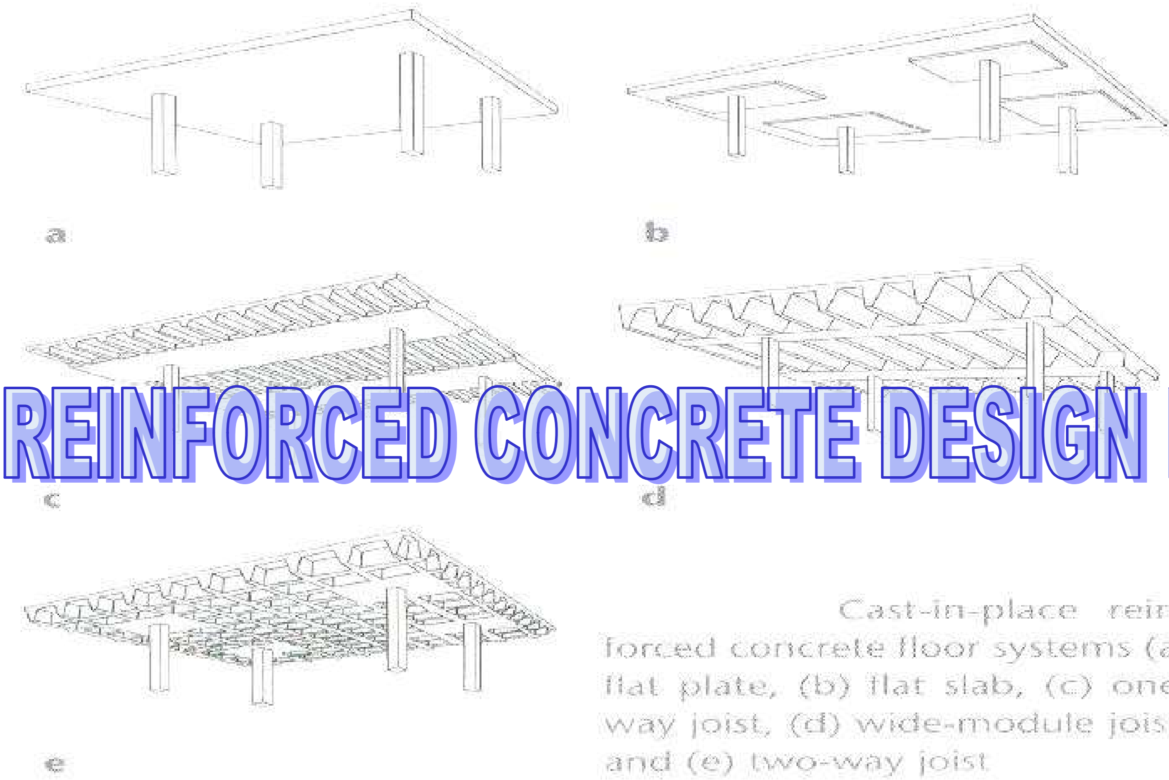
University of Baghdad

College of Engineering

Department of Civil Engineering



REINFORCED CONCRETE DESIGN II

Cast-in-place reinforced concrete floor systems (a) flat plate, (b) flat slab, (c) one-way joist, (d) wide-module joist, and (e) two-way joist

FOURTH YEAR CLASS

1

2018 - 2019

MHND

COURSE SYLLABUS

- Structural system and load paths
- Types of slabs
- Design of one-way slab
- Minimum slab thickness of two-way slabs
- Design of two-way slab
- General design concept of ACI Code
- Direct design method
- Total static moment in flat slab
- Equivalent frame method
- Shear in slab system with beams
- Shear strength in flat plate and flat slab (one way and punching shear)
- Transfer of moment at columns
- Yield line theory
- Prestressed concrete
- Design of stair case

TEXT BOOK AND REFERENCES

- Design of Concrete Structures; Arthur H. Nilson, David Darwin, and Charles W. Dolan.

- Reinforced Concrete Mechanics and Design; James K. Wight and James G. Macgregor.

- Building Design and Construction Handbook; Frederick S. Merritt and Jonathan T. Ricketts

- Structural details in Concrete; M. Y. H. Bangash.

- Manual for the Design of Reinforced Concrete Building Structures; the Institute of Structural Engineers.

- Reinforced Concrete Analysis and Design; S. S. Ray.

- ACI 318, Building Code Requirements for Structural Concrete (ACI 318M-14) and Commentary (ACI 318RM-14), ACI Committee 318, American Concrete Institute, Farmington Hills, MI, 2014.

- Shear Reinforcement for Slabs; Reported by ACI-ASCE Committee 421.

Structural system and load paths

The structural system shall include (a) through (g), as applicable:

(a) Floor construction and roof construction, including one-way and two-way slabs

(b) Beams and joists

(c) Columns

(d) Walls

(e) Diaphragms

(f) Foundations

(g) Joints, connections, and anchors as required to transmit forces from one component to another.

Load factors and combinations

According to ACI 318M-14, the required strength (U) shall be at least equal to the effects of factored loads in Table 5.3.1, with exceptions and additions in 5.3.3 through 5.3.12.

Table 5.3.1—Load combinations

Load combination	Equation	Primary load
$U = 1.4D$	(5.3.1a)	D
$U = 1.2D + 1.6L + 0.5(L_r \text{ or } S \text{ or } R)$	(5.3.1b)	L
$U = 1.2D + 1.6(L_r \text{ or } S \text{ or } R) + (1.0L \text{ or } 0.5W)$	(5.3.1c)	L_r or S or R
$U = 1.2D + 1.0W + 1.0L + 0.5(L_r \text{ or } S \text{ or } R)$	(5.3.1d)	W
$U = 1.2D + 1.0E + 1.0L + 0.2S$	(5.3.1e)	E
$U = 0.9D + 1.0W$	(5.3.1f)	W
$U = 0.9D + 1.0E$	(5.3.1g)	E

All members and structural systems shall be analyzed for the maximum effects of loads including the arrangements of live load in accordance with 6.4.

Strength

Design strength of a member and its joints and connections, in terms of moment, axial force, shear, torsion, and bearing, shall be taken as the nominal strength (S_n) multiplied by the applicable strength reduction factor (ø).

Structures and structural members shall have design strength at all sections (ø S_n) greater than or equal to the required strength (U) calculated for the factored loads and forces in such combinations as required by ACI-Code.

design strength $\geq$ required strength

$ø\, S_n \geq U$

Types of slabs

1. One-way slab: Slabs may be supported on two opposite sides only, in such case, the structural action of the slab is essentially "one-way", and the loads are carried by the slab in the direction perpendicular to the supporting beams, Figure (1-a).
2. Tow-way slab: Slabs have beam or support on all four sides. The loads are carried by the slab in two perpendicular directions to the supporting beams, Figure (1-b).
3. If the ratio of length to width of one slab panel is larger than 2, most of the load is carried by the short direction to the supporting beams, and one-way action is obtained in effect, even though supports are provided on all sides, Figure (1-c).
4. Concrete slab carried directly by columns, without the use of beams or girders, such slab is described by flat plates, and are commonly used where spans are not large and loads are not heavy, Figure (1-d).
5. Flat slabs are also beamless slab with column capitals, drop panels, or both, Figure (1-e).
6. Two–way joist systems (grid slab), to reduce the dead load of solid-slab, voids are formed in a rectilinear pattern through use of metal or fiberglass form inserts. A two–way ribbed construction results (waffle slab). Usually inserts are omitted near the columns, Figure (1-f).

One-way slabs: slabs reinforced to resist flexural stresses in only one direction.

Two-way slabs: reinforced for flexure in two directions.

Column capital: enlargement of the top of a concrete column located directly below the slab or drop panel that is cast monolithically with the column.

Drop panel: projection below the slab used to reduce the amount of negative reinforcement over a column or the minimum required slab thickness, and to increase the slab shear strength.

Panel: slab portion bounded by column, beam, or wall centerlines on all sides.

Column strip: a design strip with a width on each side of a column centerline equal to the lesser of 0.25 ℓ_2 and 0.25 ℓ_1. A column strip shall include beams within the strip, if present.

Middle strip: a design strip bounded by two column strips.

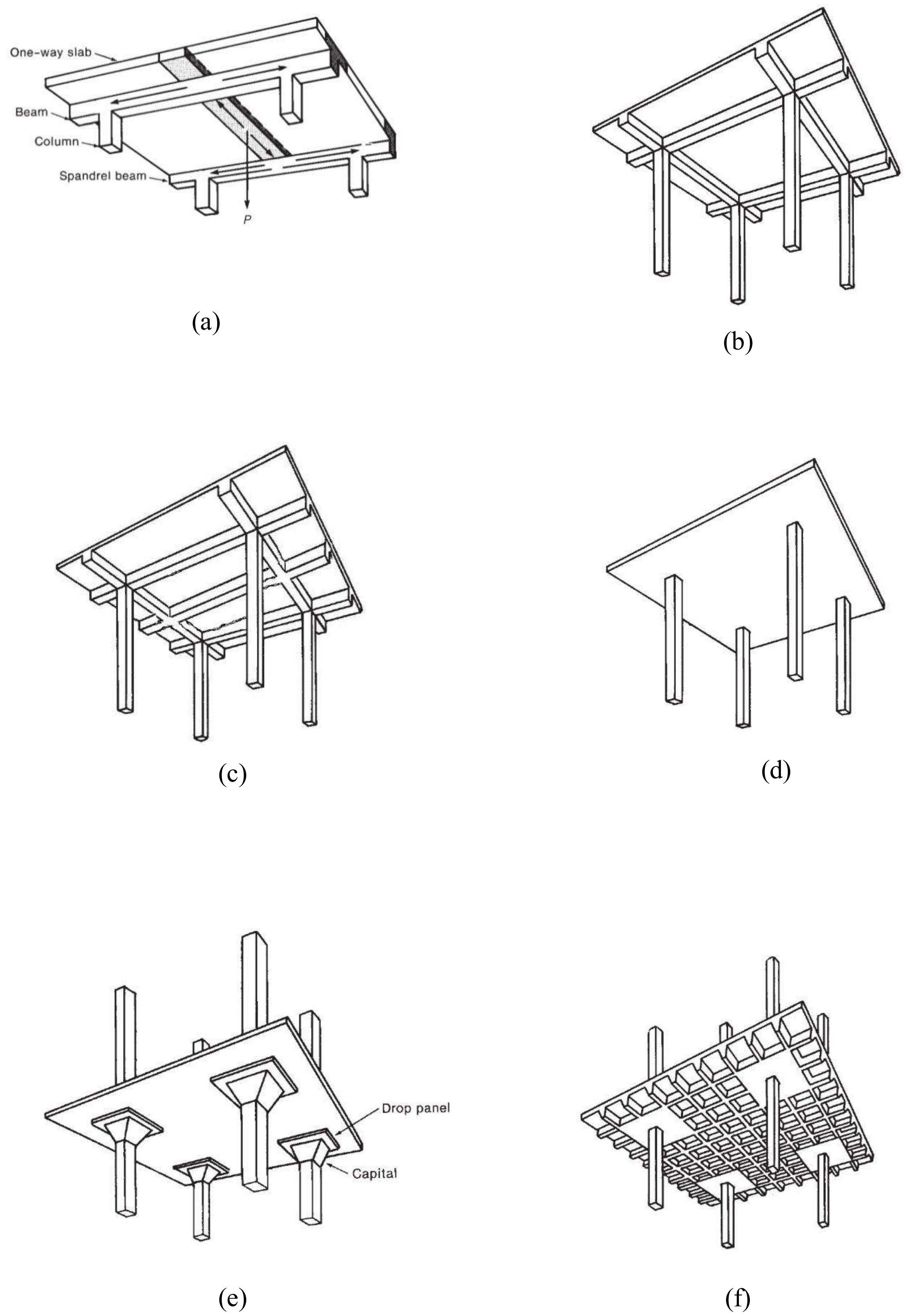
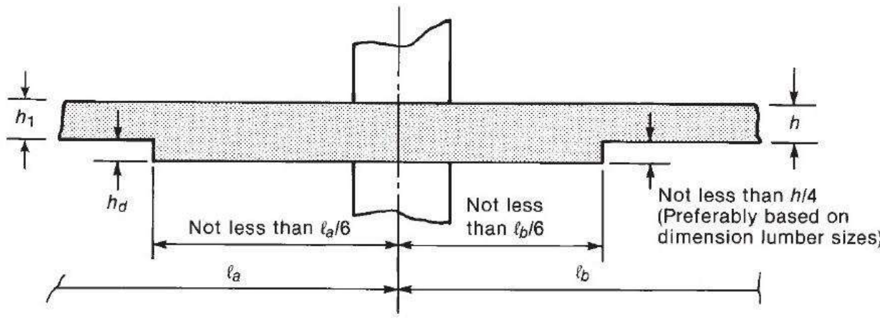


