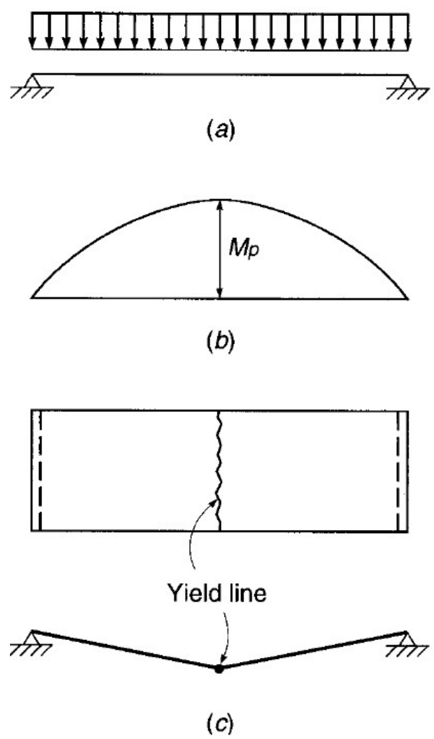
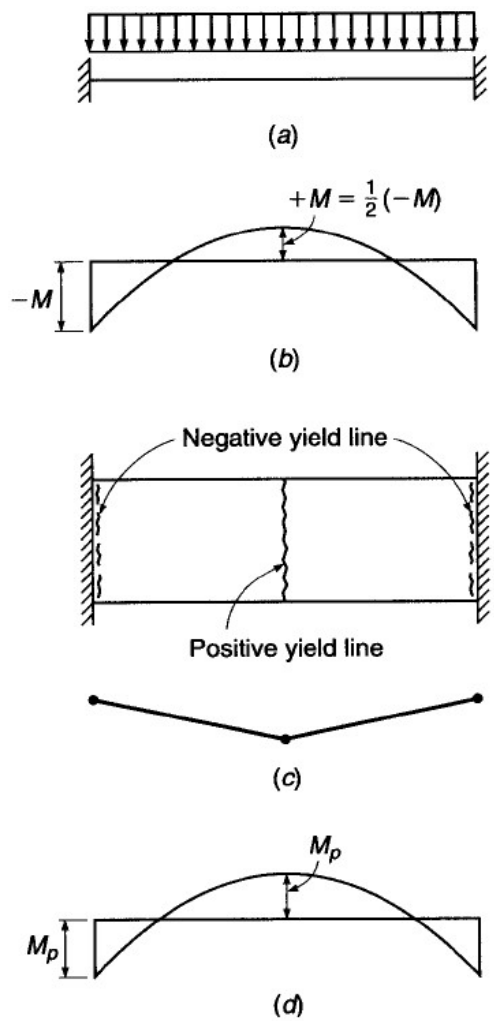
Yield Line Analysis for Slabs

In a slab failing in flexure, the reinforcement will yield first in a region of high moment. When that occurs, this portion of the slab acts as a plastic hinge, only able to resist its hinging moment. When the load is increased further, the hinging region rotates plastically, and the moments due to additional loads are redistributed to adjacent sections, causing them to yield. The bands in which yielding has occurred are referred to as yield lines and divide the slab into a series of elastic plates. Eventually, enough yield lines exist to form a plastic mechanism in which the slab can deform plastically without an increase in the applied load.

In the yield-line method for slabs, the loads required to develop a plastic mechanism are compared directly to the plastic resistance (nominal strength) of the member.



Simply supported uniformly loaded one-way slab.



Fixed-end uniformly loaded one-way slab.

Axes of rotations:

Yield lines form in regions of maximum moment and divide the slab into a series of elastic plate segments. When the yield lines have formed, all further deformations are concentrated at the yield lines, and the slab deflects as a series of stiff plates joined together by long hinges. The pattern of deformation is controlled by axes that pass along support lines, over columns, and by the yield lines. Because the individual plates rotate about the axes and/or yield lines, these axes and lines must be straight.

Location of Axes of rotations and yield-lines:

a- Axes of rotation generally lie along lines of support (the support line may be a real hinge as in simple supported, or it may establish the location of a yield line, which acts as a plastic hinge and in continuous or fixed support).

b- Axes of rotation pass over any columns.

c- The slab segments can be considered to rotate as right bodies in space about these axes of rotation.

d- Yield lines are generally straight.

e- A yield line passes through the intersection of the axes of rotation of adjacent slab.

f- A yield line passes under the point load (concentrated force).

Notations:

	Axis of rotation
	Positive yield line
	Negative yield line
	Simply supported
	Fixed or continuous support
	Free edge
	Beam
● ■	Column
⊗	Point load (concentrated force)
	Line load

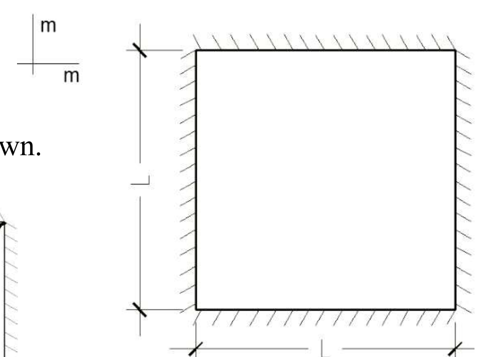
Isotropic slab: The slab is reinforced identically in all directions. The resisting moment, is the same along any line regardless of its location and orientation.
Orthotropic slab: The resisting moments are different in two perpendicular directions.

<u>Methods of solution</u>:
Once the general pattern of yielding and rotation has been established by applying the guid lines the location and the orientation of axes of rotation and the failure load for the slab can be established by either of two methods.

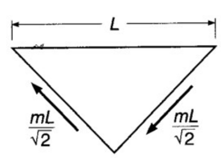
- Equilibrium method.
- Virtual-work method.

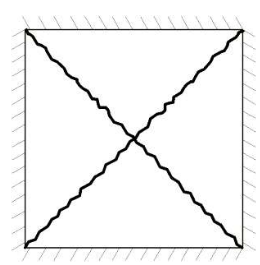
Equilibrium method:
By this method, the correct axes of rotation and the collapse load for the correspond mechanism can be found considering equilibrium of the slab segments. Each segment, studied as a free body, must be in equilibrium under the action of the applied load, the moments along the yield lines, and the reactions or shear force along the support line. Zero shear force and twisting moment along the positive yield line, and only moment per linear length (m) is considered in writing equilibrium equation.

Example
A square slab is simply supported along all sides and is to be isotropically reinforced. Determine the ultimate resisting moment (m) per linear meter required just to sustain a uniformly distributed load (q) in kN/m^2.



Solution
Conditions of symmetry indicate the yield line pattern as shown.





Consider the moment equilibrium of any one of the identical slab segments about its support:

$$\sum \mathrm{M} = 0$$

$$\mathrm{q} \times \mathrm{L} \times \frac{\mathrm{L}}{2} \times \frac{1}{2} \times \frac{1}{3} \times \frac{\mathrm{L}}{2} = \frac{\mathrm{m\,L}}{\sqrt{2}} \times \frac{1}{\sqrt{2}} \times 2$$

$$\mathrm{m} = \frac{\mathrm{q\,L}^2}{24}$$

Virtual-work method:

Since the moment and load are in equilibrium when the yield line pattern has formed, an increase in load will cause the structure to deflect further. The external work done by the loads to cause a small arbitrary virtual deflection must equal the internal work done as the slab rotates at the yield line to accommodate this deflection.

External work done by loads:

External work (EW or W_e) equals to the product of external load and the distance through which the point of application of the load moves. If the load is distributed over a length or an area rather than concentrated, the work can be calculated as the product of the total load and the displacement of the point of application of its resultant.

More complicated shapes may always be subdivided into components of triangles and rectangles. The total external work calculated by summing the work done by loads on the individual point of the failure mechanism.

Internal work done by resisting moment:

The internal work (IW or W_i) done during the assigned virtual displacement is found by summing the products of bending moment per unit length of yield line (m), the length of the yield line, and the angle change at that yield line corresponding to the virtual displacement (θ).

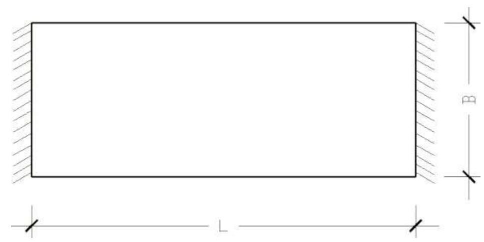
$$IW = \sum [m\,\ell\,\theta]$$

For orthotropic slab ($m_x \neq m_y$) it is necessary to choose the axes of moment parallel to the edges if possible

$$IW = \sum \left[(m_x\,\ell_x\,\theta_x) + \left(m_y\,\ell_y\,\theta_y\right)\right]$$

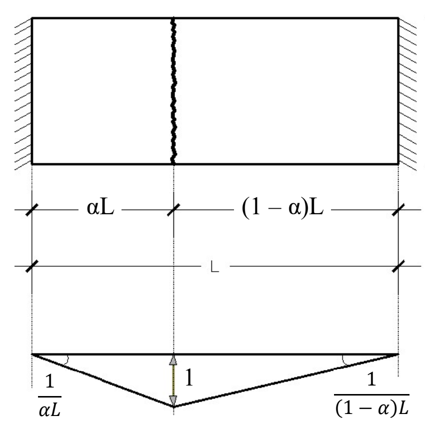
Example

Find the ultimate moment for the slab shown using the yield line theory. The slab is one way and simply supported of length (L) and normally loaded by a uniformly distributed load (w).



