$$\Rightarrow\ t_{min} = \frac{l_n\left(0.8 + \frac{f_y}{1400}\right)}{36 + 5\beta(\alpha_m - 0.2)}$$

$$\beta = \frac{L_n}{S_n} = \frac{5.8}{5.8} = 1.0$$

$$t_{min} = \frac{5800 \times \left(0.8 + \frac{420}{1400}\right)}{36 + 5 \times 1.0 \times (1.5 - 0.2)} = 150.118\ \text{mm} \quad > 125\ \text{mm} \quad \text{O.K.}$$

$\Rightarrow$ Use t = 160 mm

**Example 5**

Find the minimum thickness of a slab for an interior panels due to deflection control for the following: Use $f_y$ = 420 MPa.

- **a-** Flat slab with drop panels (7.0 × 5.6) m clear span.
- **b-** Slab with beams (5.0 × 6.3) m clear span with **$\alpha_m$= 2.3**
- **c-** Slab with beams (5.0 × 5.5) m clear span with **$\alpha_m$= 1.7**
- **d-** Flat plate (4.2 × 4.5) m clear span.
- **e-** Flat slab without drop panels (5.9 × 4.2) m clear span.

Solution

a) Flat slab with drop panels (7.0 × 5.6) m clear span.

From table

$$t = \frac{\ell_n}{36} = \frac{7000}{36} = 194.444\ \text{mm} \qquad > 100\ \text{mm} \quad \text{O.K.}$$

$\Rightarrow$ Use t = 200 mm

b) Slab with beams (5.0 × 6.3) m clear span with **$\alpha_m$= 2.3**

$\alpha_m = 2.3 > 2.0$

$$\Rightarrow\ t_{min} = \frac{\ell_n\left(0.8 + \frac{f_y}{1400}\right)}{36 + 9\beta}$$

$$\beta = \frac{\ell_n}{s_n} = \frac{6.3}{5.0} = 1.26$$

$$t_{min} = \frac{6300 \times \left(0.8 + \frac{420}{1400}\right)}{36 + 9 \times 1.26} = 146.388\ \text{mm} \quad > 90\ \text{mm} \quad \text{O.K.}$$

$\Rightarrow$ Use t = 150 mm

c) Slab with beams (5.0 × 5.5) m clear span with **$\alpha_m$= 1.7**

$0.2 < \alpha_m = 1.7 < 2.0$

$$\Rightarrow\ t_{min} = \frac{\ell_n\left(0.8 + \frac{f_y}{1400}\right)}{36 + 5\beta(\alpha_m - 0.2)}$$

$$\beta = \frac{\ell_n}{s_n} = \frac{5.5}{5.0} = 1.10$$

$$t_{min} = \frac{5500 \times \left(0.8 + \frac{420}{1400}\right)}{36 + 5 \times 1.1 \times (1.7 - 0.2)} = 136.723 \text{ mm} \quad > 125 \text{ mm} \quad \text{O.K.}$$

$\Rightarrow$ Use t = 140 mm

d) Flat plate (4.2 × 4.5) m clear span.

From table

$$t = \frac{\ell_n}{33} = \frac{4500}{33} = 136.364 \text{ mm} \qquad > 125 \text{ mm} \quad \text{O.K.}$$

$\Rightarrow$ Use t = 140 mm

e) Flat slab without drop panels (5.9 × 4.2) m clear span.

From table

$$t = \frac{\ell_n}{33} = \frac{5900}{33} = 178.788 \text{ mm} \qquad > 125 \text{ mm} \quad \text{O.K.}$$

$\Rightarrow$ Use t = 180 mm

**Example 6**

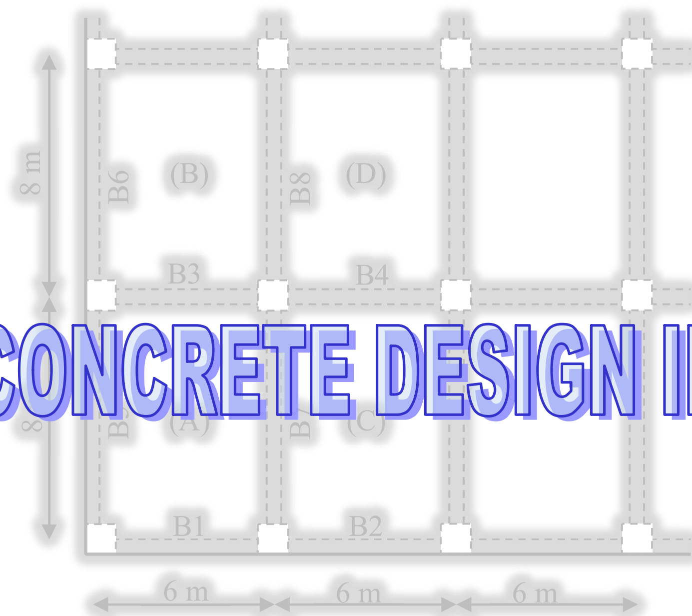
Find the minimum thickness of a slab for an interior panels due to deflection control for the following: Use **$f_y$ = 420 MPa**. (60000 psi).

- **a)** Flat slab with drop panels (6.4 × 6.0) m clear span.
- **b)** Flat plate (4.4 × 4.0) m clear span.
- **c)** Slab with beams (5.8 × 5.6) m clear span with **$\alpha_m$ = 1.7**
- **d)** Slab with beams (8.0 × 6.5) m clear span with **$\alpha_m$ = 3.4**
- **e)** Slab without drop panels (5.5 × 4.8) m clear span with **$\alpha_m$ = 0.19**

**University of Baghdad**

**College of Engineering**

**Department of Civil Engineering**


# REINFORCED CONCRETE DESIGN II

## FOURTH YEAR CLASS

# 2

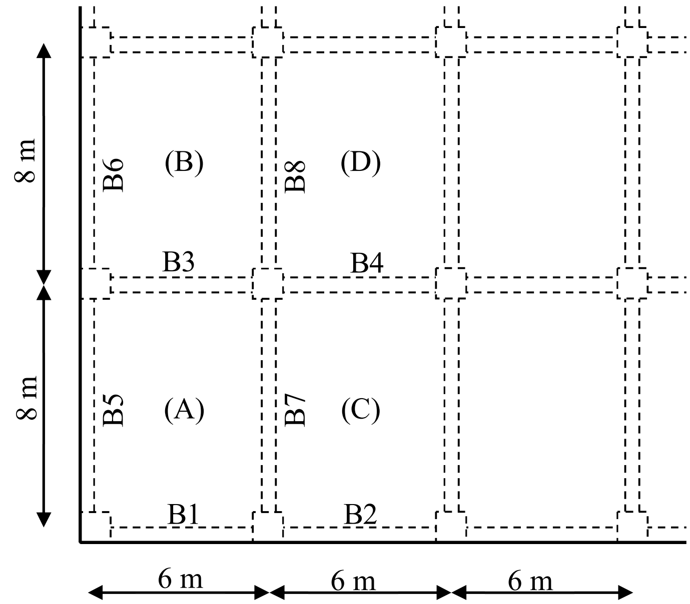
# 2018 - 2019

MHND

## General Example 1

Slab with beams

- All interior beams are 300 × 600 mm
- B1 & B2 are 300 × 600 mm
- B5 & B6 are 300 × 700 mm
- All columns are 600 × 600 mm
- Slab thickness = 180 mm
- Live load = 4.25 kN/m$^2$
- $\gamma_{concrete}$ = 25 kN/m$^3$



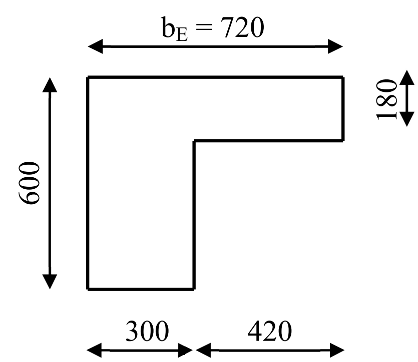

Solution

(1) Computing $\alpha_f$

Compute the ratio of the flexural stiffness of the longitudinal beams to that of the slab ($\alpha_f$) in the equivalent rigid frame, for all beams around panels A, B, C, and D.

Beam sections

B1 and B2




$$\left.\begin{aligned} 2 &< \frac{b_E}{b_w} = \frac{720}{300} = 2.40 < 4 \\ 0.2 &< \frac{h_f}{h} = \frac{180}{600} = 0.3 < 0.5 \end{aligned}\right\} \text{O.K.}$$

$$k = 1 + 0.2\,\frac{b_E}{b_w} = 1 + 0.2\,(2.4) = 1.48$$

$$I_b = k\,\frac{b_w h^3}{12} = 1.48\left(\frac{300\,(600)^3}{12}\right) = 7.992 \times 10^9 \text{mm}^4$$

$$I_s = \frac{1}{12}\,b\,t^3 = \frac{1}{12} \times 4300(180)^3 = 2.090 \times 10^9 \text{mm}^4$$

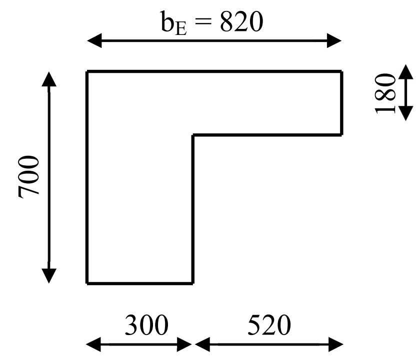
$$b = \frac{8000}{2} + 300 = 4300 \text{ mm}$$

$$\alpha_{fB1} = \alpha_{fB2} = \frac{E_{cb} I_b}{E_{cs} I_s} = \frac{I_b}{I_s} = \frac{7.992 \times 10^9}{2.090 \times 10^9} = 3.823$$