Figure (1) Types of slabs (a) one-way slab, (b) two-way slab, (c) one-way slab, (d) flat plate, (e) flat slab, (f) two–way joist

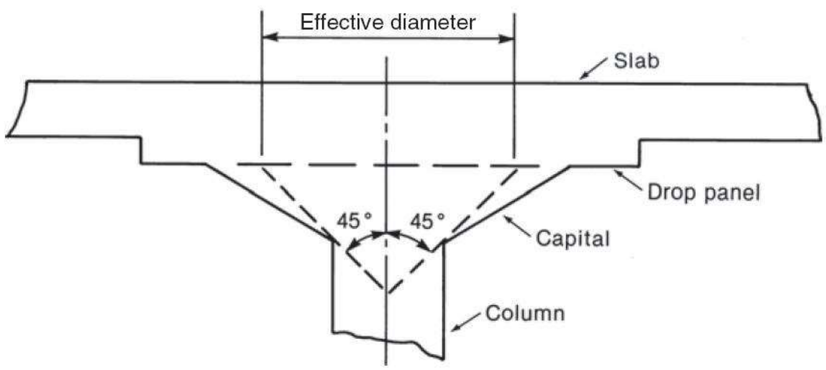
Size and projection of drop panel



Minimum size of drop panels

In computing required slab reinforcement, the thickness of drop panel below the slab shall not be assumed greater than one – quarter the distance from edge of drop panel to edge of column or column capital.

The column capital is normally 20 to 25% of the average span length.



(a) Effective diameter of column capital.

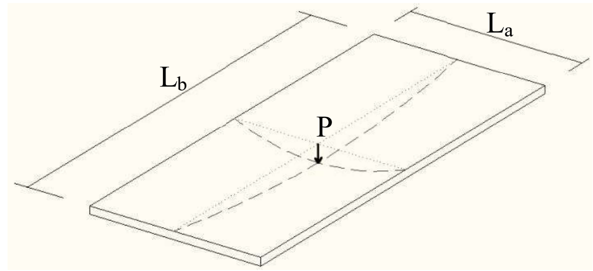
Design of one–way slab systems

At point of intersection (P) the deflection must be the same

$$\Delta = \frac{5\,w\,L^4}{384E\,I} \Rightarrow \frac{5\,w_a\,L_a{}^4}{384\,E\,I} = \frac{5\,w_b\,L_b{}^4}{384\,E\,I}$$

$$\therefore \frac{w_a}{w_b} = \frac{L_b{}^4}{L_a{}^4}$$

$$\text{if } \frac{L_b}{L_a} = 2 \qquad \therefore \frac{w_a}{w_b} = 16 \qquad \Rightarrow w_a = 16\,w_b$$



For purposes of analysis and design a unit strip of such a slab cut out at right angles to the supporting beam may be considered as a rectangular beam of unit width (1.0 m) with a depth (h) equal to the thickness of the slab and a span (L_a) equal to the distance between supported edges.

Simplified method of analysis for one-way slabs

It shall be permitted to calculate M_u and V_u due to gravity loads in accordance with Section 6.5 for one-way slabs satisfying (a) through (e):

(a) Members are prismatic

(b) Loads are uniformly distributed

(c) $L \leq 3D$

(d) There are at least two spans

(e) The longer of two adjacent spans does not exceed the shorter by more than 20 percent

M_u due to gravity loads shall be calculated in accordance with Table 6.5.2. Moments calculated shall not be redistributed.

For slabs built integrally with supports, M_u at the support shall be permitted to be calculated at the face of support.

Floor or roof level moments shall be resisted by distributing the moment between columns immediately above and below the given floor in proportion to the relative column stiffnesses considering conditions of restraint.

Table 6.5.2—Approximate moments for nonprestressed continuous beams and one-way slabs

Moment	Location	Condition	M_u
Positive	End span	Discontinuous end integral with support	$w_u \ell_n^2/14$
		Discontinuous end unrestrained	$w_u \ell_n^2/11$
	Interior spans	All	$w_u \ell_n^2/16$
Negative[1]	Interior face of exterior support	Member built integrally with supporting spandrel beam	$w_u \ell_n^2/24$
		Member built integrally with supporting column	$w_u \ell_n^2/16$
	Exterior face of first interior support	Two spans	$w_u \ell_n^2/9$
		More than two spans	$w_u \ell_n^2/10$
	Face of other supports	All	$w_u \ell_n^2/11$
	Face of all supports satisfying (a) or (b)	(a) slabs with spans not exceeding 10 ft (b) beams where ratio of sum of column stiffnesses to beam stiffness exceeds 8 at each end of span	$w_u \ell_n^2/12$

[1]To calculate negative moments, ℓ_n shall be the average of the adjacent clear span lengths.

A minimum area of flexural reinforcement ($A_{s,min}$) shall be provided in accordance with Table 7.6.1.1.

Table 7.6.1.1—$A_{s,min}$ for nonprestressed one-way slabs

Reinforcement type	f_y (MPa)	$A_{s,min}$ (mm)	
Deformed bars	< 420	0.0020 A_g	
Deformed bars or welded wire reinforcement	≥ 420	Greater of:	$\frac{0.0018 \times 420}{f_y} A_g$
			0.0014 A_g

Reinforcement for shrinkage and temperature stresses normal to the principal reinforcement should be provided in a structural slab. ACI Code specifies the minimum ratios of reinforcement area to gross concrete area, as shown in Table 24.4.3.2.

Table 24.4.3.2—Minimum ratios of deformed shrinkage and temperature reinforcement area to gross concrete area

<table>
<tr><th>Reinforcement type</th><th>f_y
(MPa)</th><th colspan="2">Minimum reinforcement ratio</th></tr>
<tr><td>Deformed bars</td><td>< 420</td><td colspan="2">0.0020</td></tr>
<tr><td rowspan="2">Deformed bars or welded wire reinforcement</td><td rowspan="2">≥ 420</td><td rowspan="2">Greater of:</td><td>$\dfrac{0.0018 \times 420}{f_y}$</td></tr>
<tr><td>0.0014</td></tr>
</table>

The spacing of deformed shrinkage and temperature reinforcement shall not exceed the lesser of 5h and 450 mm.

V_u due to gravity loads shall be calculated in accordance with Table 6.5.4.

Table 6.5.4—Approximate shears for nonprestressed continuous beams and one-way slabs

Location	V_u
Exterior face of first interior support	$1.15w_u\ell_n/2$
Face of all other supports	$w_u\ell_n/2$

For slabs built integrally with supports, V_u at the support shall be permitted to be calculated at the face of support.

Minimum slab thickness

For solid nonprestressed slabs not supporting or attached to partitions or other construction likely to be damaged by large deflections, over all slab thickness (h) shall not be less than the limits in Table 7.3.1.1, unless the calculated deflection limits of 7.3.2 are satisfied.

Table 7.3.1.1—Minimum thickness of solid nonprestressed one-way slabs

Support condition	Minimum h[1]
Simply supported	$\ell/20$
One end continuous	$\ell/24$
Both ends continuous	$\ell/28$
Cantilever	$\ell/10$

For f_y other than 420 MPa, the expressions in Table 7.3.1.1 shall be modified.

For nonprestressed slabs not satisfying 7.3.1 and for prestressed slabs, immediate and time-dependent deflections shall be calculated in accordance with 24.2 and shall not exceed the limits in 24.2.2.

Table 24.2.2—Maximum permissible calculated deflections

<table>
<tr><th>Member</th><th colspan="2">Condition</th><th>Deflection to be considered</th><th>Deflection limitation</th></tr>
<tr><td>Flat roofs</td><td colspan="2" rowspan="2">Not supporting or attached to nonstructural elements likely to be damaged by large deflections</td><td>Immediate deflection due to maximum of L_r, S, and R</td><td>$\ell/180$[1]</td></tr>
<tr><td>Floors</td><td>Immediate deflection due to L</td><td>$\ell/360$</td></tr>
<tr><td rowspan="2">Roof or floors</td><td rowspan="2">Supporting or attached to nonstructural elements</td><td>Likely to be damaged by large deflections</td><td rowspan="2">That part of the total deflection occurring after attachment of nonstructural elements, which is the sum of the time-dependent deflection due to all sustained loads and the immediate deflection due to any additional live load[2]</td><td>$\ell/480$[3]</td></tr>
<tr><td>Not likely to be damaged by large deflections</td><td>$\ell/240$[4]</td></tr>
</table>

[1]Limit not intended to safeguard against ponding. Ponding shall be checked by calculations of deflection, including added deflections due to ponded water, and considering time-dependent effects of sustained loads, camber, construction tolerances, and reliability of provisions for drainage.

[2]Time-dependent deflection shall be calculated in accordance with 24.2.4, but shall be permitted to be reduced by amount of deflection calculated to occur before attachment of nonstructural elements. This amount shall be calculated on basis of accepted engineering data relating to time-deflection characteristics of members similar to those being considered.

[3]Limit shall be permitted to be exceeded if measures are taken to prevent damage to supported or attached elements.

[4]Limit shall not exceed tolerance provided for nonstructural elements.

Design of two-way slab systems

Reinforced concrete slabs (R. C. Slabs) are usually designed for loads assumed to be uniformly distributed over on entire slab panel, bounded by supporting beam or column center-lines.

General design concept of ACI Code

1- Imagining vertical cuts are made through the entire building along lines midway between columns.
2- The cutting creates a series of frames whose width center lines lie along the column lines.
3- The resulting series of rigid frames, taken separately in the longitudinal and transverse directions of the building.
4- A typical rigid frame would consist of:
    a- The columns above and below the floor.
    b- The floor system, with or without beams, bounded laterally between the center lines of the two panels.
5- Two methods of design are presented by the ACI Code:
    a- Direct design method (DDM): An approximants method using moment and shear coefficients, Section 8.10 in ACI Code.
    b- Equivalent Frame method (EFM): More accurate using structural analysis after assuming the relative stiffness of the members, Section 8.11 in ACI Code.

