University of Al Maarif

SIGNAL PROCESSING

Department of Medical Instruments
Techniques Engineering

Class: 3rd

Lecture 1

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Introduction

DSP is the mathematics, the algorithms, and the techniques used to manipulate these signals after they have been converted into a digital form. This includes a wide variety of goals, such as: enhancement of visual images, recognition and generation of speech, compression of data for storage and transmission, etc.

Signal: is defined as any physical quantity that varies with time, space and any other independent variable or variables. e.g.

• A speech signal would be represented by acoustic pressure as a function of time.

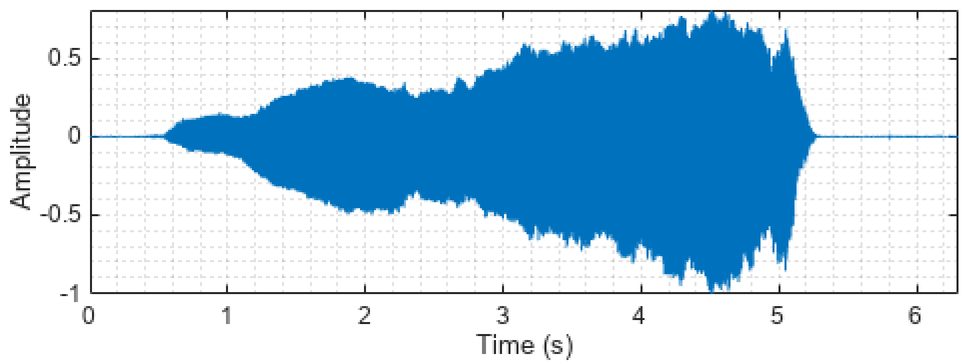

Figure 1 speech signal

• A monochromatic picture would be represented by brightness as a function of two spatial variable.

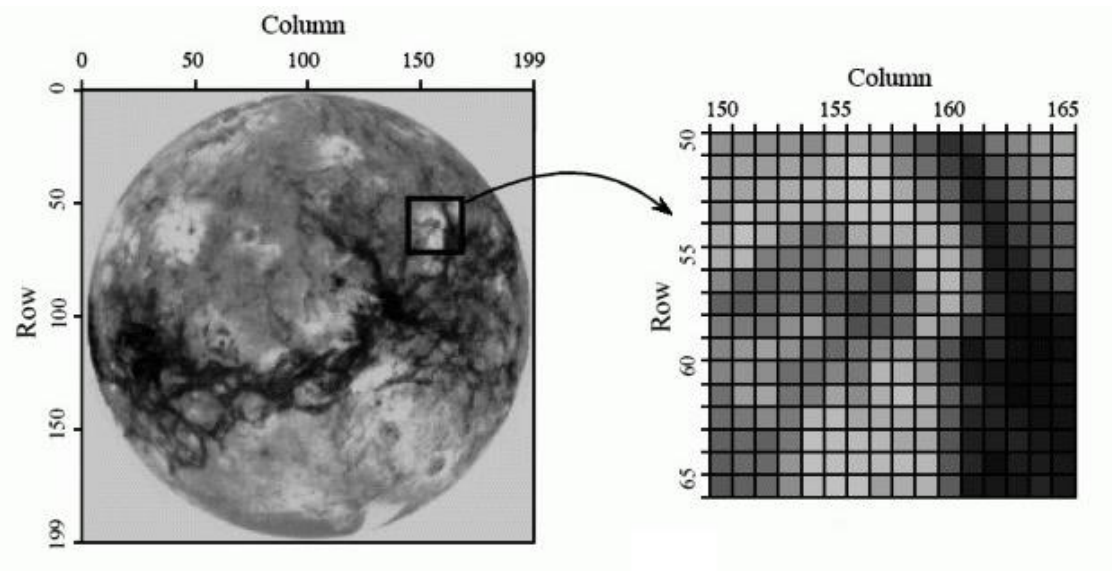

Figure 2 monochromatic picture

• An electrocardiogram (ECG) is an electrophysiological signal that contains a large amount of valuable information about the electrical activity of the heart.

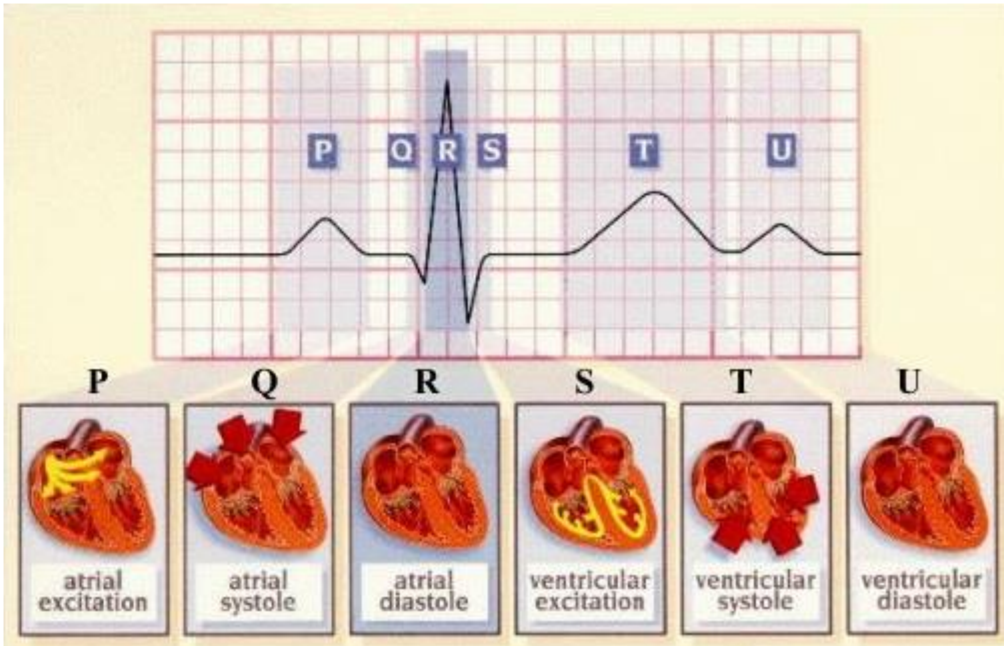

Figure 3 electrophysiological signal

Mathematically, signal described by functions with one or more independent variables. For example, the function below varies with one independent variable of time

$$S_1(t) = 5t$$

While the function below varies with two independent variables of space x and y

$$S_1(x, y) = 3x + 2xy + 10y^2$$

System: is physical or mathematical (hardware or Software) that perform operation on Signal to extract or modify information for example a low-pass filter is a system to remove a high frequencies content from a signal.