Solution

$$W_e = w \times B \times \alpha L \times \tfrac{1}{2} + w \times B \times (1 - \alpha)L \times \tfrac{1}{2}$$



$$W_e = \frac{w\,B\,L}{2} \times [\alpha + (1 - \alpha)]$$

$$W_e = \frac{w\,B\,L}{2}$$

$$W_i = \left[m \times B \times \frac{1}{\alpha L}\right] + \left[\left(m \times B \times \frac{1}{(1 - \alpha)L}\right)\right]$$

$$W_i = m\,B\left[\frac{1}{\alpha L} + \frac{1}{(1 - \alpha)L}\right]$$

$$W_e = W_i$$

$$\frac{w\,B\,L}{2} = m\,B\left[\frac{1}{\alpha L} + \frac{1}{(1 - \alpha)L}\right]$$

$$w = \frac{2\,m}{L^2}\left[\frac{1}{\alpha} + \frac{1}{(1 - \alpha)}\right]$$

to find the value of α, drive w with respect to α and equate the result to zero

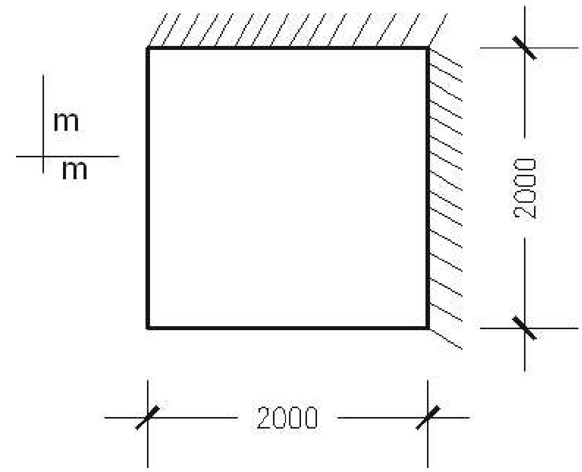
$$\frac{d\,w}{d\,\alpha} = \frac{2\,m}{L^2}\left[\frac{-1}{\alpha^2} + \frac{1}{(1 - \alpha)^2}\right] = 0$$

$\Rightarrow \quad \alpha = 0.5$

$$\therefore\ w = \frac{2\,m}{L^2}\left[\frac{1}{0.5} + \frac{1}{(1 - 0.5)}\right] \quad \rightarrow \quad m = \frac{w\,L^2}{8}$$

Example

By using the yield line theory, determine the moment (m) for an isotropic reinforced concrete two-way slab shown in figure under a uniformly distributed load (w).



Solution

$$EW = \left(w \times 2.0 \times 2.0 \times \frac{1}{2} \times \frac{1}{3} \right) \times 2 = \frac{4w}{3}$$

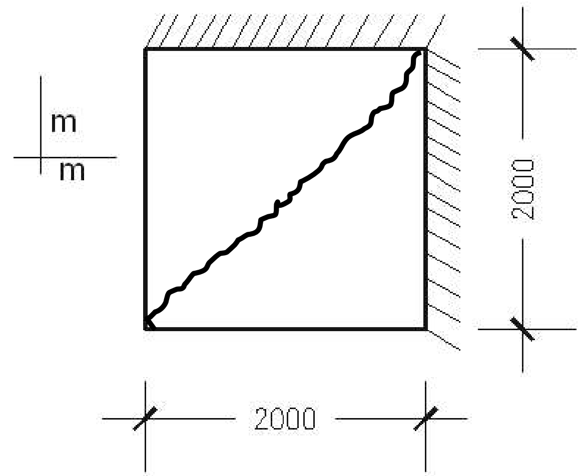
$$IW = \left(m \times 2 \times \frac{1}{2} \right) \times 2 = 2m$$

$$EW = IW$$

$$\frac{4w}{3} = 2\,m$$

$$\Rightarrow \ m = \frac{2w}{3}$$

$$m = 0.667\,w$$



Example

By using the yield line theory, determine the moment (m) for an isotropic reinforced concrete two-way slab shown in figure under a consentrated force (P) on the free corner.

Solution

$$EW = P \times 1 = P$$

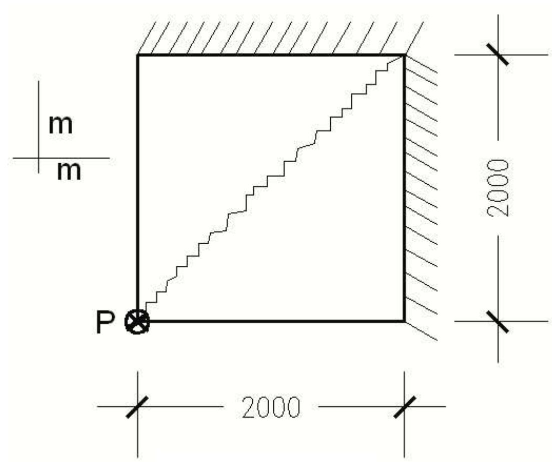
$$IW = \left(m \times 2 \times \frac{1}{2} \right) \times 2 = 2m$$

$$EW = IW$$

$$P = 2\,m$$

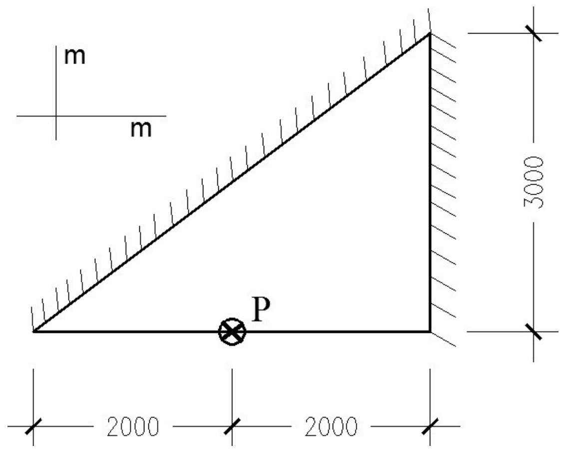
$$\Rightarrow \ m = \frac{P}{2}$$

$$m = 0.5\,P$$



Example

By using the yield line theory, determine the moment (m) for an isotropic reinforced concrete two-way slab shown in figure under the load (P) (all dimensions are in mm).



Solution

$W_e = P \times 1 = P$

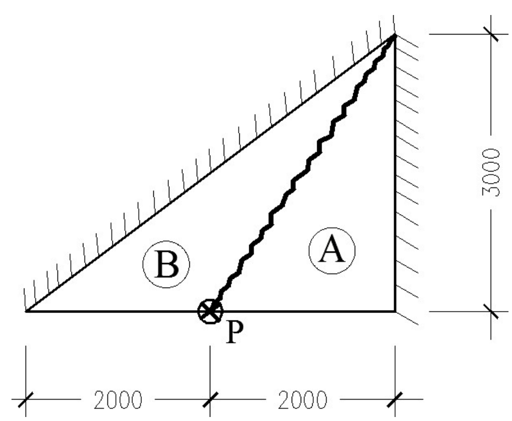
$$W_i = \left[m \times 3 \times \frac{1}{2}\right] + \left[\left(m \times 2 \times \frac{1}{1.5}\right) + \left(m \times 3 \times \frac{1}{2}\right)\right]$$

$$W_i = \left[\frac{3\ m}{2}\right] + \left[\left(\frac{4\ m}{3}\right) + \left(\frac{3\ m}{2}\right)\right]$$

$$W_i = \frac{26\ m}{6}$$

$$W_i = \frac{13\ m}{3}$$

$$W_i = 4.333\ m$$



$$W_i = W_e$$

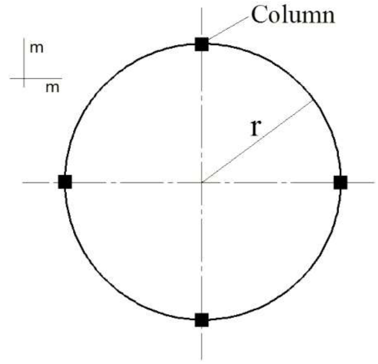
$$\frac{13\ m}{3} = P$$

$$m = \frac{3\ P}{13}$$

$$m = 0.231\ P$$

Example

The circular slab of radius r supported by four columns, as shown in figure, is to be isotropically reinforced. Find the ultimate resisting moment (m) per linear meter required just to sustain a concentrated factored load of P kN applied at the center of the slab.