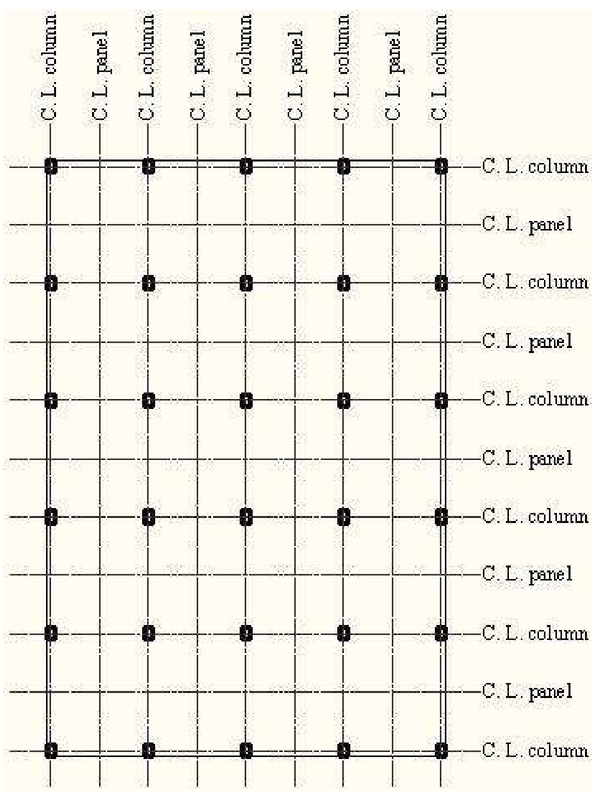


Figure (2) Floor plan.

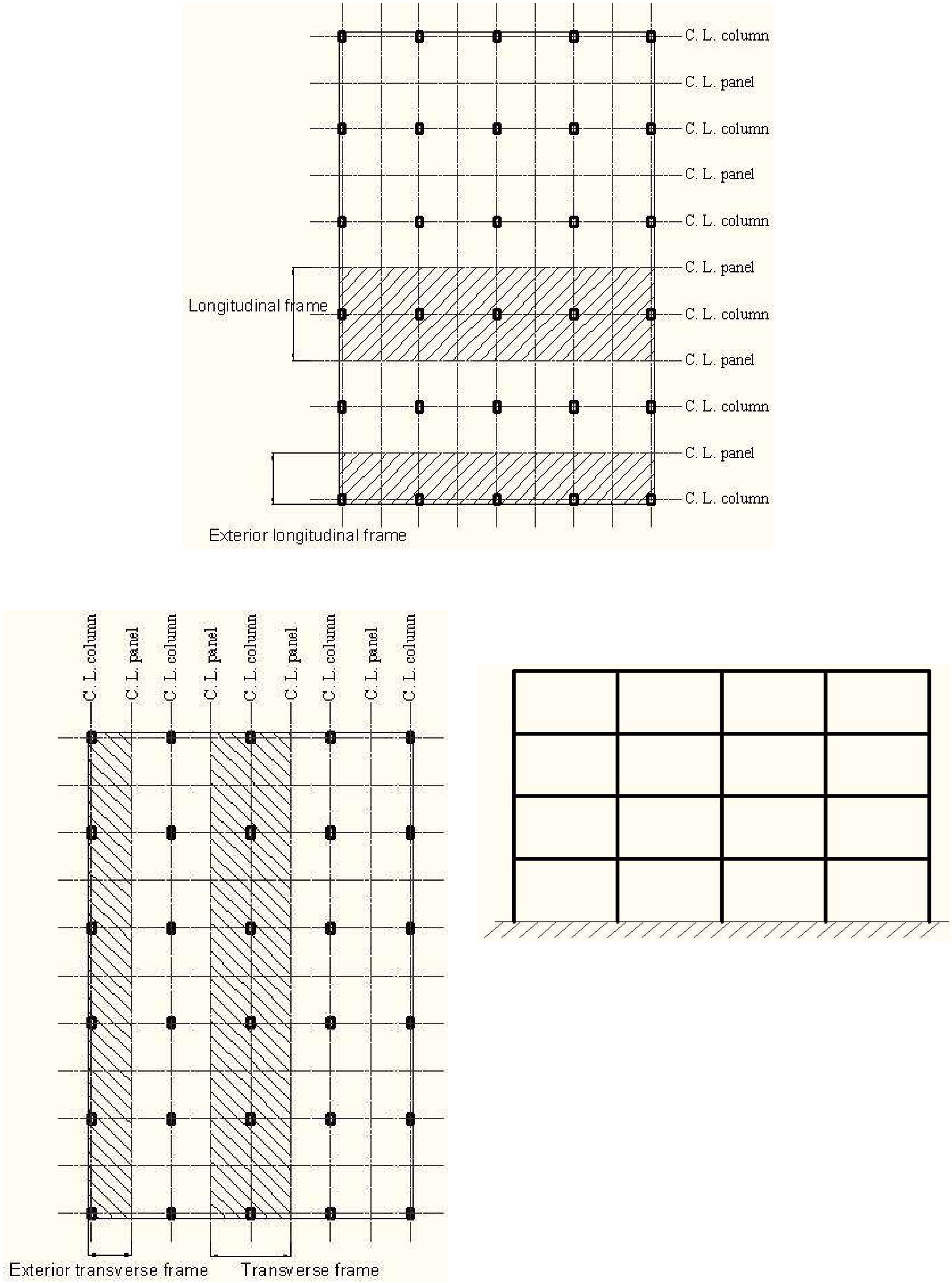


Figure (3) Location of longitudinal and transverse frames.

Direct design method (DDM)

Moments in two-way slabs can be found using a semi-empirical direct design method subject to the following Limitations:

1- There shall be at least three continuous spans in each direction.
2- Successive span lengths measured center-to-center of supports in each direction shall not differ by more than one-third the longer span.
3- Panels shall be rectangular, with the ratio of longer to shorter panel dimensions, measured center-to-center of supports, not to exceed 2.
4- Column offset shall not exceed 10 percent of the span in direction of offset from either axis between centerlines of successive columns.
5- All loads shall be due to gravity only and uniformly distributed over an entire panel.
6- Unfactored live load shall not exceed two times the unfactored dead load.
7- For a panel with beams between supports on all sides, Eq. (8.10.2.7a) shall be satisfied for beams in the two perpendicular directions.

$$0.2 \le \frac{\alpha_{f1}\ell_2^{\,2}}{\alpha_{f2}\ell_1^{\,2}} \le 5.0 \qquad (8.10.2.7a)$$

ℓ_1: is defined as the span in the direction of the moment analysis, and
ℓ_2: as the span in lateral direction.
Spans ℓ_1& ℓ_2 are measured to column centerlines.
α_{f1} and α_{f2} are calculated by:

$$\alpha_f = \frac{E_{cb} I_b}{E_{cs} I_s}$$

The direct design method consists of a set of rules for distributing moments to slab and beam sections to satisfy safety requirements and most serviceability requirements simultaneously. Three fundamental steps are involved as follows:

(1) Determination of the total factored static moment (Section 8.10.3).
(2) Distribution of the total factored static moment to negative and positive sections (Section 8.10.4).
(3) Distribution of the negative and positive factored moments to the column and middle strips and to the beams, if any (Sections 8.10.5 and 8.10.6). The distribution of moments to column and middle strips is also used in the equivalent frame method (Section 8.11).

(1) Total static moment of factored loads (M_o)

M_o: Total static moment in a panel (absolute sum of positive and average negative factored moments in each direction).

$$M_o = \frac{q_u \ell_2 {\ell_n}^2}{8}$$

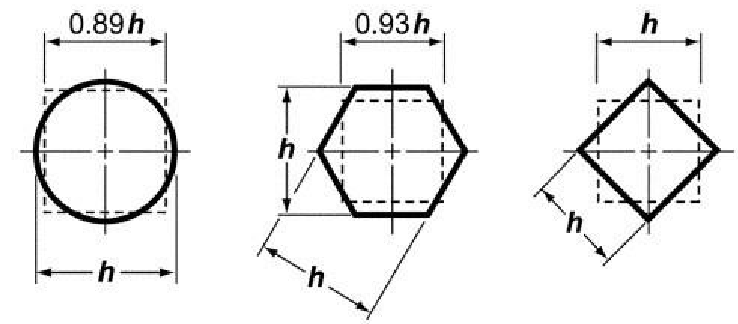
Where ℓ_n: Clear span in the direction of moment used.

ℓ_n is defined to extend from face to face of the columns, capitals, brackets, or walls but is not to be less than 0.65 ℓ_1.

M_o for a strip bounded laterally by the centerlines of the panel on each side of the centerline of support.

ℓ_2: Width of the frame.

Circular or regular polygon-shaped supports shall be treated as square supports with the same area.



(2) Longitudinal distribution of M_o

(a) Interior spans: M_o is apportioned between the critical positive and negative bending sections according to the following ratios:-

Neg. $M_u = 0.65\ M_o$

Pos. $M_u = 0.35\ M_o$

The critical section for a negative bending is taken at the face of rectangular supports, or at the face of an equivalent square support having the same sectional area.

(b) End span: In end spans, the apportionment of the total static moment (M_o) among the three critical moment sections (interior negative, positive, and exterior negative) depends upon the flexural restraint provided for the slab by the exterior column or the exterior wall and upon the presence or absence of beams on the column lines. End span, M_o shall be distributed in accordance with Table 8.10.4.2.

Table 8.10.4.2—Distribution coefficients for end spans

	Exterior edge unrestrained	Slab with beams between all supports	Slab without beams between interior supports		Exterior edge fully restrained
			Without edge beam	With edge beam	
Interior negative	0.75	0.70	0.70	0.70	0.65
Positive	0.63	0.57	0.52	0.50	0.35
Exterior negative	0	0.16	0.26	0.30	0.65

Note: At interior supports, negative moment may differ for spans framing into the common support. In such a case the slab should be designed to resist the larger of the two moments.

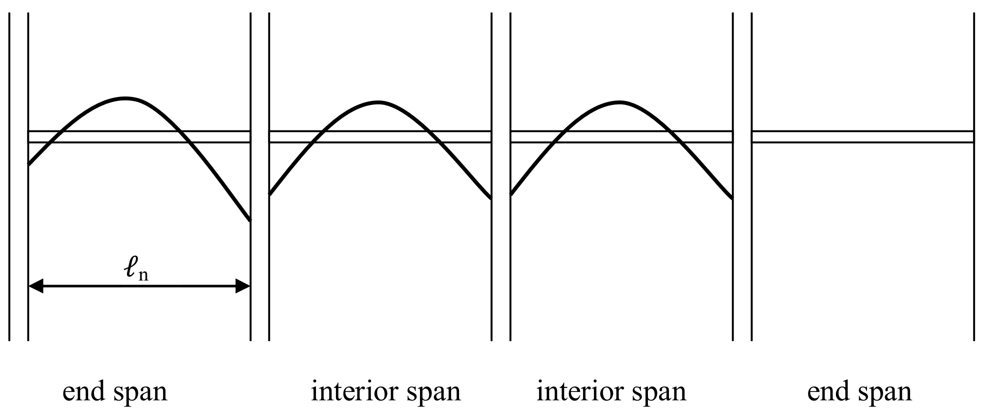


Figure (4) Longitudinal distribution of M_o

(3) Lateral distribution of moments

After the moment M_o distributed on long direction to the positive and negative moments, then these moments must distribute in lateral direction across the width, which consider the moments constant within the bounds of a middle strip or column strip. The distribution of moments between middle strips and column strip and beams depends upon:

1. The ratio ℓ_2/ℓ_1.
2. The relative stiffness of the beam and the slab.
3. The degree of torsional restraint provided by the edge beam.

The column strip shall resist the portion of interior negative M_u in accordance with Table 8.10.5.1.