Basic elements of DSP:

Most signals in nature are analog such as voltage, temperature, pressure, ECG, speech signals etc. Usually a transducer (sensor) is used to convert the non-electrical signal to the analog electrical signal (voltage). In its most general form, a DSP system consists of three main components, as illustrated in Figure 4.

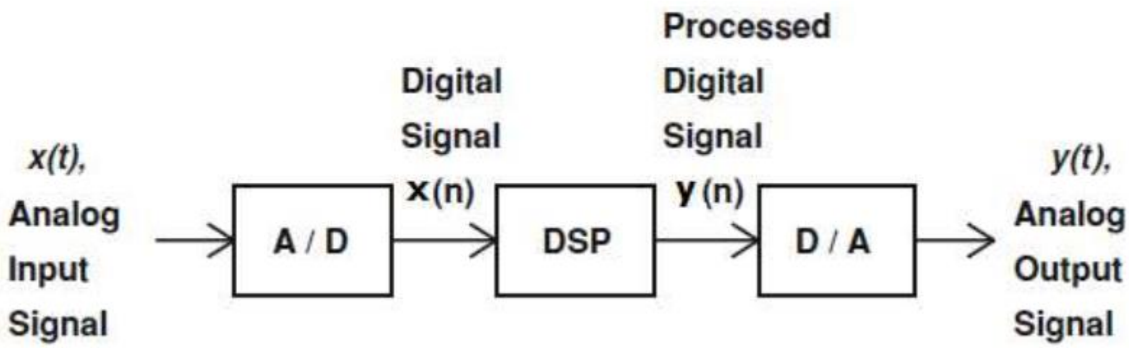

Figure 4 Block diagram of a generic signal processing system

• The analog-to-digital (A/D) converter transforms the analog signal x(t) at the system input into a digital signal x[n]. An A/D converter can be thought of as consisting of a sampler (creating a discrete-time signal), followed by a quantizer (creating discrete levels).

• The digital system performs the desired operations on the digital signal x[n] and produces a corresponding output y [n] also in digital form.

• The digital-to-analog (D/A) converter transforms the digital output y[n] into an analog signal y(t) suitable for interfacing with the outside world.

Digital Signal Processing Versus Analog Signal Processing

DSP has a number of advantages over analog signal processing (ASP), namely:

Advantages:

1- More flexible.

2- Often easier system upgrade.

3- Data easily stored in memory.

4- Better control over accuracy requirements.

5- DSP is less susceptible to noise and power supply disturbances than ASP.

Limitations:

1- A/D & signal processors speed: wide-band signals still difficult to treat

(real-time systems).

2- Finite word-length effect.

3- Cost/complexity added by A/D and D/A conversion.

DSP Applications:

The figure below shows the DSP applications. Many more areas are increasingly being explored by engineers and scientists. Applications of DSP techniques will continue to have profound impacts and improve our lives.