Where $E_{cb} = E_{cs}$

B5 and B6

$$\left.\begin{aligned} 2 < \frac{b_E}{b_w} &= \frac{820}{300} = 2.73 < 4 \\ 0.2 < \frac{t}{h} &= \frac{180}{700} = 0.26 < 0.5 \end{aligned}\right\} \text{O.K.}$$




$$k = 1 + 0.2\,\frac{b_E}{b_w} = 1 + 0.2\,(2.73) = 1.546$$

$$I_b = k\,\frac{b_w h^3}{12} = 1.546\left(\frac{300\,(700)^3}{12}\right) = 13.26 \times 10^9 \text{mm}^4$$

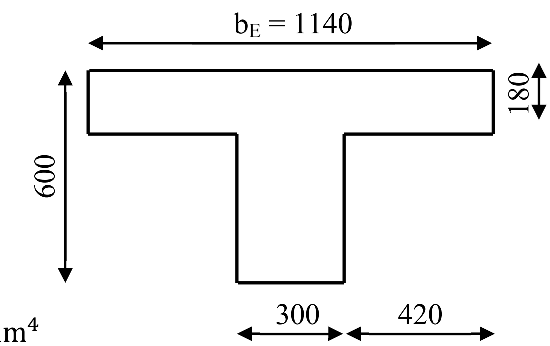
$$I_s = \frac{1}{12}\,b\,t^3 = \frac{1}{12} \times 3300(180)^3 = 1.604 \times 10^9 \text{mm}^4$$

$$b = \frac{6000}{2} + 300 = 3300 \text{ mm}$$

$$\alpha_{fB5} = \alpha_{fB6} = \frac{E_{cb} I_b}{E_{cs} I_s} = \frac{I_b}{I_s} = \frac{13.26 \times 10^9}{1.604 \times 10^9} = 8.267$$

B3 and B4

$$\left.\begin{aligned} 2 < \frac{b_E}{b_w} &= \frac{1140}{300} = 3.8 < 4 \\ 0.2 < \frac{t}{h} &= \frac{180}{600} = 0.3 < 0.5 \end{aligned}\right\} \text{O.K.}$$




$$k = 1 + 0.2\,\frac{b_E}{b_w} = 1 + 0.2\,(3.8) = 1.76$$

$$I_b = k\,\frac{b_w h^3}{12} = 1.76\left(\frac{300\,(600)^3}{12}\right) = 9.504 \times 10^9 \text{mm}^4$$

$$I_s = \frac{1}{12}\,b\,t^3 = \frac{1}{12} \times 8000(180)^3 = 3.888 \times 10^9 \text{mm}^4$$

$$b = 8000 \text{ mm}$$

$$\alpha_{fB3} = \alpha_{fB4} = \frac{E_{cb} I_b}{E_{cs} I_s} = \frac{I_b}{I_s} = \frac{9.504 \times 10^9}{3.888 \times 10^9} = 2.444$$

B7 and B8

$I_b = 9.504 \times 10^9$ same as $B_3$ and $B_4$

$$I_s = \frac{1}{12}\,b\,t^3 = \frac{1}{12} \times 6000 \times (180)^3 = 2.916 \times 10^9 \text{mm}^4$$

b = 6000 mm

$$\alpha_{fB7} = \alpha_{fB8} = \frac{E_{cb} I_b}{E_{cs} I_s} = \frac{I_b}{I_s} = \frac{9.504 \times 10^9}{2.916 \times 10^9} = 3.259$$

Note: for slab without beams, $\alpha_f$ = zero.

To use the DDM, first checking the seven limitations

Limitations 1 to 5 are satisfied by inspections.

Limitation 6:- L.L. shall not exceed 2 times D.L.

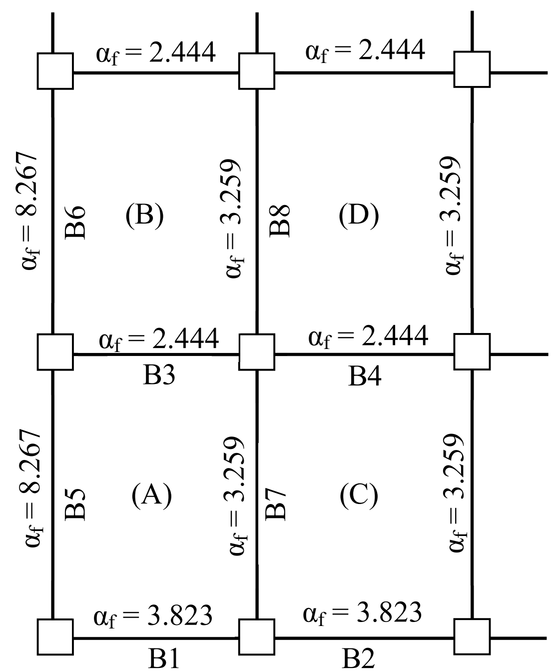
D.L. of the slab = 0.18 × 25 = 4.50 kN/m$^2$
D.L. of tiles = 0.10 × 20 = 2.00 kN/m$^2$
D.L. of partition = 1.00 kN/m$^2$
D.L. of fall ceiling = 0.08 kN/m$^2$

7.58 kN/m$^2$

$$\frac{\text{L. L.}}{\text{D. L.}} = \frac{4.25}{7.58} = 0.56 \ < \ 2.0 \qquad \text{O. K.}$$

Limitation 7:- For each panel

$$0.2 \ \leq \ \frac{\alpha_{f1}\ell_2{}^2}{\alpha_{f2}\ell_1{}^2} \ \leq \ 5.0$$




<u>Panel A</u>

$$\frac{\alpha_{f1}\ell_2{}^2}{\alpha_{f2}\ell_1{}^2} = \frac{\frac{1}{2}(\alpha_{fB1} + \alpha_{fB}\ \ ) \times (8000)^2}{\frac{1}{2}(\alpha_{fB5} + \alpha_{fB7}) \times (6000)^2} = \frac{\frac{1}{2}(3.823 + 2.444) \times (8000)^2}{\frac{1}{2}(8.267 + 3.259) \times (6000)^2} = 0.97$$

$0.2 \ < 0.97 < 5.0 \qquad \text{O. K.}$

<u>Panel B</u>

$$\frac{\alpha_{f1}\ell_2{}^2}{\alpha_{f2}\ell_1{}^2} = \frac{\frac{1}{2}(\alpha_{fB3} + \alpha_{fB3}) \times (8000)^2}{\frac{1}{2}(\alpha_{fB}\ \ + \alpha_{fB8}) \times (6000)^2} = \frac{\frac{1}{2}(2.444 + 2.444) \times (8000)^2}{\frac{1}{2}(8.267 + 3.259) \times (6000)^2} = 0.754$$

$0.2 \ < 0.754 < 5.0 \qquad \text{O. K.}$

Panel C

$$\frac{\alpha_{f1}{\ell_2}^2}{\alpha_{f2}{\ell_1}^2} = \frac{\frac{1}{2}(\alpha_{fB2} + \alpha_{fB4}) \times (8000)^2}{\frac{1}{2}(\alpha_{fB7} + \alpha_{fB7}) \times (6000)^2} = \frac{\frac{1}{2}(3.823 + 2.444) \times (8000)^2}{\frac{1}{2}(3.259 + 3.259) \times (6000)^2} = 1.71$$

$0.2 < 1.71 < 5.0$ O.K.

Panel D

$$\frac{\alpha_{f1}{\ell_2}^2}{\alpha_{f2}{\ell_1}^2} = \frac{\frac{1}{2}(\alpha_{fB} + \alpha_{fB4}) \times (8000)^2}{\frac{1}{2}(\alpha_{fB8} + \alpha_{fB}) \times (6000)^2} = \frac{\frac{1}{2}(2.444 + 2.444) \times (8000)^2}{\frac{1}{2}(3.259 + 3.259) \times (6000)^2} = 1.333$$

$0.2 < 1.333 < 5.0$ O.K.

Computing $\alpha_{fm}$

Panel A

$$\alpha_{fmA} = \frac{1}{4}(\alpha_{fB1} + \alpha_{fB3} + \alpha_{fB5} + \alpha_{fB7}) = \frac{1}{4}(3.823 + 2.444 + 8.267 + 3.259) = 4.448$$

$\alpha_{fmB} = 4.104$

$\alpha_{fmC} = 3.196$

$\alpha_{fmD} = 2.852$

Computing or checking slab thickness

Panel A

$\ell_n = 8000 - 600 = 7400$ mm ; $S_n = 6000 - 600 = 5400$ mm

$$\beta = \frac{\ell_n}{S_n} = \frac{7400}{5400} = 1.37$$

$\alpha_{mA} = 4.448$ ; here $\alpha_m > 2.0$ ; use Eq. (2)

$$t = \frac{\ell_n\,(0.8 + \frac{f_y}{1400})}{36 + 9\beta} = \frac{7400 \times (0.8 + \frac{350}{1400})}{36 + 9 \times 1.37} = 158.2 \text{ mm} \qquad \text{say } 160 \text{ mm} > 90 \text{ mm}$$

Edge beam (B1 and B5) have $\alpha > 0.8$ ∴ t = 160 mm

Summary of required slab thickness

| A | B | C | D |
|---|---|---|---|
| 160 | 160 | 160 | 160 |

t = 160 mm > 90 mm ∴ O.K. $t_{min} = 160$ mm

$t_{actual} = 180$ mm > 160 mm ∴ O.K.