Table 8.10.5.1—Portion of interior negative M_u in column strip

$\alpha_{f1}\ell_2/\ell_1$	ℓ_2/ℓ_1		
	0.5	**1.0**	**2.0**
0	0.75	0.75	0.75
≥1.0	0.90	0.75	0.45

Note: Linear interpolations shall be made between values shown.

The column strip shall resist the portion of exterior negative M_u in accordance with Table 8.10.5.2.

Table 8.10.5.2—Portion of exterior negative M_u in column strip

$\alpha_{f1}\ell_2/\ell_1$	β_t	ℓ_2/ℓ_1		
		0.5	**1.0**	**2.0**
0	0	1.0	1.0	1.0
	≥2.5	0.75	0.75	0.75
≥1.0	0	1.0	1.0	1.0
	≥2.5	0.90	0.75	0.45

Note: Linear interpolations shall be made between values shown. β_t is calculated using Eq. (8.10.5.2a), where C is calculated using Eq. (8.10.5.2b).

$$\beta_t = \frac{E_{cb}C}{2E_{cs}I_s} \qquad (8.10.5.2a)$$

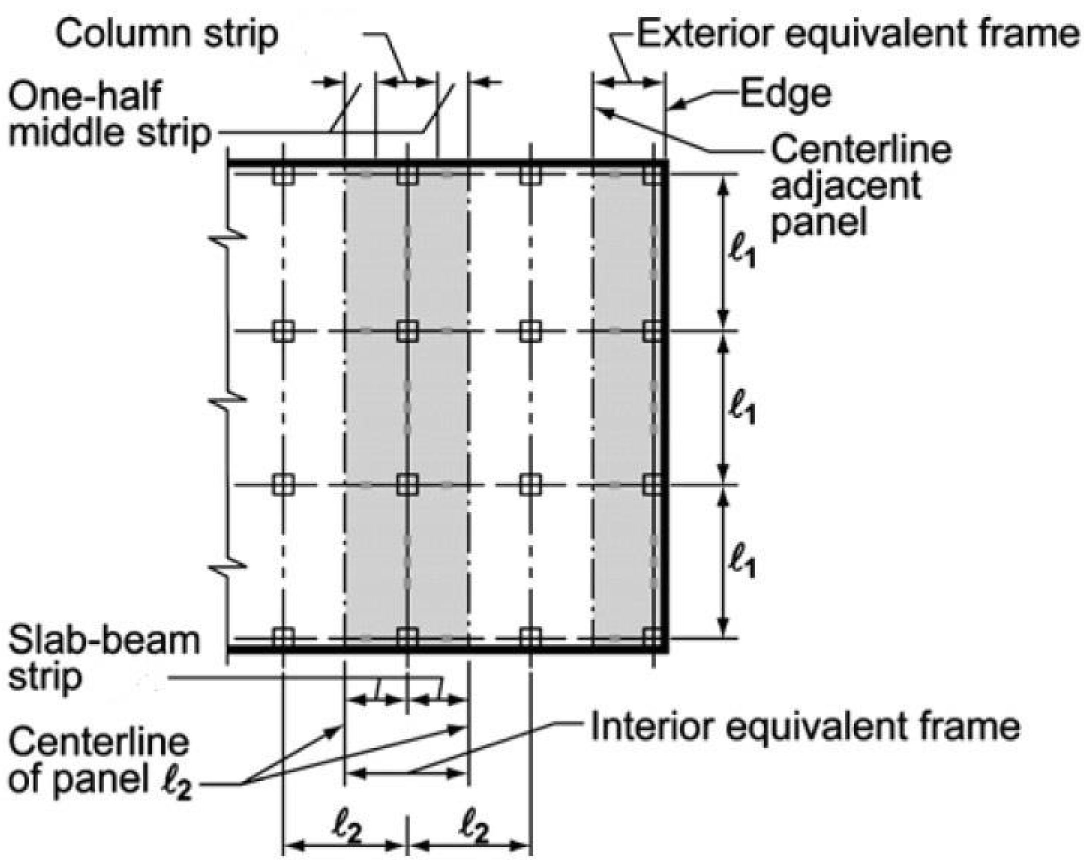
$$C = \Sigma\left(1 - 0.63\frac{x}{y}\right)\frac{x^3 y}{3} \qquad (8.10.5.2b)$$

The column strip shall resist the portion of positive M_u in accordance with Table 8.10.5.5.

Table 8.10.5.5—Portion of positive M_u in column strip

	ℓ_2/ℓ_1		
$\alpha_{f1}\ell_2/\ell_1$	**0.5**	**1.0**	**2.0**
0	0.60	0.60	0.60
≥1.0	0.90	0.75	0.45

Note: Linear interpolations shall be made between values shown.

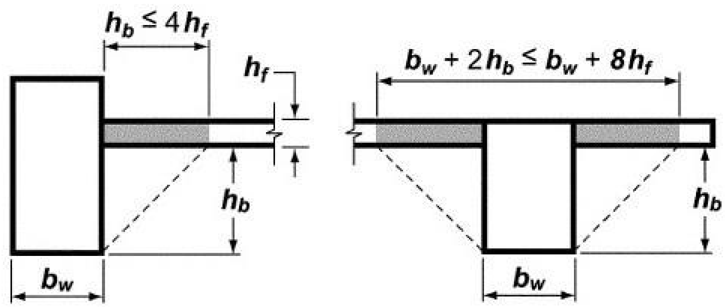


A convenient parameter defining the relative stiffness of the beam and slab spanning in either direction is:

$$\alpha_f = \frac{E_{cb}I_b}{E_{cs}I_s} = \frac{I_b}{I_s}$$

Where E_{cb}, E_{cs} are the moduli of elasticity of beam and slab concrete (usually the same), respectively. I_b and I_s are the moment of inertia of the effective beam and slab, respectively. The flexural stiffnesses of the beam and slab are based on the gross concrete section. Variation due to column capitals and drop panels are neglected (in applying DDM).

For monolithic or fully composite construction supporting two-way slabs, a beam includes that portion of slab, on each side of the beam extending a distance equal to the projection of the beam above or below the slab, whichever is greater, but not greater than four times the slab thickness.



The moment of inertia of flanged section

$$I_b = k\,\frac{b_w h^3}{12}$$

$$k \approx 1.0 + 0.2\left(\frac{b_E}{b_w}\right) \qquad \text{for} \quad 2 < \frac{b_E}{b_w} < 4 \quad \& \quad 0.2 < \frac{h_f}{h} < 0.5$$

The relative restraint provided by the torsional resistance of the effective transverse edge beam is reflected by the parameter β_t, defined by:

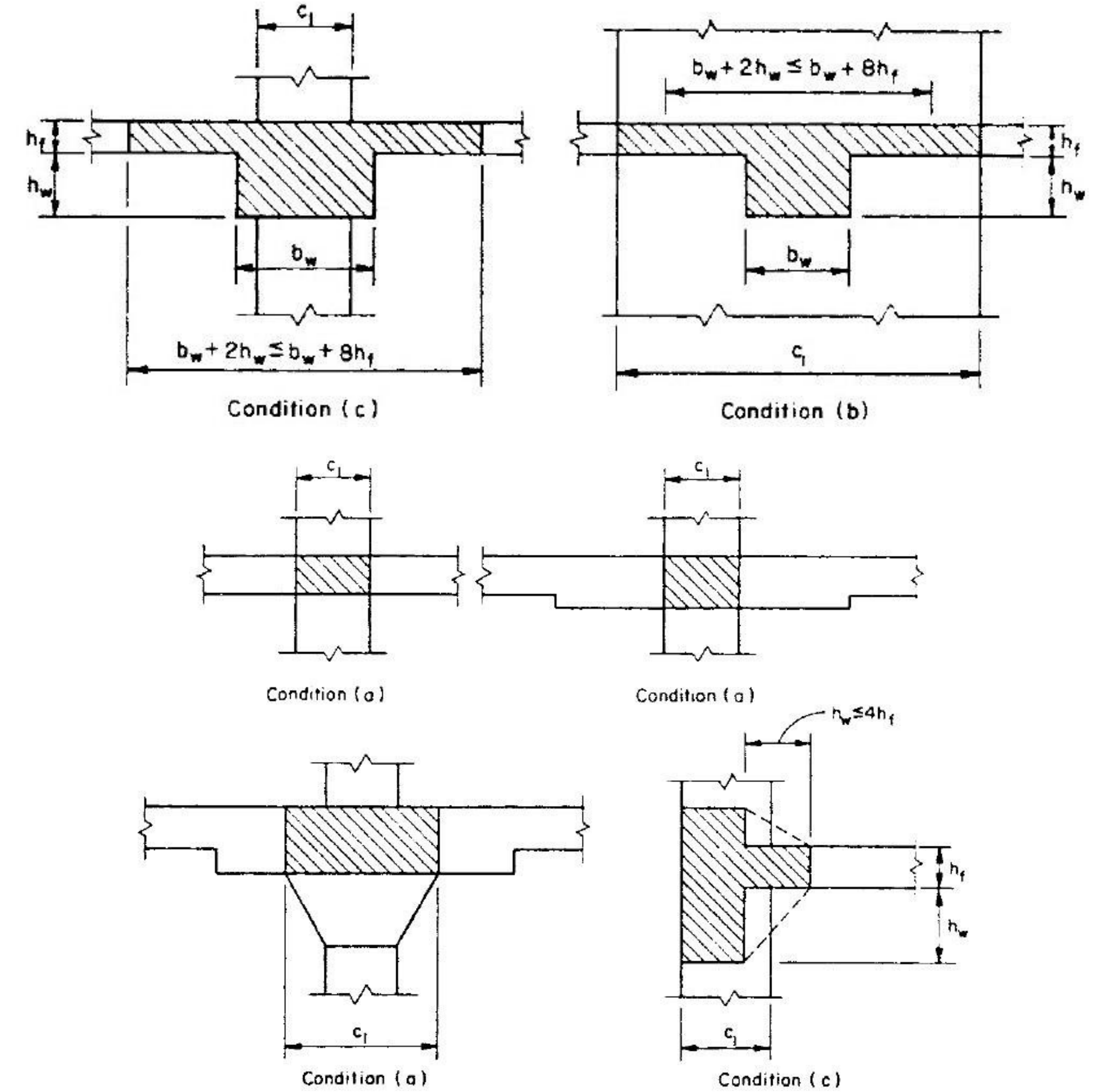
$$\beta_t = \frac{E_{cb} C}{2\,E_{cs} I_s} = \frac{C}{2\,I_s}$$

C: The torsional rigidity of the effective transverse beam, which is defined as the largest of the following three items:-

a- A portion of the slab having a width equal to that of the column, bracket, or capital in the direction in which moment are taken, c_1 (case of no actual beam).

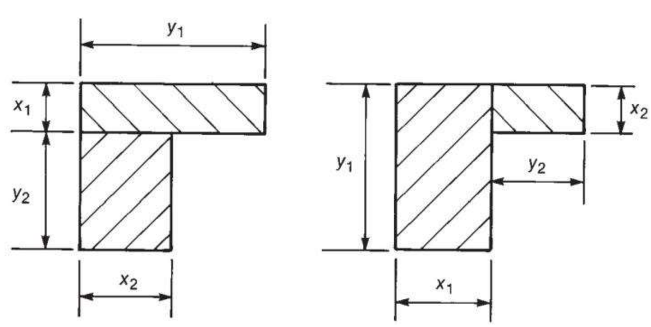
b- The portion of the slab specified in (a) plus that part of any transverse beam above and below the slab.

c- The transverse beam defined as before (in calculating α_f).