Solution

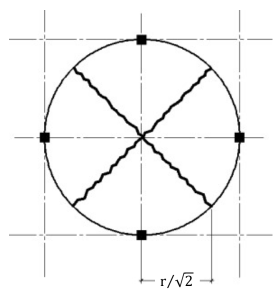
$$W_e = P \times 1 = P$$

$$W_i = \left(m \times \frac{r}{\sqrt{2}} \times 2 \times \frac{1}{r}\right) \times 4 = \sqrt{2}\, m$$

$$W_i = W_e$$

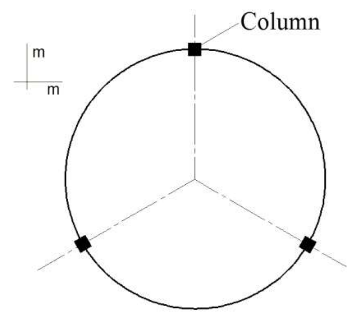
$$\sqrt{2}\, m = P$$

$$m = \frac{P}{\sqrt{2}}$$



Example

The circular slab of radius 2 m supported by three columns, as shown in figure, is to be isotropically reinforced. Find the ultimate resisting moment per linear meter (m) required just to sustain a uniformly distributed load (q) equals 16 kN/m^2.



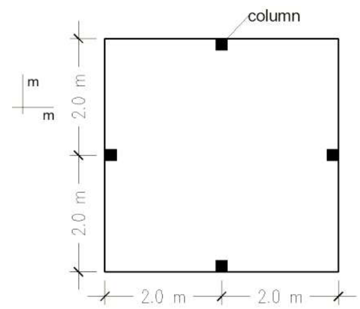
Solution

$W_e =$

$W_i =$

Example

By using the yield line theory, determine the ultimate resisting moment (m) for an isotropic reinforced concrete two-way slab shown in figure under a uniform load (q).



Solution

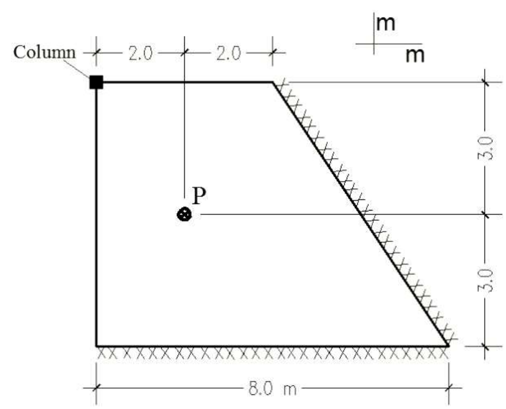
$W_e =$

$W_i =$

$W_e = W_i$

Example

By using the yield line theory, determine the ultimate resisting moment per linear meter (m) for an isotropic reinforced concrete two-way slab to sustain a concentrated factored load of P kN applied as shown in figure.



Solution

$W_e = P \times 1 = P$

$W_i =$

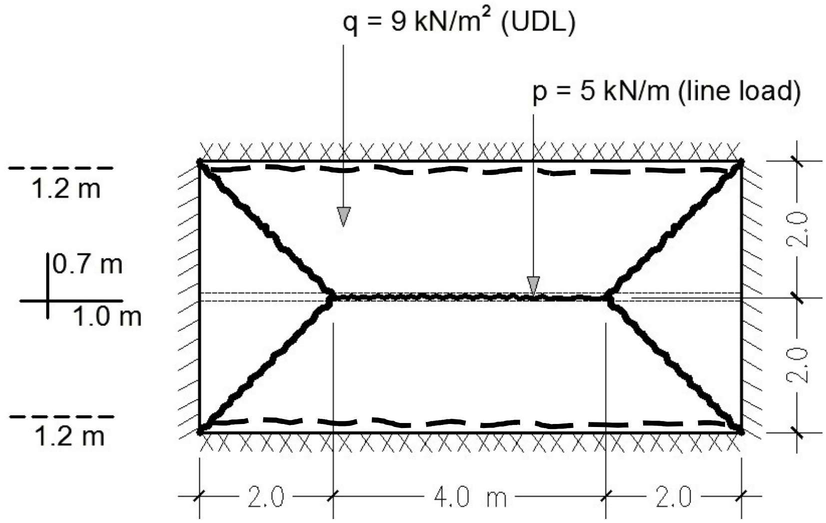
$W_e = W_i$

$P = 11.333\ m$

Example

By using the yield line theory, determine the ultimate resisting moment per linear meter (m) for an orthotropic reinforced concrete two-way slab to sustain a uniformly distributed load and line load applied as shown in figure.

Solution



$$W_e = 9 \times \left[\left(2 \times 2 \times \frac{1}{2} \times \frac{1}{3} \times 8\right) + \left(4 \times 2 \times \frac{1}{2} \times 2\right)\right] + 5 \times \left[\left(2 \times \frac{1}{2} \times 2\right) + (4 \times 1)\right]$$

$$W_e = 150 \text{ kN.m}$$

$$W_i = \left[0.7\, m \times 4 \times \frac{1}{2}\right] \times 2 + \left[\left(m \times 8 \times \frac{1}{2}\right) + \left(1.2\, m \times 8 \times \frac{1}{2}\right)\right] \times 2$$

$$W_i = 20.4\, m$$

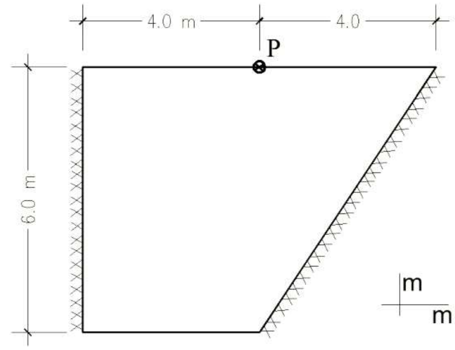
$$W_e = W_i$$

$$150 = 20.4\, m$$

$$m = 7.353 \text{ kN.m/m}$$

Example

By using the yield line theory, determine the ultimate resisting moment per linear meter (m) for an isotropic reinforced concrete two-way slab to sustain a concentrated factored load of P kN applied as shown in figure.

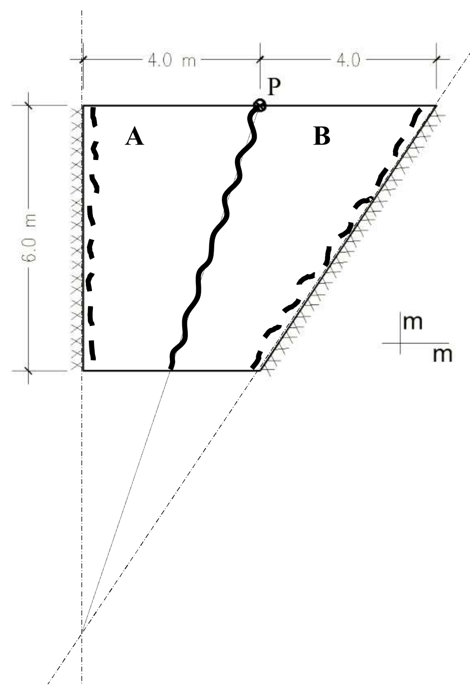


Solution

$W_e = P \times 1 = P$

$$W_i = \left[\left(m \times 6 \times \frac{1}{4}\right) + \left(m \times 6 \times \frac{1}{4}\right)\right] + \left[\left(m \times 2 \times \frac{1}{6} + m \times 6 \times \frac{1}{4}\right) + \left(m \times 4 \times \frac{1}{6} + m \times 6 \times \frac{1}{4}\right)\right]$$

$W_i = 7\ m$

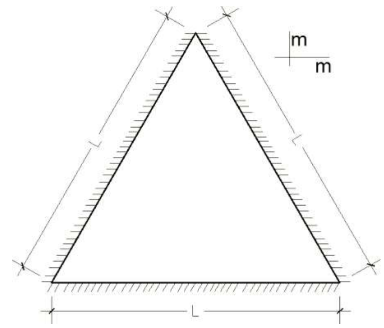


$W_e = W_i$

$P = 7\ m$

Example

By using the yield line theory, determine the ultimate resisting moment per linear meter (m) for an isotropic reinforced concrete two-way simply supported triangle slab shown in figure under a uniform load (q).



Solution

$$W_e = q \times \left(L \times x \times \frac{1}{2} \times \frac{1}{3}\right) \times 3 \qquad x = \frac{L}{3}$$

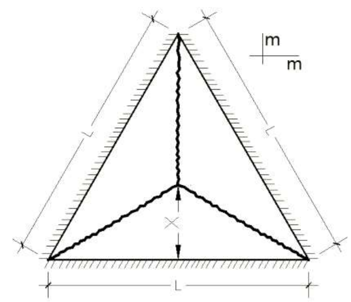
$$W_e = \frac{q\,L\,x}{2}$$

$$W_i = \left(m \times L \times \frac{1}{x}\right) \times 3 = \frac{3\,m\,L}{x}$$

$$W_e = W_i$$

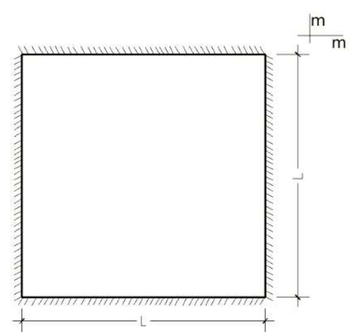
$$\frac{q\,L\,x}{2} = \frac{3\,m\,L}{x}$$

$$m = \frac{q\,x^2}{6} = \frac{q\,L^2}{54}$$



Example

By using the yield line theory, determine the ultimate resisting moment per linear meter (m) for an isotropic reinforced concrete two-way simply supported square slab shown in figure under a uniform load (q).



