
Probability Mass Function(P.M. f): (دالة الكتلة الاحتمالة)

Let X be a Discrete Random Variables (d .v . r)

A function f is a P. M .f if f (x)= p (X=x) , and satisfy the following condition :

A) f(x)≥ 0 , ∀ x ∈ X B) $\sum_{\forall x \in X} f(x) = 1$

Note : 1- condition (A) shows the graph of f (x) above of the X- axis .

2- also .if A ⊂ S then P $(x \in A) = \sum_{\forall x \in A} f(x) = 1$

Example(1) : Given f(x) = $\begin{cases} \frac{x}{10}, & \text{for } x = 0,1,2\ 3\ 4 \end{cases}$

Show that f (x) is a M. P. f

Solution :

To satisfy a Condition (A) T.P: f(x)≥ 0 , ∀ x ∈ X

f(0) = 0 , f(1) = $\frac{1}{10}$, f(2) = $\frac{2}{10}$, f(3) = $\frac{3}{10}$, f(4) = $\frac{4}{10}$

∴ f(x)≥ 0 , ∀ x ∈ X

∴ Cond (1) is satisfied .

$$cond\ ''2''\text{T.P.}\ \sum_{\forall x=0} f(x) = 1$$

$$\sum_{\forall x=0} f(x) = 0 + \frac{1}{10} + \frac{2}{10} + \frac{3}{10} + \frac{4}{10} = 1$$

∴ f (x) is a P. M .f .

- IF P(x=1) =(1) $= \frac{1}{10}$
- IF P(x ≥ 3) = P[(x=3) ∪(x=4)]= P(x=3) + P (x=4)

p (3)+ p(4) = $\frac{3}{10} + \frac{4}{10} = \frac{7}{10}$

or by Note(2) P(x ≥ 3)= $\sum_{x=3}^{x=4} p\,(x) =$ p (3)+p(4) = $\frac{7}{10}$.

* P(x ≤ 2) = P[(x=2) ∪(x=1)]= P(x=2) + P (x=1)

p (2)+p(1) = $\frac{2}{10} + \frac{1}{10} = \frac{3}{10}$

Example(2):

Given a P. m. f and p(x)= $\begin{cases} \frac{x}{k}, & \text{for } x = 0,1,2\ 3\ 4 \end{cases}$

Find the value of (k)

Sol:

∵ f (x) is a p. m .f

$$By\ cond\ (B\ )\ we\ got \sum_{x=1}^{x=5} f(x) = 1$$

f(1) + f(2) + f(3) + f(4) + f(5) = 1

$\frac{1}{k} + \frac{2}{k} + \frac{3}{k} + \frac{4}{k} + \frac{5}{k} = 1 \Rightarrow \therefore k = 15$

Hence : (x)= $\begin{cases} \frac{x}{15} & \text{for } x = 0,1,2\ 3\ 4 \end{cases}$

Example(3):

Toss a coin 3-times .

Let X = number of H : Find the p, m ,f of X.

Sol:

S= {HHH, HHT ,HTH ,THH ,THT , TTH, TTT, HTT }

∴ S has (8) elements,

X = no. of H $x \in X$; x = 0,1 2 3

Rx= { x:x =0,1,2,3 }, Rx is countable , hence x is a d, r, v.

Event (X=x) : to get xH ,x = 0,1,2,3 when Toss a coin 3-times

$\binom{3}{x}$: number of sample in event (X=x) when Toss a coin 3-yimes

$$f(x) = P(X=x) = \begin{cases} \frac{\binom{3}{x}}{8} & \text{for } x = 0,1,2,3 \end{cases}$$

Hence ;

x	F (x)= $\frac{\binom{3}{x}}{8}$
0	1/8
1	3/8
2	3/8
3	3/8

$$\sum_{X=0}^{3} f(x) = 1$$

Continue Random Variable (C.R.V) [2.2] **(المتغير العشوائي المتصل)**

Definition تعريف

المتغير العشوائي المستمر(المتصل)يعرف بشكل مبسط على أنه متغير عشوائي مجموعة القيم الممكنة له عبارة عن فترة أو إتحاد عدد من ا لفترات.ومن أمثلة الكميات التي يمكن تمثيلها بواسطة متغيرات عشوائية متصلة:

- درجة حرارة تفاعل كيميائي معين.
- نسبة تركيز مركب ما في محلول كيميائي.
- الفترة الزمنية بين الإصابة بمرض الإيدز والوفاة.
- طول الشخص.
- المسافة المقطوعة لجسم معين خلال وحدة الزمن.
-

