Control: measuring the value of the Controlled Variable of the system and applying the control signal to the system to correct or limit deviation of the measured value from a desired value.

Plants: Any physical object to be controlled, such as mechanical device, heating furnace, chemical reactor, or a spacecraft.

Process: A continuing operation marked by a series of gradual changes and lead to a particular result. it consists of a series of controlled actions.

Systems: A system is a combination of components that act together and perform a certain task.

Disturbance: A disturbance is a signal that tends to affect the value of the output of the system.

Feedback control: It refers to an operation that tends to reduce the difference between the output of a system and same reference input.

Open-loop Control system: is one in which the control action is independent of the output.

closed-loop control system: is one in which the control action is somehow dependent on the output.

Bandwidth: The bandwidth of a system is a frequency response measure of how well the system responds to (or filters) variations (or frequencies) in the input signal.

Stable system: The definition of a stable system can be based upon the response of the system to bounded inputs, that is, inputs whose magnitudes are less than some finite value for all time.

A continuous or discrete_time system is said to be stable if every bounded input produces a bounded output.

Major advantages of Open-loop Control system:

- Simple construction and ease of maintenance.
- less expensive than closed loop system.
- no stability problems.
- Convenient when output is hard to measure.

Major disadvantages of open-loop system:

- Disturbances cause errors and output may differ from the desired.
- recalibration is necessary from time to time.

Advantages of Closed-loop Control system:

- Ability to minimize the effect of external disturbance & noise.
- the use of feedback makes the system response insensitive to external disturbance.
- Can use inacurate components to obtain the accurate control of a given plant.

Disadvantages of closed-loop System:

- Stability is a major problem.
- more expensive than open loop system.

The LAPLACE TRANSFORM:

The laplace transform method substitutes relatively easily solved algebric equations for the more difficult differential equations. The time-response solution is obtained by the following operations:

1- obtain the linearized differential equations.

2- obtain the laplace transformation of the differential equations.

3- Solve the resulting algebric equation for the transform of the variable of interest.

The Laplace transformation for a function of time, $f(t)$, is:

$$\mathcal{L}\{f(t)\} = F(s) = \int_0^\infty f(t)\, e^{-st}\, dt$$

The inverse Laplace transform is written as:

$$\mathcal{L}^{-1}\{F(s)\} = f(t) = \frac{1}{2\pi j} \int_{\sigma - j\infty}^{\sigma + j\infty} F(s)\, e^{st}\, ds$$

σ : is real Positive

example: Find $F(s)$ for $f(t) = e^{-t}$

<u>sol</u>: $\mathcal{L}\{e^{-t}\} = \int_0^\infty e^{-t}\, e^{-st}\, dt = \frac{-1}{S+1}\, e^{-(S+1)t}\Big|_0^\infty = \frac{1}{S+1}$

Some properties of Laplace Transform

1- if a_1 and a_2 constants,

$$\mathcal{L}(a_1 f_1(t) + a_2 f_2(t)) = a_1 F_1(s) + a_2 F_2(s)$$

2- Transfor of a derivative df/dt,

$$\mathcal{L}\left\{\frac{df(t)}{dt}\right\} = SF(s) - f(0)$$

where $f(0)$ is the initial value of $f(t)$

3- for integral $\int_0^t f(\tau)\, d\tau$

$$\mathcal{L}\left\{\int_0^t f(\tau)\, d\tau\right\} = \frac{F(s)}{S}$$

4- The initial value $f(0)$ of a function $f(t)$,

$$f(0) = \lim_{t \to 0} f(t) = \lim_{S \to \infty} SF(s) \quad , \ t > 0$$

This relation is called the initial value Theorem

5- The final value $f(\infty)$ of the function $f(t)$,

$$f(\infty) = \lim_{t \to \infty} f(t) = \lim_{s \to 0} sF(s)$$

which is called final value theorem

6- for a Time scaling function, $f(t/a)$,

$$\mathcal{L}\left[f\left(\frac{t}{a}\right)\right] = a\,F(as)$$

7- for a frequency scaling function $F(s/a)$,

$$\mathcal{L}^{-1}\left\{ F\left(\frac{s}{a}\right)\right\} = a f(at)$$

8- Time delay function, $f(t-T)$

$$\mathcal{L}\left\{ f(t-T)\right\} = e^{-sT} F(s)$$

9- for a function $e^{-at} f(t)$,

$$\mathcal{L}\left\{ e^{-at} f(t)\right\} = F(s+a)\text{ , Complex translation}$$

10- if $\mathcal{L}\, f_1(t) = F_1(s)$ and $\mathcal{L}\, f_2(t) = F_2(s)$

then:

$$\mathcal{L}^{-1}\left\{ F_1(s) . F_2(s)\right\} = \int_0^t f_1(t) f_2(t-\tau)\, d\tau$$

$$= \int_0^t f_2(\tau) f_1(t-\tau)\, d\tau$$

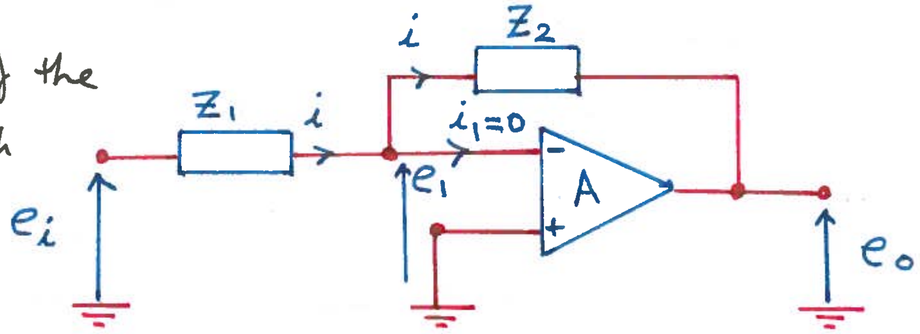
which is convolution integrals

3- Zero output impedance

4- infinite Bandwidth

Let's take the following circuit for example:

input impedance of the op amp is very high therefore $i_1 = 0$



$$I(s) = \frac{E_i(s) - E_1(s)}{Z_1(s)} \qquad \frac{E_1(s) - E_o(s)}{Z_2(s)}$$

E_1 is virtually Zero, because $i_1 = 0$ & infinite gain of op amp

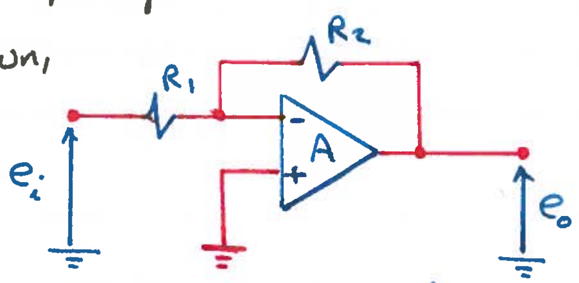
$$\rightarrow I(s) = \frac{E_i(s)}{Z_1(s)} = -\frac{E_o(s)}{Z_2(s)}$$

or $$\frac{E_o(s)}{E_i(s)} = -\frac{Z_2}{Z_1}$$

where Z_2 is the impedance connected between the output and input of the op amp., and Z_1 is the impedance connected to the input of the op amp.