The constant C is calculated by dividing the section into its component rectangles, each having smaller dimension x and larger dimension y and summing the contributions of all the parts by means of the equation:

$$C = \sum\left(1 - 0.63\,\frac{x}{y}\right)\frac{x^3\,y}{3}$$



The subdivision can be done in such a way as to maximize C.

For slabs with beams between supports, the slab portion of column strips shall resist column strip moments not resisted by beams. Beams between supports shall resist the portion of column strip M_u in accordance with Table 8.10.5.7.1.

Table 8.10.5.7.1—Portion of column strip M_u in beams

$\alpha_{f1}\ell_2/\ell_1$	Distribution coefficient
0	0
≥1.0	0.85

Note: Linear interpolation shall be made between values shown.

The portion of the moment not resisted by the column strip is proportionately assigned to the adjacent half-middle strips. Each middle strip is designed to resist the sum of the moment assigned to its two half-middle strips. A middle strip adjacent and parallel to wall is designed for twice the moment assigned to the half-middle strip corresponding to the first row of interior support.

If the width of the column or wall is at least (¾)ℓ_2, negative M_u shall be uniformly distributed across ℓ_2.

Minimum flexural reinforcement in nonprestressed slabs, $A_{s,min}$, shall be provided near the tension face in the direction of the span under consideration in accordance with Table 8.6.1.1.

Table 8.6.1.1—$A_{s,min}$ for nonprestressed two-way slabs

Reinforcement type	f_y (MPa)	$A_{s,min}$ (mm)	
Deformed bars	< 420	$0.0020\ A_g$	
Deformed bars or welded wire reinforcement	≥ 420	Greater of:	$\dfrac{0.0018 \times 420}{f_y} A_g$
			$0.0014\ A_g$

Minimum spacing of reinforcement

For parallel nonprestressed reinforcement in a horizontal layer, clear spacing shall be at least the greatest of 25 mm, d_b, and $(4/3)_{dagg}$.

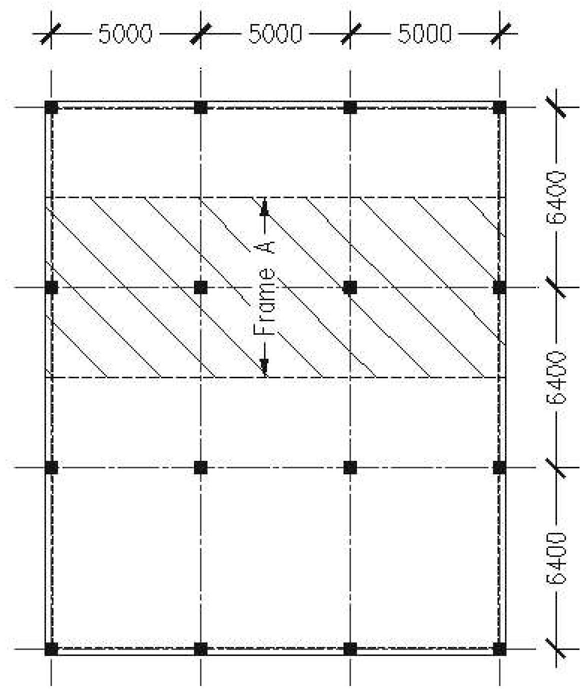
For nonprestressed solid slabs, maximum spacing (s) of deformed longitudinal reinforcement shall be the lesser of 2h and 450 mm at critical sections, and the lesser of 3h and 450 mm at other sections.

Example 1

For the the longitudinal interior frame (Frame A) of the falt plate floor shown in Figure,by using the Direct Design Method, find:

a. Longitudinal distribution of the total static moment at factored loads.

b. Lateral distribution of the moment at exterior support.



Slab thickness = 200 mm, d = 165 mm

$q_u = 15.0$ kN/m^2

All edge beams = 250×500 mm

All columns = 500×500 mm

$f_c' = 25$ MPa, $f_y = 400$ MPa

Solution

a.)

for Frame A

$\ell_1 = 5000$ mm

$\ell_2 = 6400$ mm

$\ell_n = \ell_1 - 500 = 5000 - 500 = 4500$ mm

$$M_o = \frac{q_u \ell_2 \ell_n^{\,2}}{8}$$

$$M_o = \frac{15 \times 6.4 \times (4.5)^2}{8}$$

$= 243$ kN.m

Longitudinal distribution of total static moment at factored loads

Interior span:

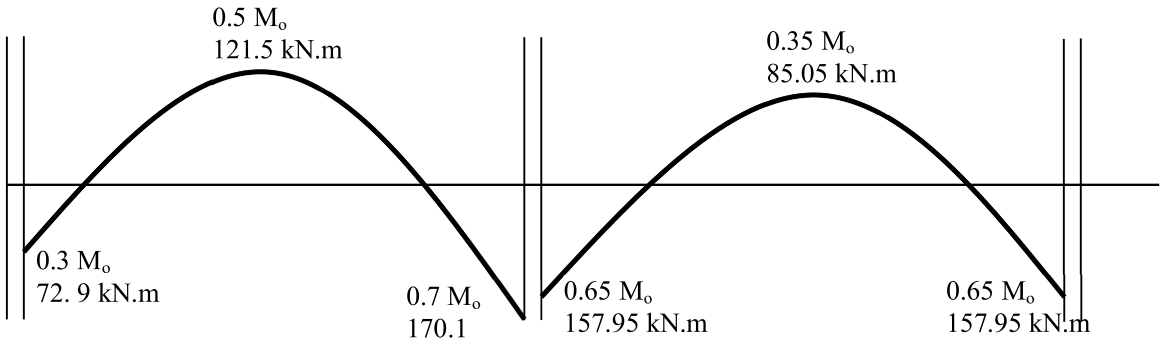
Neg. $M_u = 0.65\ M_o$

Pos. $M_u = 0.35\ M_o$

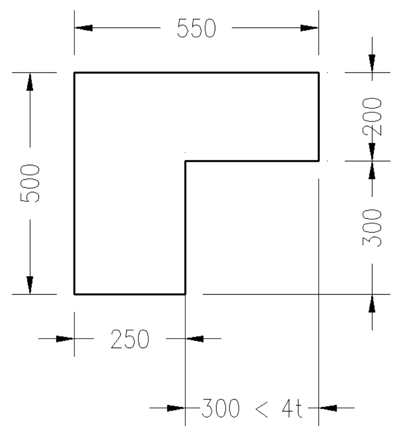
End span:

Table 8.10.4.2—Distribution coefficients for end spans

	Exterior edge unrestrained	Slab with beams between all supports	Slab without beams between interior supports: Without edge beam	Slab without beams between interior supports: With edge beam	Exterior edge fully restrained
Interior negative	0.75	0.70	0.70	0.70	0.65
Positive	0.63	0.57	0.52	0.50	0.35
Exterior negative	0	0.16	0.26	0.30	0.65



Longitudinal distribution of total static moment at factored loads



b.)

Negative moment at exterior support

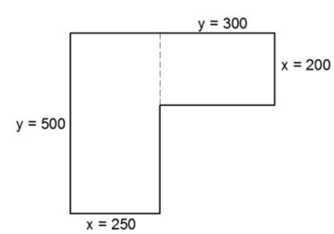
Total moment = 72.9 kN.m

$\alpha_f = 0$

for edge beam

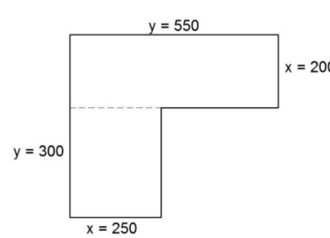
choose the section of edge beam

$$C = \sum \left(1 - 0.63 \times \frac{x}{y}\right) \cdot \frac{x^3 \cdot y}{3}$$



$$C_1 = \left(1 - 0.63 \times \frac{200}{300}\right) \times \frac{(200)^3 \times 300}{3} + \left(1 - 0.63 \times \frac{250}{500}\right) \times \frac{(250)^3 \times 500}{3}$$

$$C_1 = 2247854166667 \ \ \text{mm}^4$$



$$C_2 = \left(1 - 0.63 \times \frac{200}{550}\right) \times \frac{(200)^3 \times 550}{3} + \left(1 - 0.63 \times \frac{250}{300}\right) \times \frac{(250)^3 \times 300}{3}$$

$$C_2 = 1872854166667 \ \ \text{mm}^4$$

$$\therefore\ C = 2247854166667 \ \ \text{mm}^4$$

$$I_s = \frac{1}{12} \times \ell_2 \times t^3 = \frac{1}{12} \times 6400 \times (200)^3 = 4266666666.667 \ \text{mm}^4$$

$$\beta_t = \frac{E_{cb} \cdot C}{2 \times E_{cs} \cdot I_s} \quad ; \quad E_{cb} = E_{cs}$$

$$\beta_t = \frac{C}{2 \times I_s} = \frac{2247854166667}{2 \times 4266666666667} = 0.263$$

$$\frac{\ell_2}{\ell_1} = \frac{6.4}{5.0} = 1.28$$

ℓ_2/ℓ_1	1.0	1.28	2.0
$\beta_t = 0.0$	1.00	1.00	1.00
$\beta_t = 0.263$		**0.9737**	
$\beta_t \geq 2.5$	0.75	0.75	0.75

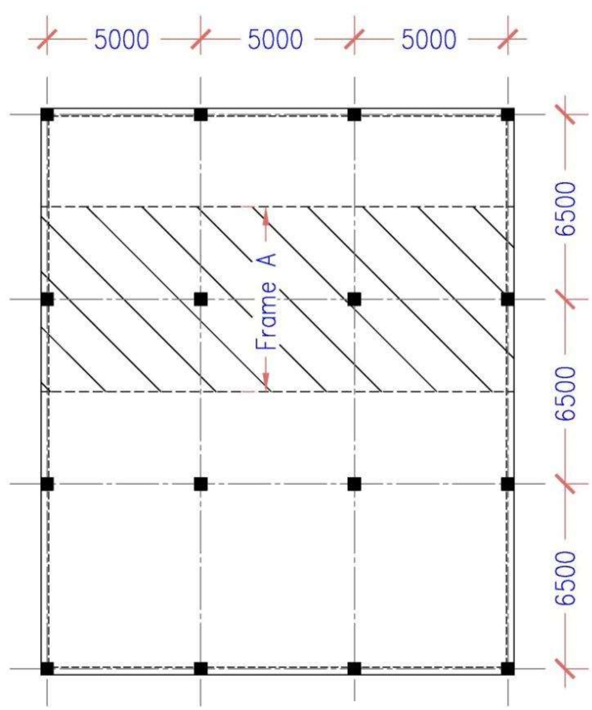
Negative moment at column strip = 72.9×0.9737 = 70.983 kN.m

Negative moment at middle strip = 72.9-70.983 = 1.917 kN.m

Example 2

For the the longitudinal interior frame of the falt plate floor shown in Figure, by using the Direct Design Method, find:

- **a.** Longitudinal distribution of total static moment at factored loads.
- **b.** Lateral distribution of moment at exterior panel.