Solution

$$W_e = q \times \left(L \times x \times \frac{1}{2} \times \frac{1}{3}\right) \times 4 \qquad x = \frac{L}{2}$$

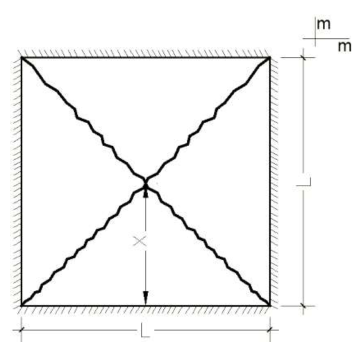
$$W_e = \frac{2\,q\,L\,x}{3}$$

$$W_i = \left(m \times L \times \frac{1}{x}\right) \times 4 = \frac{4\,m\,L}{x}$$

$$W_e = W_i$$

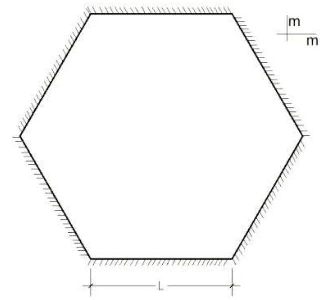
$$\frac{2\,q\,L\,x}{3} = \frac{4\,m\,L}{x}$$

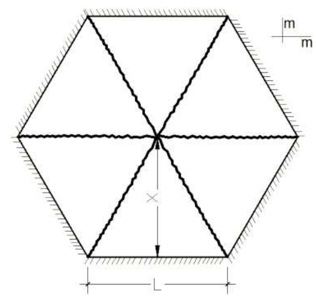
$$m = \frac{q\,x^2}{6} = \frac{q\,L^2}{24}$$



Example

By using the yield line theory, determine the ultimate resisting moment per linear meter (m) for an isotropic reinforced concrete two-way simply supported polygon slab shown in figure under a uniform load (q).





Solution

$$W_e = q \times \left(L \times x \times \frac{1}{2} \times \frac{1}{3}\right) \times 6 \qquad x = \frac{\sqrt{3}\,L}{2}$$

$$W_e = q\,L\,x$$

$$W_i = \left(m \times L \times \frac{1}{x}\right) \times 6 = \frac{6\,m\,L}{x}$$

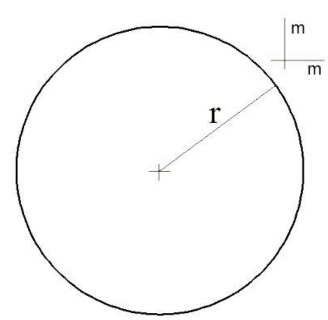
$$W_e = W_i$$

$$q\,L\,x = \frac{6\,m\,L}{x}$$

$$m = \frac{q\,x^2}{6} = \frac{q\,L^2}{8}$$

Example

By using the yield line theory, determine the ultimate resisting moment per linear meter (m) for an isotropic reinforced concrete two-way simply supported circular slab shown in figure under a uniform load (q).



Solution

$$W_e =$$

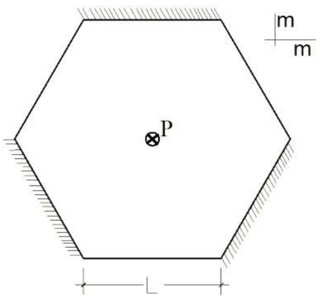
$$W_i =$$

$$W_e = W_i$$

$$m = \frac{q\,r^2}{6}$$

Example

By using the yield line theory, determine the ultimate resisting moment per linear meter (m) for an isotropic reinforced concrete two-way simply supported polygon slab shown in figure under a concentrated factored load of P.

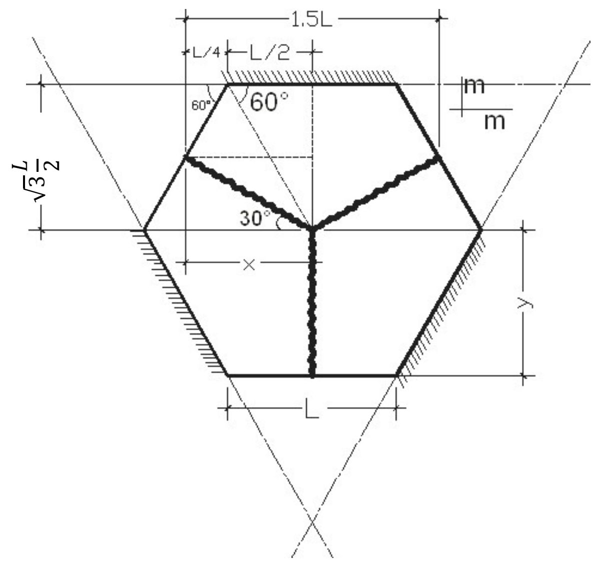


Solution

$W_e = P \times 1 = P$

$y = \sqrt{3}\,\frac{L}{2}$

$x = y \cdot \cos 30 = \frac{\sqrt{3}}{2}\,y$

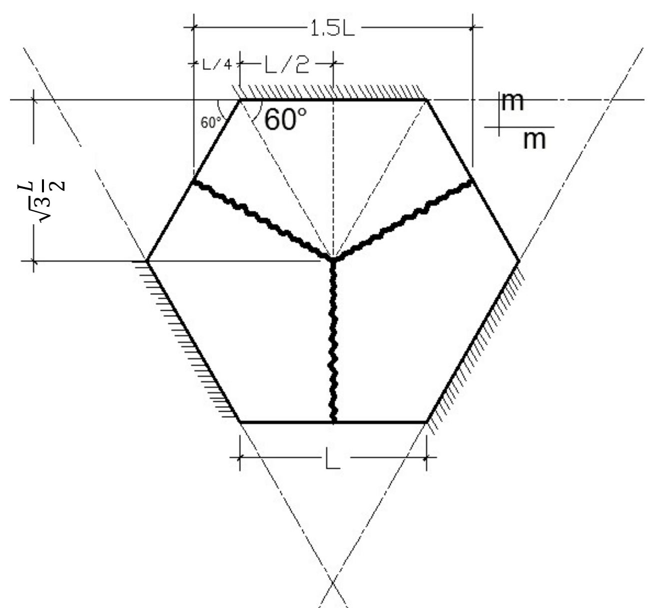


$$W_i = \left(m \times 1.5\,L \times \frac{1}{\sqrt{3}\frac{L}{2}}\right) \times 3 = 3\sqrt{3}\,m$$

$W_e = W_i$

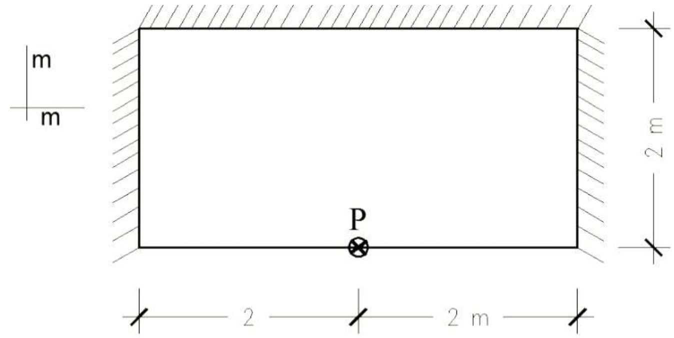
$P = 3\sqrt{3}\,m$

$m = \frac{P}{3\sqrt{3}} = 0.192\,P$



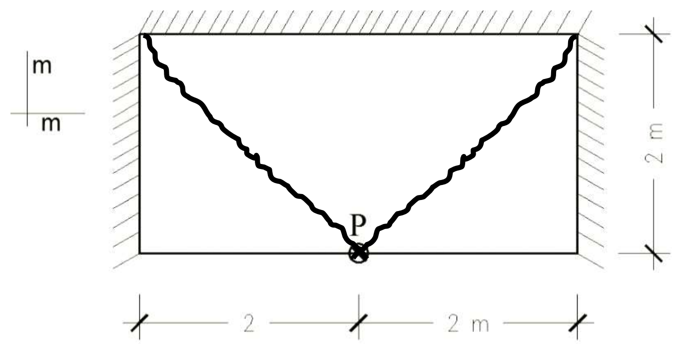
Example

By using the yield line theory, determine the moment (**m**) for an isotropic reinforced concrete two-way slab shown in Figure under a concentrated factored load of **P**.