Computing C

For B5 and B6

$$C = \sum \left(1 - 0.63\ \frac{x}{y}\right)\frac{x^3\, y}{3}$$

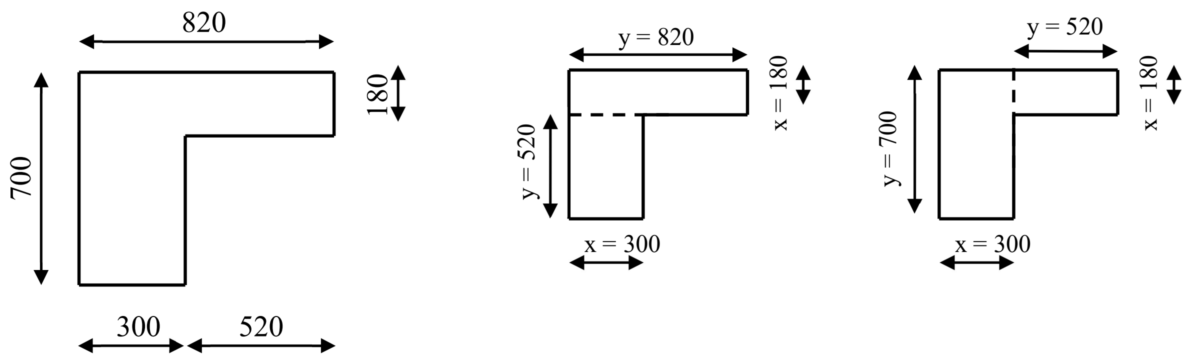
$$C_1 = \left(1 - 0.63\ \times \frac{180}{820}\right)\frac{(180)^3 \times 820}{3} + \left(1 - 0.63\ \times \frac{300}{520}\right)\frac{(300)^3 \times 520}{3}$$

$$= 4.353 \times 10^9\ \ \mathrm{mm}^4$$

$$C_2 = \left(1 - 0.63\ \times \frac{300}{700}\right)\frac{(300)^3 \times 700}{3} + \left(1 - 0.63\ \times \frac{180}{520}\right)\frac{(180)^3 \times 520}{3}$$

$$= 5.191 \times 10^9\ \ \mathrm{mm}^4$$

∴ For beam B5 and B6 $C = 5.191 \times 10^9\ \mathrm{mm}^4$




B1 and B2

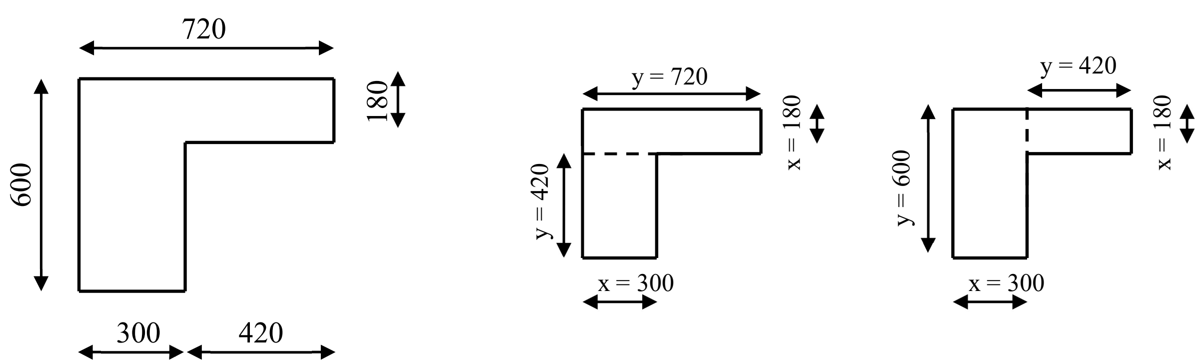
$$C_1 = \left(1 - 0.63\ \times \frac{180}{720}\right)\frac{(180)^3 \times 720}{3} + \left(1 - 0.63\ \times \frac{300}{420}\right)\frac{(300)^3 \times 420}{3}$$

$$= 3.258 \times 10^9\ \ \mathrm{mm}^4$$

$$C_2 = \left(1 - 0.63\ \times \frac{180}{420}\right)\frac{(180)^3 \times 420}{3} + \left(1 - 0.63\ \times \frac{300}{600}\right)\frac{(300)^3 \times 600}{3}$$

$$= 4.295 \times 10^9\ \ \mathrm{mm}^4$$

∴ For beam B1 and B2 $C = 4.295 \times 10^9\ \mathrm{mm}^4$



Computing $\beta_t$

$$\beta_t = \frac{E_{cb} C}{2\,E_{cs} I_s} = \frac{C}{2\,I_s} \qquad ; \qquad E_{cb} = E_{cs}$$

For B5 and B6

$$I_s = \frac{1}{12}\ell_2 t^3 = \frac{1}{12} \times 8000 \times (180)^3 = 3.888 \times 10^9 \ \text{mm}^4$$

$$\beta_t = \frac{C}{2\,I_s} = \frac{5.191 \times 10^9}{2 \times 3.888 \times 10^9} = 0.693$$

For beam B1 and B2

$$I_s = \frac{1}{12} \times 6000 \times (180)^3 = 2.916 \times 10^9 \ \text{mm}^4$$

$$\beta_t = \frac{4.295 \times 10^9}{2 \times 2.916 \times 10^9} = 0.736$$

<u>Exterior longitudinal frame</u>

D.L. = 4.5 (slab) + 2.0 (tiles) + 1.0 (partition) + 0.08 (fall ceiling) = 7.58 kN/$m^2$

L.L. = 4.25 kN/$m^2$

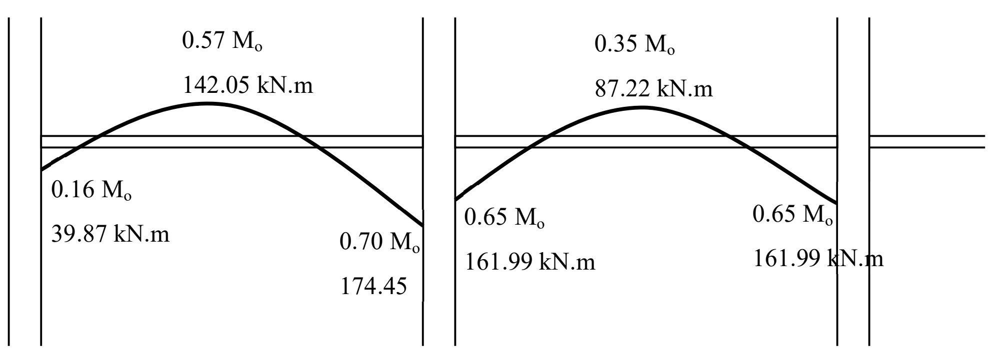
$q_u$ = 1.2×7.58 + 1.6× 4.25 = 15.9 kN/$m^2$

$$\ell_2 = \frac{8000}{2} + \frac{600}{2} = 4300 \ \text{mm}$$

$$\ell_n = 6000 - 600 = 5400 \ \text{mm}$$

$$M_o = \frac{1}{8} q_u \ell_2 \ell_n^2 = \frac{1}{8} \times 15.9 \times 4.3 \times (5.4)^2 = 249.21 \ \text{kN.m}$$

<u>Longitudinal distribution of moments</u>:



Transverse distribution of longitudinal moments

End span

Negative moment at exterior support (total = -0.16 $M_o$= -39.87 kN.m)

need $\frac{\alpha_{f1}\ell_2}{\ell_1}$ , $\beta_t$ , and $\frac{\ell_2}{\ell_1}$

Here $\alpha_{f1} = \alpha_{fB1} = 3.823$, $\ell_2 = 8000$ mm, $\ell_1 = 6000$ mm

$$\frac{\ell_2}{\ell_1} = \frac{8000}{6000} = 1.333 \quad \& \quad \frac{\alpha_1\ell_2}{\ell_1} = \frac{3.823 \times 8000}{6000} = 5.10 > 1.0$$

$\beta_t = \beta_{tB5} = 0.693 \approx 0.69$

| $\ell_2/\ell_1$ | | 1.0 | 1.333 | 2.00 |
|---|---|---|---|---|
| $\frac{\alpha_{f1}\ell_2}{\ell_1} > 1.0$ | $\beta_t = 0$ | 100 | 100 | 100 |
| | $\beta_t = 0.69$ | | **90.34** | |
| | $\beta_t \geq 2.5$ | 75 | 65 | 45 |

$$\frac{y}{0.667} = \frac{30}{1} \rightarrow \quad y = 20$$

∴ Neg. moment in column strip = 39.87 × 0.903 = 36.02 kN.m

Neg. moment in beam = 36.02 × 0.85 = 30.62 kN.m

Neg. moment in column strip slab = 36.02 – 30.62 = 5.4 kN.m

Neg. moment in middle strip = 39.87 - 36.02 = 3.85 kN.m

Positive moments (total = 0.57 $M_o$ = 142.05 kN.m)

| $\ell_2/\ell_1$ | 1.0 | 1.333 | 2.0 |
|---|---|---|---|
| $\frac{\alpha_{f1}\ell_2}{\ell_1} > 1.0$ | 75 | **65** | 45 |

Moment in column strip = 142.05 × 0.65 = 92.33 kN.m

Moment in beam = 92.33 × 0.85 = 78.48 kN.m

Moment in column strip slab = 92.33 – 78.48 = 13.85 kN.m

Moment in middle strip = 142.05 – 92.33 = 49.72 kN.m

Interior negative moment (total = 0.70 $M_o$ = -174.45 kN.m)

| $\ell_2/\ell_1$ | 1.0 | 1.333 | 2.0 |
|---|---|---|---|
| $\frac{\alpha_{f1}\ell_2}{\ell_1} > 1.0$ | 75 | 65 | 45 |

Moment in column strip = 174.45 × 0.65 = -113.39 kN.m
Moment in beam = 113.39 × 0.85 = -96.38 kN.m
Moment in column strip slab = 113.39 – 96.38 = -17.01 kN.m
Moment in middle strip = 174.45 – 113.39 = -61.06 kN.m

