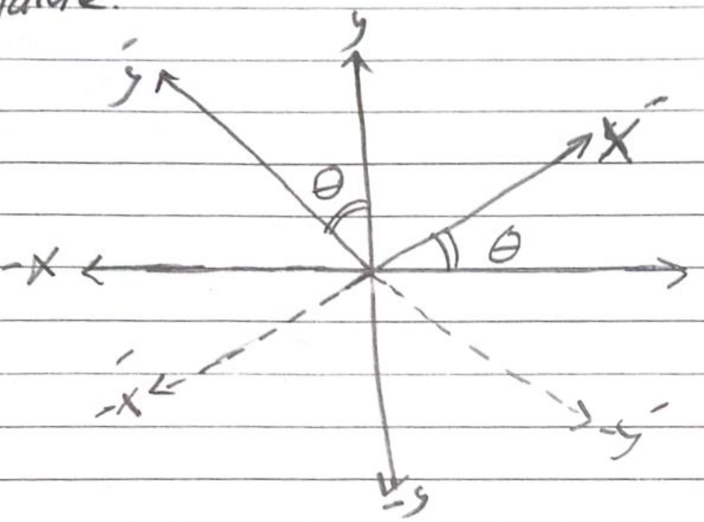
change Coordinat axis

Rotation axis

دوران محاور الاحداثيات.
دوران المحاور بزاوية قدرها θ في اتجاه ضد عقارب الساعة.

The rotation of axis about the origin Point opposite clockwise with angle θ value.



Rotation Equations — معادلات الدوران

$$\boxed{\begin{aligned} x &= x'\cos\theta - y'\sin\theta \\ y &= y'\cos\theta + x'\sin\theta \end{aligned}}$$

or

$$\boxed{\begin{aligned} x' &= x\cos\theta + y\sin\theta \\ y' &= y\cos\theta - x\sin\theta \end{aligned}}$$

المعادلات العكسية
The Inverse Equations

Example (1): Find the image of the equation by rotate insert front each equation.

$$x^2 - 4xy + y^2 = 0 \text{ , with } \theta = \pi/2$$

ترجمة السؤال
أوجد شكل المعادلة التالية بعد دوران المحاور بزاوية
$\theta = \pi/2$

sol) عند الدوران بزاوية 90° ستصبح معادلات الدوران بالصورة الآتية

$$x = x'\cos\theta - y'\sin\theta$$
$$= x'\cos\pi/2 - y'\sin\pi/2$$
$$= x'(0) - y'(1)$$

$\therefore x = -y'$

and

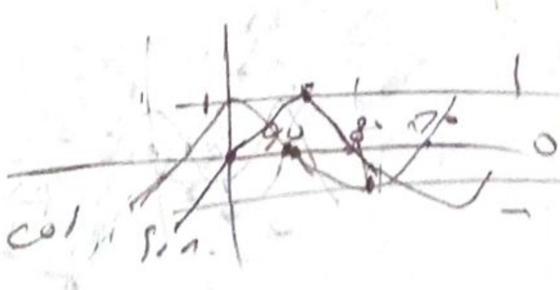
$$y = y'\cos\theta + x'\sin\theta$$
$$= y'\cos\pi/2 + x'\sin\pi/2$$
$$= y'(0) + x'(1)$$

$\therefore y = x'$

Now substitute x, y values in equation we get

$$(-y')^2 - 4(y')(x') + (x')^2 = 0$$

$$y'^2 - 4y'x' + x'^2 = 0$$



Example (2): Find the image of the equation after rotation

$$xy = 3/2 , \theta = \pi/4$$

Solution

اوجد شكل المعادلة بعد الدوران

After a rotate by angle value $\pi/4$, the equation will be

بعد الدوران بزاوية مقدارها $\pi/4$ ستصبح معادلات الدوران :

$$x = \left(\frac{x'}{\sqrt{2}}\right) - \left(\frac{y'}{\sqrt{2}}\right) \text{ and } y = \frac{y'}{\sqrt{2}} + \frac{x'}{\sqrt{2}}$$

when substitute x, y values in certain equation we get the following.

$$\left(\frac{x'}{\sqrt{2}} - \frac{y'}{\sqrt{2}}\right)\left(\frac{x'}{\sqrt{2}} + \frac{y'}{\sqrt{2}}\right) = 3/2$$

$$\Rightarrow \frac{x'^2}{2} - \frac{y'^2}{2} = 3/2$$

Example (3): Find an image of a point $(1, \sqrt{3})$ by rotation around the origin point ~~measured~~ by angle $\pi/3$.

ترجمة السؤال : اوجد النقطة P بعد دوران المحاور حول نقطة الاصل بزاوية مقدارها $\pi/3$

Sol) The point P is old point dented by $P(x,y) = (1, \sqrt{3})$ but we want to find new point.

from the oppsit equations formula, we get

$x' = x\cos(\pi/3) + y\sin(\pi/3)$
$= 1(1/2) + \sqrt{3}(\sqrt{3}/2)$
$\therefore x' = 2$

and

$y' = y\cos(\pi/3) - x\sin(\pi/3)$
$= \sqrt{3}(1/2) - 1(\sqrt{3}/2)$
$\therefore y' = 0$