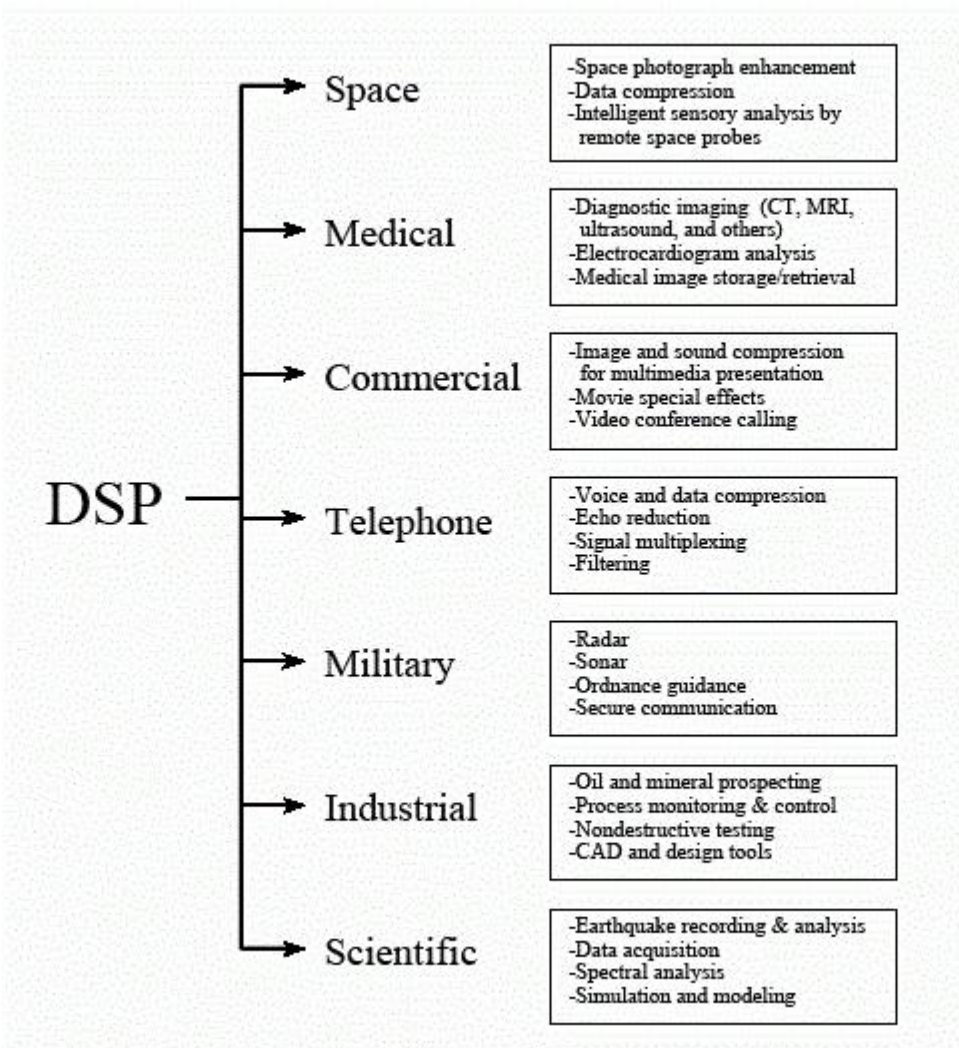

Figure 5 The DSP Applications

DISCRETE-TIME SIGNALS

In digital signal processing, signals are represented as sequences of numbers, called samples. A sample value of a typical discrete–time signal or sequence is denoted as x(n) with the argument n being an integer in the range - ∞ and ∞. It should be noted that x(n) is defined only for integer values of n. The graphical representation of a sequence x(n) with real–valued samples is illustrated in figure below.

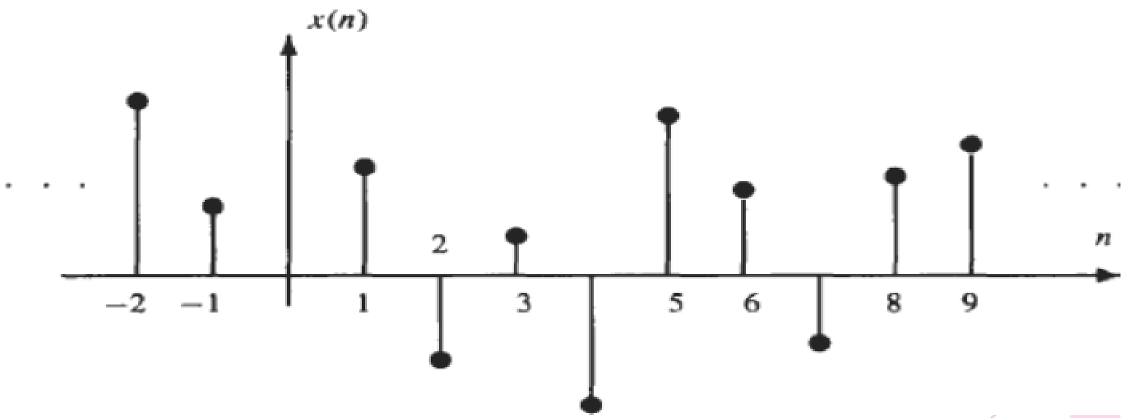

Figure 6 graphical representation of a discrete-time signal x(n)

In some problems and applications, it is convenient to view x(n) as a vector. Thus, the sequence values x (0) to x (N - 1) may often be considered to be the elements of a column vector as follows:

$$X = [x(0), x(1), \dots, x(N-1)]$$

CLASSIFICATION OF SIGNALS

A signal is a function representing a physical quantity or variable, and typically it contains information about the behavior or nature of the phenomenon. For instance, in a RC circuit the signal may represent the voltage across the capacitor or the current flowing in the resistor. Mathematically, a signal is represented as a function of an independent variable t. Usually t represents time. Thus, a signal is denoted by x(t).

A. Continuous-Time and Discrete-Time Signals:

A signal x(t) is a continuous-time signal if t is a continuous variable. If t is a discrete variable, that is, x(t) is defined at discrete times, then x(t) is a discrete-time signal. Since a discrete-time signal is defined at discrete times, a discrete-time signal is often identified as a sequence of numbers, denoted by $\{x_n\}$ or x[n], where n = integer.

Illustrations of a continuous-time signal x(t) and of a discrete-time signal x[n] are shown in Figure 7.

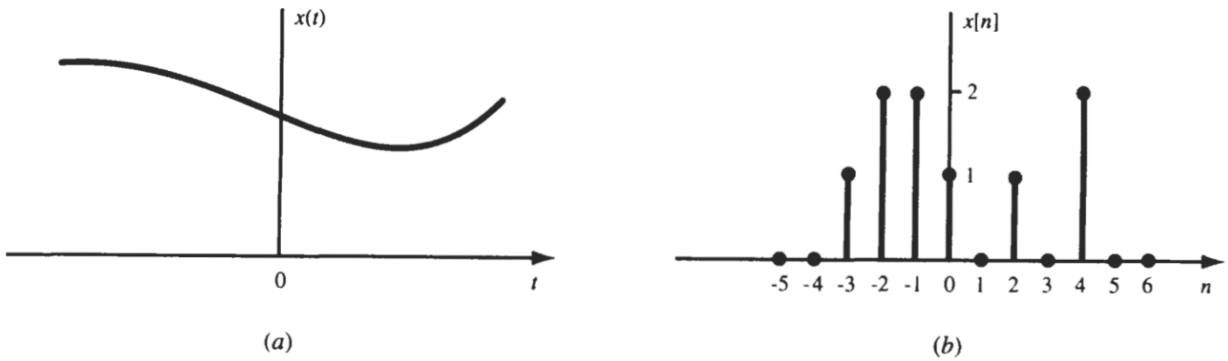

Figure 7 Graphical representation of (a) continuous-time and (b) discrete-time signals

A discrete-time signal x[n] may represent a phenomenon for which the independent variable is inherently discrete. For instance, the daily closing stock market average is by its nature a signal that evolves at discrete points in time (that is, at the close of each day). On the other hand, a discrete-time signal x[n] may be obtained by sampling a continuous-time signal x(t).

A discrete-time signal x[n] can be defined in two ways:

1. We can specify a rule for calculating the nth value of the sequence. For example,

$$X[n] = \begin{cases} (\frac{1}{2})^n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

$$\{X_n\} = \left\{1, \frac{1}{2}, \frac{1}{4}, \cdots, (\frac{1}{2})^n, \cdots\right\}$$

2. We can also explicitly list the values of the sequence. For example, the sequence shown in Figure 7 (b) can be written as

$$\{X_n\} = \{\cdots, 0,0,1,2,2, \underset{\uparrow}{1}, 0,1,0,2,0,0, \cdots\}$$

or

$$\{X_n\} = \{1,2,2, \underset{\uparrow}{1}, 0,1,0,2\}$$

We use the arrow to denote the n = 0 term. We shall use the convention that if no arrow is indicated, then the first term corresponds to n = 0 and all the values of the sequence are zero for n < 0.