Interior span
Negative moment (total = -0.65 $M_o$ = -161.99 kN.m)
Negative moment in column Strip = 161.99 × 0.65 = 105.29 kN.m
Negative moment in beam = 105.29 × 0.85 = 89.50 kN.m
Negative moment in column strip slab = 105.29 – 89.5 = 15.79 kN.m
Negative moment in middle strip = 161.99 – 105.29 = 56.7 kN.m

Positive moment (total = 0.35 $M_o$ = 87.22 kN.m)
Moment in column strip = 87.22 × 0.65 = 56.69 kN.m
Moment in beam = 56.69 × 0.85 = 48.19 kN.m
Moment in column strip slab = 56.69 – 48.19 = 8.5 kN.m
Moment in middle strip = 87.22 – 56.69 = 30.53 kN.m

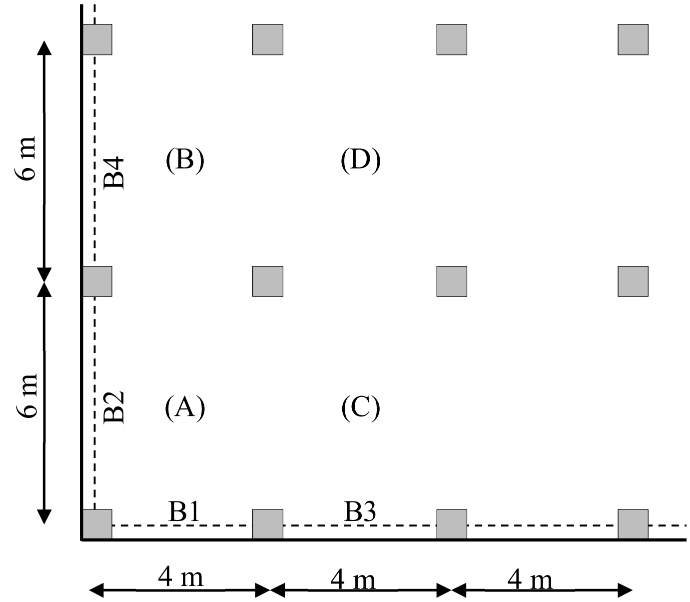
Moments in Exterior longitudinal frame
Total width = 4.3 m, column strip width = 1.8 m, & half middle strip width = 2.5 m.

| | Exterior span | | | Interior span | |
|---|---|---|---|---|---|
| | Exterior negative | Positive | Interior negative | Negative | Positive |
| Total moment (kN.m) | -39.87 | +142.05 | -174.45 | -161.99 | +87.22 |
| Moment in beam (kN.m) | -30.62 | +78.48 | -96.38 | -89.50 | +48.19 |
| Moment in column strip slab (kN.m) | -5.4 | +13.85 | -17.01 | -15.79 | +8.50 |
| Moment in middle strip slab (kN.m) | -3.85 | +49.72 | -61.06 | -56.70 | +30.53 |

## General Example 2

Flat plate with edge beams

- Edge beams are 250 × 500 mm
- All columns are 500 × 500 mm
- Slab thickness = 200 mm
- Live load = 4.0 kN/m$^2$
- $\gamma_{concrete}$ = 24 kN/m$^3$




Solution

(1) Computing $\alpha_f$

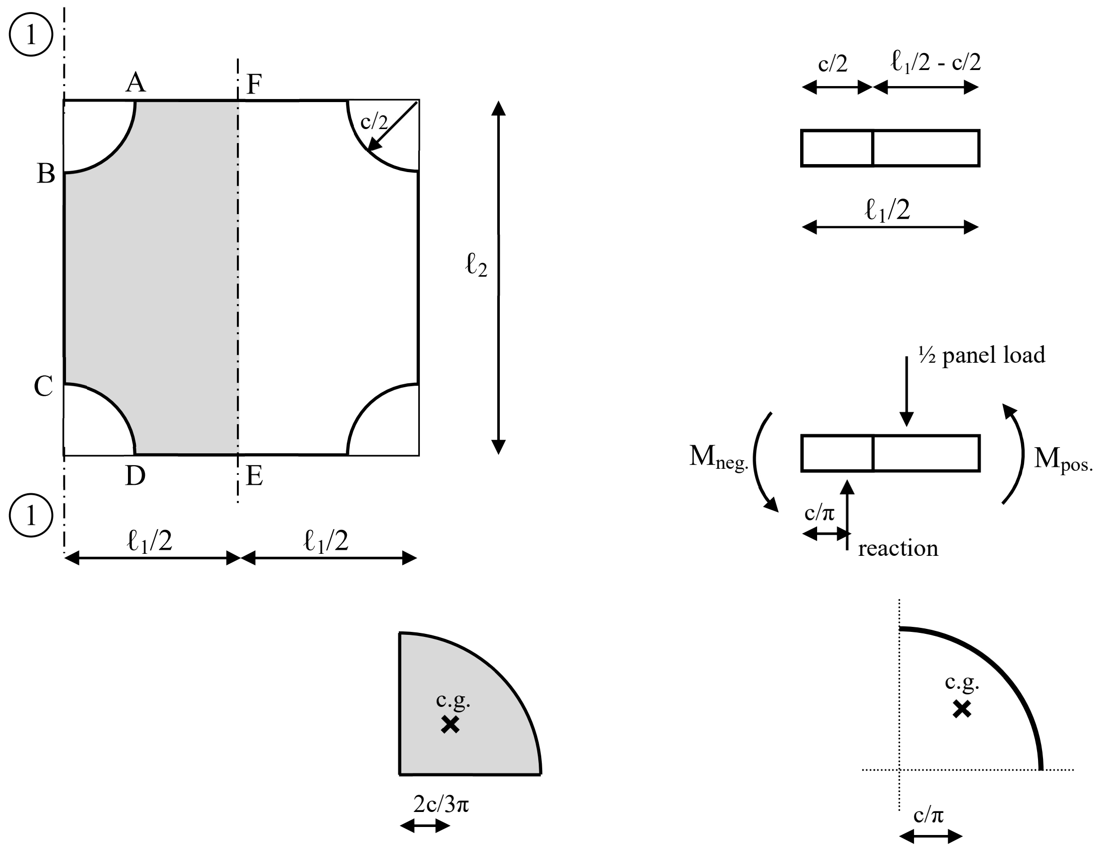
Compute the ratio of the flexural stiffness of the longitudinal beams to that of the slab ($\alpha_f$) in the equivalent rigid frame, for all edge beams.

Beam sections

B1 and B3

## Total static moment in flat slab

c = diameter of column capital




Sum of reactions on arcs AB and CD = load on area ABCDEF

$$= q_u \left\{ \ell_2 \,\frac{\ell_1}{2} - 2\left(\frac{1}{4}\,\pi\left(\frac{c}{2}\right)^2\right)\right\}$$

$$= q_u \left\{ \frac{\ell_2\,\ell_1}{2} - \frac{\pi\, c^2}{8} \right\}$$

No shear along lines AF, BC, DE, EF

$\sum M_{1\text{-}1} = 0$

$$M_{neg.} + M_{pos.} + q_u \left\{ \frac{\ell_2\,\ell_1}{2} - \frac{\pi\, c^2}{8} \right\} \frac{c}{\pi} - \frac{q_u\,\ell_2\,\ell_1}{2}\left(\frac{\ell_1}{4}\right) + q_u \times 2\left(\frac{1}{4}\,\frac{\pi\, c^2}{4} \times \frac{2\,c}{3\,\pi}\right) = 0$$

previously $\qquad M_o = \dfrac{q_u\,\ell_2\,\ell_n{}^2}{8} \qquad \ldots\ldots(1)$

Letting $M_o = M_{neg.} + M_{pos.}$

$$M_o = \frac{q_u\,\ell_2\,\ell_1{}^2}{8}\left(1 - \frac{4\,c}{\pi\,\ell_1} + \frac{c^3}{3\,\ell_2\,\ell_1{}^2}\right)$$

$$M_o \approx \frac{q_u\,\ell_2\,\ell_1{}^2}{8}\left(1 - \frac{2\,c}{3\,\ell_1}\right)^2 \qquad \ldots\ldots(2)$$

Eq. (1) is useful for flat plate floor or two – way slab with beams, while Eq. (2) is more suitable for flat slab, where in round column capitals are used.

Example:

Compute the total factored static moment in the long and short directions for an interior panel in flat slab 6 × 7 m, given $q_u = 15$ kN/m$^2$, column capital = 1.40 m.