For $Z_2 = R_2$, and $Z_1 = R_1$, as shown,

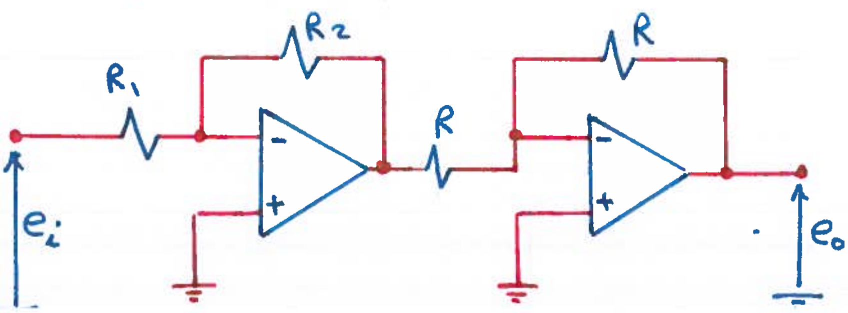
thus $$\frac{E_o(s)}{E_i(s)} = -\frac{R_2}{R_1}$$



-Inverting Amplifier-

and the circuit is called an Inverting Amplifier

example: Find the Transfer function for the below circuit,

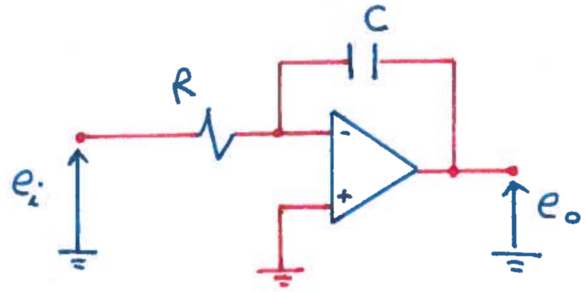


sol

$$\frac{E_o}{E_i} = -\frac{R_2}{R_1} \cdot \left(-\frac{R}{R}\right)$$

$$\rightarrow \frac{E_o}{E_i} = \frac{R_2}{R_1}$$

example: Find $e_o(t)$ for the circuit shown below,



Sol

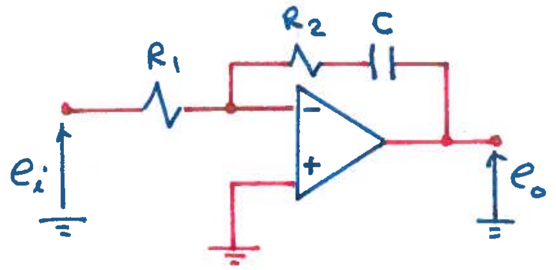
$$\frac{E_o(s)}{E_i(s)} = -\frac{Z_2}{Z_1} = -\frac{1/Cs}{R} = -\frac{1}{RCs}$$

$$E_o(s) = -\frac{1}{RCs}E_i(s)$$

$$\rightarrow e_o(t) = \mathcal{L}^{-1}E_o(s) = -\frac{1}{RC}\int_0^t e_i(\tau)\,d\tau$$

The circuit is an integrator

example: Find $e_o(t)$ for the below circuit and mention how it Processes the input Signal.



Sol

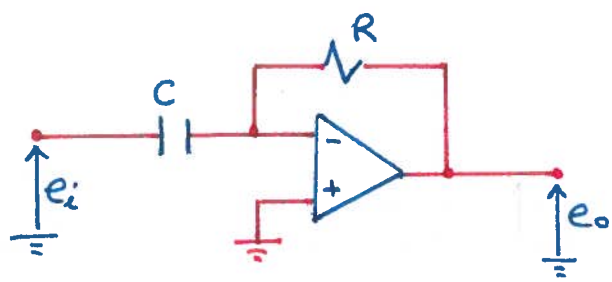
$$\frac{E_o(s)}{E_i(s)} = -\frac{R_2 + \frac{1}{Cs}}{R_1} = -\frac{R_2}{R_1} - \frac{1}{R_1Cs}$$

$$E_o(s) = -\frac{R_2}{R_1}E_i(s) - \frac{1}{R_1Cs}E_i(s)$$

$$\rightarrow e_o(t) = \mathcal{L}^{-1}E_o(s) = -\frac{R_2}{R_1}e_i(t) - \frac{1}{R_1C}\int_0^t e_i(\tau)\,d\tau$$

The circuit processes the input signal by Proportional plus integral.

example: Derive the output signal with respect to the input signal for the circuit shown,



Sol

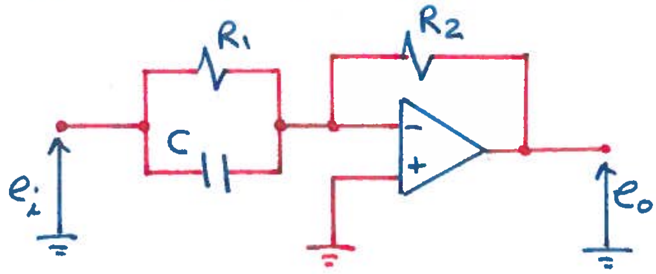
$$\frac{E_o(s)}{E_i(s)} = -\frac{R}{1/Cs} = -RCs$$

$$E_o(s) = -RCs\,E_i(s)$$

$$\rightarrow e_o(t) = -RC\frac{de_i(t)}{dt}$$

The circuit is a differentiator

example: Find the output Voltage with respect to time and Comment on how it processes the input signal, for the below circuit.



Sol

$$\frac{E_o(s)}{E_i(s)} = -\frac{R_2}{\dfrac{R_1 \cdot \frac{1}{Cs}}{R_1 + \frac{1}{Cs}}}$$

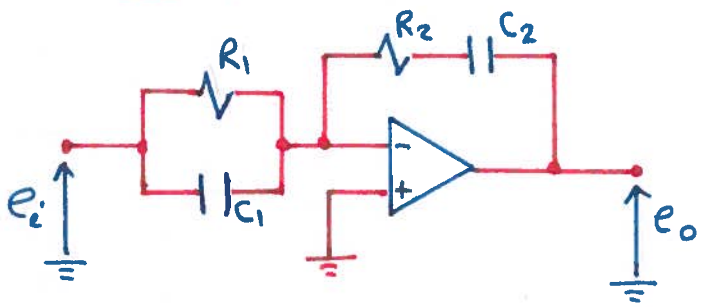
$$= -\frac{R_2}{R_1}(R_1 C s + 1)$$

$$E_o(s) = -\frac{R_2}{R_1} E_i(s) - R_2 C s\, E_i(s)$$

$$\rightarrow e_o(t) = -\frac{R_2}{R_1} e_i(t) - R_2 C \frac{d e_i(t)}{dt}$$

The circuit processes the input signal by proportional and derivative actions.

example: Find $e_o(t)$ for the below circuit then Comment.