B. Analog and Digital Signals:

If a continuous-time signal x(t) can take on any value in the continuous interval (a, b), where a may be - ∞ and b may be + ∞, then the continuous-time signal x(t) is called an analog signal. If a discrete-time signal x[n] can take on only a finite number of distinct values, then we call this signal a digital signal.

C. Real and Complex Signals:

A signal x(t) is a real signal if its value is a real number, and a signal x(t) is a complex signal if its value is a complex number. A general complex signal x(t) is a function of the form

$$x(t) = x_1(t) + jx_2(t) \qquad (1)$$

where $x_1(t)$ and $x_2(t)$ are real signals and $j = \sqrt{-1}$. Note that in Eq. (1) t represents either a continuous or a discrete variable.

D. Deterministic and Random Signals:

Deterministic signals are those signals whose values are completely specified for any given time. Thus, a deterministic signal can be modeled by a known function of time t. Random signals are those signals that take random values at any given time and must be characterized statistically. Random signals will not be discussed in this text.

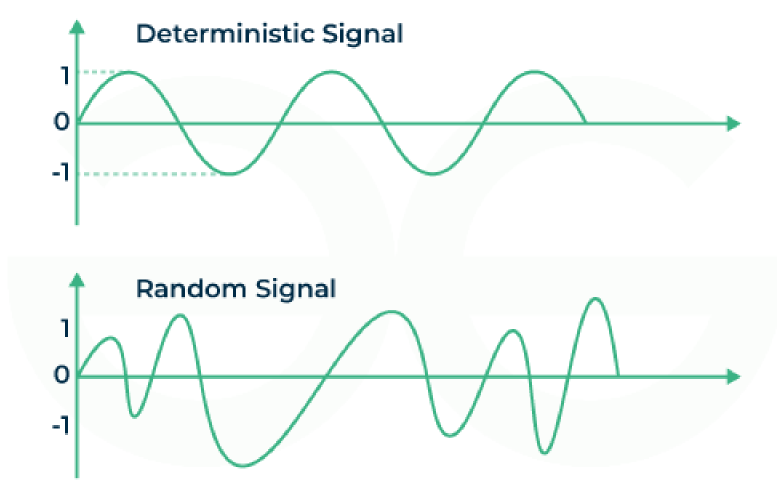

Figure 8 Deterministic and Random Signal Graph

E. Even and Odd Signals:

A signal x(t) or x[n] is referred to as an even signal if

$$x(-t) = x(t) \qquad (2)$$

$$x[-n] = x[n] \qquad (3)$$

A signal x(t) or x[n] is referred to as an odd signal if

$$x(-t) = -x(t) \qquad (4)$$

$$x[-n] = -x[n] \qquad (5)$$

Examples of even and odd signals are shown in Figure 9.

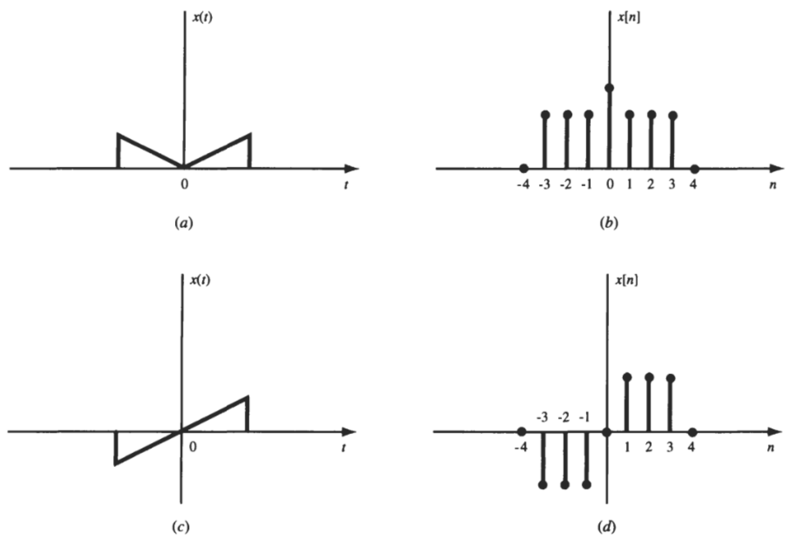

Figure 9 Examples of even signals (a and b) and odd signals (c and d)

F. Periodic and Nonperiodic Signals:

A signal is said to be periodic if it repeats at regular intervals. Non – periodic signal does not repeat at regular intervals.

Condition for periodicity of signal:

• CT signal: if x(t) = x (t + T), then x(t) is periodic.

- Smallest T = Fundamental period
- Fundamental frequency $f_o = 1/T_o$ (Hz or cycles/second)
- Angular frequency: $\omega_o = 2\pi/T_o$ (rad/seconds)

• DT signal: if x[n] = x [n + N], then x[n] is periodic.

- Min (No): fundamental period
- $f_o = 1/N_o$ (cycles/sample)
- $\Omega = 2\pi/N$ (rads/sample).

Basic / Elementary signals

These signals serve as basic building block for construction of somewhat more complex signals. The list of elementary signals mainly contains singularity functions and exponential functions. These elementary signals are also known as basic signals/standard signals. Let us discuss these basic signals one-by-one.

1. **The Unit Step Function:**

The unit step function u(t), also known as the Heaviside unit function, is defined as

$$u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases} \quad (16)$$

which is shown in Figure 10 (a). Note that it is discontinuous at t = 0 and that the value at t = 0 is undefined. Similarly, the shifted unit step function $u(t - t_o)$ is defined as

$$u(t - t_o) = \begin{cases} 1 & t > t_o \\ 0 & t < t_o \end{cases} \quad (17)$$

which is shown in Figure 10 (b).