Solution:-

a- In long direction

$$\mathrm{M_o} = \frac{q_u\,\ell_2\,{\ell_1}^2}{8}\left(1-\frac{2\,\mathrm{c}}{3\,\ell_1}\right)^2 = \frac{15\ \times 6\ \times\ (7)^2}{8}\left(1-\frac{2\ \times 1.4}{3\ \times 7}\right)^2 = 414\ \ \mathrm{kN.m}$$

b- In short direction

$$\mathrm{M_o} = \frac{15\ \times 7\ \times\ (6)^2}{8}\left(1-\frac{2\ \times 1.4}{3\ \times 6}\right)^2 = 337\ \ \mathrm{kN.m}$$

To compare with previous method:-

a- In long direction

$\ell_n = 7.0 - 0.89 \times 1.4 = 5.754$ m

$$\mathrm{M_o}\ = \frac{q_u\,\ell_2\,{\ell_\mathrm{n}}^2}{8} = \frac{15\ \times 6\ \times\ (5.754)^2}{8} = 372.4\ \ \mathrm{kN.m}$$

b- In short direction

$\ell_n = 6.0 - 0.89 \times 1.4 = 4.754$ m

$$\mathrm{M_o}\ = \frac{15\ \times 7\ \times\ (4.754)^2}{8} = 296.6\ \ \mathrm{kN.m}$$

| | Eq. 1 (kN.m) | Eq. 2 (kN.m) | Error (%) |
|---|---|---|---|
| long direction | 414 | 372.4 | 10 |
| short direction | 337 | 296.4 | 12 |

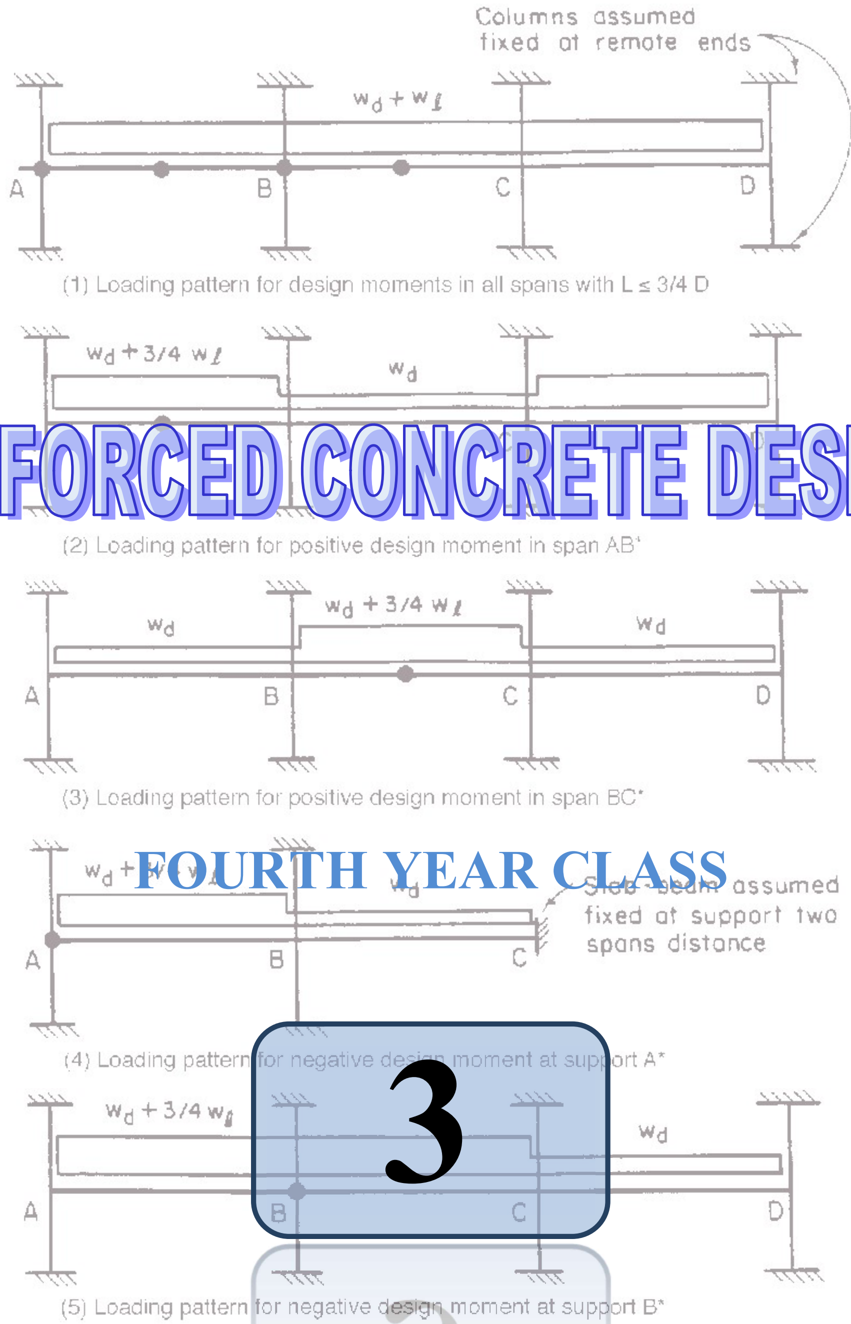
**University of Baghdad**

**College of Engineering**

**Department of Civil Engineering**

# REINFORCED CONCRETE DESIGN II

## FOURTH YEAR CLASS




# 3

# 2018 - 2019

MHND

## Equivalent frame method (EFM)

The equivalent frame method involves the representation of the three-dimensional slab system by a series of two-dimensional frames that are then analyzed for loads acting in the plane of the frames. The negative and positive moments so determined at the critical design sections of the frame are distributed to the slab sections (column strip, middle strip and beam).

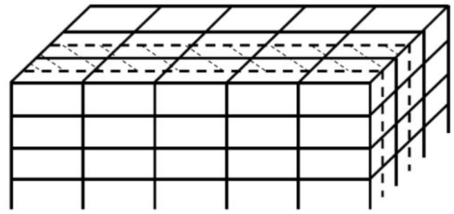
Limitations:

1) Panels shall be rectangular, with a ratio of longer to shorter panel dimensions, measured center-to-center of supports, not to exceed 2.
2) Live load shall be arranged in accordance with arrangement of live loads.
3) Complete analysis must include representative interior and exterior equivalent frames in both the longitudinal and transverse directions of the floor.

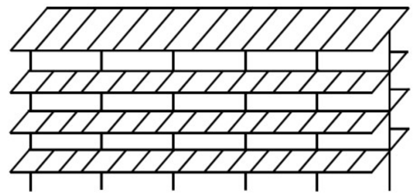
Procedure:-

1- Divide the structure into longitudinal and transverse frames centered on column and bounded by panels.
2- Each frame shall consist of a row of columns and slab-beam strips, bounded laterally by of panels.
3- Columns shall be assumed to be attached to slab-beam strips by torsional members transverse to the direction of the span for which moment are being determined.
4- Frames adjacent and parallel to an edge shall be bounded by that edge and the centerline of adjacent panel.
5- The slab–beam may be assumed to be fixed at any support two panels distance from the support of the span where critical moments are being obtained, provided the slab is continuous beyond that point.

Selected frame in 3-D building

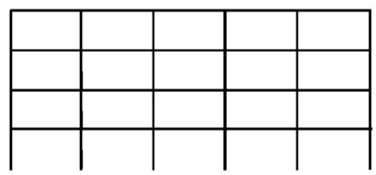


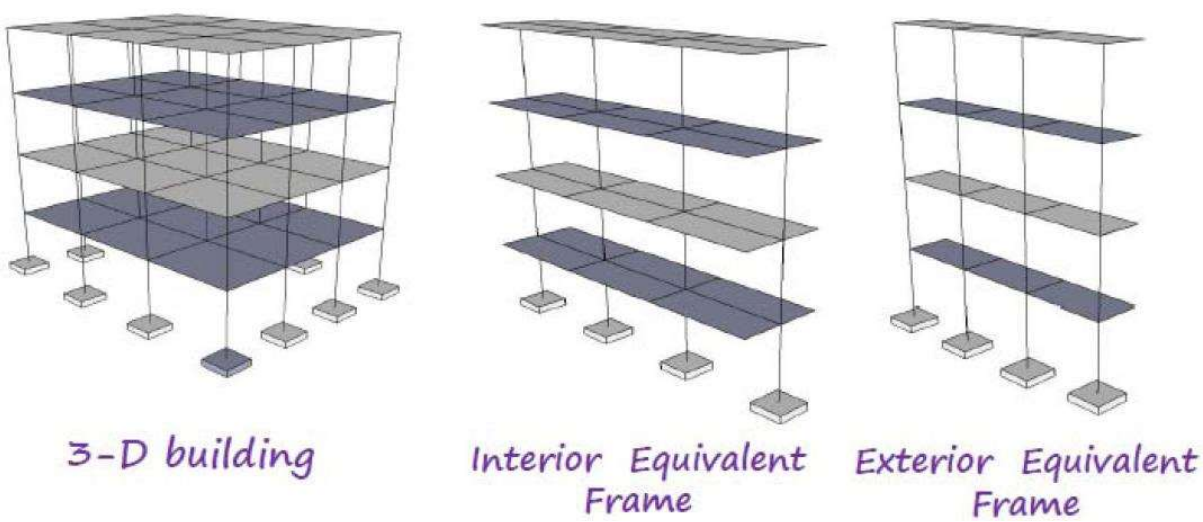
The detached frame alone



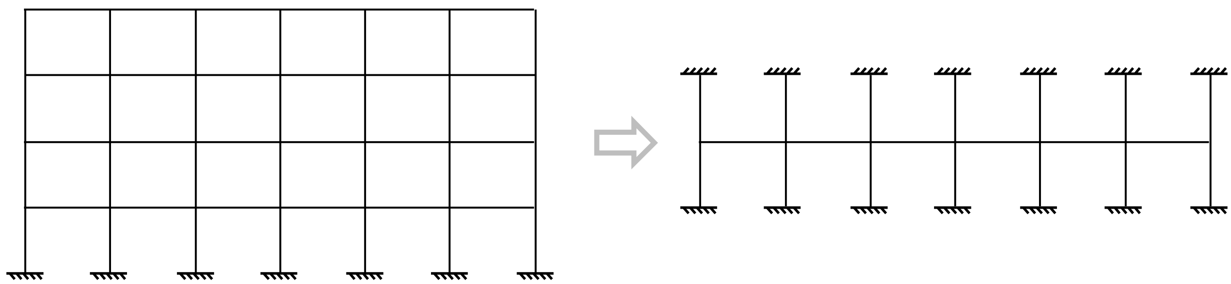
The width of the frame is same as mentioned in DDM. The length of the frame extends up to full length of 3-D system and the frame extends the full height of the building.