Solution

$$WE = P \times 1 = P$$



$$WI = \left(2 \times m \times \frac{1}{2}\right) \times 2 + \left(4 \times m \times \frac{1}{2}\right) = 4m$$

$$WE = WI$$

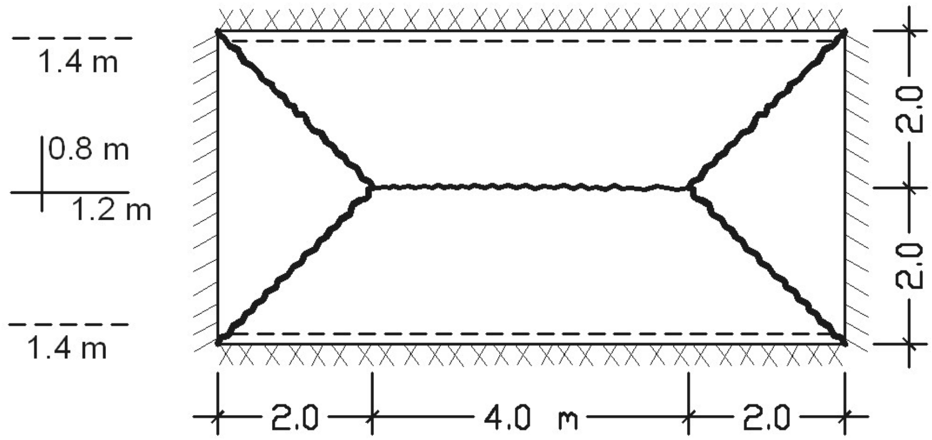
$$P = 4\,m$$

$$\Rightarrow \quad m = \frac{P}{4}$$

$$m = 0.25\,P$$

Example

By using the yield line theory, determine the ultimate resisting moment per linear meter (m) for an orthotropic rectangulare reinforced concrete two-way slab, shown in Figure, to sustain a uniformly distributed load equals 12 kN/m^2. Use the proposed positions for the positive and negative yield lines as shown in Figure.



Solution

$$W_e = 12 \times \left[\left(2 \times 2 \times \frac{1}{2} \times \frac{1}{3} \times 8\right) + \left(4 \times 2 \times \frac{1}{2} \times 2\right)\right]$$

$$W_e = 160 \ \text{kN.m}$$

$$W_i = \left[0.8\ \text{m} \times 4 \times \frac{1}{2}\right] \times 2 + \left[\left(1.2\ \text{m} \times 8 \times \frac{1}{2}\right) + \left(1.4\ \text{m} \times 8 \times \frac{1}{2}\right)\right] \times 2$$

$$W_i = 24\ \text{m}$$

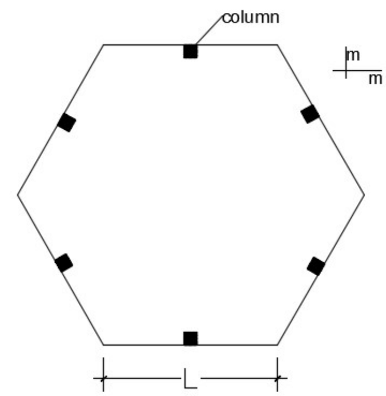
$$W_e = W_i$$

$$160 = 24\ \text{m}$$

$$\text{m} = 6.667\ \text{kN.m/m}$$

Example

By using the yield line theory, determine the ultimate resisting moment per linear meter (m) for an isotropic reinforced concrete two-way polygon slab shown in figure under a uniform load (q).



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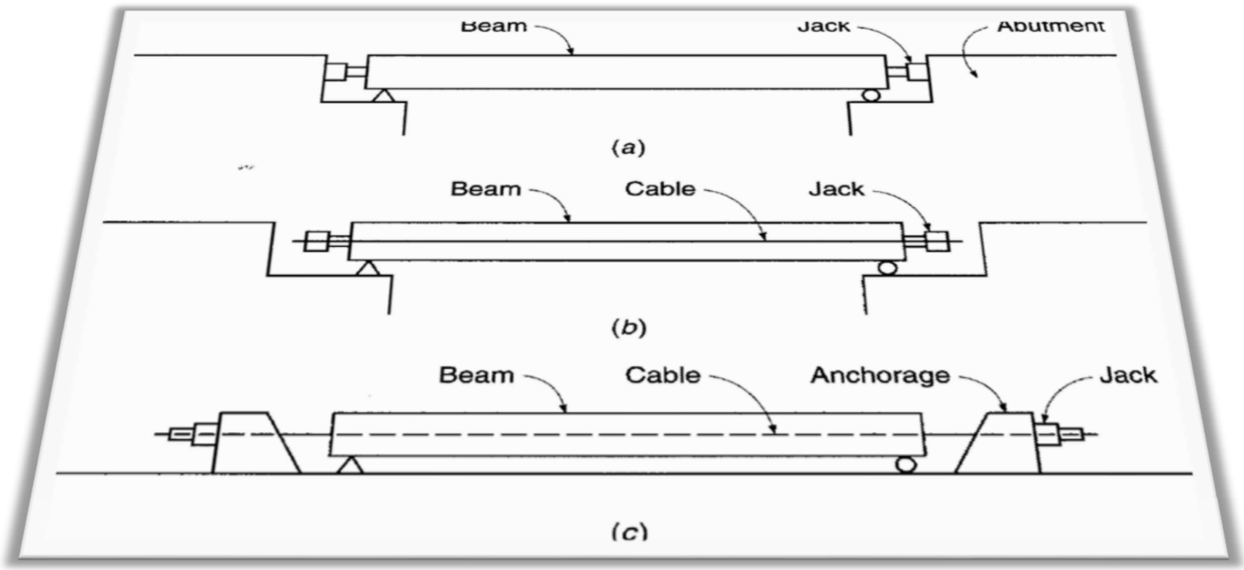


REINFORCED CONCRETE DESIGN II

FOURTH YEAR CLASS

6

2018 - 2019

MHND

Prestressed Concrete

Prestressed concrete member can be defined as one in which there have been introduced internal stresses of such magnitude and distribution that the stresses resulting from the given external loading are counteracted to a designed degree.

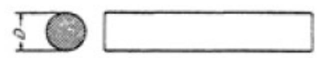
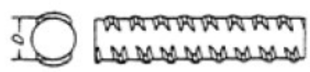
Advantage of prestressed concrete

1- High strength steel and concrete.
2- Eliminated cracks in concrete.
3- Prestressed concrete more suitable for structure of long span and those carrying heavy loads.
4- Under dead load, the deflection is reduced, owing to the cambering effected of prestress (useful for bridges and long cantilevers).

Disadvantage of prestressed concrete

1- Higher cost of materials.
2- More complicated formwork may be necessitated.
3- End anchorages and bearing plates are usually required.
4- Labor costs are greater.

Tendon: A stretched element used in a concrete member of structure to impart prestress to the concrete. Generally, high tensile steel wires, bars, cables or strands are used as tendons.
Strand: A group of wires (7 wires).
Wires: individually drawn wires of 7 mm diameter;
Bar: a specially formed bar of high strength steel of greater than 20 mm diameter
Anchorage: A device generally used to enable the tendon to impart and maintain prestress the concrete.

Type	Size (Diameter)		Shape
	mm	in.	
Plain round wire	2.0 ~ 9.0	0.06 ~ 0.360	
Indented wire	5.0 ~ 7.0	0.200 ~ 0.276	
Sumi - Twist	7.3 ~ 13.0	0.276 ~ 0.512	
Two-wire strand	2.9 x 2	0.114 x 2	
Seven-wire strand	6.2 ~ 15.2	0.250 ~ 0.600	
Nineteen-wire strand	17.8 ~ 21.8	0.700 ~ 0.860	
Round bar	9.2 ~ 32.0	0.362 ~ 1.260	
Threaded bar (Dywidag)	23.0 ~ 32.0	0.906 ~ 1.260	

Classifications and types

a- Externally and internally prestressed

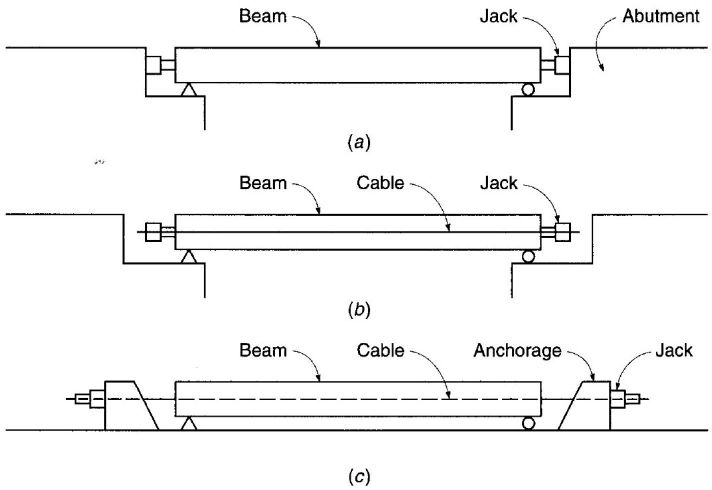
- Externally by jacking against abutments, this cannot be accomplished in practice, because even if abutment is stiff, shrinkage and creep in concrete y completely offset the strain.
- Internally accomplished by pretensioiny of steel.

b- Linear and circular prestressing

- Linear for beam and slabs, can be curved.
- Circular used for round tanks, silos, and pipes.

c- Pretensioning and postensioning

- Pretensioning: tendons tensioned before the concrete is placed, used in prestressing plants where permanent beds are provided for such tensioning.
- Posttensioning: tendons are tensioned after the concrete has hardened.



Prestressing methods: (a) post-tensioning by jacking against abutments; (b) post-tensioning with jacks reacting against beam; (c) pretensioning with tendon stressed between fixed external anchorages.

d- End-anchored and non-end-anchored tendons

- End-anchored: used in post tensioned, the tendons are anchored at their ends by means of mechanical devices to transmit the prestress to the concrete.
- Non-end-anchored: used in pretensioned where the tendons have their perstress transmitted to the concrete by their bond action near the ends. This type is limited to wires and strand of small size.

e- Bounded and unbounded tendons

Bounded: denote those bounded throughout their length to the surrounding concrete.