Slab thickness = 180 mm, d = 150 mm

q_u = 14.0 kN/m^2

All edge beams = 250×500 mm

All columns = 400×400 mm

f_c' = 24 MPa, f_y = 400 MPa

Solution

a.)

for Frame A

ℓ_1 = 5000 mm

ℓ_2 = 6500 mm

$\ell_n = \ell_1 - 400 = 5000 - 400 = 4600$ mm

$$M_o = \frac{q_u \ell_2 \ell_n^2}{8}$$

$M_o = \frac{14 \times 6.5 \times (4.6)^2}{8}$

$= 240.695$ kN.m

<u>Longitudinal distribution of total static moment at factored loads</u>

Interior span:

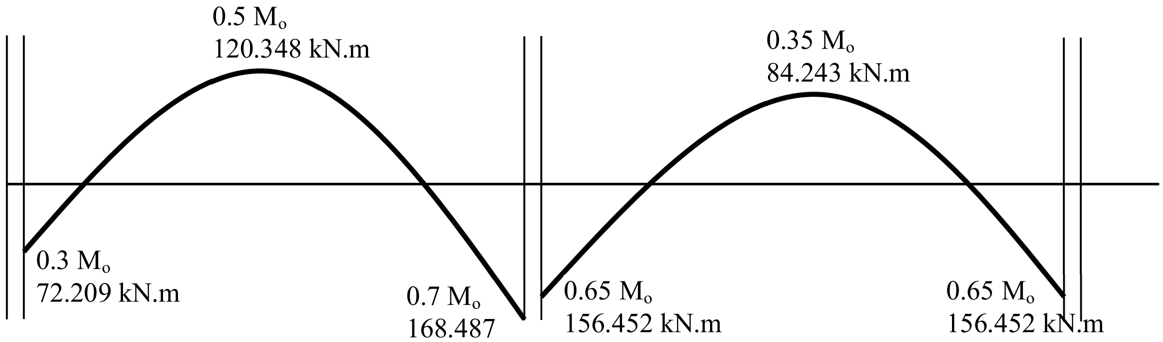
Neg. $M_u = 0.65\ M_o$

Pos. $M_u = 0.35\ M_o$

End span:

Table 8.10.4.2—Distribution coefficients for end spans

	Exterior edge unrestrained	Slab with beams between all supports	Slab without beams between interior supports: Without edge beam	Slab without beams between interior supports: With edge beam	Exterior edge fully restrained
Interior negative	0.75	0.70	0.70	0.70	0.65
Positive	0.63	0.57	0.52	0.50	0.35
Exterior negative	0	0.16	0.26	0.30	0.65



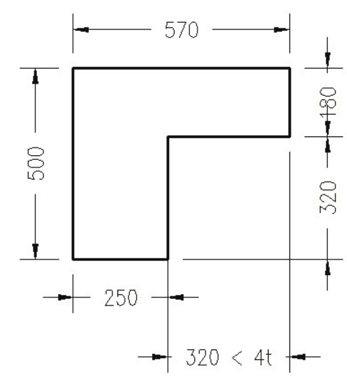
Longitudinal distribution of total static moment at factored loads

b.)

exterior panel

1- Negative moment at exterior support

Total moment = 72.209 kN.m



$\alpha_f = 0$

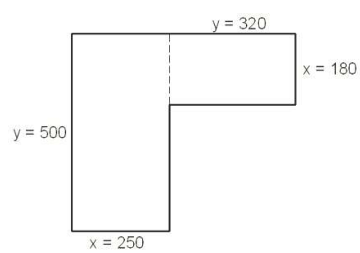
for edge beam

choose the section of edge beam

$$C = \sum \left(1 - 0.63 \times \frac{x}{y}\right) \cdot \frac{x^3 \cdot y}{3}$$

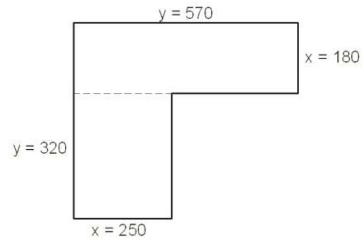
$$C_1 = \left(1 - 0.63 \times \frac{180}{320}\right) \times \frac{(180)^3 \times 320}{3} + \left(1 - 0.63 \times \frac{250}{500}\right) \times \frac{(250)^3 \times 500}{3}$$

$$C_1 = 21854845667 \quad mm^4$$



$$C_2 = \left(1 - 0.63 \times \frac{180}{570}\right) \times \frac{(180)^3 \times 570}{3} + \left(1 - 0.63 \times \frac{250}{320}\right) \times \frac{(250)^3 \times 320}{3}$$

$$C_2 = 17339845667 \quad mm^4$$



$$\therefore \quad C = 21854845667 \quad mm^4$$

$$I_s = \frac{1}{12} \times \ell_2 \times t^3 = \frac{1}{12} \times 6500 \times (180)^3 = 3159000000 \text{ mm}^4$$

$$\beta_t = \frac{E_{cb} \cdot C}{2 \times E_{cs} \cdot I_s} \qquad ; \qquad E_{cb} = E_{cs}$$

$$\beta_t = \frac{C}{2 \times I_s} = \frac{21854845667}{2 \times 3159000000} = 0.346$$

$$\frac{\ell_2}{\ell_1} = \frac{6.5}{5.0} = 1.3$$

ℓ_2/ℓ_1	1.0	1.3	2.0
$\beta_t = 0.0$	1.00	1.00	1.00
$\beta_t = 0.346$			
$\beta_t \geq 2.5$	0.75	0.75	0.75

Negative moment at column strip = 72.209 × = kN.m

Negative moment at middle strip = 72.209 - = kN.m

2- Positive moment

Total moment = 120.348 kN.m

$\alpha_f = 0$

3- Negative moment at interior support
Total moment = 168.487 kN.m

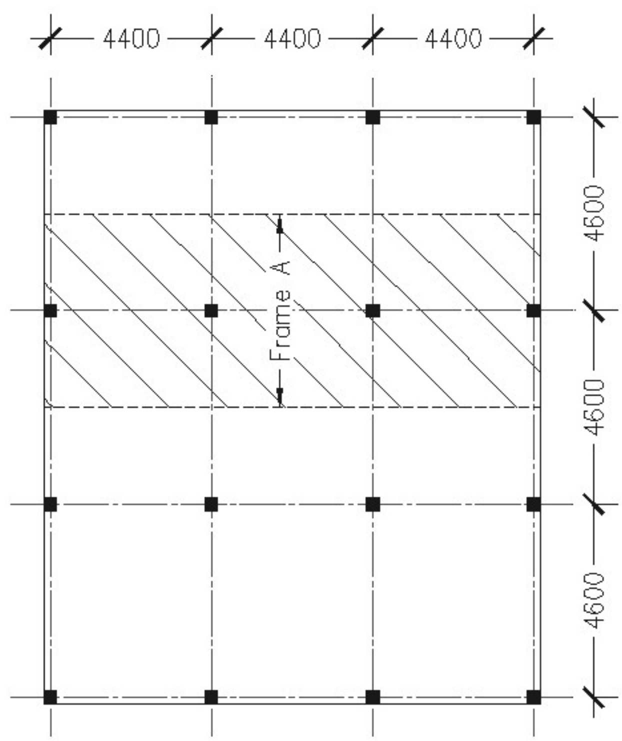
$\alpha_f = 0$

Example 3

For the the longitudinal interior frame (Frame A) of the falt plate floor shown in Figure, by using the Direct Design Method, find:

a- Longitudinal distribution of the total static moment at factored loads.

b- Lateral distribution of moment at interior panel (column and middle strip moments at negative and positive moments).



Slab thickness = 180 mm, d = 150 mm

$q_u = 15.0\,kN/m^2$

All columns = 400 × 400 mm

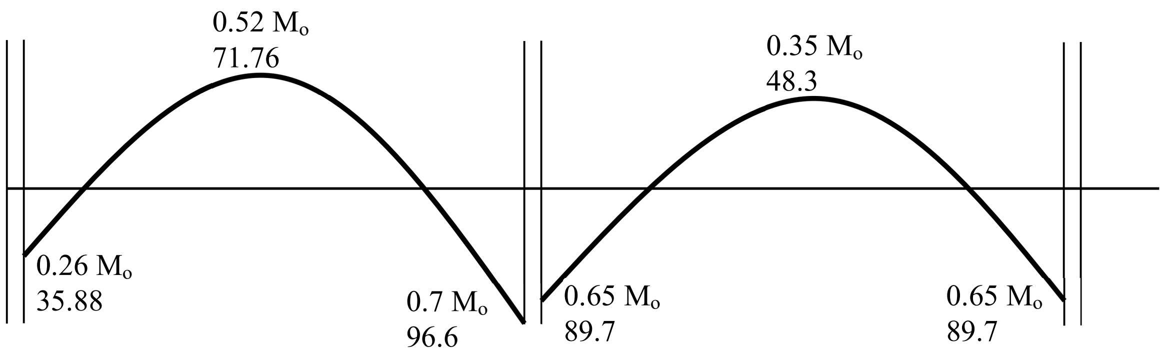
Solution

a.

$$L_n = L_1 - 0.5$$
$$= 4.4 - 0.4$$
$$= 4.0\text{ m}$$

$$M_o = \frac{w_u \cdot L_2 \cdot L_n^2}{8}$$

$$M_o = \frac{15 \times 4.6 \times (4.0)^2}{8}$$
$$= 138\text{ kN.m}$$



Longitudinal distribution of total static moment at factored loads

b.

interior panel

1) Negative moment

Total moment = 89.7 kN.m

ℓ_2/ℓ_1	0.5	1.0	2.0
$(\alpha_{f1}\ell_2/\ell_1) = 0$	75	75	75
$(\alpha_{f1}\ell_2/\ell_1) \geq 1.0$	90	75	45

$\alpha_1 = 0$

Negative moment at column strip = 89.7×0.75 = 67.275 kN.m

Negative moment at midlle strip = 89.7-67.275 = 22.425 kN.m

2) Positive moment

Total moment = 48.3 kN.m

ℓ_2/ℓ_1	0.5	1.0	2.0
$(\alpha_{f1}\ell_2/\ell_1) = 0$	60	60	60
$(\alpha_{f1}\ell_2/\ell_1) \geq 1.0$	90	75	45

$\alpha_1 = 0$

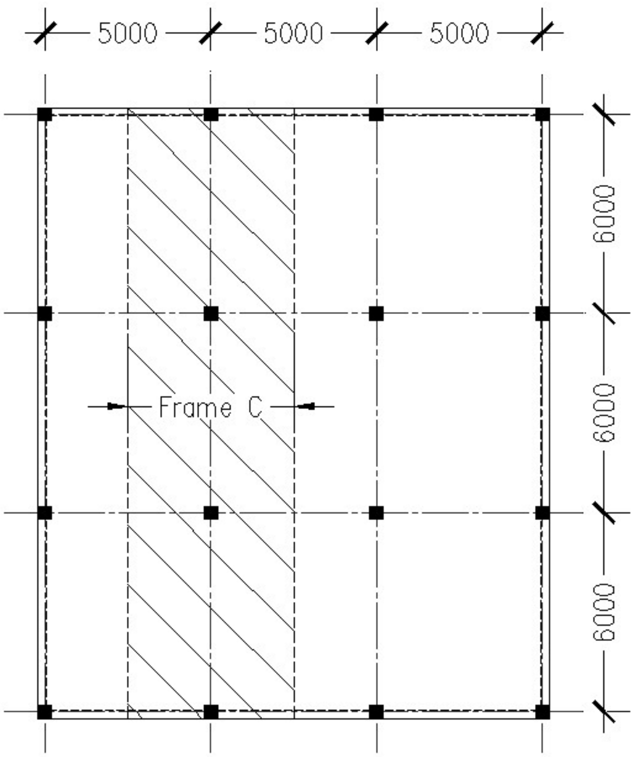
Negative moment at column strip = 48.3×0.60 = 28.98 kN.m

Negative moment at midlle strip = 48.3-28.98 = 19.32 kN.m

Example 4

For the the transverse interior frame (Frame C) of the flat plate floor with edge beams shown in Figure, by using the Direct Design Method, find:

1) Longitudinal distribution of total static moment at factored loads.
2) Lateral distribution of moment at interior panel (column and middle strip moments at negative and positive moments).
3) Lateral distribution of moment at exterior panel (column and middle strip moments at negative and positive moments).