2-D frame

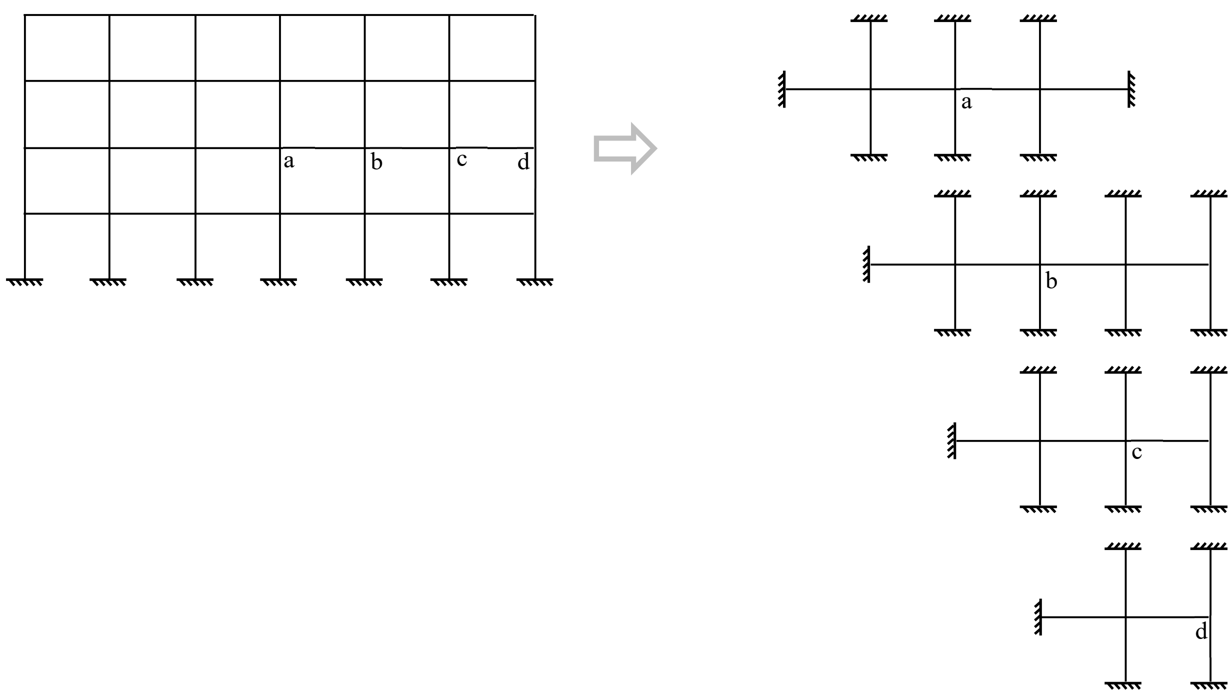




Analysis of each equivalent frame in its entirety shall be permitted. Alternatively, for gravity loading, a separate analysis of each floor or roof with the far ends of columns considered fixed is permitted.

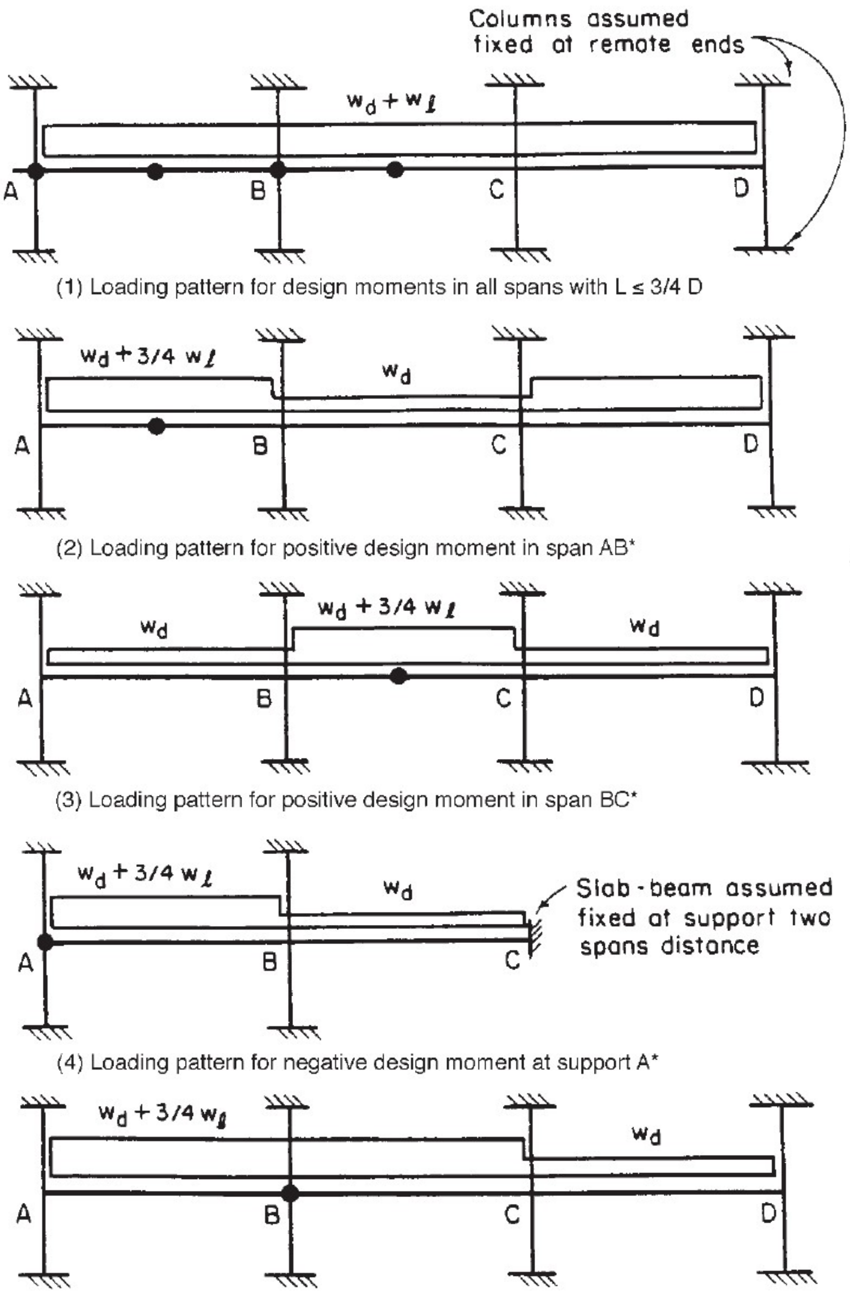


If slab-beams are analyzed separately, it shall be permitted to calculate the moment at a given support by assuming that the slab-beam is fixed at supports two or more panels away, provided the slab continues beyond the assumed fixed supports.



Arrangement of live loads:

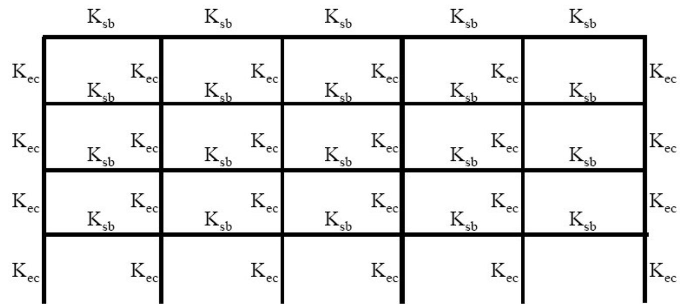
1- If the arrangement of L is known, the slab system shall be analyzed for that arrangement.
2- If all panels will be loaded with L, the slab system shall be analyzed when full factored L on all spans.
3- If the arrangement of L is unknown:
   - a- $L \leq 0.75\ D \Rightarrow$ Maximum factored moment when full factored L on all spans.
   - b- $L > 0.75\ D \Rightarrow$ Pattern live loading using 0.75(factored L) to determine maximum factored moment.




Partial frame analysis for vertical loading

Stiffness calculation:

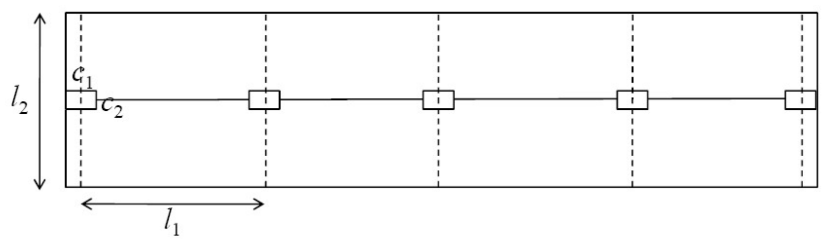
- Stiffness of Slab-Beam Member
- Stiffness of Equivalent Column
    - Stiffness of Column
    - Stiffness of Torsional Member




$K_{sb}$ represents the combined stiffness of slab and longitudinal beam (if any).
$K_{ec}$ represents the modified column stiffness. The modification depends on lateral members (slab, beams etc.) and presence of column in the story above.

Once a 2-D frame is obtained, the analysis can be done by any method of 2-D frame analysis.

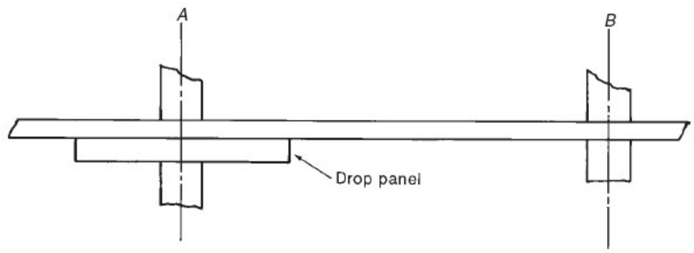
Stiffness of slab beam member ($K_{sb}$):
The stiffness of slab beam ($K_{sb}$= $kEI_{sb}/\ell$) consists of combined stiffness of slab and any longitudinal beam present within.
For a span, the k factor is a direct function of ratios $c_1/\ell_1$ and $c_2/\ell_2$.
Tables are available in literature for determination of k for various conditions of slab systems.