Non-end-anchored: tendons may be either bounded or unbounded to the concrete by grouting.

f- Precast, cast-in-place, composite construction

- Precasting: involves the placing of concrete away from its final position. This permits better control on mass production, and it is economical.
- Cast-in-place: concrete requires more form and false work.
- Composite: to precast pant of a member, erect it, casting the remaining portion in place.

g- Partial and full prestressing

- Full prestressing: the member is designed, so that, under working loads (service) there are no tensile stresses in it.
- Partial: tension is produced under working load. Addition, mild steel bars are provided to reinforce the tension zone.

Stages of Loading:

1- Initial stage: the member is under prestress, but is not subjected to any superimposed external loads.
2- Intermediate stage: during transportation and erection.
3- Final (service) stage: when the actual working load come on the structure.

Concrete:

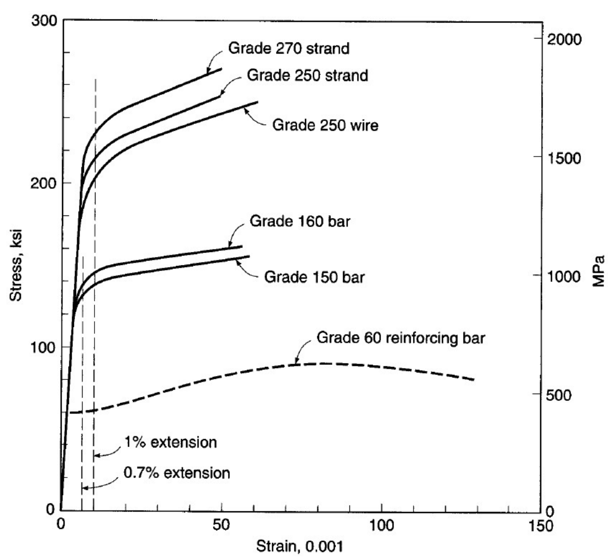
High strength concrete is used ($f_c' > 40$ MPa) for the following reasons:

1- High bearing stresses needed at end anchorage in post-tensioned.
2- High bond offered by high strength concrete in pretension.
3- A smaller cross sectional area can be used to carry a given load.
4- Higher modulus of elasticity, this means a reduction in initial elastic strain under application of prestress force and a reduction in creep strain. This results in a reduction in loss of prestress.

Steel:

The tensile strengths of prestressing steels range from about 2.5 to 6 times the yield strengths of commonly used reinforcing bars. The grade designations correspond to the minimum specified tensile strength in ksi (MPa). For the widely used seven-wire strand, two grades are recognized in ASTM A416: Grade 250 ksi (1725 MPa) and Grade 270 ksi (1860 MPa). For alloy steel bars, two grades are used: Grade 150 ksi (regular) and Grade 160 ksi. Round wires may be obtained in Grades 235, 240, and 250 ksi.

High strength steel must be used due to the low prestressing force obtained by using ordinary steel is quickly lost due to shrinkage and creep.



Typical stress-strain curves for prestressing steels.

Losses in prestressing force

The magnitude of prestress force will gradually decrease. The most significant causes are:-

1- Elastic shortening of concrete.
2- Concrete creep under sustained load.
3- Concrete shrinkage.
4- Relaxation of stress in steel.
5- Friction loss between the tendons and the concrete during stressing operation.
6- Loss due to slip of steel strands.

Summary of losses:

Pretensioned beam		**Post-tensioned beam**	
a- Before transfer			
- Shrinkage	3%	————	
b- At transfer			
- Elastic shortening	3%	- Elastic shortening	1%
		- Anchor slip	2%
		- Friction	2%
c- After transfer			
- Shrinkage	4%	- Shrinkage	4%
- Creep	7%	- Creep	4%
- Steel relation	3%	- Steel relation	3%
total	20%		16%

Analysis: to determine the stresses in the steel and concrete when the shape and size of a section are already given or assumed.

Design: to determine a suitable section for a given loading and stresses.

The analysis is a simpler operation than design.

The f_{pu} is the ultimate strength of the steel and f_{py} is the yield strength.

Stages of investigation of prestressed beam:

Initial stage

Initial force (P_i) plus beam weight (w_g):

Stress at top	$f_{ti} = \frac{-P_i}{A} + \frac{P_i \cdot e \cdot c_t}{I} - \frac{M_g \cdot c_t}{I}$
Stress at bottom	$f_{bi} = \frac{-P_i}{A} - \frac{P_i \cdot e \cdot c_b}{I} + \frac{M_g \cdot c_b}{I}$

Service stage

The beam under effective prestressing force (P_e) plus weight of the beam plus service load (live load plus weight of cast-in-situ concrete):

Stress at top	$f_{ts} = \frac{-P_e}{A} + \frac{P_e \cdot e \cdot c_t}{I} - \frac{M_g \cdot c_t}{I} - \frac{M_s \cdot c_t}{I}$
Stress at bottom	$f_{bs} = \frac{-P_e}{A} - \frac{P_e \cdot e \cdot c_b}{I} + \frac{M_g \cdot c_b}{I} + \frac{M_s \cdot c_b}{I}$

Permissible stresses in prestressed concrete flexural members

For calculation of stresses at transfer of prestress, at service loads, and at cracking loads, elastic theory shall be used with assumptions (a) and (b):

(a) Strains vary linearly with distance from neutral axis.

(b) At cracked sections, concrete resists no tension.

Classification of prestressed flexural members

Prestressed flexural members shall be classified as Class U, T, or C in accordance with Table 24.5.2.1, based on the extreme fiber stress in tension $\mathbf{f_t}$ in the precompressed tension zone calculated at service loads assuming an uncracked section.

Table 24.5.2.1 - Classification of prestressed flexural members based on $\mathbf{f_t}$

Assumed behavior	**Class**	**Limits of f_t**
uncracked	U	$f_t \leq 0.62\sqrt{f'_c}$
Transition between uncracked and cracked	T	$0.62\sqrt{f'_c} < f_t \leq 1.0\sqrt{f'_c}$
cracked	C	$f_t > 1.0\sqrt{f'_c}$

Prestressed two-way slabs shall be designed as Class U

Three classes of behavior of prestressed flexural members are defined. Class U members are assumed to behave as uncracked members. Class C members are assumed to behave as cracked members. The behavior of Class T members is assumed to be in transition between uncracked and cracked. The serviceability requirements for each class are summarized in Table R24.5.2.1. For comparison, Table R24.5.2.1 also shows corresponding requirements for nonprestressed members.

Table R24.5.2.1—Serviceability design requirements

	Prestressed			Nonprestressed
	Class U	Class T	Class C	
Assumed behavior	Uncracked	Transition between uncracked and cracked	Cracked	Cracked
Section properties for stress calculation at service loads	Gross section 24.5.2.2	Gross section 24.5.2.2	Cracked section 24.5.2.3	No requirement
Allowable stress at transfer	24.5.3	24.5.3	24.5.3	No requirement
Allowable compressive stress based on uncracked section properties	24.5.4	24.5.4	No requirement	No requirement
Tensile stress at service loads 24.5.2.1	$\leq 0.62\sqrt{f_c'}$	$0.62\sqrt{f_c'} < f_t \leq 1.0\sqrt{f_c'}$	No requirement	No requirement
Deflection calculation basis	24.2.3.8, 24.2.4.2 Gross section	24.2.3.9, 24.2.4.2 Cracked section, bilinear	24.2.3.9, 24.2.4.2 Cracked section, bilinear	24.2.3, 24.2.4.1 Effective moment of inertia
Crack control	No requirement	No requirement	24.3	24.3
Computation of Δf_{ps} or f_s for crack control	—	—	Cracked section analysis	$M/(A_s \times$ lever arm), or $2/3f_y$
Side skin reinforcement	No requirement	No requirement	9.7.2.3	9.7.2.3

For Class U and T members, stresses at service loads shall be permitted to be calculated using the uncracked section.

For Class C members, stresses at service loads shall be calculated using the cracked transformed section.

Permissible concrete stresses at transfer of prestress

Calculated extreme concrete fiber stress in compression immediately after transfer of prestress, but before time-dependent prestress losses, shall not exceed the limits in Table 24.5.3.1.

Table 24.5.3.1—Concrete compressive stress limits immediately after transfer of prestress

Location	Concrete compressive stress limits
End of simply-supported members	$0.70f_{ci}'$
All other locations	$0.60f_{ci}'$

Calculated extreme concrete fiber stress in tension immediately after transfer of prestress, but before time-dependent prestress losses, shall not exceed the limits in Table 24.5.3.2, unless permitted by 24.5.3.2.1.

