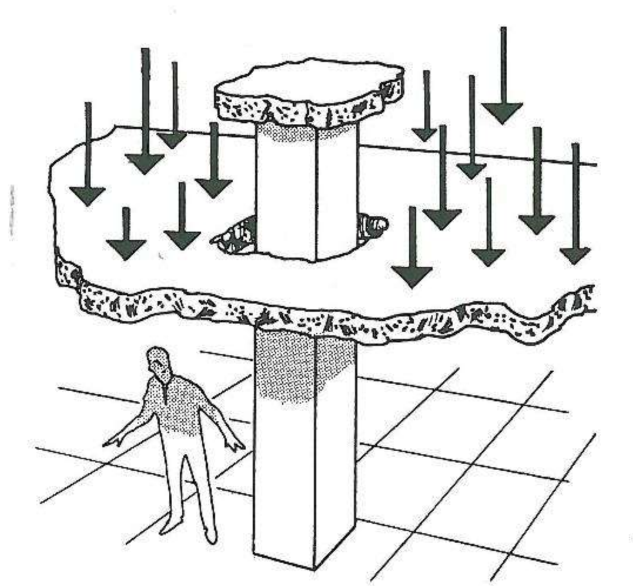
1- The face of the rectangular column in flat plate.

2- The face of the equivalent square column capital or from the face of drop panel, if any in flat slab.

The short direction is controlling because it has a wider area and short critical section:-

$$V_u = q_u \cdot S \cdot \left[\frac{L}{2} - \left(\frac{c}{2} + d\right)\right] \quad ; \quad v_n = \frac{V_n}{b \cdot d} = \frac{V_n}{S \cdot d}$$

Factored two-way shear (punching)



Critical section:

Slabs shall be evaluated for two-way shear in the vicinity of columns, concentrated loads, and reaction areas at critical sections.

Two-way shear shall be resisted by a section with a depth (**d**) and an assumed critical perimeter ($b_o$).

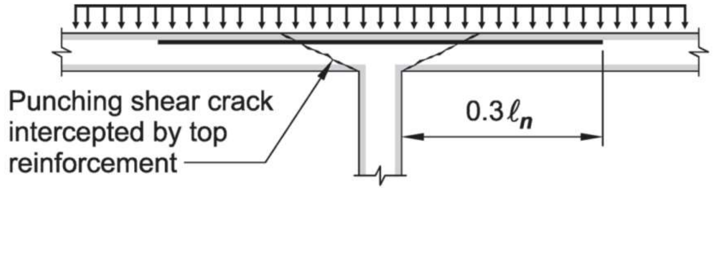
For calculation of $\mathbf{v_c}$ and $\mathbf{v_s}$ for two-way shear, **d** shall be the average of the effective depths in the two orthogonal directions.

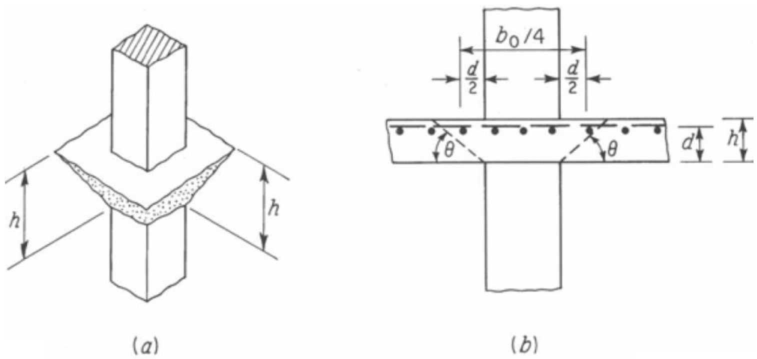
For two-way shear, critical sections shall be located so that the perimeter ($b_o$) is a minimum but need not be closer than **d/2** to (a) and (b):

(a) Edges or corners of columns, concentrated loads, or reaction areas.

(b) Changes in slab or footing thickness, such as edges of capitals, drop panels, or shear caps.

For a circular or regular polygon-shaped column, critical sections for two-way shear shall be permitted to be defined assuming a square column of equivalent area.






Failure surface defined by punching shear

Nominal shear strength for two-way members without shear reinforcement shall be calculated by:

$v_n = v_c$

$v_c$ for two-way shear shall be calculated in accordance with Table 22.6.5.2.

Table 22.6.5.2 - Calculation of $v_c$ for two-way shear

| $v_c$ | | |
|---|---|---|
| Least of (a), (b), and (c): | $0.33\sqrt{f'_c}$ | (a) |
| | $0.17\left(1+\frac{2}{\beta}\right)\sqrt{f'_c}$ | (b) |
| | $0.083\left(2+\frac{\alpha_s d}{b_o}\right)\sqrt{f'_c}$ | (c) |

Note: $\beta$ is the ratio of long side to short side of the column, concentrated load, or reaction area.

$\alpha_s$ = 40 for interior columns

= 30 for edge columns

= 20 for corner columns

Nominal shear strength for two-way members with shear reinforcement other than shearheads shall be calculated by:

$v_n = v_c + v_s$

For two-way members with shear reinforcement, $v_c$ shall not exceed the limits:

$v_c = 0.17\sqrt{f'_c}$

For two-way members with shear reinforcement, effective depth shall be selected such that $v_u$ calculated at critical sections does not exceed the value:

$v_u \le \phi 0.5\sqrt{f'_c}$

For two-way members reinforced with headed shear reinforcement or single- or multi-leg stirrups, a critical section with perimeter $b_o$ located d/2 beyond the outermost peripheral line of shear reinforcement shall also be considered. The shape of this critical section shall be a polygon selected to minimize $b_o$.

<u>Effective depth</u>

For calculation of $v_c$ and $v_s$ for two-way shear, d shall be the average of the effective depths in the two orthogonal directions.

<u>Two-way shear strength provided by single- or multiple-leg stirrups:</u>

Single- or multiple-leg stirrups fabricated from bars or wires shall be permitted to be used as shear reinforcement in slabs and footings satisfying (a) and (b):

(a) d is at least 150 mm.

(b) d is at least $16d_b$, where $d_b$ is the diameter of the stirrups.

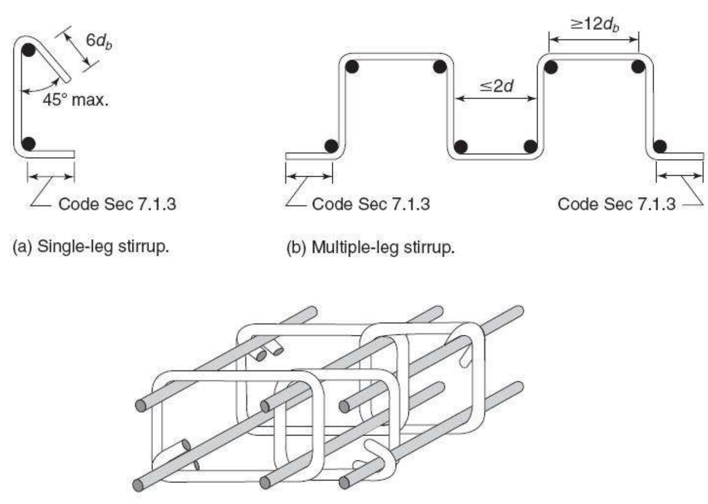
For two-way members with stirrups, $v_s$ shall be calculated by:

$$v_s = \frac{A_v\, f_y}{b_o\, s}$$

Where

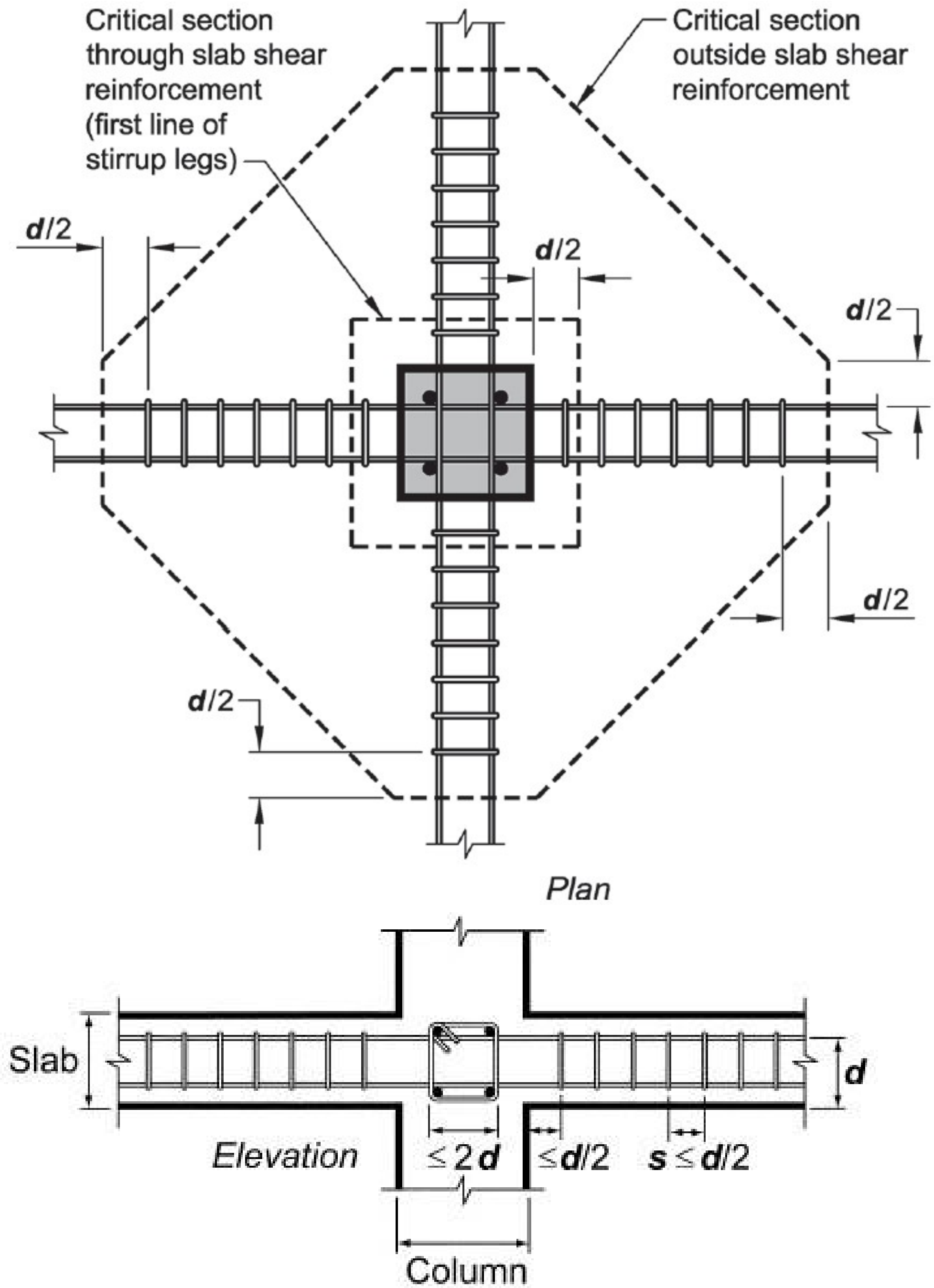
$A_v$: is the sum of the area of all legs of reinforcement on one peripheral line that is geometrically similar to the perimeter of the column section.

s: is the spacing of the peripheral lines of shear reinforcement in the direction perpendicular to the column face.