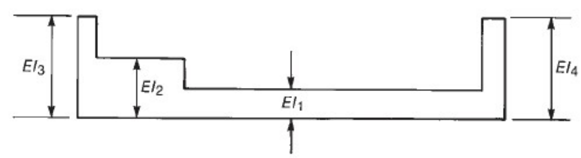

In the moment-distribution method, it is necessary to compute *flexural stiffnesses, K; carryover factors, COF; distribution factors, DF*; and *fixed-end moments, FEM*, for each of the members in the structure. For a prismatic member fixed at the far end and with negligible axial loads, the flexural stiffness is:

$$K = k\,\frac{E\,I}{l}$$

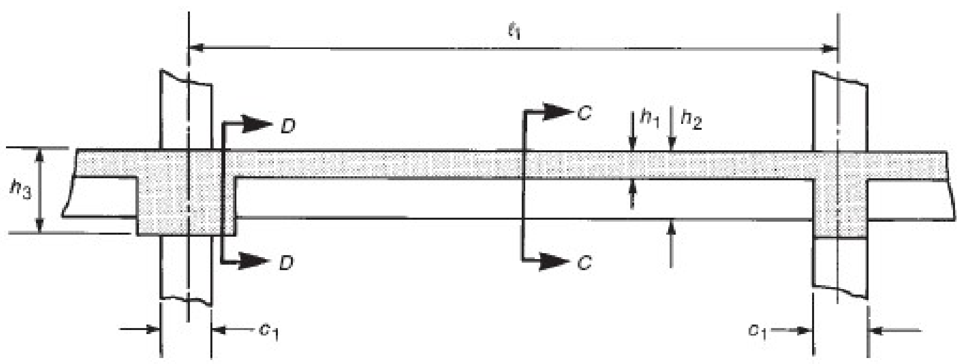
where k = 4 and the carryover factor is 0.5, the sign depending on the sign convention used for moments. For a prismatic, uniformly loaded beam, the fixed-end moments are $w\ell^2/12$.
In the equivalent-frame method, the increased stiffness of members within the column–slab joint region is accounted for, as is the variation in cross section at drop panels. As a result, all members have a stiffer section at each end, as shown in Figure. If the *EI* used is that at the midspan of the slab strip, *k* will be greater than 4; similarly, the carryover factor will be greater than 0.5, and the fixed-end moments for a uniform load (w) will be greater than $w\ell^2/12$.



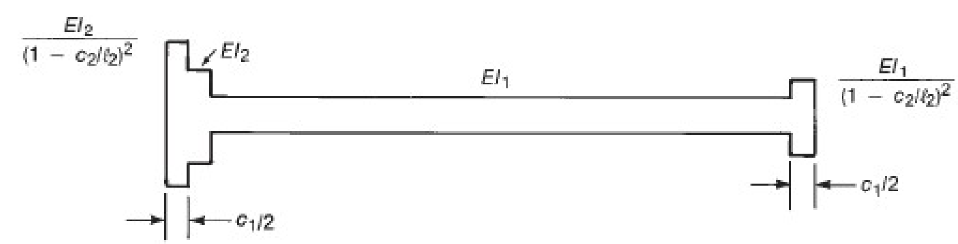

(a) Slab $A$–$B$.



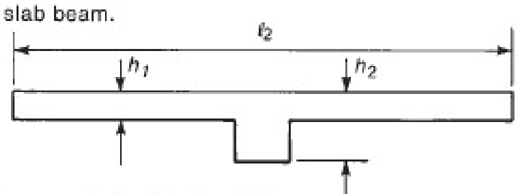

(b) Distribution of $EI$ along slab.



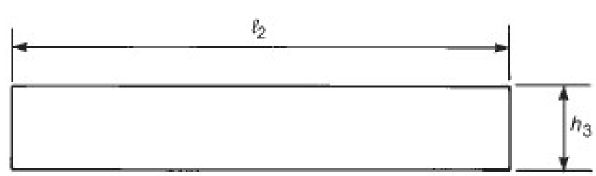

(a) Slab with beams in two directions.



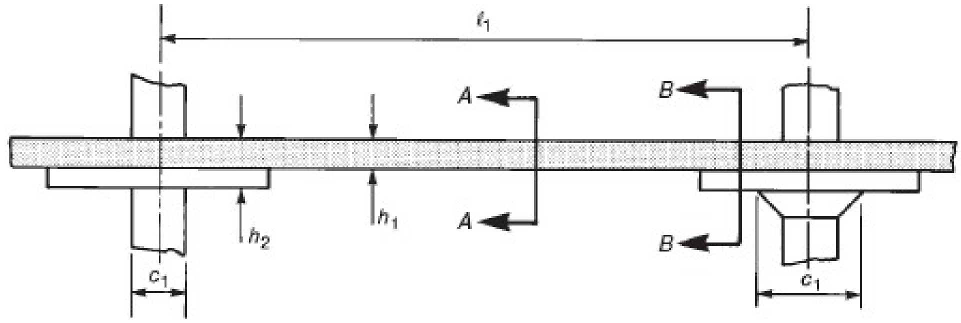

(b) Variation in $EI$ along slab beam.



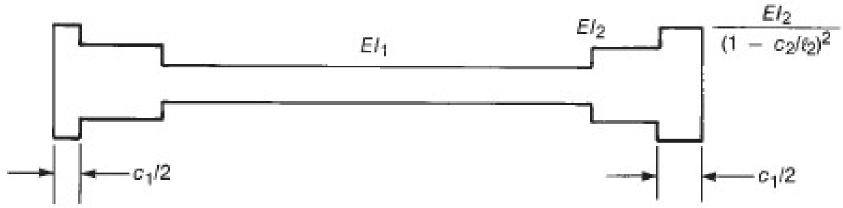

(c) Cross section used to compute $I_1$—Section $C$–$C$



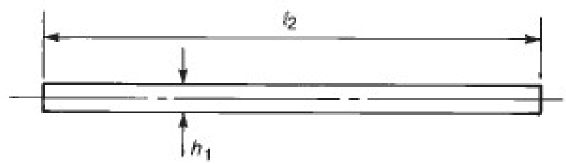

(d) Cross section used to compute $I_2$—Section $D$–$D$



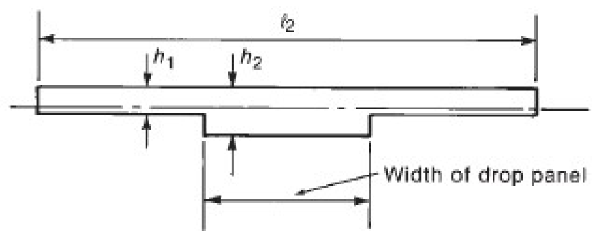

(a) Slab with drop panels.




(b) Variation in $EI$ along slab-beam.




(c) Cross section used in compute $I_1$—Section $A$-$A$.




(d) Cross section used to compute $I_2$—Section $B$-$B$.

Several methods are available for computing values of *k, COF*, and *FEM*. Originally; these were computed by using the column analogy.

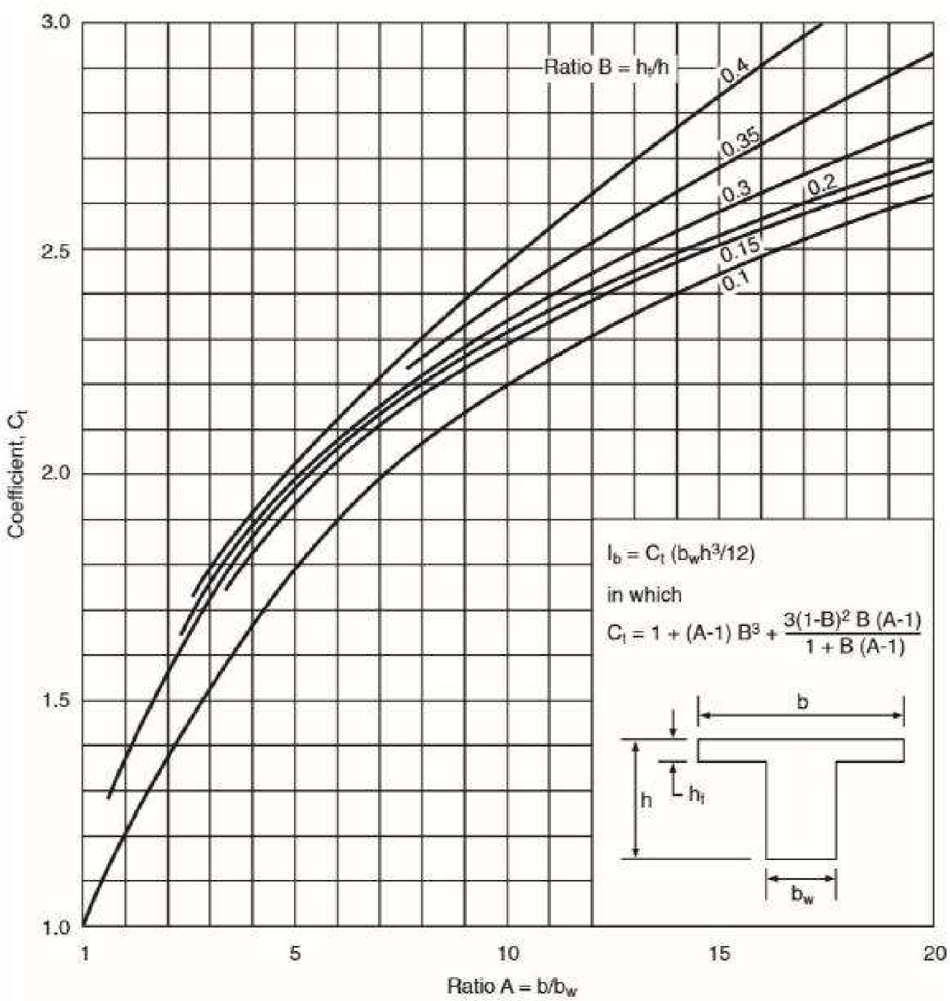
Properties of Slab–Beams

The horizontal members in the equivalent frame are referred to as *slab-beams*. These consist of either only a slab, or a slab and a drop panel, or a slab with a beam running parallel to the equivalent frame.