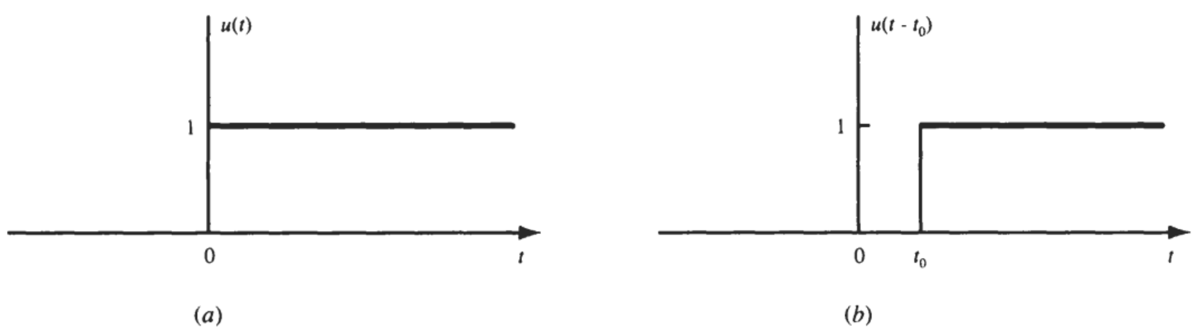

Figure 10 (a) Unit step function; (b) shifted unit step function

DISCRETE-TIME:

The unit step sequence u[n] is defined as

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases} \quad (18)$$

which is shown in Figure 11(a). Note that the value of u[n] at n = 0 is defined [unlike the continuous-time step function u(f) at t = 0] and equals unity. Similarly, the shifted unit step sequence u[n - k] is defined as

$$u[n-k] = \begin{cases} 1 & n \geq k \\ 0 & n < k \end{cases} \quad (19)$$

which is shown in Figure 11(b)

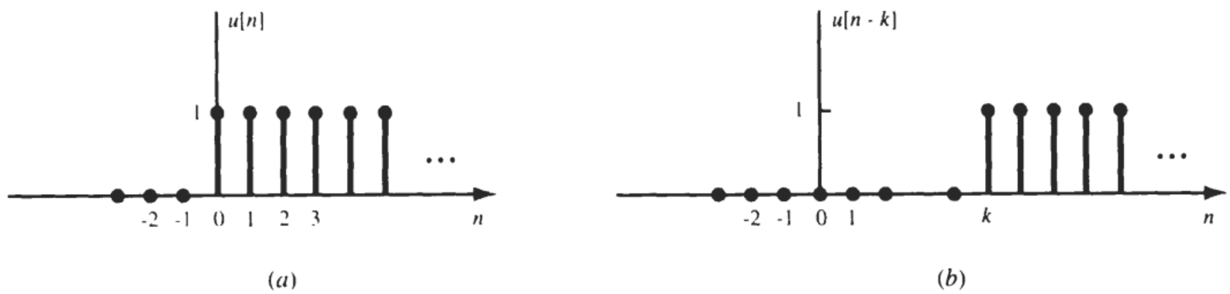

Figure 11 (a) Unit step sequence; (b) shifted unit step sequence

2. The Unit Impulse Function:

The unit impulse function δ(t), also known as the Dirac delta function, plays a central role in system analysis. Traditionally, δ(t) is often defined as the limit of a suitably chosen conventional function having unity area over an infinitesimal time interval as shown in Figure 12.

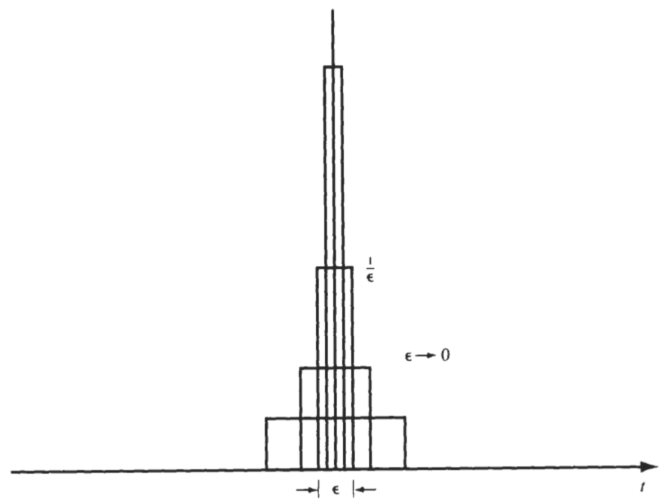

Figure 12

And possesses the following properties:

$$\delta(t) = \begin{cases} 0 & t \neq 0 \\ \infty & t = 0 \end{cases} \quad (20)$$

$$\int_{-\infty}^{\infty} \delta(t) dt = 1 \quad (21)$$

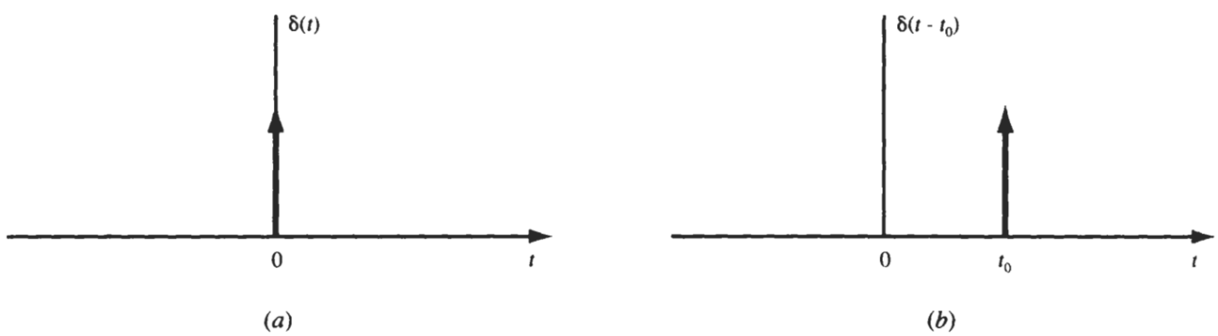

Figure 13 (a) Unit impulse function; (b) shifted unit impulse function

DISCRETE-TIME:

The unit impulse (or unit sample) sequence δ[n] is defined as

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases} \quad (22)$$

which is shown in Figure 14(a). Similarly, the shifted unit impulse (or sample) sequence δ[n - k] is defined as

$$\delta[n-k] = \begin{cases} 1 & n = k \\ 0 & n \neq k \end{cases} \quad (23)$$

which is shown in Figure 14(b).