Table 24.5.3.2—Concrete tensile stress limits immediately after transfer of prestress, without additional bonded reinforcement in tension zone

Location	Concrete tensile stress limits
Ends of simply-supported members	$0.50\sqrt{f'_{ci}}$
All other locations	$0.25\sqrt{f'_{ci}}$

Permissible concrete compressive stresses at service loads

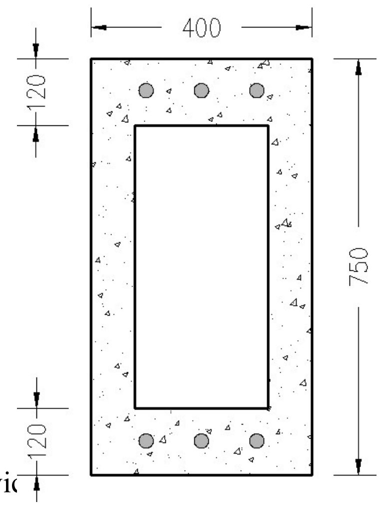
For Class U and T members, the calculated extreme concrete fiber stress in compression at service loads, after allowance for all prestress losses, shall not exceed the limits in Table 24.5.4.1.

Table 24.5.4.1—Concrete compressive stress limits at service loads

Load condition	Concrete compressive stress limits
Prestress plus sustained load	$0.45f_c'$
Prestress plus total load	$0.60f_c'$

Example

A prestress rectangular box beam post-tensioned by straight high tensile steel wires of total area A_s mm^2, equally divided between the top and bottom flanges and placed on center of flanges. The forces are initially stressed to 850 N/mm^2 and the total losses of prestress is 15%. The beam is required to carry a uniformly distributed superimposed load of 4.5 kN/m in addition to its own weight, over a span of 15 m. If the concrete stresses are not to exceed 17.5 N/mm^2 in compression and 1 N/mm^2 in tension (during the prestressing operation and working load). Calculate the max. and min. A_s of steel, which may be used. Use $\gamma_c = 25\ kN/m^3$



Solution

$A = 400 \times 750 - 240 \times 510 = 177600\ mm^2$

$$I = \frac{400 \times (750)^3 - 240 \times (510)^3}{12} = 1.140948 \times 10^{10}\ mm^4$$

$w_g = 177600 \times 10^{-6} \times 25 = 4.44\ kN/m$

Note:

a- check compressive stress at initial stage.

b- check compressive stress at top and tensile stress at bottom at servic

a- Immediately after prestressing

prestressing force before losses = 850 A_s

initial compressive stress =

$$f_{ti} = f_{bi} = \frac{P_i}{A}$$

$$-17.5 \le \frac{-850 \times A_s}{177600} \quad \Rightarrow \quad A_s \le 3656.5\ mm^2$$

b- Service stage (final stage)

1- Top fiber

Prestressing stress after losses =

Final stress @ top =

2- Bottom fiber:

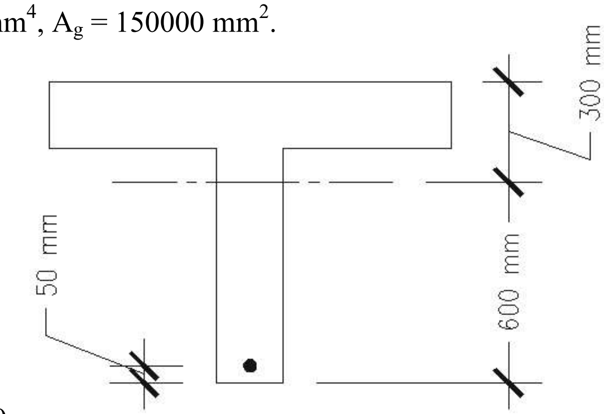
Example

A simply supported prestressed beam, of span 8 m and its cross section is shown in Figure, is carrying a live load equals to 12 kN/m. Compute the required prestressing forces for:

a) Tob fiber stress equals to zero under beam weight plus prestressing force only.

b) Bottom fiber stress equals to zero under full load.

Use $\gamma_c = 25$ kN/m^3, I = 120 × 10^8 mm^4, A_g = 150000 mm^2.



Solution

$c_t = 300$ mm $\qquad c_b = 600$ mm $\qquad e = 550$ mm

$$w_g = A \cdot \gamma = 150000 \times 10^{-6} \times 25 = 3.75 \text{ kN/m}$$

$$M_g = \frac{w_g \cdot L^2}{8} = \frac{3.75 \times (8)^2}{8} = 30.0 \text{ kN.m}$$

a) Top fiber stress equals to zero under beam weight plus prestressing force only at top fiber

$$f_{ti} = \frac{-P_e}{A} + \frac{P_e \cdot e \cdot c_t}{I} - \frac{M_g \cdot c_t}{I}$$

$$0 = \frac{-P \times 10^3}{150000} + \frac{P \times 10^3 \times 550 \times 300}{120 \times 10^8} - \frac{30 \times 10^6 \times 300}{120 \times 10^8}$$

$$0 = \frac{-P}{15} + \frac{P \times 11}{80} - \frac{30}{4}$$

$$\Rightarrow \quad P = 7.5 \times 17 = 105.882 \text{ kN}$$

b) Bottom fiber stress equals to zero under full load

$$M_s = \frac{w_s \cdot L^2}{8} = \frac{12 \times (8)^2}{8} = 96 \text{ kN.m}$$

$$f_{bs} = \frac{-P_e}{A} - \frac{P_e \cdot e \cdot c_b}{I} + \frac{M_g \cdot c_b}{I} + \frac{M_s \cdot c_b}{I}$$

$$0 = \frac{-P \times 10^3}{150000} - \frac{P \times 10^3 \times 550 \times 600}{120 \times 10^8} + \frac{30 \times 10^6 \times 600}{120 \times 10^8} + \frac{96 \times 10^6 \times 600}{120 \times 10^8}$$

$$0 = \frac{-P}{15} - \frac{P \times 11}{40} + 15 + 48$$

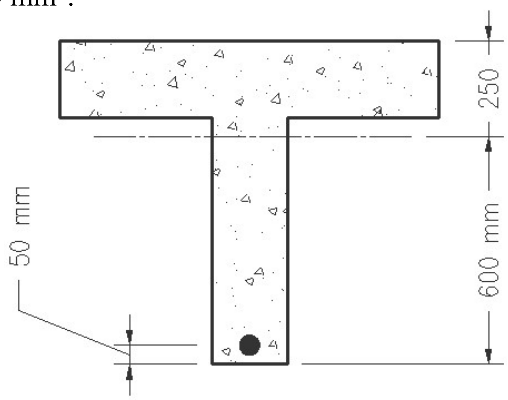
$$\Rightarrow \quad P = \frac{63 \times 120}{41} = 194.390 \text{ kN}$$

Example

A simply supported prestressed beam, of span 8 m and its cross section is shown in Figure, is carrying a live load equals to 10 kN/m. Compute the required prestressing forces for:

a) Top fiber stress equals to zero under beam weight plus prestressing force only.
b) Bottom fiber stress equals to zero under full load.

Use $\gamma_c = 25$ kN/m^3, $I = 10 \times 10^9$ mm^4, $A_g = 100000$ mm^2.



Solution

$c_t = 250$ mm $\qquad c_b = 600$ mm $\qquad e = 550$ mm

$$w_g = A \cdot \gamma = 100000 \times 10^{-6} \times 25 = 2.5 \text{ kN/m}$$

$$M_g = \frac{w_g \cdot L^2}{8} = \frac{2.5 \times (8)^2}{8} = 20.0 \text{ kN.m}$$

a) Top fiber stress equals to zero under beam weight plus prestressing force only at top fiber

$$f_{ti} = \frac{-P_e}{A} + \frac{P_e \cdot e \cdot c_t}{I} - \frac{M_g \cdot c_t}{I}$$

$$0 = \frac{-P \times 10^3}{100000} + \frac{P \times 10^3 \times 550 \times 250}{10 \times 10^9} - \frac{20 \times 10^6 \times 250}{10 \times 10^9}$$

$$0 = \frac{-P}{100} + \frac{P \times 1.375}{100} - \frac{5}{10}$$

$$\Rightarrow \quad P = 133.333 \text{ kN}$$

b) Bottom fiber stress equals to zero under full load

$$M_s = \frac{w_s \cdot L^2}{8} = \frac{10 \times (8)^2}{8} = 80 \text{ kN.m}$$

$$f_{bs} = \frac{-P_e}{A} - \frac{P_e \cdot e \cdot c_b}{I} + \frac{M_g \cdot c_b}{I} + \frac{M_s \cdot c_b}{I}$$

$$0 = \frac{-P \times 10^3}{100000} - \frac{P \times 10^3 \times 550 \times 600}{10 \times 10^9} + \frac{20 \times 10^6 \times 600}{10 \times 10^9} + \frac{80 \times 10^6 \times 600}{10 \times 10^9}$$

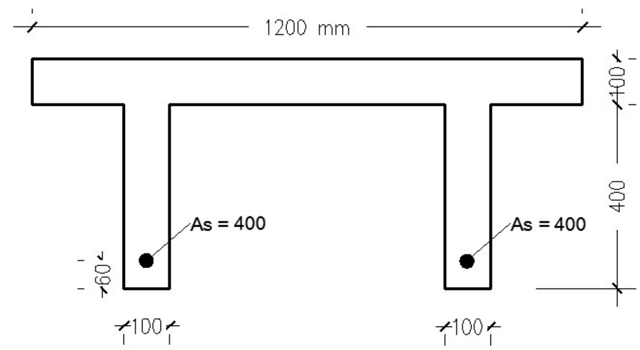
$$0 = \frac{-P}{100} - \frac{P \times 3.3}{100} + \frac{12}{10} + \frac{48}{10}$$

$$\Rightarrow \quad P = 139.535 \text{ kN}$$

Example

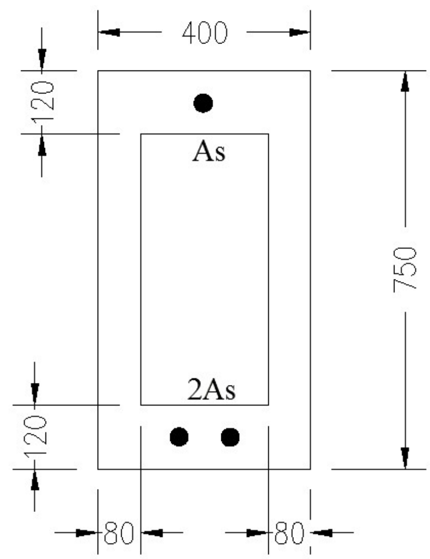
A double-T simply supported concrete beam its cross section is shown in Figure, is prestressed with 2 tendons each 400 mm^2. Determine the allowable service load.