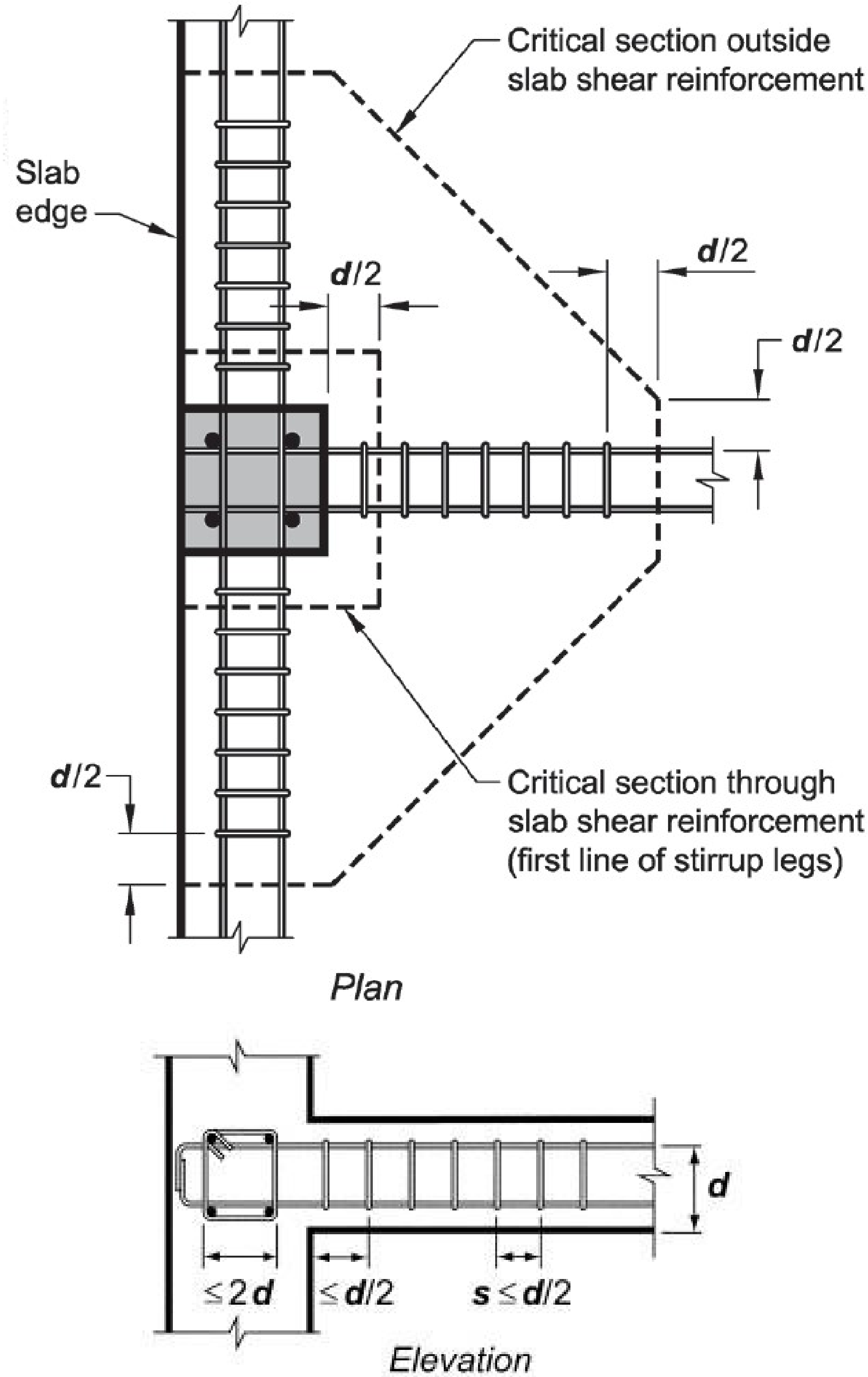


(a) Single-leg stirrup.
(b) Multiple-leg stirrup.
(c) Closed stirrups.

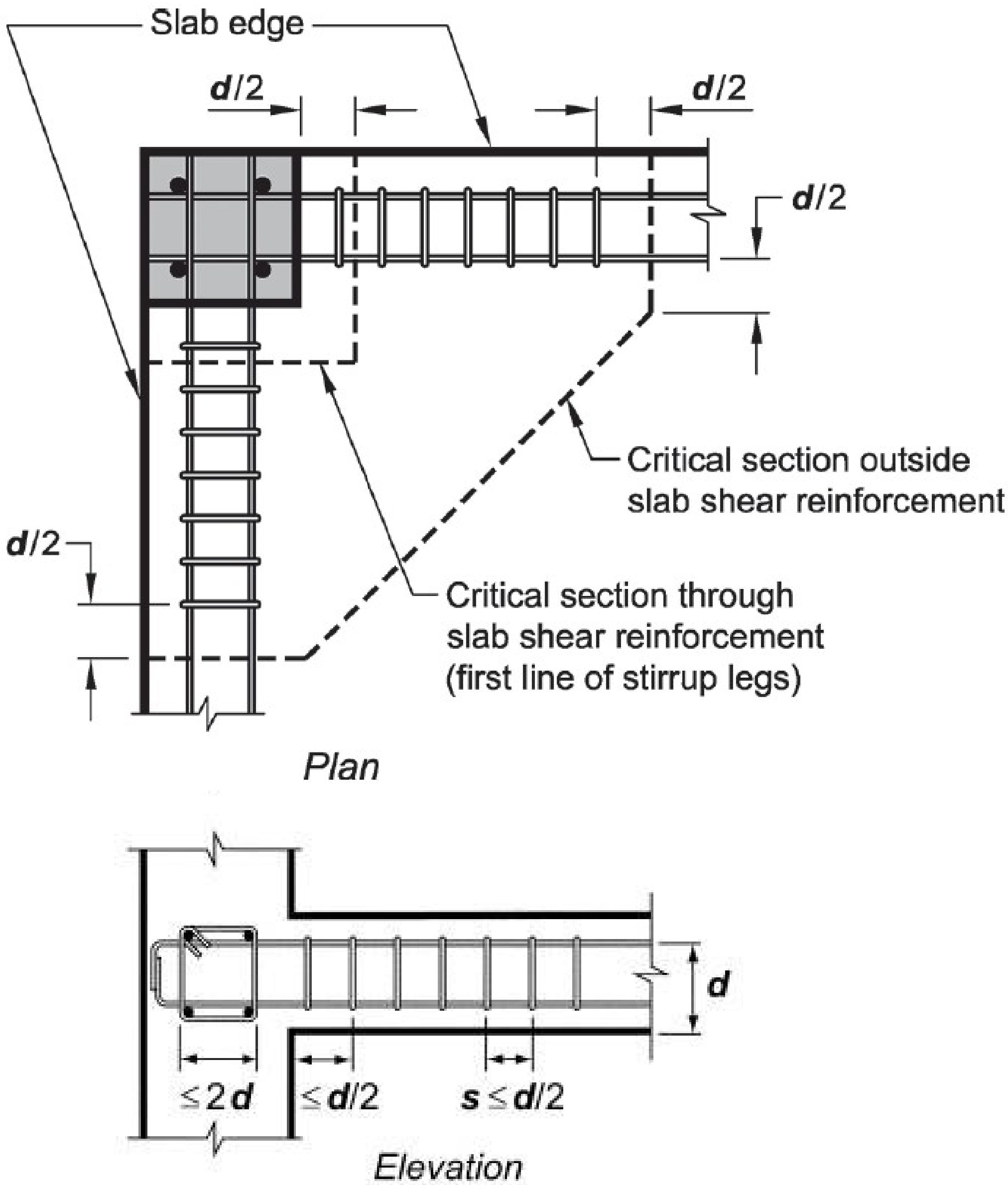
Shear reinforcement.




Arrangement of stirrup shear reinforcement, interior column.
Critical sections for two-way shear in slab with shear reinforcement at interior column.

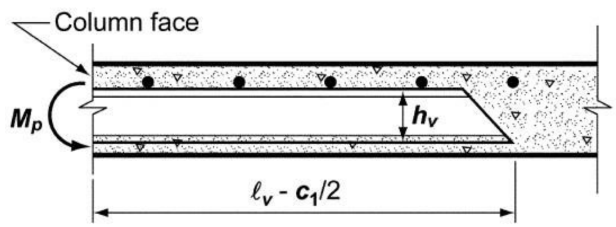



Arrangement of stirrup shear reinforcement, edge column.
Critical sections for two-way shear in slab with shear reinforcement at edge column.