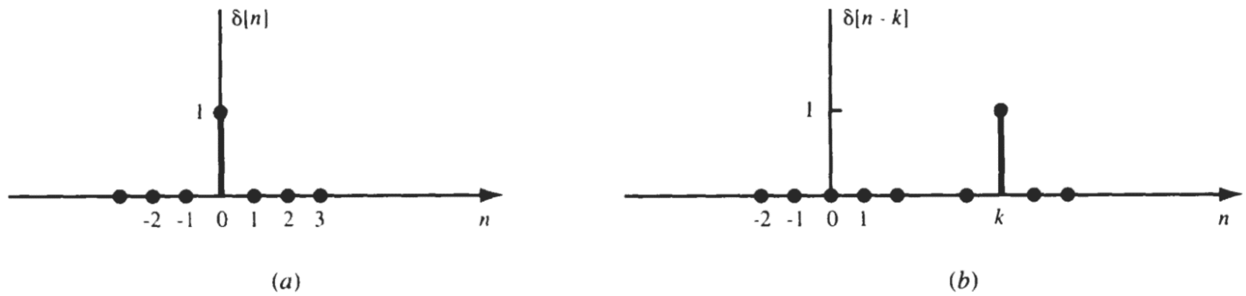

Figure 14 (a)Unit impulse (sample) sequence; (b) shifted unit impulse sequence

3. The Unit Ramp Function:

The unit ramp function is the integral of the unit step function. It is called the unit ramp function because for positive t, its slope is one amplitude unit per time.

$$r(t) = \begin{cases} t & t \geq 0 \\ 0 & t < 0 \end{cases} \quad (24)$$

$$r(t) = tu(t) \quad (25)$$

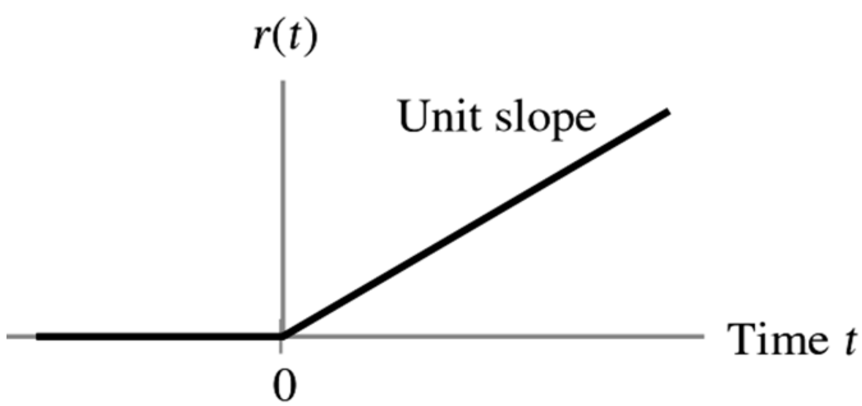

Figure 15 Unit Ramp Function

DISCRETE-TIME:

The discrete-time version of the ramp function

$$r[n] = \begin{cases} n & n \geq 0 \\ 0 & n < 0 \end{cases} \quad (26)$$

$$r[n] = nu[n] \quad (27)$$

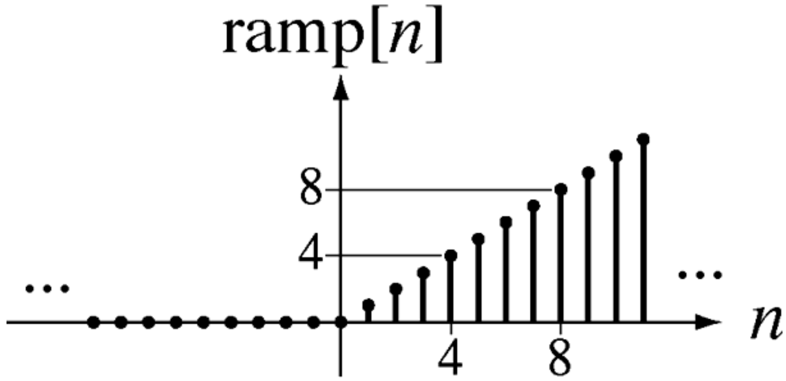

Figure 16 The discrete-time version of the ramp function

4. Real Exponential Signals:

A real exponential signal, is written as:

$$x(t) = Be^{at} \quad (28)$$

Where both B and a are real parameters. B is the amplitude of the exponential signal measured at time t = 0.

(i) Decaying exponential, for which a < 0.

(ii) Growing exponential, for which a > 0.

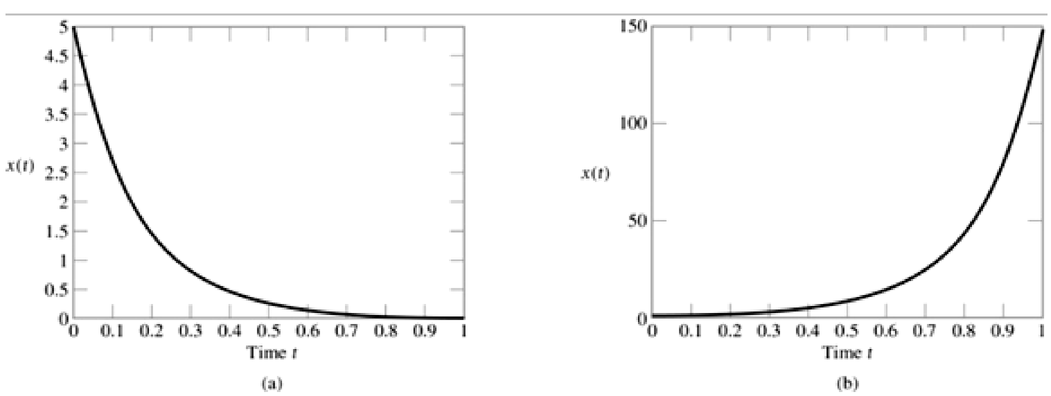

Figure 17 (a) Decaying exponential form of continuous-time signal. (b) Growing exponential form of continuous-time signal

DISCRETE-TIME:

$$x[n] = Be^{\alpha n} \quad (29)$$

where B and α are real.