Probability Density function (P. d .f):[3.2]

Definition : Let X be a c. v .r

A function f(x) is a p. d. f of x if for any interval A ⊂ Rx ,

p(x∈ A)= $\int f(x)$ =dx. And satisfy the following conditions :

A) f(x)≥ 0 , ∀ x ∈ X

B) - $\int_{-\infty}^{\infty} f(x)\ dx = 1$

X(s) =x ∈Rx , Rx is uncountable A ⊂ Rx

A is a set of real no . Suppose that A={x: a <x< b }.

p (x A)= p (a <x< b)= $\int_a^b f(x)\ dx$

= area under curve from a to b .

Example (1) :

Given f(x)= 2(1- x) and (0< x < 1) is f(x) = P, d, f .

Sol: $\int_a^b f(x)\ dx \Rightarrow$ f(x)= $\int_0^1 2(1 - x)$

$$= 2x - x^2 \Big|_0^1 = 2- 1 =1$$

∴ f(x) is a P. d.. f

Ex (2):

Given a p. d. f , $f(x) = \begin{cases} Kx & \text{for } 0 \le x \le 2 \\ K & \text{for } 2 \le x \le 4 \end{cases}$

a- find the value of K and sketch f(x).

b- find p(x> 1) , p(x <3) , p ($\frac{3}{2} < x < \frac{5}{2}$)

Sol:

By cond (B) of p. d .f $\Rightarrow \int_{-\infty}^{\infty} f(x)\ dx = 1$

$1 = \int_0^2 Kx\ dx + \int_2^4 Kx\ dx \Rightarrow 1 = \frac{k}{2} + Kx$

$\Rightarrow 1 = \frac{k}{2}$ [4-0] + K [4 – 2] $\Rightarrow$ 1= 2K +2K

$\Rightarrow 1 = 4K \Rightarrow K=\frac{1}{4}$

$$f(x) = \begin{cases} \frac{x}{4} & \text{for } 0 < x < 2 \\ \frac{1}{4} & \text{for } 2 \le x < \end{cases}$$

b) $p(x > 1) = \int_1^2 \frac{x}{4} \, dx + \int_2^4 \frac{1}{4} \, dx$

$= \frac{1}{8} x^2 \Big| + \frac{1}{4} \Big| = \frac{1}{8} (4-1) + (4-2) = \frac{3}{8} + \frac{1}{2} = \frac{7}{8}$

* $p(x > 3)$

* $p\left(\frac{3}{2} < x < \frac{5}{2}\right)$

Ex(3);

Given a p. d .f f(x) = $\begin{cases} K\, e^{-x} & \text{for } x > 0 \end{cases}$

A – find the value of K

B –find p $(x < 2)$.

Sol ;

A – by cond ''B '' of p .d. f

F(x) = $\int_{-\infty}^{\infty} f(x)\ dx = 1$

$1 = K \int_{0}^{\infty} e^{-x}\ dx \Rightarrow 1 = K\, e^{-x} \Big|$

$\Rightarrow 1 = K\,(e^{-\infty} - e^{0}) \Rightarrow K = -1$

Hence f(x) = - e^{-x} for x > 0

b) p $(x < 2) = \int_{-\infty}^{2} f(x)\ dx$

$\Rightarrow \int_{-\infty}^{2} f(x) = \int_{-\infty}^{0} f(x) + \int_{0}^{2} f(x)$

= $- e^{-x}$ = $-[e^{-2} - e^{-0}]$

= $1 - e^{-2}$ = 1- 0.13 = 0.87

Ex (4) :

Given a p.d. f

$$F(x) = \begin{cases} b \sin x \cos x & \text{for} \quad 0 < x < \frac{\pi}{2} \end{cases}$$

find the value of b

Sol:

$\because$ f(x) is a p.d,f $\Rightarrow$ cond (B) is satisfied .

$\therefore\ b \int_0^{\frac{\pi}{2}} \sin x \cos x \, dx = 1$

$1 = b \left[\int_0^{\frac{\pi}{2}} \frac{1}{2} \sin 2x \ dx\right]$ (since sin2x = 2 sin x cos x)

$1 = \frac{b}{2}$ cos 2x |

$1 = \frac{-b}{2}$ [co π - co 0]

$1 = \frac{-b}{2}$ [-1 - (1)] $\Rightarrow$ b=1

$$\therefore f(x) = \begin{cases} \sin x \cos x & \text{for} \quad 0 < x < \frac{\pi}{2} \end{cases}$$