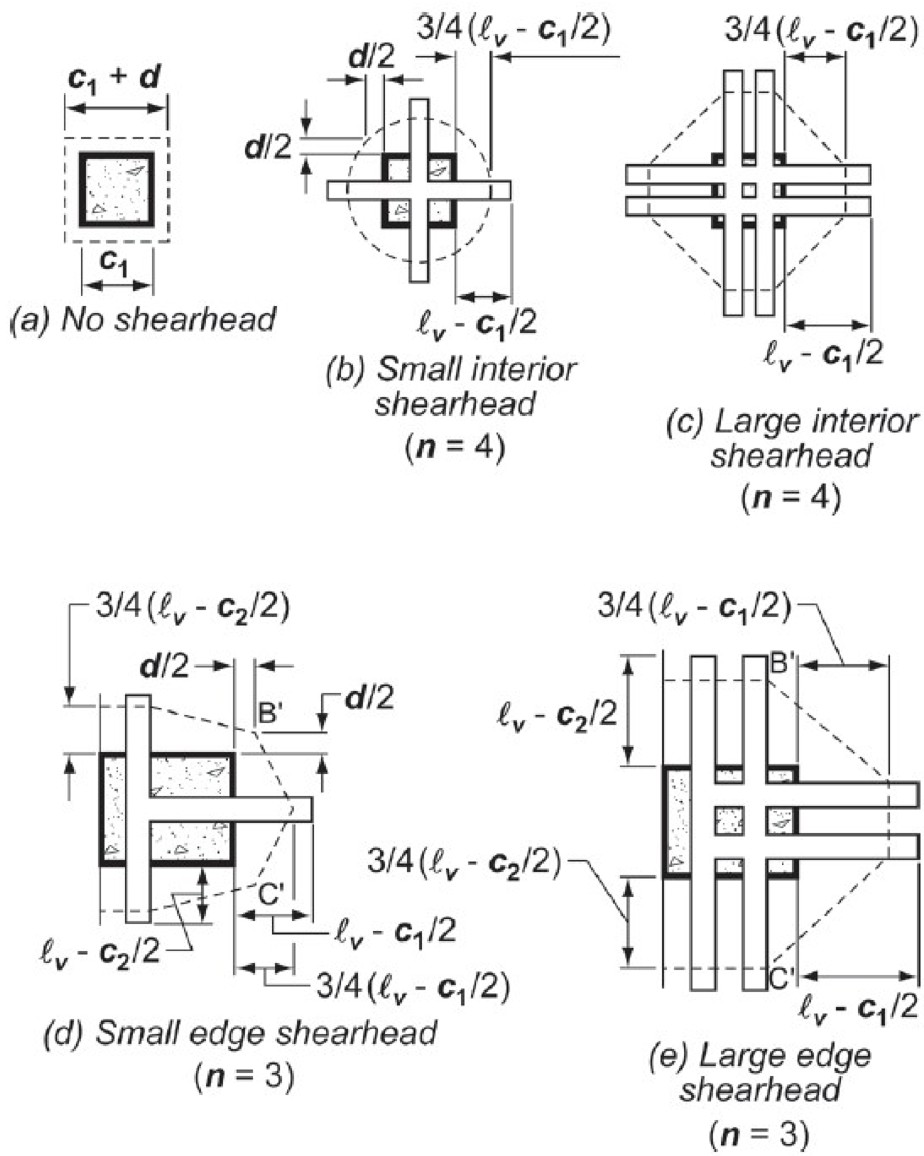



Arrangement of stirrup shear reinforcement, corner column.

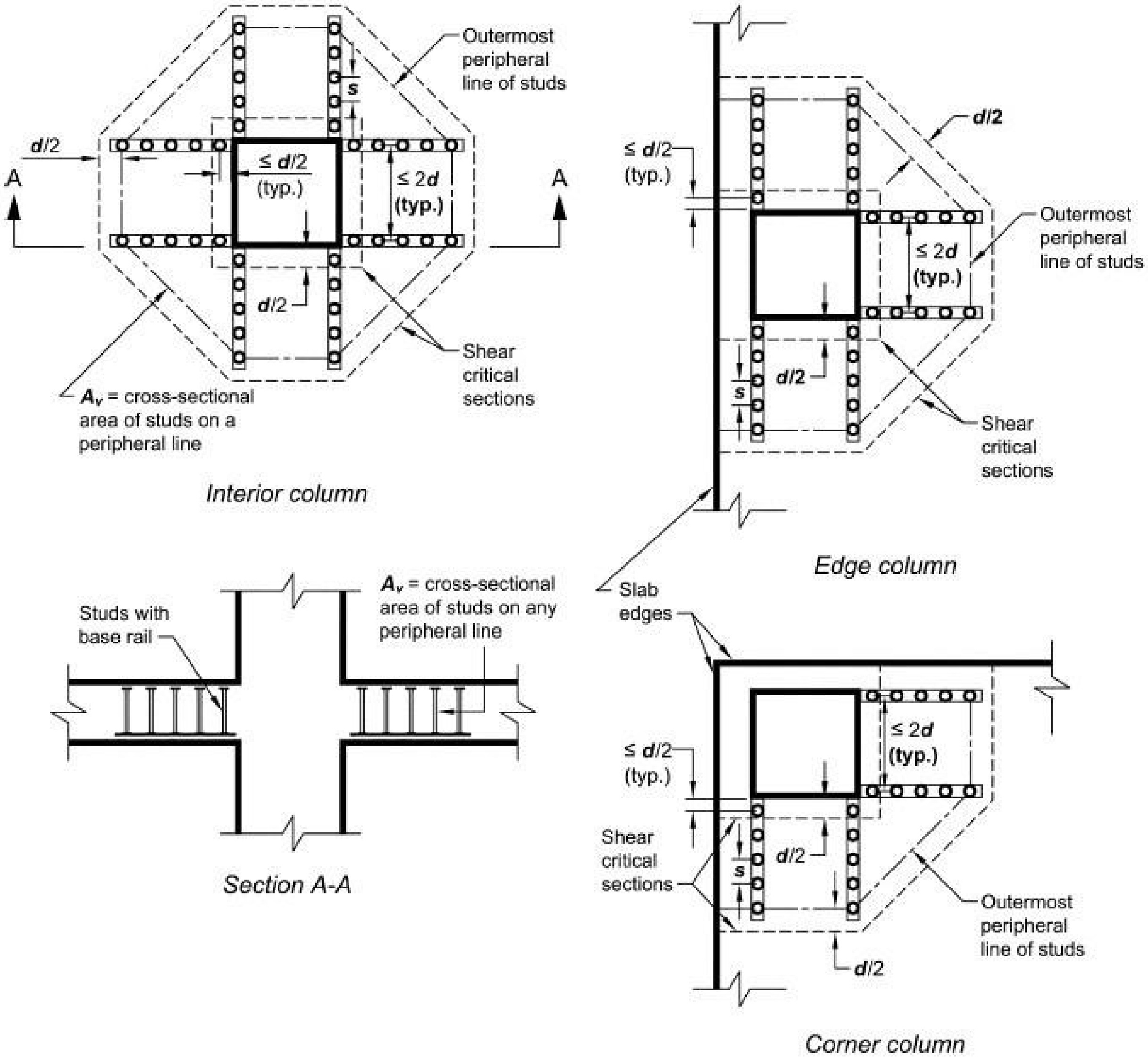
Critical sections for two-way shear in slab with shear reinforcement at corner column.




Structural shearheads.

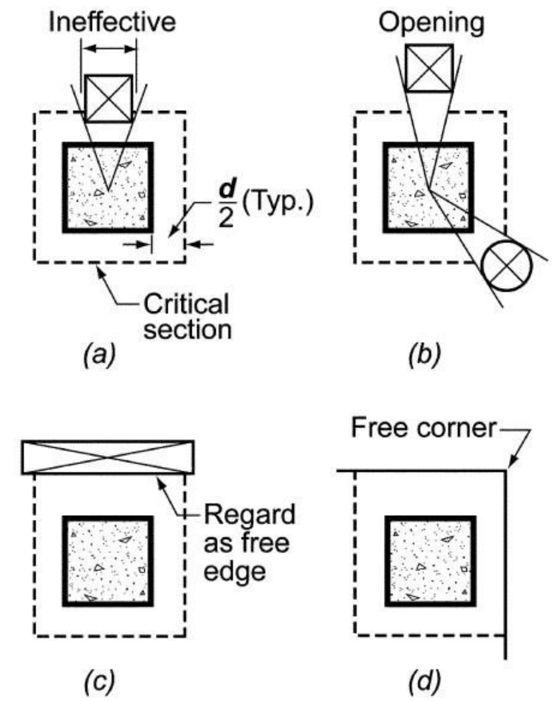



Location of critical section without and with shearheads.




Typical arrangements of headed shear stud reinforcement and critical sections.

Effect of any openings and free edges in slab shall be considered in calculating $v_n$




Effect of openings and free edges (effective perimeter shown with dashed lines).

Example:

The flat plate slab of 200 mm total thickness and 160 mm effective depth is carried by 300 mm square column 4.50 m on centers in each direction. A factored load of 580 kN must be transmitted from the slab to a typical interior column. Determine if shear reinforcement is required for the slab, and if so, design integral beams with vertical stirrups to carry the excess shear. Use $f_y$ = 414 MPa, $f_c'$ = 30 MPa.

Solution:-

Shear perimeter ($b_o$) = ( 300 + 160 ) × 4 = 1840 mm

$V_u$ = 580 kN

$$v_{ug} = \frac{V_u}{b_o \cdot d} = \frac{580 \times 10^3}{1840 \times 160} = 1.970 \ \text{MPa}$$

i) without shear reinforcement

The design shear strength of the concrete alone at the critical section d/2 from the face of column is

$$v_c = \min. \begin{cases} 0.33\sqrt{f_c'} = 0.33\sqrt{30} = 1.807 \ \text{MPa} \\ 0.17\left(1 + \frac{2}{\beta}\right)\sqrt{f_c'} = 0.17\left(1 + \frac{2}{1}\right) \times \sqrt{30} = 2.793 \ \text{MPa} \\ 0.083\left(2 + \frac{\alpha_s d}{b_o}\right)\sqrt{f_c'} = 0.083\left(2 + \frac{40 \times 160}{1840}\right) \times \sqrt{30} = 2.49 \ \text{MPa} \end{cases}$$

$$\beta_c = \frac{300}{300} = 1$$

$\therefore \ v_c = 1.807$ MPa

$v_n = v_c$

$\phi v_n = 0.75 \times 1.807 = 1.355$ MPa $< v_u = 1.97$ MPa    not O.K.

$\therefore$ Shear reinforcement is required

ii) with shear reinforcement

$v_n = v_c + v_s$

For two-way members with shear reinforcement, effective depth shall be selected such that $v_u$ calculated at critical sections does not exceed the value:

$$v_u \le \phi 0.5\sqrt{f_c'}$$

$$v_u = v_{ug} = 1.97 \ \text{MPa} \quad < \quad \phi 0.5\sqrt{f_c'} = 0.75 \times 0.5 \times \sqrt{30} = 2.054 \ \text{MPa} \qquad \text{O.K.}$$

$$v_c = 0.17\sqrt{f_c'} = 0.17 \times \sqrt{30} = 0.931 \ \text{MPa}$$

Let $\phi v_n = v_u = 1.97$ MPa

$\phi\,(v_c + v_s) = v_u$

$$\Rightarrow \quad v_s = \frac{v_u}{\phi} - v_c = \frac{1.97}{0.75} - 0.931 = 1.696 \text{ MPa}$$

$$v_s = \frac{A_v\, f_y}{b_o\, s}$$

$$\Rightarrow \quad A_v = \frac{v_s\, b_o\, s}{f_y} = \frac{1.696 \times 1840 \times 80}{414} = 603 \text{ mm}^2 \qquad s = \frac{d}{2} = 80 \text{ mm}$$

The required area of vertical shear reinforcement = 603 $mm^2$

For trial, ø10 mm vertical closed hoop stirrups will be selected and arranged along four integral beams.
effective depth = 160 mm = 16 × 10 (d is at least $16d_b$). O.K.
$A_v$ provided is 4 × 2 × 78.5 = 628 $mm^2$ at the first critical section, at distance d/2 = 80 mm from the column face.