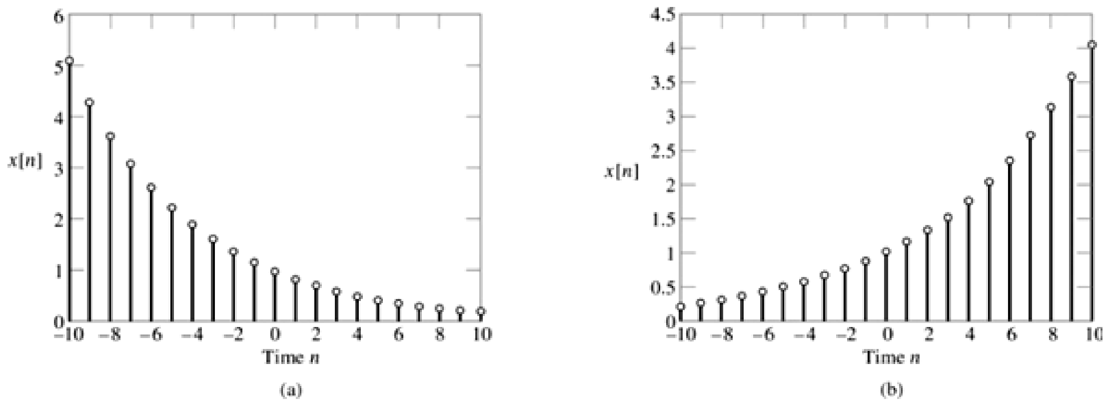

Figure 18 (a) Decaying exponential form of discrete-time signal. (b) Growing exponential form of discrete-time signal

5. Sinusoidal Signals:

A general form of sinusoidal signal is

$$x(t) = A\cos(\omega_o t + \theta) \quad (30)$$

where A is the amplitude, ω_o is the frequency in radians per second, and θ is the phase angle in radians.

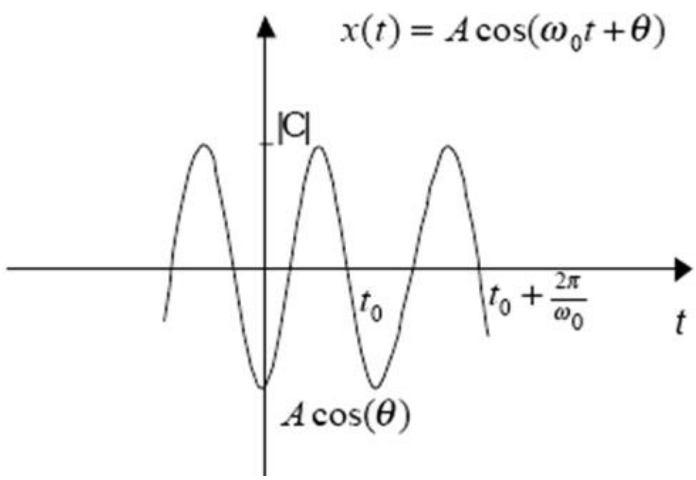

Figure 19 Continuous-Time Sinusoidal signal

DISCRETE-TIME:

Discrete time version of sinusoidal signal, written as

$$x[n] = A\cos(\Omega n + \phi) \quad (31)$$

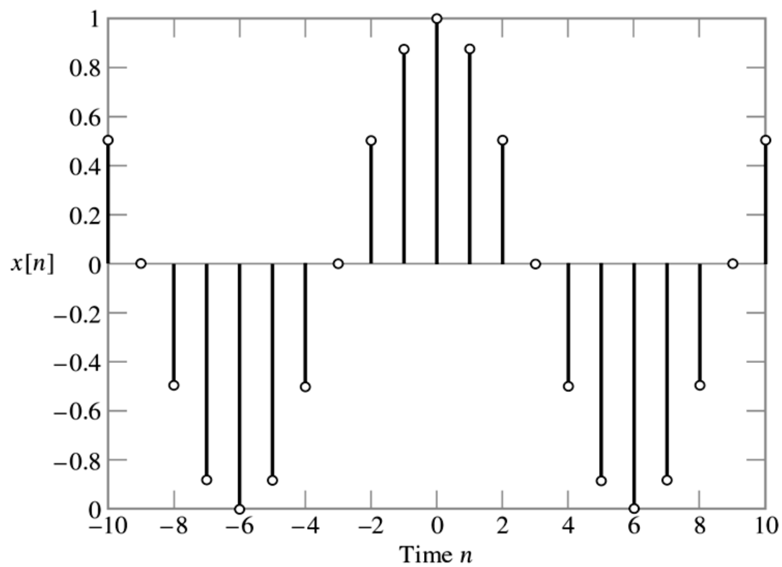

Figure 20 Discrete-Time Sinusoidal Signal

Introduction to Systems

A system is a mathematical model of a physical process that relates the input (or excitation) signal to the output (or response) signal. Let x and y be the input and output signals, respectively, of a system. Then the system is viewed as a transformation (or mapping) of x into y. This transformation is represented by the mathematical notation:

$$y = \boldsymbol{T}x \quad (32)$$

where T is the operator representing some well-defined rule by which x is transformed into y. Relationship (32) is depicted as shown in Figure 21(a). Multiple input and/or output signals are possible as shown in Figure 21(b).

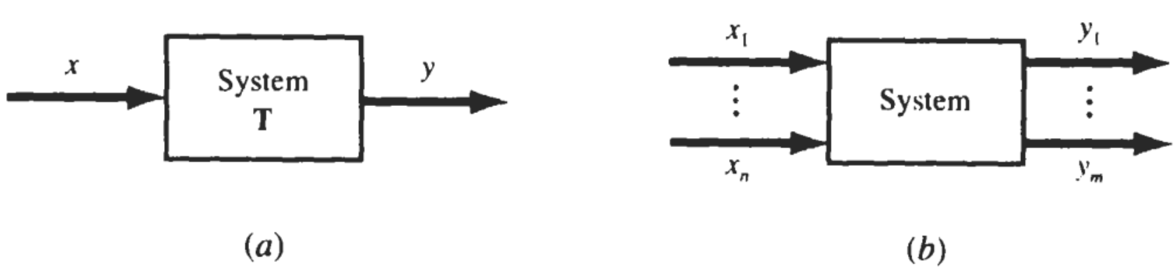

Figure 21 System with single or multiple input and output signals

Real life example of system; In communication system; the system will transport the information contained in the message over a communication channel and deliver that information to the destination.