Sol

$$\frac{E_o}{E_i} = -\frac{R_2 + \frac{1}{C_2 s}}{\dfrac{R_1 \cdot \frac{1}{C_1 s}}{R_1 + \frac{1}{C_1 s}}}$$

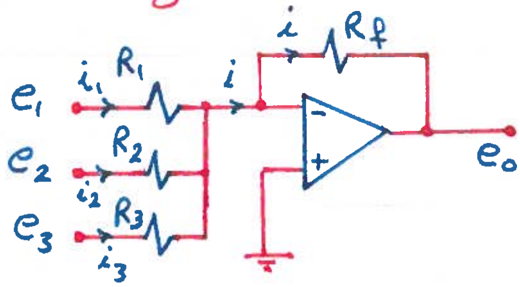
$$= -\frac{R_1 R_2 C_1 C_2 s^2 + R_1 C_1 s + R_2 C_2 s + 1}{R_1 C_2 s}$$

$$\rightarrow E_o = -\frac{R_1 C_1 + R_2 C_2}{R_1 C_2} E_i - \frac{1}{R_1 C_2 s} E_i - R_2 C_1 s\, E_i$$

$$\rightarrow e_o(t) = -\frac{R_1 C_1 + R_2 C_2}{R_1 C_2} e_i(t) - \frac{1}{R_1 C_2}\int_0^t e_i(\tau)\, d\tau - R_2 C_1 \frac{d e_i(t)}{dt}$$

∴ The circuit processes the input signal with proportional and integral and derivative actions.

example: what is the function of the following circuit?



sol

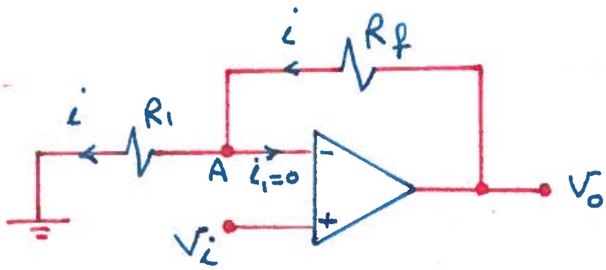
$$i = i_1 + i_2 + i_3$$
$$= \frac{e_1}{R_1} + \frac{e_2}{R_2} + \frac{e_3}{R_3}$$

$$e_o = -i R_f = -\left(\frac{e_1}{R_1} + \frac{e_2}{R_2} + \frac{e_3}{R_3}\right) R_f$$

or $$E_o = -\frac{R_f}{R_1} E_1 - \frac{R_f}{R_2} E_2 - \frac{R_f}{R_3} E_3$$

it is a summing amplifier

non Inverting Amplifier



since $i_1 = 0$

$\rightarrow V_A \approx V_i$

$\rightarrow i = \frac{V_o - V_A}{R_f} = \frac{V_A}{R_1}$

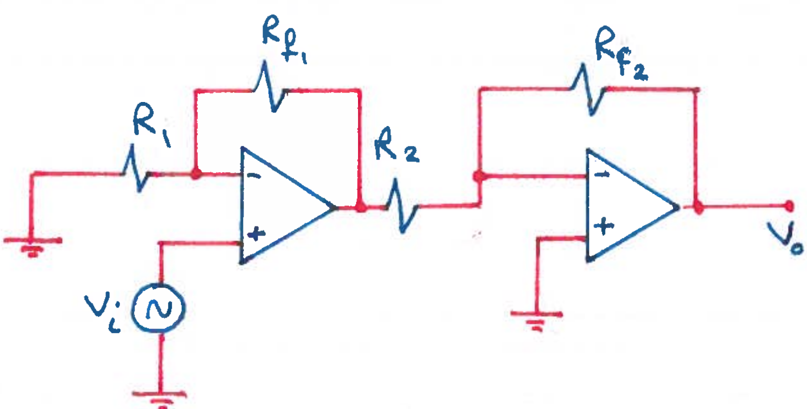
$\rightarrow \frac{V_o - V_i}{R_f} = \frac{V_i}{R_1} \rightarrow V_i (R_f + R_1) = V_o R_1$

$\rightarrow$ gain $= A = \frac{V_o}{V_i} = \frac{R_1 + R_f}{R_1} = 1 + \frac{R_f}{R_1}$, (non-inverting Amplifier)

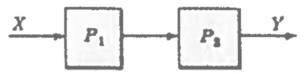
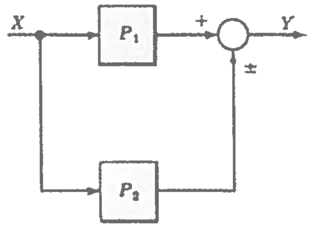
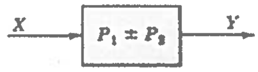
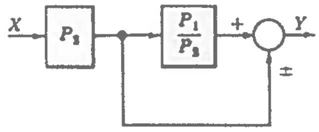
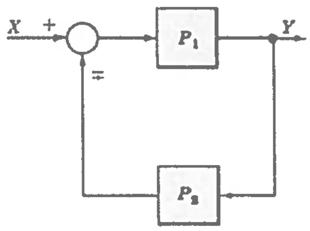
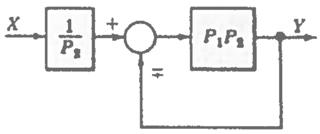
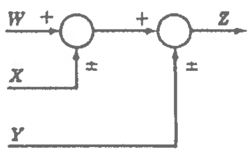
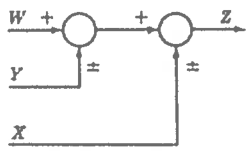
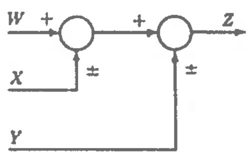
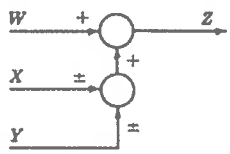
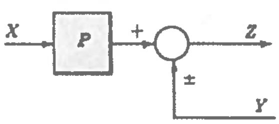
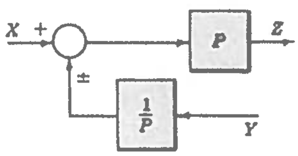
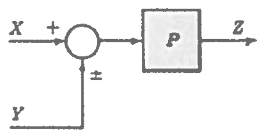
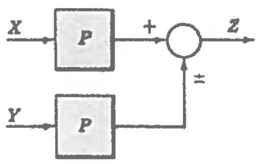
example: Find the output voltage in the below circuit,

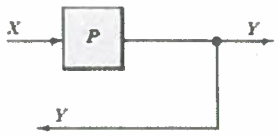
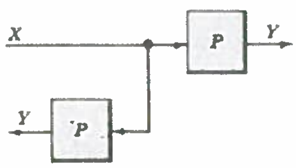
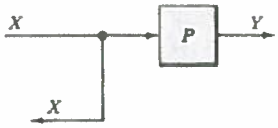
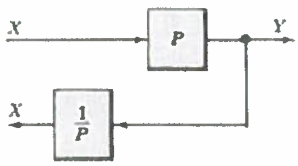
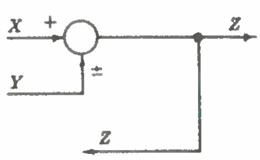
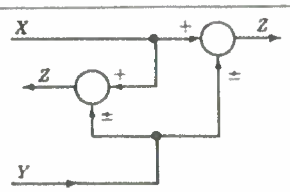
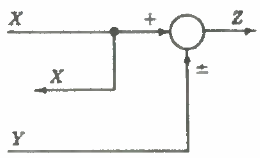
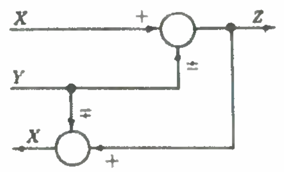
sol

$$V_o = V_i \left(1 + \frac{R_{f_1}}{R_1}\right) \cdot \left(-\frac{R_{f_2}}{R_2}\right)$$