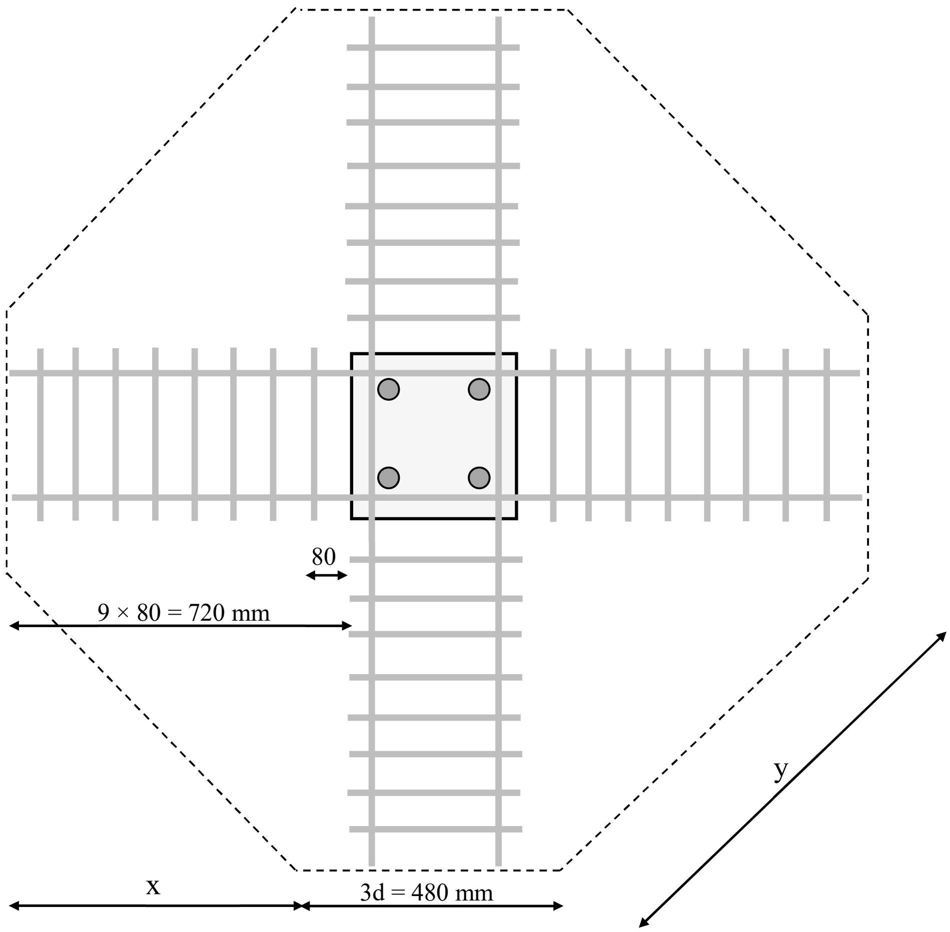
The required perimeter of the second critical section, at which the concrete alone can carry the shear, is found from the controlling equation as follows:

$$v_u = \phi v_n = \phi v_c = \phi\, 0.17 \sqrt{f_c'} = 0.75 \times 0.17 \times \sqrt{30} = 0.698 \text{ MPa}$$

$$v_u = v_{ug} = 0.698 = \frac{580 \times 10^3}{b_o \times 160} \quad \Rightarrow \quad b_o = 5193.4 \text{ mm}$$

5193.4 = 4 × (3d + y )
⇒ y = 818.35 mm
x = 818.35 × sin 45 = 578.7 mm
8 stirrups at constant 80 mm spacing will be sufficient, the first placed at 80 mm from the column face, this provides a shear perimeter ($b_o$) at second critical section of:
9 × 80 + 150 = 870 mm > x + 240 = 818.7 mm O.K.

It is essential that this shear reinforcement engage longitudinal reinforcement at both the top and bottom of the slab, so 4 longitudinal ø16 bars will be provided inside the corners of each closed hoop stirrup. Alternatively, the main slab reinforcement could be used.


80
9 × 80 = 720 mm
y
x
3d = 480 mm

Example:

Check the two way shear action (punching shear) only around an edge column (400×400) mm in a flat plate floor of a span (6.0 × 6.0) m. Find the area of vertical shear reinforcement if required. Assume d = 158 mm. Total $q_u$ = 16.0 kPa (including slab weight), $f_c$' = 25 MPa, $f_y$ = 400 MPa.

Solution:-

Shear perimeter ($b_o$) = (400 + 79) × 2 + (400 + 158) = 1516 mm

$V_u$ = 16 × (6 × 3.2 – 0.558 × 0.479) = 302.923 kN

$$v_{ug} = \frac{V_u}{b_o \cdot d} = \frac{302.923 \times 10^3}{1516 \times 158} = 1.265 \text{ MPa}$$

i) without shear reinforcement

The design shear strength of the concrete alone at the critical section d/2 from the face of column is

$$v_c = \min. \begin{cases} 0.33\sqrt{f_c'} = 0.33\sqrt{25} = 1.65 \text{ MPa} \\ 0.17\left(1 + \frac{2}{\beta}\right)\sqrt{f_c'} = 0.17\left(1 + \frac{2}{1}\right) \times \sqrt{25} = 2.55 \text{ MPa} \\ 0.083\left(2 + \frac{\alpha_s d}{b_o}\right)\sqrt{f_c'} = 0.083\left(2 + \frac{30 \times 158}{1516}\right) \times \sqrt{25} = 2.128 \text{ MPa} \end{cases}$$

$$\beta_c = \frac{400}{400} = 1$$

$\therefore \; v_c = 1.65$ MPa

$v_n = v_c$

$\phi v_n = 0.75 \times 1.65 = 1.238$ MPa $< v_u = 1.265$ MPa     not O.K.

∴ Shear reinforcement is required

ii) with shear reinforcement

$v_n = v_c + v_s$

For two-way members with shear reinforcement, effective depth shall be selected such that $v_u$ calculated at critical sections does not exceed the value:

$v_u \le \phi 0.5\sqrt{f_c'}$

$v_u = v_{ug} = 1.265 \text{ MPa} < \phi 0.5\sqrt{f_c'} = 0.75 \times 0.5 \times \sqrt{25} = 1.875 \text{ MPa}$     O.K.

$$v_c = 0.17\sqrt{f_c'} = 0.17 \times \sqrt{25} = 0.85 \text{ MPa}$$

Let $\phi v_n = v_u = 1.265$ MPa

$\phi\,(v_c + v_s) = v_u$

$$\Rightarrow \quad v_s = \frac{v_u}{\phi} - v_c = \frac{1.265}{0.75} - 0.85 = 0.837 \text{ MPa}$$

$$v_s = \frac{A_v f_y}{b_o s}$$

$$\Rightarrow \quad A_v = \frac{v_s\, b_o\, s}{f_y} = \frac{0.837 \times 1516 \times 79}{400} = 250.6\ \text{mm}^2 \qquad s = \frac{d}{2} = 79\ \text{mm}$$

The required area of vertical shear reinforcement = 250.6 mm$^2$

To design the integral beams with the vertical stirrups to carry the excess shear:

For trial, ø8 mm vertical closed hoop stirrups will be selected and arranged along three integral beams.
effective depth = 158 mm > 16 × 8 = 128 mm (d is at least 16$d_b$). O.K.

$A_v$ provided is 3 × 2 × 50.2 = 301 mm$^2$ at the first critical section, at distance d/2 ≈ 75 mm from the column face.

The required perimeter of the second critical section, at which the concrete alone can carry the shear, is found from the controlling equation as follows:

$$v_u = \phi v_n = \phi v_c = \phi\, 0.17 \sqrt{f_c'} = 0.75 \times 0.17 \times \sqrt{25} = 0.638\ \text{MPa}$$

$$v_u = v_{ug} = 0.638 = \frac{302.923 \times 10^3}{b_o \times 158} \quad \Rightarrow \quad b_o = 3005.1\ \text{mm}$$

Example:

Check the two way shear action (punching shear) only around a corner column (400×400) mm in a flat plate floor of a span (6.0×6.0) m. Find the area of vertical shear reinforcement if required. Assume d =158 mm. Total $q_u$ = 19.0 kPa (including slab weight), $f_c$'= 25 MPa, $f_y$ = 400 MPa.

Solution:-

Shear perimeter ($b_o$) = (400 + 79) × 2 = 958 mm

$V_u$ = 19 × (3.2 × 3.2 – 0.479 × 0.479) = 190.201 kN

$$v_{ug} = \frac{V_u}{b_o \cdot d} = \frac{190.201 \times 10^3}{958 \times 158} = 1.257 \text{ MPa}$$

i) without shear reinforcement

The design shear strength of the concrete alone at the critical section d/2 from the face of column is

$$v_c = \min.\begin{cases} 0.33\sqrt{f_c'} = 0.33\sqrt{25} = 1.65 \text{ MPa} \\ 0.17\left(1+\frac{2}{\beta}\right)\sqrt{f_c'} = 0.17\left(1+\frac{2}{1}\right)\times\sqrt{25} = 2.55 \text{ MPa} \\ 0.083\left(2+\frac{\alpha_s d}{b_o}\right)\sqrt{f_c'} = 0.083\left(2+\frac{20\times 158}{958}\right)\times\sqrt{25} = 2.199 \text{ MPa} \end{cases}$$

$$\beta_c = \frac{400}{400} = 1$$

$\therefore\ v_c = 1.65$ MPa

$v_n = v_c$

$\phi v_n = 0.75 \times 1.65 = 1.238$ MPa $<$ $v_u = 1.257$ MPa  not O.K.