Figure 22 Elements of a communication system

Continuous-Time and Discrete-Time Systems:

If the input and output signals x and y are continuous-time signals, then the system is called a continuous-time system Figure 23(a). If the input and output signals are discrete-time signals or sequences, then the system is called a discrete-time system Figure 23(b).

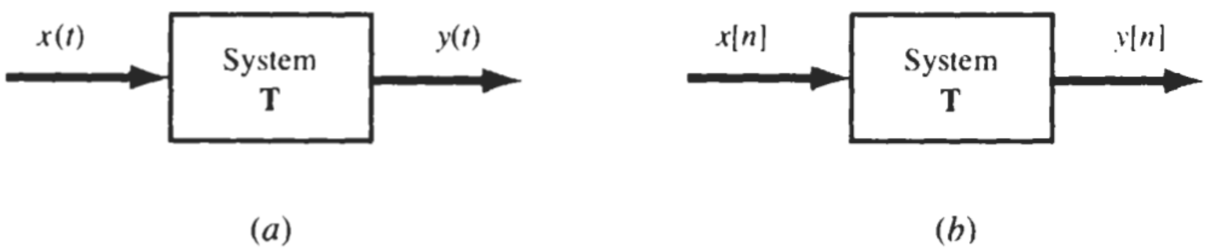

Figure 23 (a) Continuous-time system; (b) discrete-time system

Properties of Systems

The properties of a system describe the characteristics of the operator **T** representing the system. Basic properties of the system:

A. Stability
B. Memory
C. Linearity
D. Time Invariance
E. Causality
F. Invertibility

A. Stable and Unstable Systems

A system is bounded-input/bounded-output (BIBO) stable if for any bounded input x defined by

$$|x| \leq k_1 \quad (33)$$

the corresponding output y is also bounded defined by

$$|y| \leq k_2 \quad (34)$$

where k_1 and k_2 are finite real constants. Note that there are many other definitions of stability.

B. Systems with Memory and without Memory

A system is said to be memoryless if the output at any time depends on only the input at that same time. Otherwise, the system is said to have memory. An example of a memoryless system is a resistor R with the input x (t) taken as the current and the voltage taken as the output y (t). The input-output relationship (Ohm's law) of a resistor is

$$y(t) = Rx(t) \quad (35)$$

An example of a system with memory is a capacitor C with the current as the input x(t) and the voltage as the output y (t); then

$$y(t) = \frac{1}{C}\int_{-\infty}^{t} x(\tau)\, d\tau \quad (36)$$

A second example of a system with memory is a discrete-time system whose input and output sequences are related by

$$y[n] = \sum_{k=-\infty}^{n} x[k] \quad (37)$$

C. Linear Systems and Nonlinear Systems

If the operator **T** in Eq. (32) satisfies the following two conditions, then **T** is called a linear operator and the system represented by a linear operator **T** is called a linear system:

(i) Additivity:

Given that **T**$x_1 = y_1$ and **T**$x_2 = y_2$ then

$$\boldsymbol{T}\{x_1 + x_2\} = y_1 + y_2 \quad (38)$$

for any signals x_1 and x_2.

(ii) Homogeneity (or Scaling):

$$\boldsymbol{T}\{\alpha x\} = \alpha y \quad (39)$$

for any signals x and any scalar α.

Any system that does not satisfy Eq. (38) and/or Eq. (39) is classified as a nonlinear system. Equations (38) and (39) can be combined into a single condition as

$$\boldsymbol{T}\{\alpha_1 x_1 + \alpha_2 x_2\} = \alpha_1 y_1 + \alpha_2 y_2 \quad (40)$$

where α_1 and α_2 are arbitrary scalars. Equation (40) is known as the superposition property. Examples of linear systems are the resistor Eq. (35) and the capacitor Eq. (36).

D. Time-Invariant and Time-Varying Systems

A system is called time-invariant if a time shift (delay or advance) in the input signal causes the same time shift in the output signal. Thus, for a continuous-time system, the system is time-invariant if

$$\boldsymbol{T}\{x(t-\tau)\} = y(t-\tau) \quad (41)$$

for any real value of τ. For a discrete-time system, the system is time-invariant (or shift- invariant) if

$$\boldsymbol{T}\{x[n-k]\} = y[n-k] \quad (42)$$

for any integer k. A system which does not satisfy Eq. (41) (continuous-time system) or Eq. (42) (discrete-time system) is called a time-varying system. To check a system for time-invariance, we can compare the shifted output with the output produced by the shifted input.

Note: If the system is linear and also time-invariant, then it is called a **linear rime-invariant (LTI) system**.

E. Causal and Noncausal Systems

A system is called causal if its output y (t) at an arbitrary time t = t_o depends on only the input x (t) for $t \leq t_o$. That is, the output of a causal system at the present time depends on only the present and/or past values of the input, not on its future values. Thus, in a causal system, it is not possible to obtain an output before an input is applied to the system. A system is called noncausal if it is not causal. Examples of noncausal systems are

$$y(t) = x(t+1) \quad (43)$$

$$y[n] = x[-n] \quad (44)$$

Note that all memoryless systems are causal, but not vice versa.

F. Invertible and Non-invertible Systems

A system is invertible if different inputs lead to different outputs i.e., two-different inputs for a given system should not produce same output. In other words, if input can be recovered from the system output, the system is said to be invertible.

$$y(t) = \boldsymbol{T}\{x(t)\} \quad (45)$$

$$z(t) = \boldsymbol{T}^{-1}\{y(t)\} = \boldsymbol{T}^{-1}\{\boldsymbol{T}\{x(t)\}\} = x(t) \qquad (46)$$