Slab thickness $= 180$ mm, $d = 150$ mm

$q_u = 16.0\,\text{kN}/\text{m}^2$

All edge beams $= 250 \times 500$ mm

All columns $= 500 \times 500$ mm

Example 5

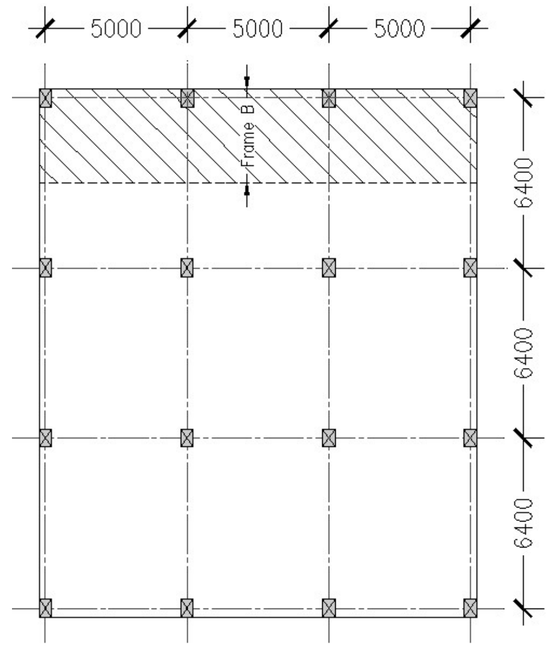
For the exterior longitudinal frame (Frame B) of the flat plate floor shown in figure, and by using the Direct Design Method, find:

- **a.** Longitudinal distribution of the total static moment at factored loads.
- **b.** Lateral distribution of moment at exterior panel (column and middle strip moments at exterior support)

Slab thickness $= 175$ mm, $d = 140$ mm

$q_u = 14.0\,\text{kN/m}^2$

All columns $= 600 \times 400$ mm



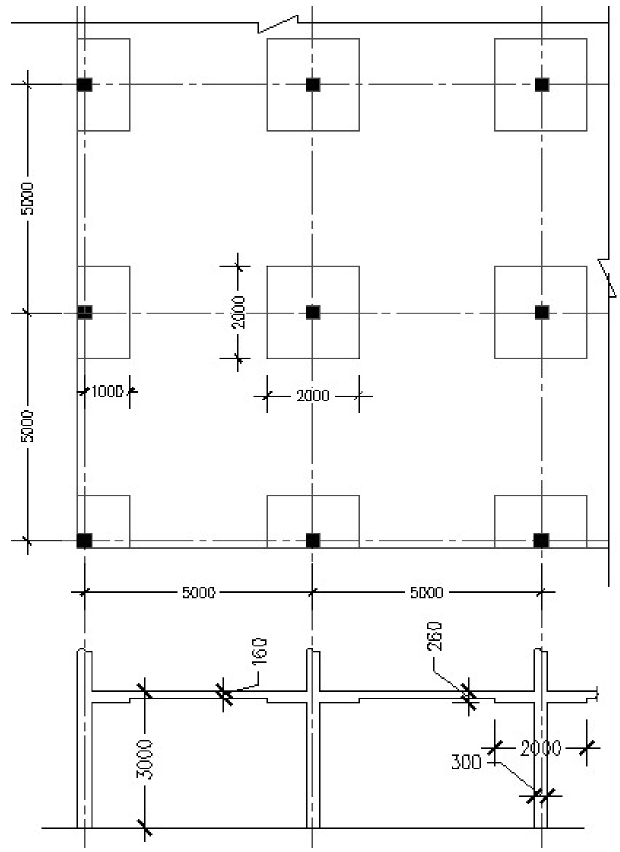
Example 6

For the exterior transverse frame of the flat slab floor shown in figure, and by using the Direct Design Method, find:

- **a.** Longitudinal distribution of the total static moment at factored loads.
- **b.** Lateral distribution of moment at exterior panel (column and middle strip moments at exterior support)

D = 6.5 kN/m^2

L = 5.0 kN/m^2



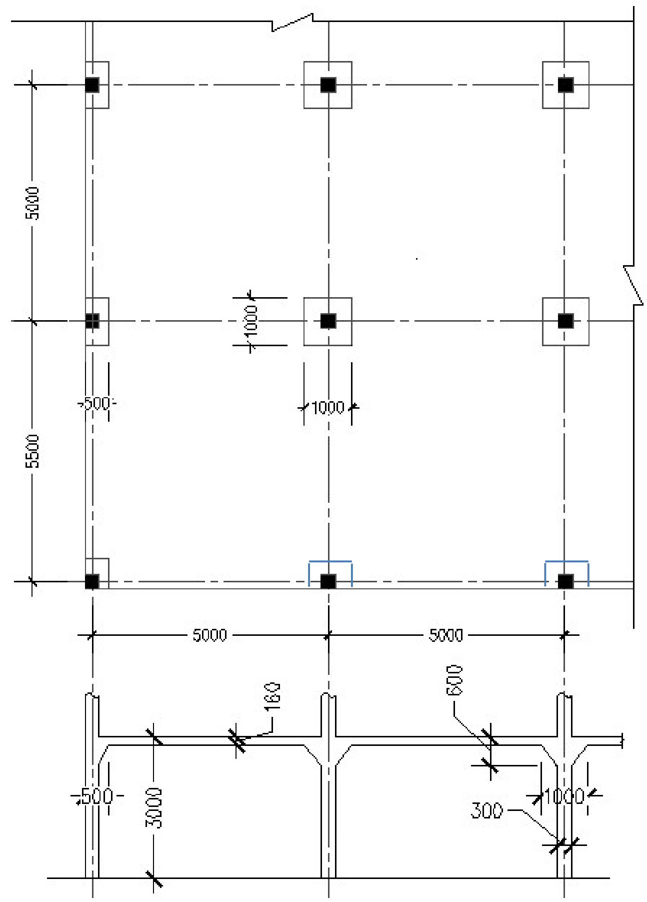
Example 7

For the transverse frame of the flat slab floor shown in figure, and by using the Direct Design Method, find:

- **a.** Longitudinal distribution of the total static moment at factored loads.
- **b.** Lateral distribution of moment at exterior panel (column and middle strip moments at exterior support)

D = 7.0 kN/m^2

L = 4.0 kN/m^2



Example 8

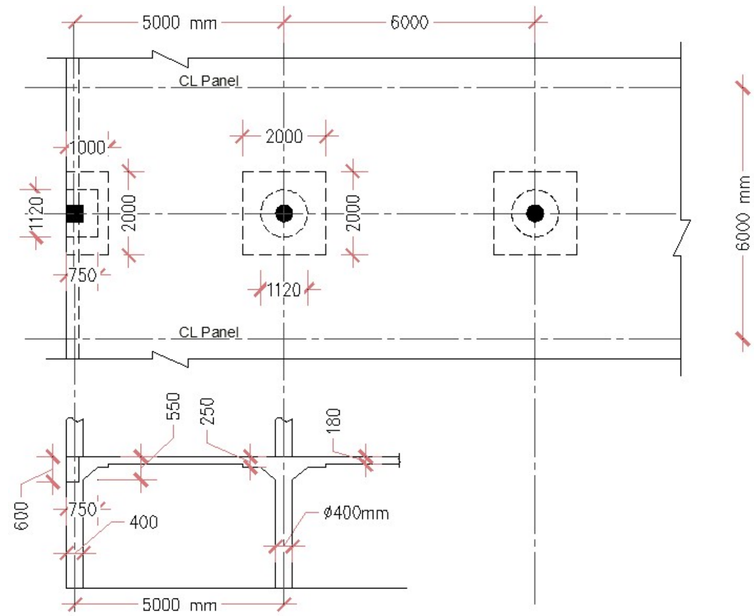
For the longitudinal frame of the flat slab floor shown in figure, and by using the Direct Design Method, find:

a. Longitudinal distribution of the total static moment at factored loads.

b. Lateral distribution of moment at exterior panel (column and middle strip moments at exterior support)

$q_u = 18.0\ kN/m^2$

edge beams: 300×600 mm



Example 9

For the the transverse extiror frame (Frame D) of the falt plate floor, without edge beams, shown in Figure, and by using the Direct Design Method, find:

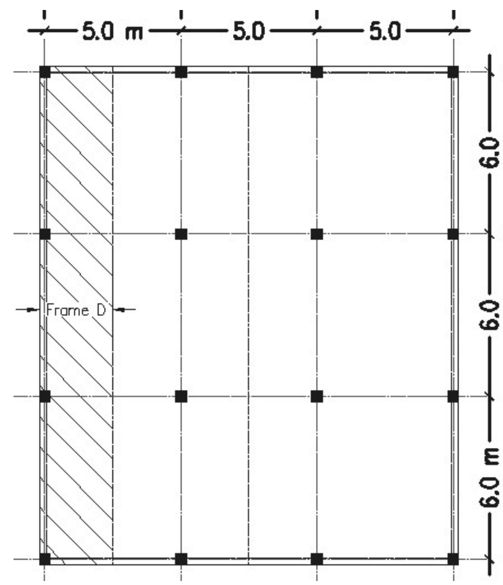
a. Longitudinal distribution of the total static moment at factored loads.

b. Lateral distribution of moment at interior panel (column and middle strip moments at negative and positive moments).

Slab thickness = 180 mm,   d = 150 mm

q_u = 15.0 kN/m^2

All columns = 400×400 mm



Minimum slab thickness for two-way slabs

To ensure that slab deflection service will not be troublesome, the best approach is to compute deflections for the total load or load component of interest and to compare the computed deflections with limiting values.

Alternatively, deflection control can be achieved indirectly by adhering to more or less arbitrary limitations on minimum slab thickness, limitations developed from review of test data and study of the observed deflections of actual structures.

Simplified criteria are used to slabs without interior beams (provided by table), flat plates and flat slabs with or without edge beams. While equations are to be applied to slabs with beams spanning between the supports on all sides. In both cases, minimum thicknesses less than the specified value may be used if calculated deflections are within code specified limits.

Slab without interior beams (Flat plates and flat slabs with or without edge beams)

For nonprestressed slabs without interior beams spanning between supports on all sides, having a maximum ratio of long-to-short span of 2, overall slab thickness (h) shall not be less than the limits in Table 8.3.1.1, and shall be at least the value in (a) or (b), unless the calculated deflection limits of 8.3.2 (ACI 318) are satisfied:

(a) Slabs without drop panels as given in 8.2.4........... 125 mm.

(b) Slabs with drop panels as given in 8.2.4................ 100 mm.

Table 8.3.1.1—Minimum thickness of nonprestressed two-way slabs without interior beams (mm)[1]

f_y, MPa[2]	Without drop panels[3]			With drop panels[3]		
	Exterior panels		Interior panels	Exterior panels		Interior panels
	Without edge beams	With edge beams[4]		Without edge beams	With edge beams[4]	
280	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$	$\ell_n/36$	$\ell_n/40$	$\ell_n/40$
420	$\ell_n/30$	$\ell_n/33$	$\ell_n/33$	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$
520	$\ell_n/28$	$\ell_n/31$	$\ell_n/31$	$\ell_n/31$	$\ell_n/34$	$\ell_n/34$

[1]ℓ_n is the clear span in the long direction, measured face-to-face of supports (mm).