Use span = 12 m, f_{se} = 1300 MPa, f_c' = 40 MPa, γ_c = 25 kN/m^3.



Example

A prestressed simply supported 15 m span beam with rectangular box section is post-tensioned by straight high tensile steel wires as shown in Figure. The prestressing wires are placed at the center line of the flanges and initially stressed to 850 N/mm^2. The beam is required to carry a uniformly distributed superimposed load of 4.5 kN/m in addition to its weight. If the concrete stresses are not to exceed 17 MPa in compression and 1 MPa in tension at service stage, calculate the range of the total prestressing wires area required. Ignore prestressing force losses in your answer. (γ_c = 24 kN/m^3).

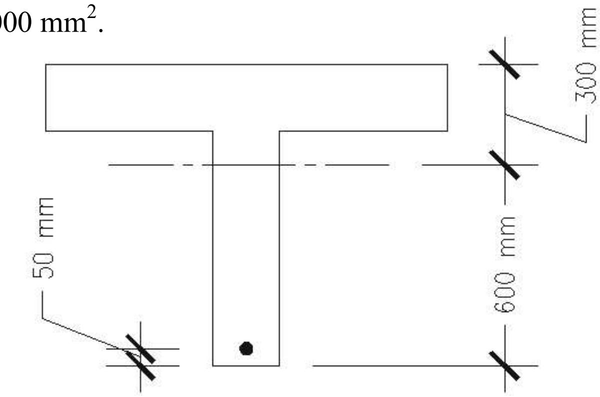


Example

A simply supported prestressed beam, of span 10 m and its cross section is shown in Figure, is carrying a live load equals to 10 kN/m. Compute the required prestressing forces for:

a) Top fiber stress equals to zero under beam weight plus prestressing force only.
b) Bottom fiber stress equals to zero under full load.

Use $\gamma_c = 25$ kN/m^3, I = 150 × 10^8 mm^4, A_g = 100000 mm^2.

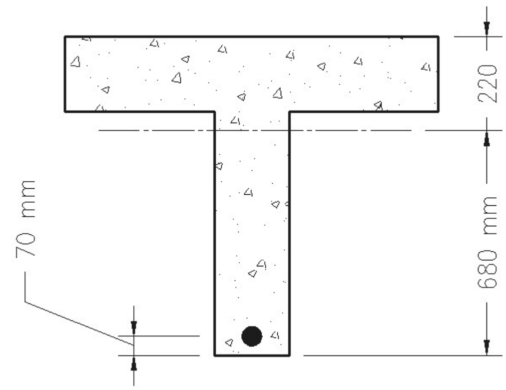


Example

A simply supported prestressed concrete beam, of span 10 m and its cross section is shown in Figure, is carrying a service load equals to 12 kN/m. Compute the required prestressing forces for:

a) Top fiber stress equals to zero under beam weight plus prestressing force only.
b) Bottom fiber stress equals to zero under full loads.

Use γ_c= 24 kN/m^3, I = 12×10^9 mm^4, A_g = 120000 mm^2.



Solution

$c_t = 220$ mm $\quad c_b = 680$ mm $\quad e = 610$ mm

$$w_g = A \cdot \gamma = 100000 \times 10^{-6} \times 24 = 2.4 \text{ kN/m}$$

$$M_g = \frac{w_g \cdot L^2}{8} = \frac{2.4 \times (10)^2}{8} = 30.0 \text{ kN.m}$$

a) Top fiber stress equals to zero under beam weight plus prestressing force only at top fiber

$$f_{ti} = \frac{-P}{A} + \frac{P \cdot e \cdot c_t}{I} - \frac{M_g \cdot c_t}{I}$$

$$0 = \frac{-P \times 10^3}{120000} + \frac{P \times 10^3 \times 610 \times 220}{12 \times 10^9} - \frac{30 \times 10^6 \times 220}{12 \times 10^9}$$

$$0 = \frac{-P}{120} + \frac{P \times 1.342}{120} - \frac{6.6}{12}$$

$\Rightarrow \quad P = 192.982$ kN

b) Bottom fiber stress equals to zero under full loads

$$M_s = \frac{w_s \cdot L^2}{8} = \frac{12 \times (10)^2}{8} = 150 \text{ kN.m}$$

$$f_{bs} = \frac{-P_e}{A} - \frac{P_e \cdot e \cdot c_b}{I} + \frac{M_g \cdot c_b}{I} + \frac{M_s \cdot c_b}{I}$$

$$0 = \frac{-P \times 10^3}{120000} - \frac{P \times 10^3 \times 610 \times 680}{12 \times 10^9} + \frac{30 \times 10^6 \times 680}{12 \times 10^9} + \frac{150 \times 10^6 \times 680}{12 \times 10^9}$$

$$0 = \frac{-P}{120} - \frac{P \times 4.148}{120} + \frac{20.4}{12} + \frac{102}{12}$$

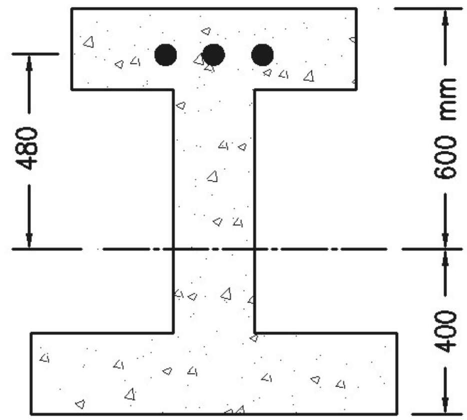
$\Rightarrow \quad P = 237.762$ kN

Example

A cantilever prestressed concrete beam, of span 6 m and its cross section is shown in Figure, is carrying a service load equals to 12 kN/m. Compute the required prestressing forces for:

a) Bottom fiber stress equals to zero under beam weight plus prestressing force only.
b) Top fiber stress equals to zero under full loads.

Use γ_c= 25 kN/m^3, I = 18×10^9 mm^4, A_g = 120000 mm^2.



Solution

$c_t = 600$ mm $c_b = 400$ mm $e = 480$ mm

$$w_g = A \cdot \gamma = 120000 \times 10^{-6} \times 25 = 3.0 \text{ kN/m}$$

$$M_g = \frac{w_g \cdot L^2}{2} = \frac{3.0 \times (6)^2}{2} = 54.0 \text{ kN.m}$$

a) Bottom fiber stress equals to zero under beam weight plus prestressing force only at bottom fiber

$$f_{bi} = \frac{-P}{A} + \frac{P \cdot e \cdot c_b}{I} - \frac{M_g \cdot c_b}{I}$$

$$0 = \frac{-P \times 10^3}{120000} + \frac{P \times 10^3 \times 480 \times 400}{18 \times 10^9} - \frac{54 \times 10^6 \times 400}{18 \times 10^9}$$

$$\Rightarrow \quad P = 514.286 \text{ kN}$$

b) Top fiber stress equals to zero under full loads

$$M_s = \frac{w_s \cdot L^2}{2} = \frac{12 \times (6)^2}{2} = 216 \text{ kN.m}$$

$$f_{ts} = \frac{-P}{A} - \frac{P \cdot e \cdot c_t}{I} + \frac{M_g \cdot c_t}{I} + \frac{M_s \cdot c_t}{I}$$

$$0 = \frac{-P \times 10^3}{120000} - \frac{P \times 10^3 \times 480 \times 600}{18 \times 10^9} + \frac{54 \times 10^6 \times 600}{18 \times 10^9} + \frac{216 \times 10^6 \times 600}{18 \times 10^9}$$

$$\Rightarrow \quad P = 369.863 \text{ kN}$$

Example

A simply supported rectangular prestressed concrete beam, of span 13 m and its cross section as shown in figure, is carrying a live load equals to 30 kN/m in addition to its weight, compute the following stresses and compare it with ACI allowable stress:

a) Bottom fiber stress at support in initial stage.
b) Top fiber stress at mid span in final stage.

Use $\gamma_c = 24\,\mathrm{kN/m^3}$, $\mathrm{As} = 600\ \mathrm{mm^2}$, initial stress of the prestressed steel = 1200 MPa, total losses is 20%, $\mathrm{f_{ci}} = 22\ \mathrm{MPa}$, and $\mathrm{f_c}' = 28\ \mathrm{MPa}$