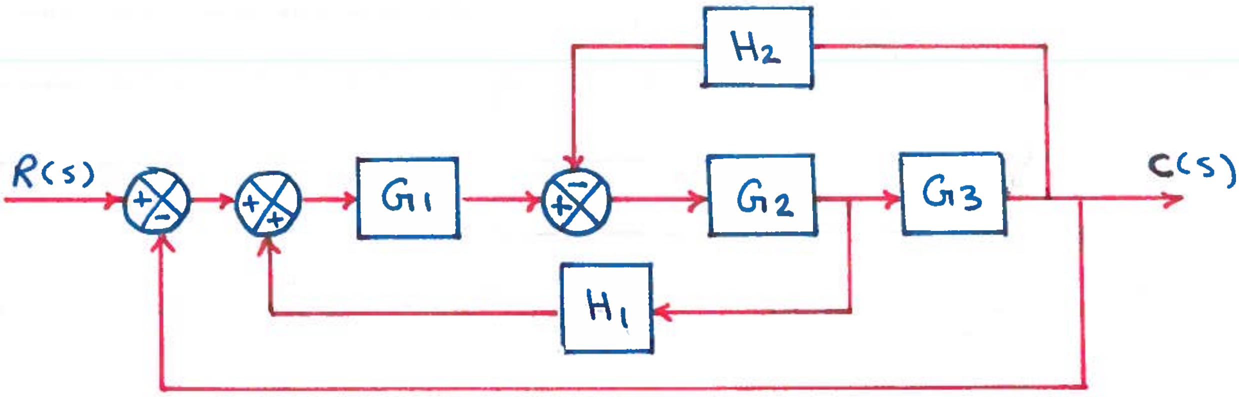


BLOCK DIAGRAM ALGEBRA AND TRANSFER FUNCTIONS OF SYSTEMS

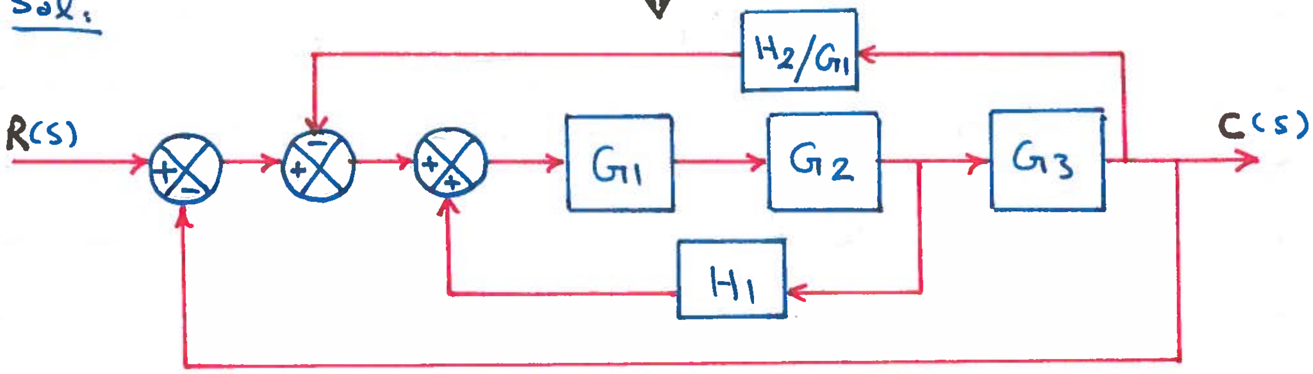
	Transformation	Equation	Block Diagram	Equivalent Block Diagram
1	Combining Blocks in Cascade	$Y = (P_1P_2)X$		
2	Combining Blocks in Parallel; or Eliminating a Forward Loop	$Y = P_1X \pm P_2X$		
3	Removing a Block from a Forward Path	$Y = P_1X \pm P_2X$		
4	Eliminating a Feedback Loop	$Y = P_1(X \mp P_2Y)$		
5	Removing a Block from a Feedback Loop	$Y = P_1(X \mp P_2Y)$		
6a	Rearranging Summing Points	$Z = W \pm X \pm Y$		
6b	Rearranging Summing Points	$Z = W \pm X \pm Y$		
7	Moving a Summing Point Ahead of a Block	$Z = PX \pm Y$		
8	Moving a Summing Point Beyond a Block	$Z = P[X \pm Y]$		

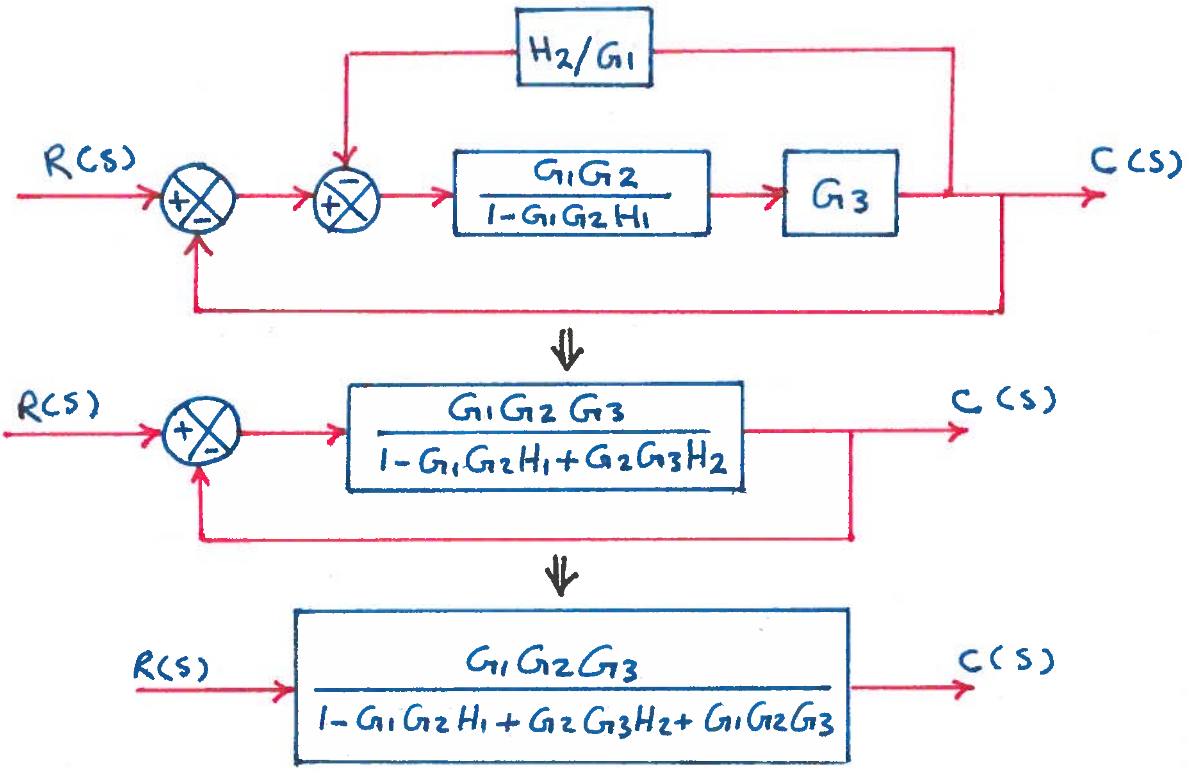
	Transformation	Equation	Block Diagram	Equivalent Block Diagram
9	Moving a Takeoff Point Ahead of a Block	$Y = PX$		
10	Moving a Takeoff Point Beyond a Block	$Y = PX$		
11	Moving a Takeoff Point Ahead of a Summing Point	$Z = X \pm Y$		
12	Moving a Takeoff Point Beyond a Summing Point	$Z = X \pm Y$		

example: Simplify the below diagram and obtain the transfer function,



sol.