∴ Shear reinforcement is required

ii) with shear reinforcement

$v_n = v_c + v_s$

For two-way members with shear reinforcement, effective depth shall be selected such that $v_u$ calculated at critical sections does not exceed the value:

$v_u \le \phi 0.5\sqrt{f_c'}$

$v_u = v_{ug} = 1.257 \text{ MPa} < \phi 0.5\sqrt{f_c'} = 0.75 \times 0.5 \times \sqrt{25} = 1.875 \text{ MPa}$  O.K.

$$v_c = 0.17\sqrt{f_c'} = 0.17 \times \sqrt{25} = 0.85 \text{ MPa}$$

Let $\phi v_n = v_u = 1.257$ MPa

$\phi\,(v_c + v_s) = v_u$

$$\Rightarrow \quad v_s = \frac{v_u}{\phi} - v_c = \frac{1.257}{0.75} - 0.85 = 0.826 \text{ MPa}$$

$$v_s = \frac{A_v f_y}{b_o s}$$

$$\Longrightarrow \quad A_v = \frac{v_s b_o s}{f_y} = \frac{0.826 \times 958 \times 75}{400} = 148.4 \text{ mm}^2 \qquad s = 75 \text{ mm} < \frac{d}{2} = 79 \text{ mm}$$

The required area of vertical shear reinforcement = 148.4 $mm^2$

Example:

Check the two way shear action (punching shear) only around an interior column (450×450) mm in a flat plate floor of a span (5.8×5.6) m. Find the area of vertical shear reinforcement if required. Assume d =150 mm. Total $q_u$ = 17.5 kPa (including slab weight), $f_c$'= 32 MPa, $f_y$ = 420 MPa.

Example:

Check the two way shear action (punching shear) only around an interior column (400×500) mm in a flat plate floor of a span (5.6×5.6) m. Find the area of vertical shear reinforcement if required. Assume d =170 mm. Total $q_u$ = 18.0 kPa (including slab weight), $f_c'$= 30 MPa, $f_y$ = 420 MPa.

Solution:-

Shear perimeter ($b_o$) = (400 + 170) × 2 + (500 + 170) × 2 = 2480 mm

$V_u$ = 18 × (5.6 × 5.6 – 0.57 × 0.67) = 557.606 kN

$$v_{ug} = \frac{V_u}{b_o \cdot d} = \frac{557.606 \times 10^3}{2480 \times 170} = 1.323 \text{ MPa}$$

i) without shear reinforcement

The design shear strength of the concrete alone at the critical section d/2 from the face of column is

$$v_c = \min. \begin{cases} 0.33\sqrt{f_c'} = 0.33\sqrt{30} = 1.807 \text{ MPa} \\ 0.17\left(1 + \frac{2}{\beta}\right)\sqrt{f_c'} = 0.17\left(1 + \frac{2}{1.25}\right) \times \sqrt{30} = 2.421 \text{ MPa} \\ 0.083\left(2 + \frac{\alpha_s d}{b_o}\right)\sqrt{f_c'} = 0.083\left(2 + \frac{40 \times 170}{2480}\right) \times \sqrt{30} = 2.156 \text{ MPa} \end{cases}$$

$$\beta_c = \frac{500}{400} = 1.25$$

$\therefore\ v_c = 1.807$ MPa

$v_n = v_c$

$\phi v_n = 0.75 \times 1.807 = 1.355$ MPa $> v_u = 1.323$ MPa not O.K.

$\therefore$ Shear reinforcement is not required

Example:

Check the two way shear action (punching shear) only around an edge column (300×300) mm in a flat plate floor of a span (4.0×4.0) m. Find the area of vertical shear reinforcement if required. Assume d =165 mm. Total $q_u$ = 17.6 kPa (including slab weight), $f_c'$= 35 MPa, $f_y$ = 420 MPa.

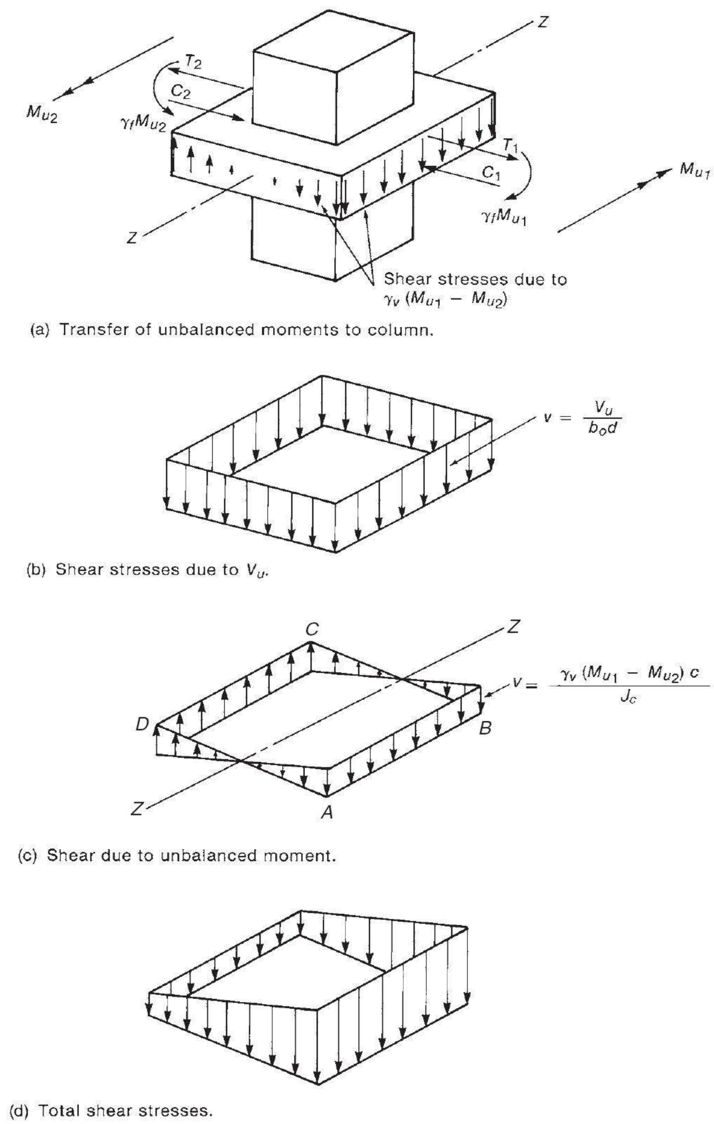
**Transfer of moments to columns**

The above analysis for punching shear in slabs assumed that the shear force ($V_u$) was uniformly distributed around the perimeter of the critical section ($b_o$), at distance d/2 from the face of supporting column and resisted by concrete shear strength ($v_c$), which was given by the minimum of three equations. If significant moment is to be transferred from the slab to the column, the shear stress on the critical section is no longer uniformly distributed. The situation is shown in figures below.

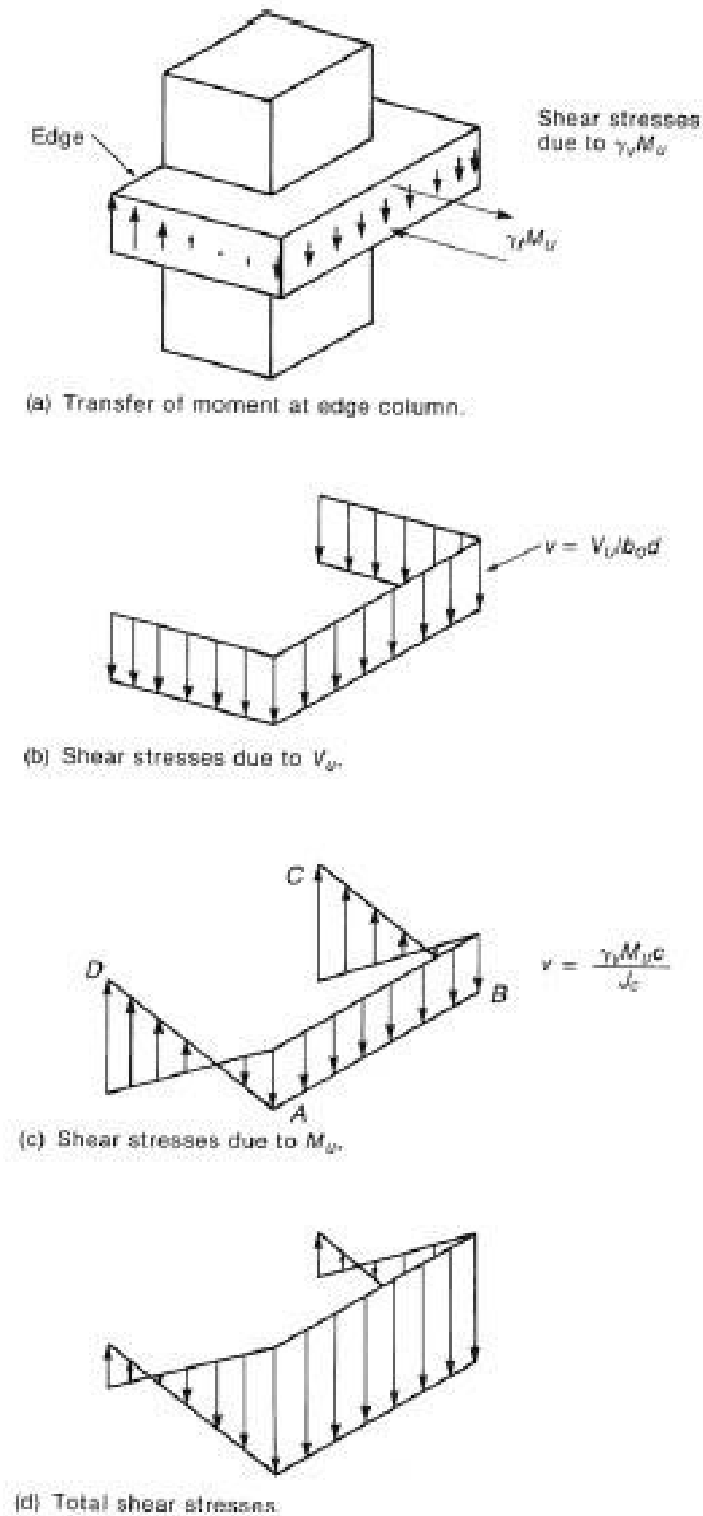
$V_u$ represents the total vertical reaction to be transferred to the column.

$M_u$ ($\gamma_v M_{sc}$) represents the unbalanced moment to be transferred by shear.