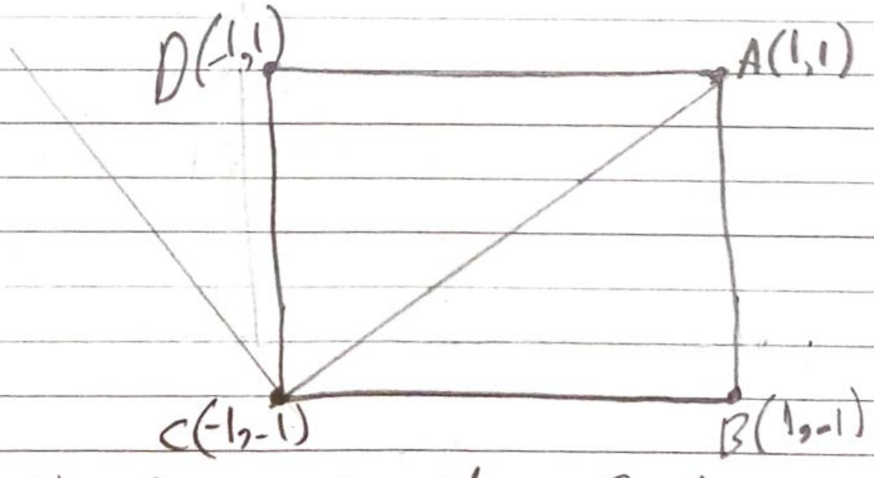
Hence new Point is $P'(x', y') = (2, 0)$

Example (4): Find an image of each vertex of the squar ABCD where

$A(1,1), B(1,-1), C(-1,-1), D(-1,1), \theta = \pi/4$

by Rotation around the origin Point such that the new axes apply with diameters square.

Solution: A rotation around the origin Point such that the new axis apply with diameters square.



To find the imag of these Points using the equation of invers rotation, where $\theta = 45°$

$$x' = \frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}} = \frac{x+y}{\sqrt{2}}$$

$$y' = \frac{y}{\sqrt{2}} - \frac{x}{\sqrt{2}} = \frac{y-x}{\sqrt{2}}$$

Now turn to find an imag of a Point $A(\overset{x}{1},\overset{y}{1})$

$$\therefore \acute{A}\left(\frac{x+y}{\sqrt{2}}, \frac{y-x}{\sqrt{2}}\right) = \left(\frac{1+1}{\sqrt{2}}, \frac{1-1}{\sqrt{2}}\right)$$

$$\therefore \acute{A}(\sqrt{2}, 0)$$

Image of a Point $B(1,-1)$

$$\acute{B}\left(\frac{x+y}{\sqrt{2}}, \frac{y-x}{\sqrt{2}}\right) = \left(\frac{1-1}{\sqrt{2}}, \frac{-1-1}{\sqrt{2}}\right)$$

$$\therefore \acute{B}(0, -\sqrt{2})$$

Image of a Point $C(-1,-1)$

$$\acute{C}\left(\frac{x+y}{\sqrt{2}}, \frac{y-x}{\sqrt{2}}\right) = \left(\frac{-1-1}{\sqrt{2}}, \frac{-1+1}{\sqrt{2}}\right)$$

$$\therefore \acute{C}(-\sqrt{2}, 0)$$

Imag of a Point $D(\overset{x}{-1},\overset{y}{1})$

$$\acute{D}\left(\frac{x+y}{\sqrt{2}}, \frac{y-x}{\sqrt{2}}\right) = \left(\frac{-1+1}{\sqrt{2}}, \frac{1+1}{\sqrt{2}}\right)$$

$$\acute{D}(0, \sqrt{2})$$

بتحويل هندسي مناسب حول المعادلة الى معادلة خالية
من الحدود الدرجة xy

Example (5): By transfer of appropriat geometric turn the equation

$$14x^2 - 4xy + 11y^2 = 0$$

To equation - free of second order limits xy

Solution: with rotation for axises around origin point and angle θ.

Now using the rotation equations we get

$$x = x' \cos\theta - y' \sin\theta$$
$$y = y' \cos\theta + x' \sin\theta$$

So by substitute x, y in last equation

$$14(x'\cos\theta - y'\sin\theta)^2 - 4(x'\cos\theta - y'\sin\theta)(y'\cos\theta + x'\sin\theta) + 11(y'\cos\theta + x'\sin\theta)^2 = 0$$

$$\therefore 14(x'^2\cos^2\theta + y'^2\sin^2\theta - 2x'y'\cos\theta\sin\theta) - 4(x'y'\cos^2\theta + x'^2\cos\theta\sin\theta - y'^2\sin\theta\cos\theta - x'y'\sin^2\theta) + 11(y'^2\cos^2\theta + x'^2\sin^2\theta + 2x'y'\sin\theta\cos\theta) = 0$$

Remember that $\boxed{2\sin\theta\cos\theta = \sin 2\theta}$

$$14x'^2\cos^2\theta + 14y'^2\sin\theta - 14x'y'\sin 2\theta - 4x'^2\cos\theta\sin\theta + 4y'^2\sin\theta\cos\theta - 4x'y'(\cos^2\theta + \sin^2\theta) + 11y'^2\cos^2\theta + 11x'^2\sin^2\theta + 11x'y'\sin 2\theta = 0$$

To be this equation is free of second order $x'y'$
Hence should be $x'y' = $ Zero

$$\therefore -14\sin 2\theta - 4 + 11\sin 2\theta = 0$$

$$\therefore -3\sin 2\theta = 4$$

$$\therefore 2\theta = \sin^{-1}\left(\frac{-4}{3}\right)$$
$$\theta = \frac{1}{2}\sin^{-1}(-4/3)$$

#