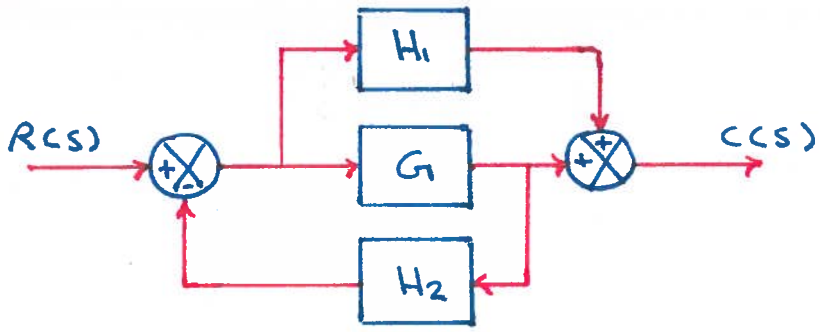
Note that the numerator of the closed_loop transfer function $C(s)/R(s)$ is the product of the transfer functions of the feed forward path, and the denominator of $C(s)/R(s)$ is equal to:

$$1 - \sum (\text{product of the T.F. around each loop})$$

$$= 1 - (G_1G_2H_1 - G_2G_3H_2 - G_1G_2G_3)$$

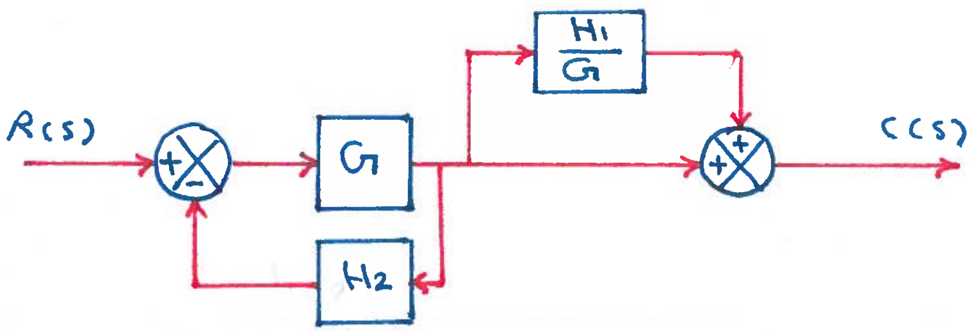
$$= 1 - G_1G_2H_1 + G_2G_3H_2 + G_1G_2G_3$$

example: Simplify the block diagram shown in figure below,

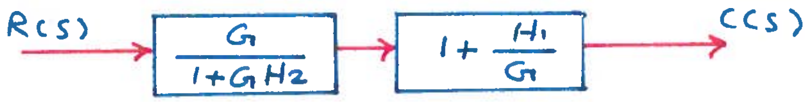


sol

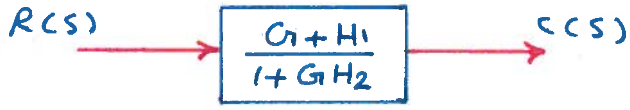
(a)



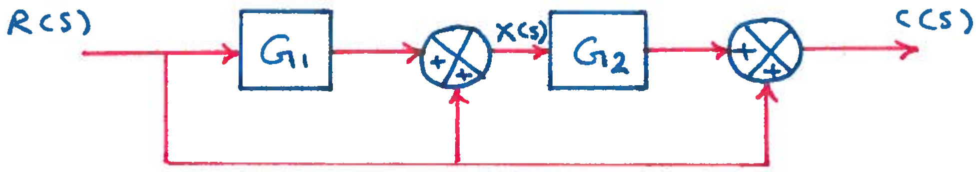
(b)



(c)

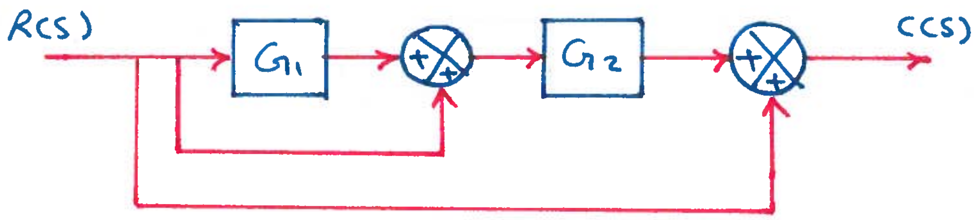


example: simplify the block diagram shown in Figure below and obtain the transfer function relating C(s) and R(s).

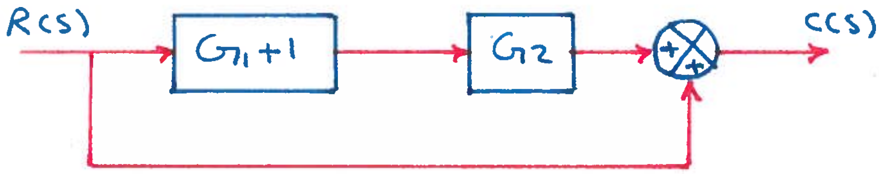


Sol

(a)



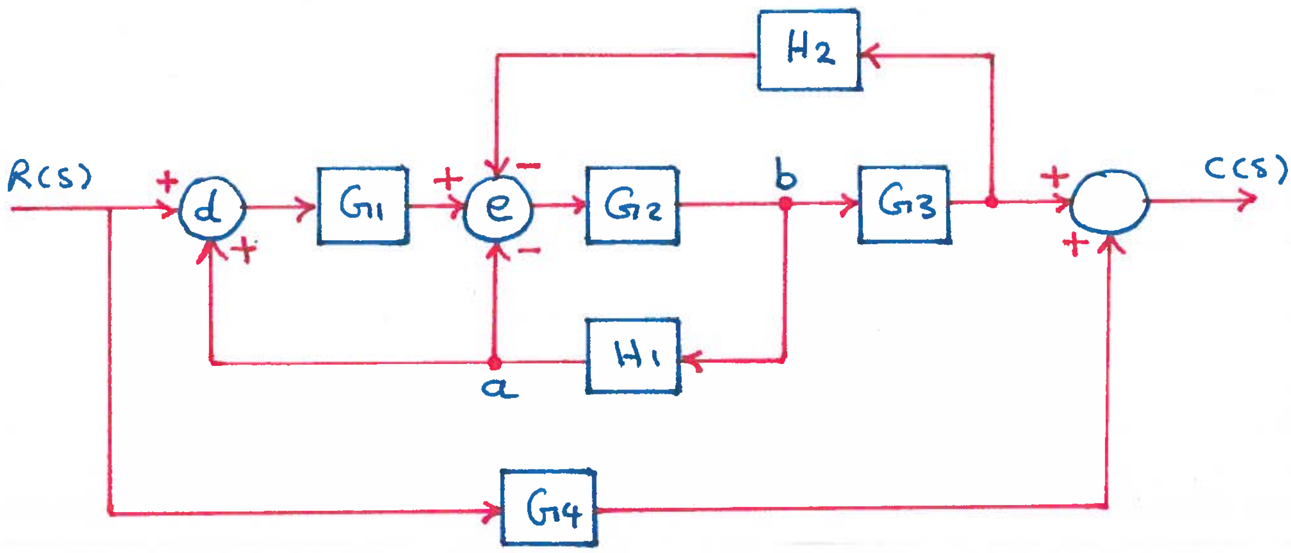
(b)