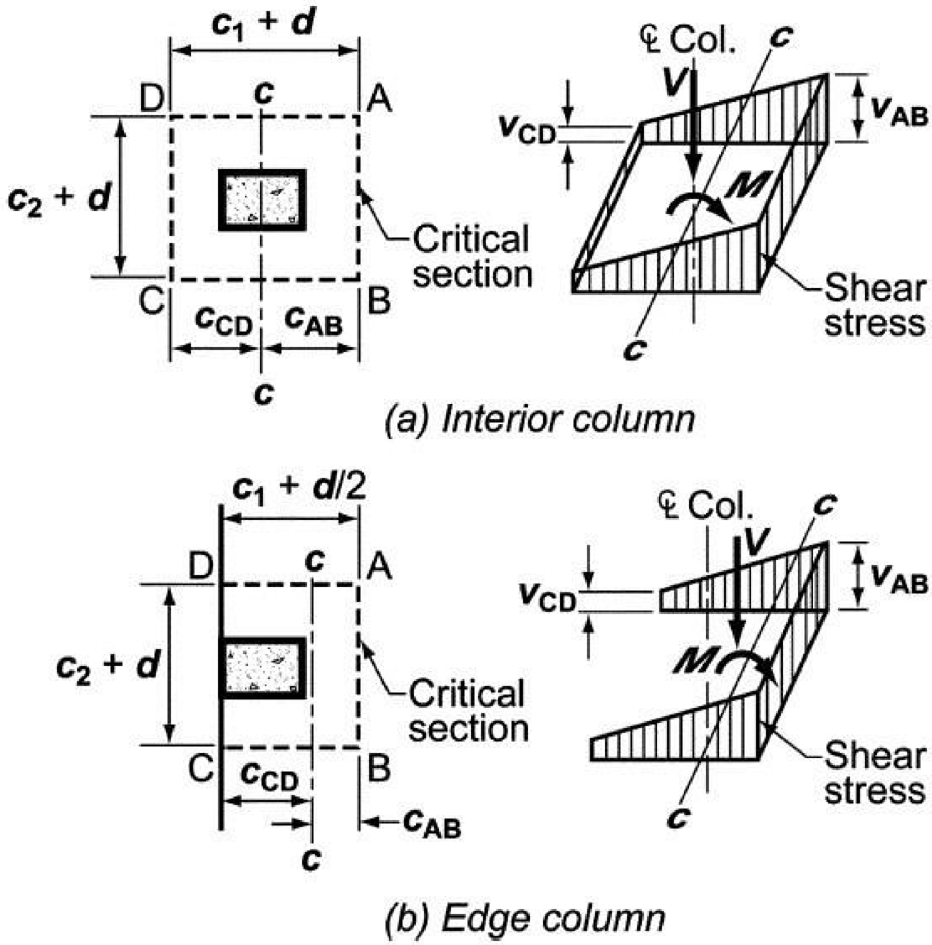
$V_u$ causes shear stress distributed uniformly around the perimeter of the critical section, which acting downward. $M_u$ causes additional loading, which add to shear stresses in one side and subtract to the other side.




Shear stresses due to shear and moment transfer at an interior column.




Shear stresses due to shear and moment transfer at an edge column.




Assumed distribution of shear stress.

If there is a transfer of moment between the slab and column, a fraction of $M_{sc}$, the factored slab moment resisted by the column at a joint, shall be transferred by flexure ($\gamma_f M_{sc}$), where $\gamma_f$ shall be calculated by:

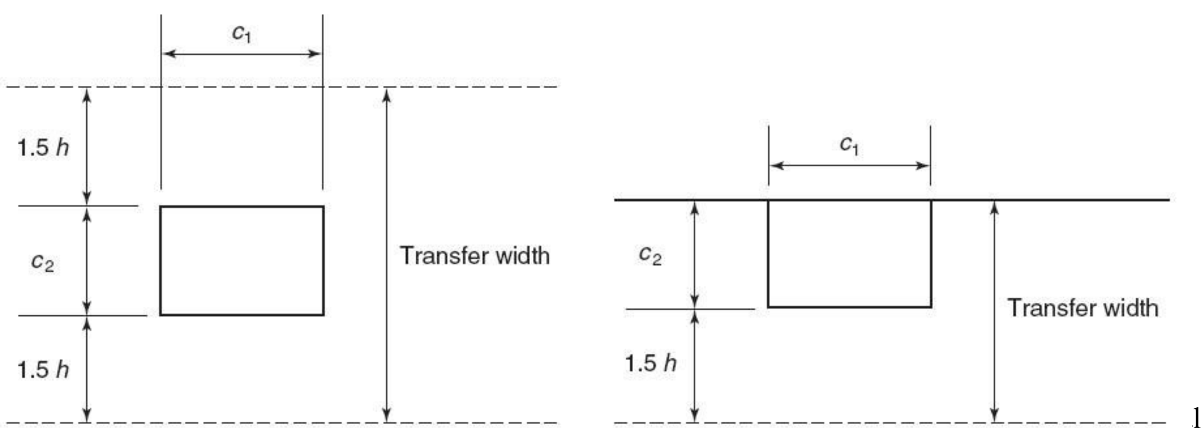
$$\gamma_f = \frac{1}{1 + \left(\frac{2}{3}\right)\sqrt{\frac{b_1}{b_2}}}$$

For nonprestressed slabs, where the limitations on $v_{ug}$ and $\varepsilon_t$ in Table 8.4.2.3.4 are satisfied, $\gamma_f$ shall be permitted to be increased to the maximum modified values provided in Table 8.4.2.3.4, where $v_c$ is calculated in accordance with Table 22.6.5.2, and $v_{ug}$ is the factored shear stress on the slab critical section for two-way action due to gravity loads without moment transfer.

**Table 8.4.2.3.4—Maximum modified values of $\gamma_f$ for nonprestressed two-way slabs**

| Column location | Span direction | $v_{ug}$ | $\varepsilon_t$ (within $b_{slab}$) | Maximum modified $\gamma_f$ |
|---|---|---|---|---|
| Corner column | Either direction | $\leq 0.5\phi v_c$ | $\geq 0.004$ | 1.0 |
| Edge column | Perpen-dicular to the edge | $\leq 0.75\phi v_c$ | $\geq 0.004$ | 1.0 |
| | Parallel to the edge | $\leq 0.4\phi v_c$ | $\geq 0.010$ | $\dfrac{1.25}{1+\left(\frac{2}{3}\right)\sqrt{\frac{b_1}{b_2}}} \leq 1.0$ |
| Interior column | Either direction | $\leq 0.4\phi v_c$ | $\geq 0.010$ | $\dfrac{1.25}{1+\left(\frac{2}{3}\right)\sqrt{\frac{b_1}{b_2}}} \leq 1.0$ |

The effective slab width ($b_{slab}$) for resisting $\gamma_f M_{sc}$ shall be the width of column or capital plus **1.5 h** of slab or drop panel on either side of column or capital.




(a) Interior column.

(b) Exterior column with moment transferred parallel to the edge.

The fraction of $M_{sc}$ not calculated to be resisted by flexure shall be assumed to be resisted by eccentricity of shear.

For two-way shear with factored slab moment resisted by the column, factored shear stress ($v_u$) shall be calculated at critical sections. $v_u$ corresponds to a combination of $v_{ug}$ and the shear stress produced by $\gamma_v M_{sc}$.

The fraction of $M_{sc}$ transferred by eccentricity of shear ($\gamma_v M_{sc}$) shall be applied at the centroid of the critical section, where:
$\gamma_v = 1 - \gamma_f$

The stress distribution is assumed as illustrated in Figure above for an interior or exterior column. The perimeter of the critical section, ABCD, is determined. The factored shear stress ($v_{ug}$) and factored slab moment resisted by the column ($M_{sc}$) are determined at the centroidal axis c-c of the critical section. The maximum factored shear stress may be calculated from:

$$v_{u,AB} = v_{ug} + \frac{\gamma_v M_{sc} c_{AB}}{J_c} \quad ; \quad v_{u,CD} = v_{ug} - \frac{\gamma_v M_{sc} c_{DB}}{J_c}$$

$J_c$ = property of assumed critical section analogous to polar moment of inertia

Interior column:

$$J_c = \frac{d(c_1+d)^3}{6} + \frac{(c_1+d)d^3}{6} + \frac{d(c_2+d)(c_1+d)^2}{2}$$

or

$$J_c = 2\left(\frac{b_1 d^3}{12} + \frac{d{b_1}^3}{12}\right) + 2(b_2 d)\left(\frac{b_1}{2}\right)^2$$

Edge column:
In case of moment about an axis parallel to the edge:

$$c_{AB} = \frac{\text{moment of area of the sides about AB}}{\text{area of the sides}}$$

$$c_{AB} = \frac{2(b_1 d)\left(\frac{b_1}{2}\right)}{2(b_1 d) + b_2 d}$$

$$J_c = 2\left[\frac{b_1 d^3}{12} + \frac{d{b_1}^3}{12} + (b_1 d)\left(\frac{b_1}{2} - c_{AB}\right)^2\right] + (b_2 d)c_{AB}^2$$

In case of moment about an axis perpendicular to the edge:

$$J_c = \left(\frac{b_2 d^3}{12} + \frac{d{b_2}^3}{12}\right) + 2(b_1 d)\left(\frac{b_2}{2}\right)^2$$

Corner column:

$$c_{AB} = \frac{(b_1 d)\left(\frac{b_1}{2}\right)}{b_1 d + b_2 d}$$

$$J_c = \left[\frac{b_1 d^3}{12} + \frac{d {b_1}^3}{12} + (b_1 d)\left(\frac{b_1}{2} - c_{AB}\right)^2\right] + (b_2 d) c_{AB}^2$$

At an interior support, columns or walls above and below the slab shall resist the factored moment calculated by the equation below in direct proportion to their stiffnesses unless a general analysis is made.

$$M_{sc} = 0.07\left[(q_{Du} + 0.5\, q_{Lu})\, \ell_2\, {\ell_n}^2 - q_{Du}'\, \ell_2'\, (\ell_n')^2\right]$$

where $q_{Du}'$, $\ell_2'$, and $\ell_n'$ refer to the shorter span.

The gravity load moment to be transferred between slab and edge column shall not be less than $0.3M_o$.

**Calculation of factored shear strength $v_u$ (ACI 421.1R-4)**

The maximum factored shear stress $v_u$ at a critical section produced by the combination of factored shear force $V_u$ and unbalanced moments $M_{ux}$ and $M_{uy}$, is:

$$v_u = \frac{V_u}{A_c} + \frac{\gamma_{vx} M_{ux}\, y}{J_x} + \frac{\gamma_{vy} M_{uy}\, x}{J_y}$$

$A_c$: area of concrete of assumed critical section.

x, y: coordinate of the point at which $v_u$ is maximum with respect to the centroidal principal axes x and y of the assumed critical section.