[2]For f_y between the values given in the table, minimum thickness shall be calculated by linear interpolation.

[3]Drop panels as given in 8.2.4.

[4]Slabs with beams between columns along exterior edges. Exterior panels shall be considered to be without edge beams if α_f is less than 0.8. The value of α_f for the edge beam shall be calculated in accordance with 8.10.2.7.

Slabs with beams on all sides:

For nonprestressed slabs with beams spanning between supports on all sides, overall slab thickness (h) shall satisfy the limits in Table 8.3.1.2, unless the calculated deflection limits of 8.3.2 are satisfied.

Table 8.3.1.2—Minimum thickness of nonprestressed two-way slabs with beams spanning between supports on all sides

α_{fm}[1]	**Minimum h, mm.**		
$\alpha_{fm} \leq 0.2$	8.3.1.1 applies		(a)
$0.2 < \alpha_{fm} \leq 2.0$	Greater of:	$\dfrac{\ell_n\left(0.8+\dfrac{f_y}{1400}\right)}{36+5\beta\left(\alpha_{fm}-0.2\right)}$	(b)[2],[3]
		125	(c)
$\alpha_{fm} > 2.0$	Greater of:	$\dfrac{\ell_n\left(0.8+\dfrac{f_y}{1400}\right)}{36+9\beta}$	(d)[2],[3]
		90	(e)

[1]α_{fm} is the average value of α_f for all beams on edges of a panel and α_f shall be calculated in accordance with 8.10.2.7.

[2]ℓ_n is the clear span in the long direction, measured face-to-face of beams (mm).

[3]β is the ratio of clear spans in long to short directions of slab.

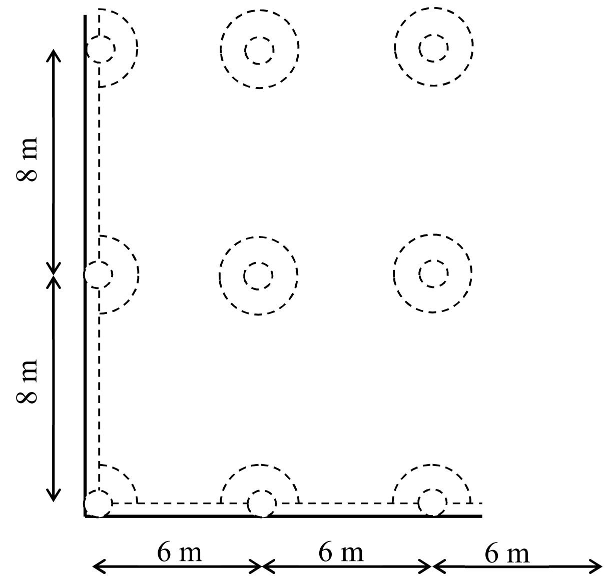
At discontinuous edges of slabs conforming to 8.3.1.2, an edge beam with $\alpha_f \geq 0.80$ shall be provided, or the minimum thickness required by (b) or (d) of Table 8.3.1.2 shall be increased by at least 10 percent in the panel with a discontinuous edge.

Example 1

Thickness of a flat slab with edge beams

Column capital diameter = 1000 mm

(f_y = 420 MPa)



Solution

exterior panel

$t = \ell_n/33$

$\ell_n = 8000 - 0.89 \times 1000 = 7110$ mm

$t = 7110/33 = 215.4$ mm $>$ 125 mm

interior panel

$t = \ell_n/33 = 215.4$ mm $>$ 125 mm

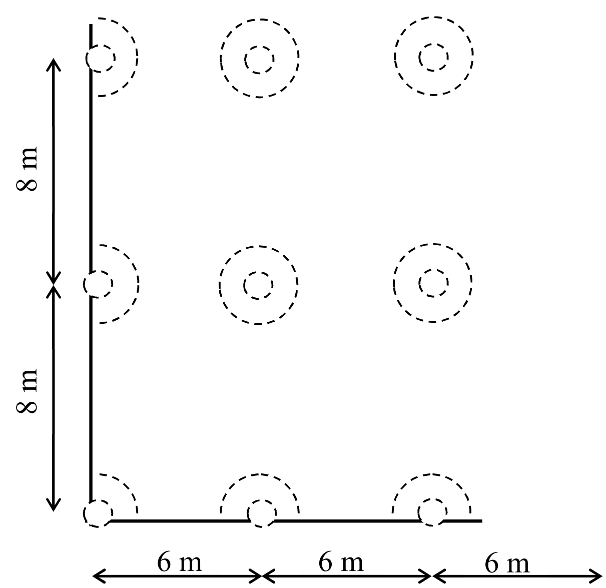
$\therefore$ use t = 220 mm

Example 2

Thickness of a flat slab without edge beams

Column capital diameter = 1000 mm

(f_y = 420 MPa)



Solution

exterior panel

$t = \ell_n/30$

$\ell_n = 8000 - 0.89 \times 1000 = 7110$ mm

$t = 7110/30 = 237.0$ mm $>$ 125 mm

interior panel

$t = \ell_n/33 = 215.4$ mm $>$ 125 mm

$\therefore$ use t = 240 mm

Example 3

Find the minimum thickness of a slab for an interior panels due to deflection control for the following: Use f_y = 350 MPa.

- **a-** Slab with beams (8.1 × 8.2) m clear span with $\boldsymbol{\alpha_m = 2.3}$
- **b-** Flat plate (4.4 × 4.6) m clear span.
- **c-** Flat slab with drop panels (6.2 × 6.2) m clear span.

Solution

a- Slab with beams (8.1 × 8.2) m clear span with $\boldsymbol{\alpha_m = 2.3}$

$\alpha_m = 2.3 > 2.0$

$$\Rightarrow t_{min} = \frac{l_n\left(0.8 + \frac{f_y}{1400}\right)}{36 + 9\beta} \quad ; \quad \beta = \frac{L_n}{S_n} = \frac{8.2}{8.1} = 1.012$$

$$t_{min} = \frac{8200 \times \left(0.8 + \frac{350}{1400}\right)}{36 + 9 \times 1.012} = 190.875 \text{ mm} \quad > 90 \text{ mm O.K.}$$

Use t = 200 mm

b- Flat plate (4.4 × 4.6) m clear span.

From table

For f_y = 280 $\quad t = \frac{L_n}{36} = \frac{4600}{36} = 127.778$ mm

For f_y = 420 $\quad t = \frac{L_n}{33} = \frac{4600}{33} = 139.394$ mm

For f_y = 350 (by linear interpolation)

$$t = \frac{127.778 + 139.394}{2} = 133.586 \text{ mm} \quad > 125 \text{ mm O.K.}$$

Use t = 140 mm

c- Flat slab with drop panels (6.2 × 6.2) m clear span.

From table

For f_y = 280 $\quad t = \frac{L_n}{40} = \frac{6200}{40} = 155$ mm

For f_y = 420 $\quad t = \frac{L_n}{36} = \frac{6200}{36} = 172.222$ mm

For f_y = 350 (by linear interpolation) $\quad t = \frac{155 + 172.222}{2} = 163.611$ mm $\quad >$ 100 mm O.K.

Use t = 170 mm

Example 4

Find the minimum thickness of a slab for an interior panels due to deflection control for the following: Use f_y = 420 MPa. (60000 psi).

a- Slab with beams (8.2 × 7.7) m clear span with **α_m= 2.3**

b- Slab without drop panels (5.4 × 4.8) m clear span with **α_m= 0.18**

c- Flat plate (4.2 × 4.6) m clear span.

d- Flat slab with drop panels (6.0 × 6.2) m clear span.

e- Slab with beams (5.8 × 5.8) m clear span with **α_m= 1.5**

<u>Solution</u>

a- Slab with beams (8.2 × 7.7) m clear span with **α_m= 2.3**

$\alpha_m = 2.3 > 2.0$

$$\Rightarrow\ t_{min} = \frac{l_n\left(0.8+\frac{f_y}{1400}\right)}{36+9\beta} \quad ; \quad \beta = \frac{L_n}{S_n} = \frac{8.2}{7.7} = 1.065$$

$$t_{min} = \frac{8200\times\left(0.8+\frac{420}{1400}\right)}{36+9\times1.065} = 197.872 \text{ mm} \quad > 90 \text{ mm O.K.}$$

$\Rightarrow$ Use t = 200 mm

b- Slab without drop panels (5.4 × 4.8) m clear span with **α_m= 0.18**

$\alpha_m = 0.18 < 0.2$

From table $t = \frac{L_n}{33} = \frac{5400}{33} = 163.636$ mm $\quad > 125$ mm O.K.

$\Rightarrow$ Use t = 170 mm

c- Flat plate (4.2 × 4.6) m clear span.

From table $t = \frac{L_n}{33} = \frac{4600}{33} = 139.394$ mm $\quad > 125$ mm O.K.

$\Rightarrow$ Use t = 140 mm

d- Flat slab with drop panels (6.0 × 6.2) m clear span.

From table $t = \frac{L_n}{36} = \frac{6200}{36} = 172.222$ mm $\quad > 100$ mm O.K.

$\Rightarrow$ Use t = 175 mm

e- Slab with beams (5.8 × 5.8) m clear span with **α_m= 1.5**

$0.2 < \alpha_m = 1.5 < 2.0$

$$\Rightarrow\ t_{min} = \frac{l_n\left(0.8 + \dfrac{f_y}{1400}\right)}{36 + 5\beta(\alpha_m - 0.2)}$$

$$\beta = \frac{L_n}{S_n} = \frac{5.8}{5.8} = 1.0$$

$$t_{min} = \frac{5800\times\left(0.8 + \dfrac{420}{1400}\right)}{36 + 5\times 1.0\times(1.5 - 0.2)} = 150.118 \text{ mm} \quad > 125 \text{ mm} \quad \text{O.K.}$$

$\Rightarrow$ Use t = 160 mm

Example 5

Find the minimum thickness of a slab for an interior panels due to deflection control for the following: Use f_y = 420 MPa.

a- Flat slab with drop panels (7.0 × 5.6) m clear span.

b- Slab with beams (5.0 × 6.3) m clear span with **α_m= 2.3**

c- Slab with beams (5.0 × 5.5) m clear span with **α_m= 1.7**

d- Flat plate (4.2 × 4.5) m clear span.

e- Flat slab without drop panels (5.9 × 4.2) m clear span.

<u>Solution</u>

a) Flat slab with drop panels (7.0 × 5.6) m clear span.

From table

$$t = \frac{\ell_n}{36} = \frac{7000}{36} = 194.444 \text{ mm} \qquad > 100 \text{ mm} \quad \text{O.K.}$$

$\Rightarrow$ Use t = 200 mm

b) Slab with beams (5.0 × 6.3) m clear span with **α_m= 2.3**

$\alpha_m = 2.3 > 2.0$

$$\Rightarrow\ t_{min} = \frac{\ell_n\left(0.8 + \dfrac{f_y}{1400}\right)}{36 + 9\beta}$$

$$\beta = \frac{\ell_n}{s_n} = \frac{6.3}{5.0} = 1.26$$

$$t_{min} = \frac{6300\times\left(0.8 + \dfrac{420}{1400}\right)}{36 + 9\times 1.26} = 146.388 \text{ mm} \quad > 90 \text{ mm} \quad \text{O.K.}$$

$\Rightarrow$ Use t = 150 mm