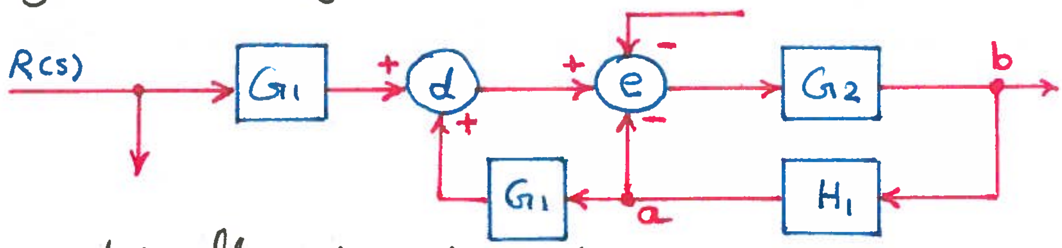
(c)

$$\rightarrow \frac{C(s)}{R(s)} = G_1G_2 + G_2 + 1$$

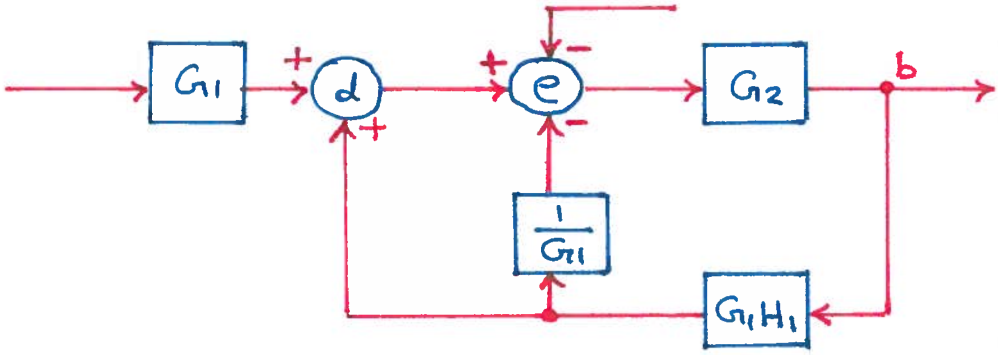
example: Reduce the block diagram givenbelow to open loop form.



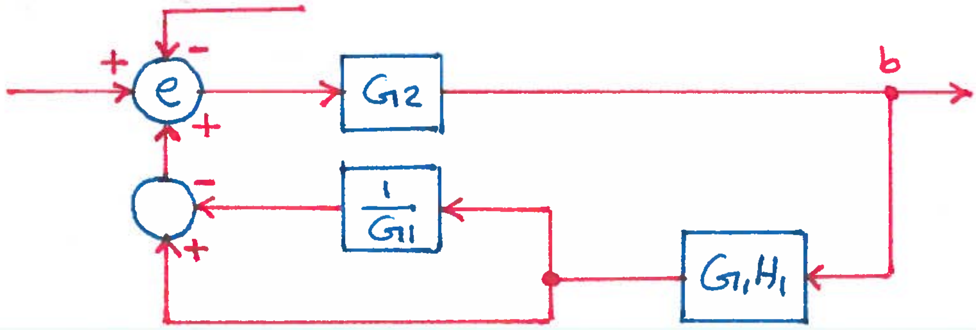
First, moving the summing point (d) beyond G_1, we get:

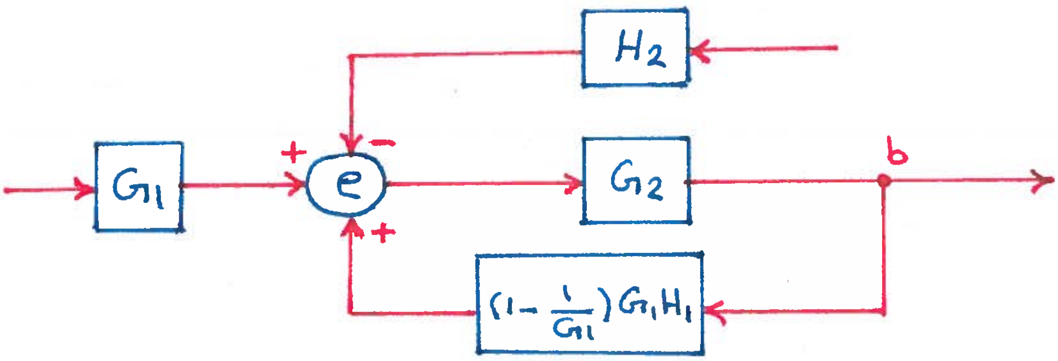


next, moving take off point (a) beyond G_1, we get:

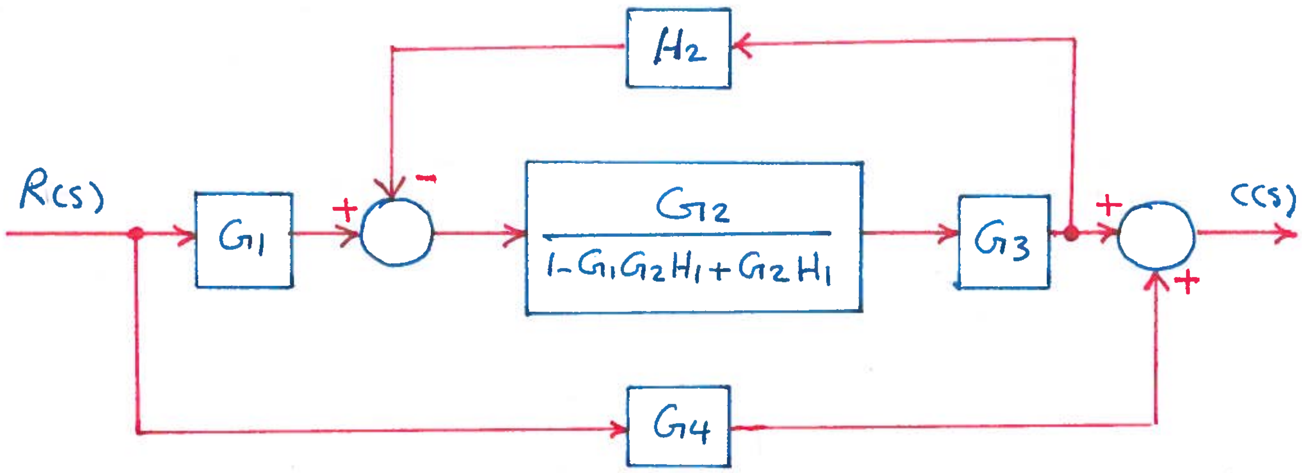


using Transformation (6b) and (2), from the tables, to combine the two lower feedback loops (from $G_1 H_1$) entering (d) and (e), we get:

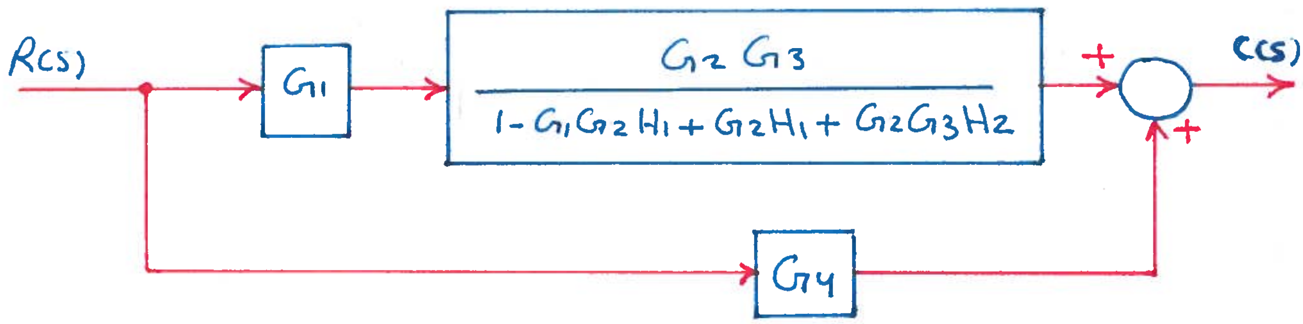




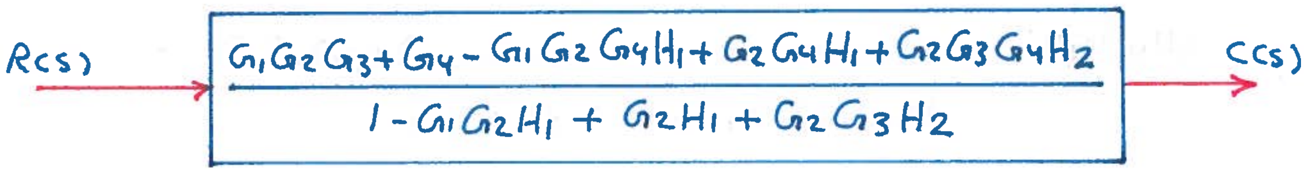
now, applying Transformation (4), from tables, to this inner loop, the system becomes:



Again, applying Transformation (4), from the tables, to the remaining feedback loop yields :



Finally, Transformation (1) and (2) from the tables, give the open-loop block diagram:



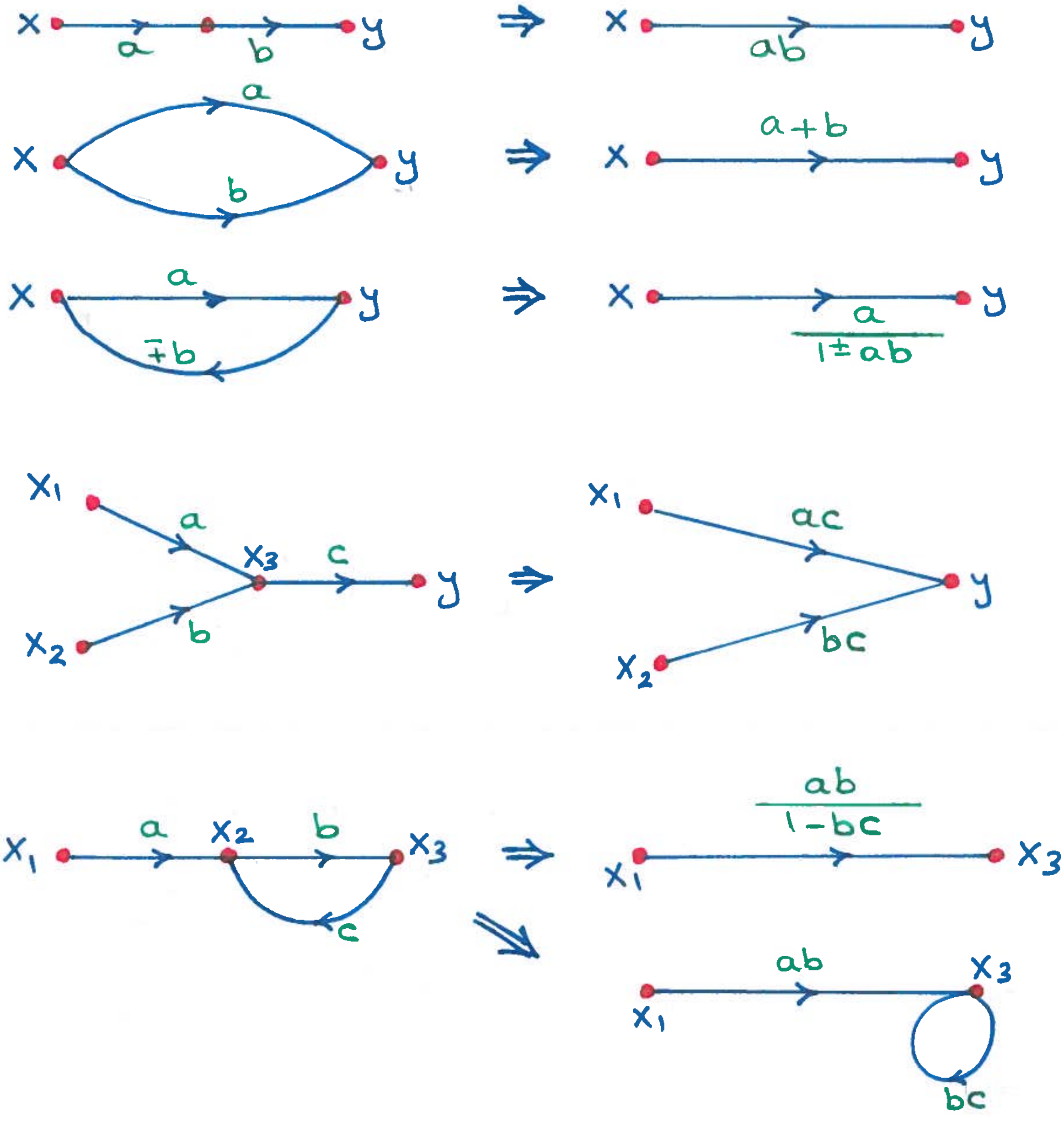
Signal Flow Graph and Mason's rule

The block diagram reduction technique is tedious and time consuming. Signal flow method gives an alternate approach for finding out transfer function of a control system. Signal flow graph is a network diagram consisting of nodes, branches, and arrows. Nodes represent variables or signals in a system. The nodes are connected by branches and arrows that indicate the direction of flow of signal.

If $y = ax$, then the signal flow graph for the equation is :

The transmittance or gain (a) is written on top of the arrow.