It shall be permitted to use the gross cross-sectional area of concrete to determine the moment of inertia of slab-beams at any cross section outside of joints or column capitals.

The moment of inertia of the slab-beams from the center of the column to the face of the column, bracket, or capital shall be taken as the moment of inertia of the slab-beam at the face of the column, bracket, or capital divided by the quantity $(1 - c_2/\ \ell_2)^2$, where $\ell_2$ is the transverse width of the equivalent frame and $c_2$ is the width of the support parallel to $\ell_2$.

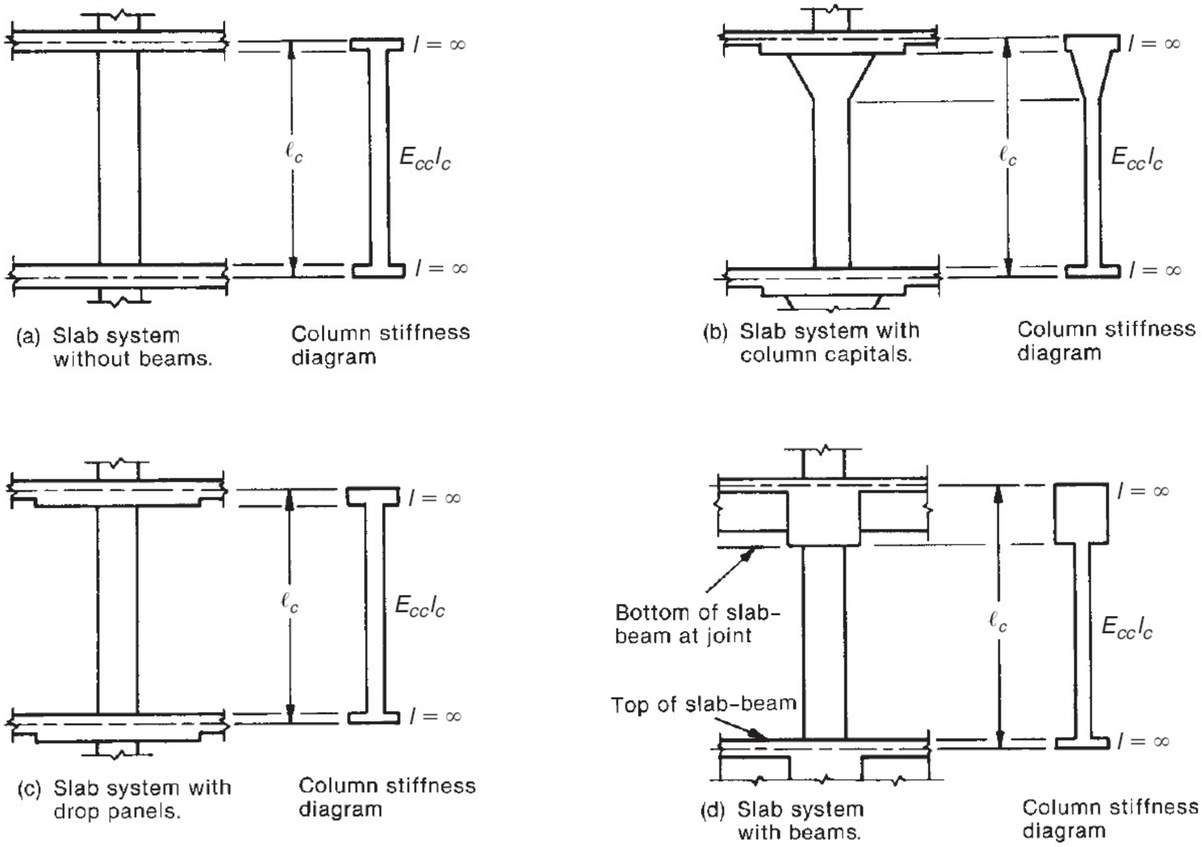
Moment of inertia of the slab-beam strip can be calculated from the following figure or equation:



Properties of Columns

The moment of inertia of columns at any cross section outside of the joints or column capitals may be based on the gross area of the concrete.

The moment of inertia of columns shall be assumed to be infinite within the depth of the slab-beam at a joint.




Sections for the calculations of column stiffness ($K_c$)

$\ell_c$ is the overall height and $\ell_u$ is the unsupported or clear height.

$$K_t = \sum \frac{9\, E_{cs}\, C}{\ell_2 \left(1 - \frac{c_2}{\ell_2}\right)^3}$$

where $\ell_2$ refers to the transverse spans on each side of the column. For a corner column, there is only one term in the summation.

If a beam parallel to the $\ell_1$ direction, multiply $K_t$ by the ratio $I_{sb}/I_s$, where $I_{sb}$ is the moment of inertia of the slab and beam together and $I_s$ is the moment of inertia of the slab neglecting the beam stem.

$$\frac{1}{K_{ec}} = \frac{1}{\sum K_c} + \frac{1}{K_t}$$

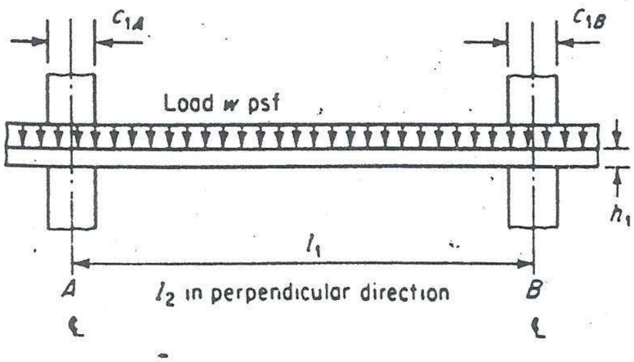
Factored moments

At interior supports, the critical section for negative $M_u$ in both column and middle strips shall be taken at the face of rectilinear supports, but not farther away than $0.175\ell_1$ from the center of a column.

At exterior supports without brackets or capitals, the critical section for negative $M_u$ in the span perpendicular to an edge shall be taken at the face of the supporting element.

At exterior supports with brackets or capitals, the critical section for negative $M_u$ in the span perpendicular to an edge shall be taken at a distance from the face of the supporting element not exceeding one-half the projection of the bracket or capital beyond the face of the supporting element.

**Table A.13a Coefficients for slabs with variable moment of inertia†**




| Column dimension | | Uniform load FEM = coeff. $(wl_2 l_1^2)$ | | Stiffness factor‡ | | Carryover factor | |
|---|---|---|---|---|---|---|---|
| $\frac{c_{1A}}{l_1}$ | $\frac{c_{1B}}{l_1}$ | $M_{AB}$ | $M_{BA}$ | $k_{AB}$ | $k_{BA}$ | $COF_{AB}$ | $COF_{BA}$ |
| 0.00 | 0.00 | 0.083 | 0.083 | 4.00 | 4.00 | 0.500 | 0.500 |
| | 0.05 | 0.083 | 0.084 | 4.01 | 4.04 | 0.504 | 0.500 |
| | 0.10 | 0.082 | 0.086 | 4.03 | 4.15 | 0.513 | 0.499 |
| | 0.15 | 0.081 | 0.089 | 4.07 | 4.32 | 0.528 | 0.498 |
| | 0.20 | 0.079 | 0.093 | 4.12 | 4.56 | 0.548 | 0.495 |
| | 0.25 | 0.077 | 0.097 | 4.18 | 4.88 | 0.573 | 0.491 |
| 0.05 | 0.05 | 0.084 | 0.084 | 4.05 | 4.05 | 0.503 | 0.503 |
| | 0.10 | 0.083 | 0.086 | 4.07 | 4.15 | 0.513 | 0.503 |
| | 0.15 | 0.081 | 0.089 | 4.11 | 4.33 | 0.528 | 0.501 |
| | 0.20 | 0.080 | 0.092 | 4.16 | 4.58 | 0.548 | 0.499 |
| | 0.25 | 0.078 | 0.096 | 4.22 | 4.89 | 0.573 | 0.494 |
| 0.10 | 0.10 | 0.085 | 0.085 | 4.18 | 4.18 | 0.513 | 0.513 |
| | 0.15 | 0.083 | 0.088 | 4.22 | 4.36 | 0.528 | 0.511 |
| | 0.20 | 0.082 | 0.091 | 4.27 | 4.61 | 0.548 | 0.508 |
| | 0.25 | 0.080 | 0.095 | 4.34 | 4.93 | 0.573 | 0.504 |
| 0.15 | 0.15 | 0.086 | 0.086 | 4.40 | 4.40 | 0.526 | 0.526 |
| | 0.20 | 0.084 | 0.090 | 4.46 | 4.65 | 0.546 | 0.523 |
| | 0.25 | 0.083 | 0.094 | 4.53 | 4.98 | 0.571 | 0.519 |
| 0.20 | 0.20 | 0.088 | 0.088 | 4.72 | 4.72 | 0.543 | 0.543 |
| | 0.25 | 0.086 | 0.092 | 4.79 | 5.05 | 0.568 | 0.539 |
| 0.25 | 0.25 | 0.090 | 0.090 | 5.14 | 5.14 | 0.563 | 0.563 |

† Applicable when $c_1/l_1 = c_2/l_2$. For other relationships between these ratios, the constants will be slightly in error.

‡ Stiffness is $K_{AB} = k_{AB}\,E(l_2 h_1^3/12l_1)$ and $K_{BA} = k_{BA}\,E(l_2 h_1^3/12l_1)$.