$M_{ux}$, $M_{uy}$: factored unbalanced moments transferred between the slab and the column about the centroidal axes x and y of the assumed critical section, respectively

$\gamma_{ux}$, $\gamma_{uy}$: fraction of moment between slab and column that is considered transferred by eccentricity of shear about the axes x and y of the assumed critical section. The coefficients $\gamma_{ux}$ and $\gamma_{uy}$ are given by:

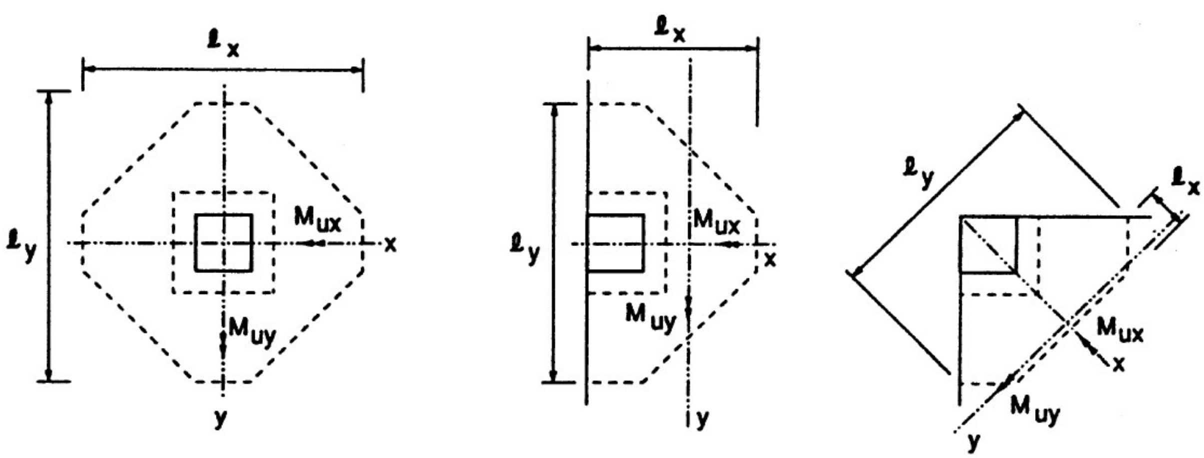
$$\gamma_{vx} = 1 - \frac{1}{1 + \left(\frac{2}{3}\right)\sqrt{\ell_{y1}/\ell_{x1}}}$$

$$\gamma_{vy} = 1 - \frac{1}{1 + \left(\frac{2}{3}\right)\sqrt{\ell_{x1}/\ell_{y1}}}$$

where $\ell_{x1}$ and $\ell_{y1}$ are lengths of the sides in the x and y directions of the critical section at d/2 from column face.

$J_x$ , $J_y$: property of assumed critical section, analogous to polar amount of inertia about the axes x and y, respectively. In the vicinity of an interior column, $J_y$ for a critical section at d/2 from column face is:

$$J_y = d\left[\frac{{\ell_{x1}}^3}{6} + \frac{\ell_{y1}{\ell_{x1}}^2}{2}\right] + \frac{\ell_{x1}d^3}{6}$$




$$\gamma_{vx} = 1 - \frac{1}{1 + \frac{2}{3}\sqrt{\ell_y/\ell_x}}$$

$\gamma_{vx}$ = same as for interior column

$\gamma_{vx} = 0.4$

$$\gamma_{vy} = 1 - \frac{1}{1 + \frac{2}{3}\sqrt{\ell_x/\ell_y}}$$

$$\gamma_{vy} = 1 - \frac{1}{1 + \frac{2}{3}\sqrt{(\ell_x/\ell_y) - 0.2}}$$

(but $\gamma_{vy} = 0$ when $(\ell_x/\ell_y) < 0.2$)

$\gamma_{vy}$ = same as for edge column

Equations for $\gamma_{vx}$ and $\gamma_{vy}$ applicable for critical sections at d/2 from column face and outside shear-reinforced zone. Note: $\ell_x$ and $\ell_y$ are projections of critical sections on directions of principal x and y axes.

**Properties of critical sections of general shape**

This section is general; it applies regardless of the type of shear reinforcement used. Figure below shows the top view of critical sections for shear in slab in the vicinity of interior column. The centroidal x and y axes of the critical sections, $V_u$, $M_{ux}$, and $M_{uy}$ are shown in their positive directions. The shear force $V_u$ is acting at the column centroid; $V_u$, $M_{ux}$, and $M_{uy}$ represent the effects of the column on the slab.

$v_u$ for a section of general shape, the parameters $J_x$ and $J_y$ may be approximated by the second moments of area $I_x$ and $I_y$ given below. The coefficients $\gamma_{vx}$ and $\gamma_{vy}$ are given in Figure, which is based on finite element studies.

The critical section perimeter is generally composed of straight segments. The values of $A_c$, $I_x$, and $I_y$ can be determined by summation of the contribution of the segments:

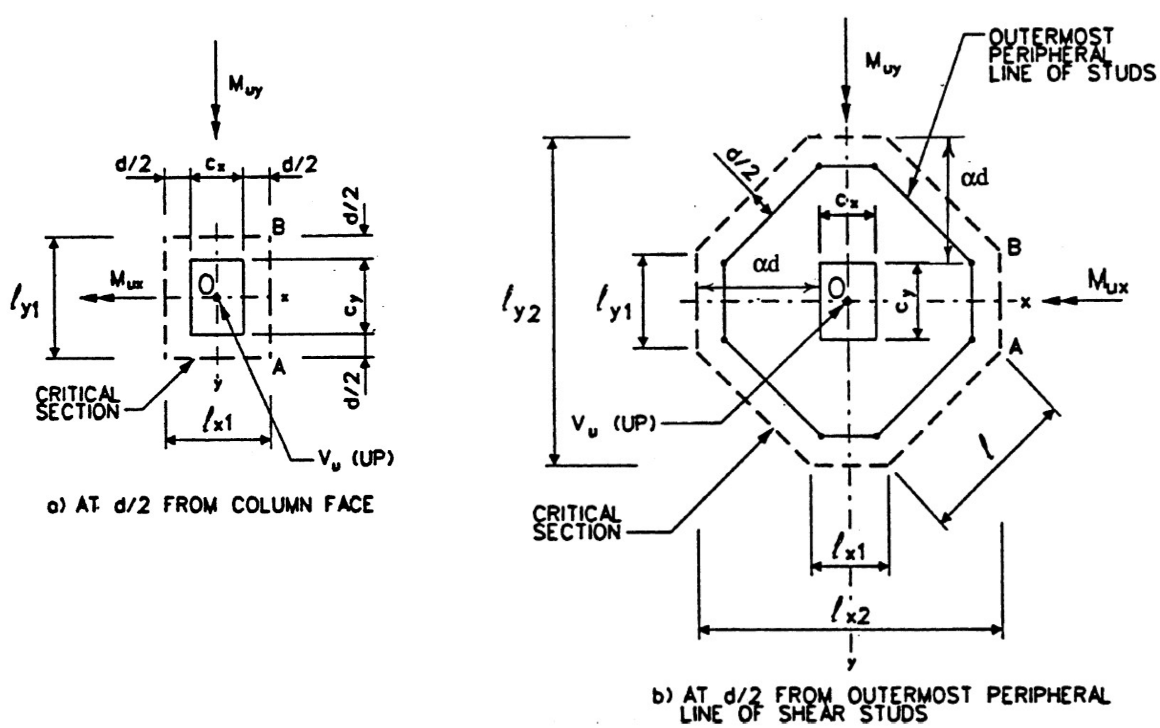
$$A_c = d\sum \ell$$

$$I_x = d\sum\left[\frac{\ell}{3}\left(y_i^2 + y_i y_j + y_j^2\right)\right]$$

$$I_y = d\sum\left[\frac{\ell}{3}\left(x_i^2 + x_i x_j + x_j^2\right)\right]$$

where $x_i$ , $y_i$ , $x_j$ , and $y_j$ are coordinates of Points i and j at the extremities of the segment whose length is $\ell$.

When the maximum $v_u$ occurs at a single point on the critical section, rather than on a side, the peak value of $v_u$ does not govern the strength due to stress redistribution. In this case, $v_u$ may be investigated at a point located at a distance 0.4d from the peak point. This will give a reduced $v_u$ value compared with the peak value; the reduction should not be allowed to exceed 15%.




Critical sections for shear in slab in vicinity of interior column.

Example:-

Check combined shear and moment transfer at an edge column 400 mm square column supporting a flat plate slab system. Use $f_c' = 28$ MPa , $f_y = 420$ MPa

Overall slab thickness (t) = 190 mm, (d = 154 mm).

Consider two loading conditions:

1- Total factored shear force $V_u = 125$ kN, the factored slab moment resisted by the column ($M_{sc}$) = 35 kN.m, and $\varepsilon_t = 0.004$
2- $V_u = 250$ kN , $M_{sc} = 70$ kN.m, and $\varepsilon_t > 0.004$

Solution:

$$b_1 = c_1 + \frac{d}{2} = 400 + \frac{154}{2} = 477 \text{ mm}$$

$b_2 = c_2 + d = 400 + 154 = 554$ mm

$b_o = 2\, b_1 + b_2 = 2 \times 477 + 554 = 1508$ mm

Edge column:

In case of moment about an axis parallel to the edge:

$$c_{AB} = \frac{2(b_1 d)\left(\frac{b_1}{2}\right)}{2(b_1 d) + b_2 d} = \frac{(b_1)^2}{2\, b_1 + b_2} = \frac{(477)^2}{2 \times 477 + 554} = 150.9 \text{ mm}$$

$$J_c = 2\left[\frac{b_1 d^3}{12} + \frac{d {b_1}^3}{12} + (b_1 d)\left(\frac{b_1}{2} - c_{AB}\right)^2\right] + (b_2 d) c_{AB}^2$$

$$J_c = 2\left[\frac{477 \times (154)^3}{12} + \frac{154 \times (477)^3}{12} + (477 \times 154)\left(\frac{477}{2} - 150.9\right)^2\right] + (554 \times 154)(150.9)^2$$

$$= 6146105085.12 \text{ mm}^4$$

$A_c = (2\, b_1 + b_2)\, d = (2 \times 477 + 554) \times 154 = 232232$ $\text{mm}^2$

$A_c$: area of critical section.

The design shear strength of the concrete alone (without shear reinforcement) at the critical section d/2 from the face of the column is:

$$v_c = \min.\begin{cases} 0.33\sqrt{f_c'} = 0.33\sqrt{28} = 1.746 \text{ MPa} \\ 0.17\left(1 + \frac{2}{\beta}\right)\sqrt{f_c'} = 0.17\left(1 + \frac{2}{1}\right) \times \sqrt{28} = 2.699 \text{ MPa} \\ 0.083\left(2 + \frac{\alpha_s d}{b_o}\right)\sqrt{f_c'} = 0.083\left(2 + \frac{30 \times 154}{1508}\right) \times \sqrt{28} = 2.224 \text{ MPa} \end{cases}$$

$$\beta_c = \frac{400}{400} = 1$$

$\therefore\ v_c = 1.746$ MPa

$\phi v_c = 0.75 \times 1.746 = 1.31$ MPa

Loading condition (1) $V_u = 125$ kN, $M_{sc} = 35$ kN.m, and $\varepsilon_t = 0.004$

$$v_{ug} = \frac{V_u}{A_c} = \frac{125 \times 10^3}{232232} = 0.538 \text{ MPa}$$

Span direction is perpendicular to the edge

$0.75\phi v_c = 0.75 \times 1.31 = 0.983$ MPa $>$ $v_{ug} = 0.538$ MPa & $\varepsilon_t = 0.004$ $\Rightarrow$ $\gamma_f = 1.0$

Therefore, all of the factored slab moment resisted by the column ($M_{sc}$) may be considered to be transferred by flexure (i.e $\gamma_f = 1.0$ and $\gamma_v = 0$).

Check shear strength of the slab without shear reinforcement. Shear stress along inside face of the critical section.

$v_n = v_c$

$\phi v_n = 1.31$ MPa $>$ $v_u = v_{ug} = 0.538$ MPa O.K.

$\therefore$ Shear reinforcement is not required

Loading condition (2) $V_u = 250$ kN, $M_{sc} = 70$ kN.m, and $\varepsilon_t > 0.004$

$$v_{ug} = \frac{V_u}{A_c} = \frac{250 \times 10^3}{232232} = 1.077 \text{ MPa}$$

Span direction is perpendicular to the edge

$0.75\phi v_c = 0.983$ MPa $<$ $v_{ug} = 1.077$ MPa $\Rightarrow$ $\gamma_f < 1.0$

$$\gamma_f = \frac{1}{1 + \left(\frac{2}{3}\right)\sqrt{\frac{b_1}{b_2}}} = \frac{1}{1 + \left(\frac{2}{3}\right)\sqrt{\frac{477}{554}}} = 0.618$$

$\gamma_v = 1 - \gamma_f = 1 - 0.618 = 0.382$

Check shear strength of the slab without shear reinforcement. Shear stress along inside face of the critical section.

$v_n = v_c$

$$v_{u,AB} = v_{ug} + \frac{\gamma_v \, M_{sc} \, c_{AB}}{J_c} = 1.077 + \frac{0.382 \times 70 \times 10^6 \times 150.9}{6146105085.12} = 1.734 \text{ MPa}$$

$\phi v_n = 1.31$ MPa $<$ $v_u = 1.734$ MPa not O.K.

$\therefore$ Shear reinforcement is required to carry excess shear stress.

Check maximum shear stress permitted with shear reinforcement.

$$v_u \le \emptyset\, 0.5 \sqrt{f'_c}$$

$$v_u = 1.734 \text{ MPa} < 0.75 \times 0.5\sqrt{28} = 1.984 \text{ MPa} \qquad \text{O.K.}$$

$$v_c = 0.17\sqrt{f'_c} = 0.17 \times \sqrt{28} = 0.9 \text{ MPa}$$

Let $\phi v_n = v_u = 1.734$ MPa

$\phi\ (v_c + v_s) = v_u$

$$\Longrightarrow \quad v_s = \frac{v_u}{\phi} - v_c = \frac{1.734}{0.75} - 0.9 = 1.412 \text{ MPa}$$

$$v_s = \frac{A_v\, f_y}{b_o\, s}$$

$$\Longrightarrow \quad A_v = \frac{v_s\, b_o\, s}{f_y} = \frac{1.412 \times 1508 \times 75}{420} = 380.2 \ \text{mm}^2$$

$$\text{here } s = \frac{d}{2} = \frac{154}{2} = 77 \ \approx \ 75 \ \text{mm}$$

The required area of vertical shear reinforcement = 380.2 $mm^2$

For trial, 3ø8 mm vertical single-leg stirrups will be selected and arranged along three integral beams.

effective depth = 154 mm > 16 × 8 = 128 mm (d is at least $16d_b$). O.K.

$A_v$ provided is 3 × 3 × 50.2 = 451.8 $mm^2$ at the first critical section, at distance d/2 ≈ 75 mm from the column face.

Example:

A flat plate floor has a thickness equals to 220 mm, and supported by 500 mm square columns spaced 6.0 m on center each way. Check the adequacy of the slab in resisting punching shear at a typical interior column, and provide shear reinforcement, if needed. The floor will carry a total factored load of 17.0 kN/m$^2$ and the factored slab moment resisted by the column is 40 kN.m.

Use effective depth = 170 mm, $f_y$ = 420 MPa, and $f_c'$ = 28.0 MPa.

Solution:-

The first critical section for punching shear is at distance d/2 = 85 mm from the column face.

$b_1 = c_1 + d = 500 + 170 = 670$ mm

$b_2 = c_2 + d = 500 + 170 = 670$ mm

Shear perimeter ($b_o$) = 2 $b_1$ + 2 $b_2$ = 2 × 670 + 2 × 670 = 2680 mm

The design shear strength of the concrete alone (without shear reinforcement) at the critical section d/2 from the face of the column is:

$$v_c = \min.\begin{cases} 0.33\sqrt{f_c'} = 0.33\sqrt{28} = 1.746 \text{ MPa} \\ 0.17\left(1+\frac{2}{\beta}\right)\sqrt{f_c'} = 0.17\left(1+\frac{2}{1}\right)\times\sqrt{28} = 2.699 \text{ MPa} \\ 0.083\left(2+\frac{\alpha_s\, d}{b_o}\right)\sqrt{f_c'} = 0.083\left(2+\frac{40\times 170}{2680}\right)\times\sqrt{28} = 1.993 \text{ MPa} \end{cases}$$

$$\beta_c = \frac{500}{500} = 1$$

$\therefore\ v_c = 1.746$ MPa

$\phi v_c = 0.75 \times 1.746 = 1.31$ MPa

$V_u = 17.0 \times [\,(6.0)^2 - (0.67)^2\,] = 604.369$ kN

$$v_{ug} = \frac{V_u}{b_o \cdot d} = \frac{604.369\times 10^3}{2680\times 170} = 1.327 \text{ MPa}$$

$$v_{u,AB} = v_{ug} + \frac{\gamma_v \cdot M_{sc} \cdot c_{AB}}{J_c}$$

$$\gamma_f = \frac{1}{1+\left(\frac{2}{3}\right)\sqrt{\frac{b_1}{b_2}}} = \frac{1}{1+\left(\frac{2}{3}\right)\sqrt{\frac{670}{670}}} = 0.6$$

$\gamma_v = 1 - \gamma_f = 1 - 0.6 = 0.4$

Example:

The flat plate slab of 200 mm total thickness and 160 mm effective depth is carried by 300 mm square column 4.50 m on centers in each direction. A factored load of 370 kN and a factored slab moment resisted by the column is 44 kN.m must be transmitted from the slab to a typical interior column. Determine if shear reinforcement is required for the slab, and if so, design integral beams with vertical stirrups to carry the excess shear. Use $f_y$ = 420 MPa, $f_c^{'}$ = 30 MPa.

Solution:-

The first critical section for punching shear is at distance d/2 = 80 mm from the column face.

$b_1 = c_1 + d =$

$b_2 = c_2 + d =$

Shear perimeter $(b_o) = 2\ b_1 + 2\ b_2 =$

The design shear strength of the concrete alone (without shear reinforcement) at the critical section d/2 from the face of the column is:

$$v_c = \min. \begin{cases} 0.33\sqrt{f_c'} = \\ 0.17\left(1+\dfrac{2}{\beta}\right)\sqrt{f_c'} = \\ 0.083\left(2+\dfrac{\alpha_s\, d}{b_o}\right)\sqrt{f_c'} = \end{cases}$$

$$\beta_c = \frac{300}{300} = 1$$

$V_u = 370$ kN

$$v_{ug} = \frac{V_u}{b_o \cdot d} = \frac{370 \times 10^3}{1840 \times 160} = 1.257 \text{ MPa}$$

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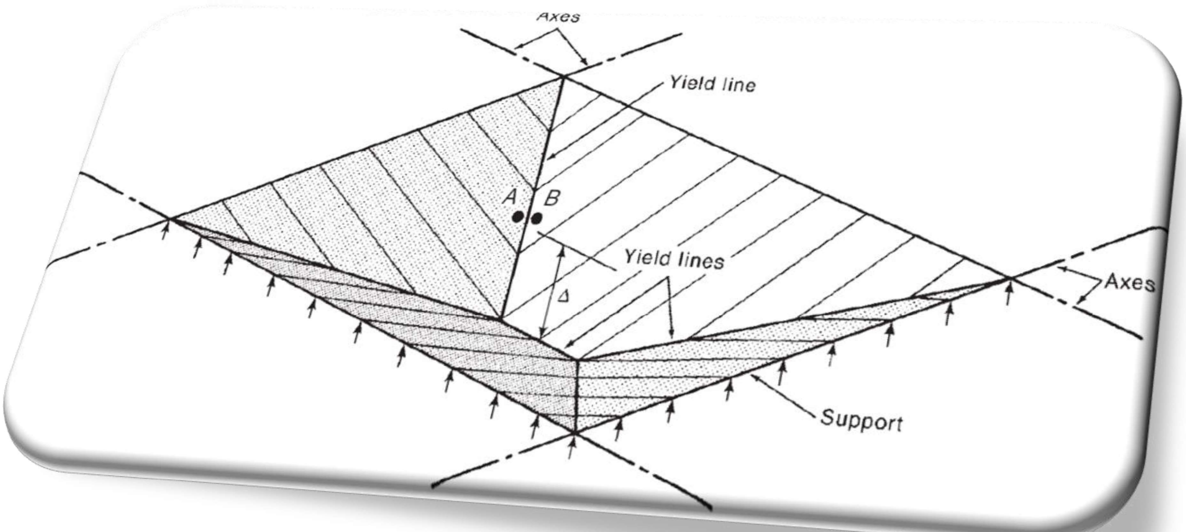
College of Engineering
Civil Engineering Department




# REINFORCED CONCRETE DESIGN II

## FOURTH YEAR CLASS

# 5

# 2018 - 2019

MHND