The following rules apply, when reducing a signal flow graph:

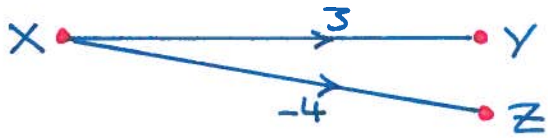


For example, Ohms low states that $V = IR$, where V is a voltage, and I a current, and R a resistance. The signal flow graph for this equation is given as:

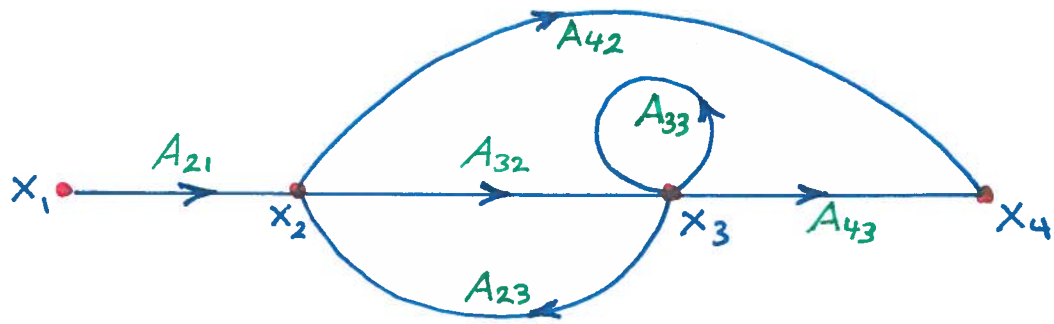
example: Represent the following equations in a signal flow graph,

$$y = 3x \;,\; z = -4x$$

Sol



Now, let's consider the below example of a signal flow graph, to define some terms,



Path: A path is a continuous unidirectional succession of branches along which no node is passed more than once. refering to the graph: x_1 to x_2 to x_3 to x_4, and x_2 to x_3 and back to x_2, and x_1 to x_2 to x_4.

input node/source: is a node with only outgoing branches, For example, x_1

output node/Sink: is a node with only incoming branches. For example, x_4

Forward path: is a path from input node to the output node. For example, x_1 to x_2 to x_3 to x_4, x_1 to x_2 to x_4 are Forward paths.

Feedback path/Feedback loop: is a path which originates and terminates on the same node. For example, x_2 to x_3, back to x_2 is a feedback path.

Self-loop: is a feedback loop consisting of a single branch. For example, A_{33} is a self loop.

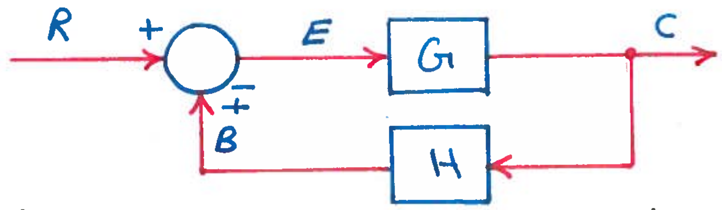
gain: is the transmission function of that branch when the transmission function is a multiplicative operator. For example, A_{33} is the gain of the self loop if A_{33} is a constant or transfer function.

path gain: is the product of the branch gains encountered in in transversing a path. For example, the path gain of the path from X_1 to X_2 to X_3 to X_4 is $A_{21} A_{32} A_{43}$.

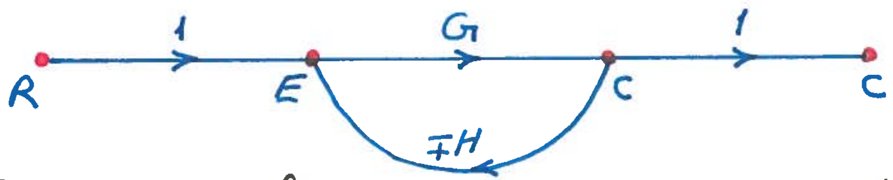
Loop gain: is the product of the branch gains of the loop. For example, the loop gain of the feedback loop from X_2 to X_3 and back to X_2 is $A_{32} A_{23}$.

Constructing the Signal flow Graph:

Consider the below block diagram,



The signal flow graph is easily constructed as shown, note that the (−) or (+) sign of the summing point is associated with H.



In general, the signal flow graph can be constructed as following:

1- write the system equations in the form,

$$X_1 = A_{11} X_1 + A_{12} X_2 + \cdots + A_{1n} X_n$$

$$X_2 = A_{21} X_1 + A_{22} X_2 + \cdots + A_{2n} X_n$$

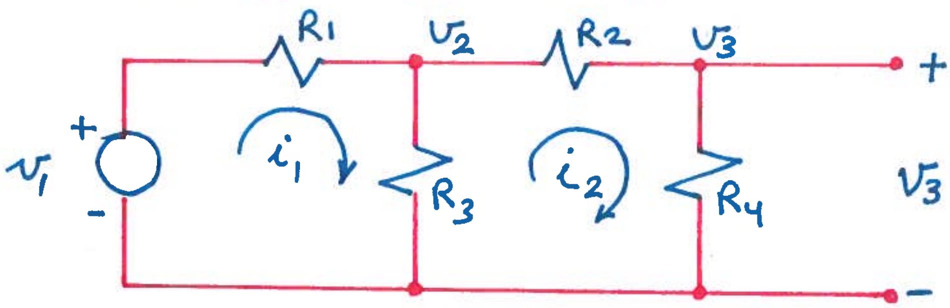
$$\vdots$$

2- arrange the nodes from left to right.

3- Connect the nodes by appropriate branches A_{11}, A_{12}, etc.

4- If the desired output node has outgoing branches, add a dummy node and a unity gain branch.

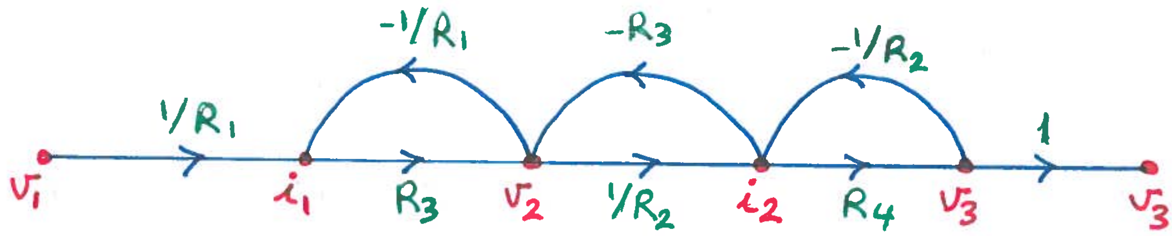
example: Consider the below circuit,



We can write four independent equations from Kirchhoff's voltage and current laws, we shall proceed from left to right,

$$i_1 = \left(\frac{1}{R_1}\right) v_1 - \left(\frac{1}{R_1}\right) v_2$$
$$v_2 = R_3 i_1 - R_3 i_2$$
$$i_2 = \left(\frac{1}{R_2}\right) v_2 - \left(\frac{1}{R_2}\right) v_3$$
$$v_3 = R_4 i_2$$

We have five variables (nodes): v_1, v_2, v_3, i_1, and i_2. laying out these five nodes in the same order, from left to right, with v_1 as an input node, and connecting nodes with the appropriate branches, we get the below graph. If we wish to consider v_3 as an output node, we must add a unity gain branch and another node,



Mason's Gain Formula:

A complicated block diagram can be reduced as

$$\frac{C}{R} = \frac{G}{1 \mp GH}$$

It is possible to simplify signal flow graphs in a manner similar to that of block diagram reduction.

Mason gave us a formula relating the output and input. The formula is:

$$T = \frac{C}{R} = \frac{1}{\Delta} \sum_i P_i \Delta_i$$

where T: overall gain of the system

P_i: gain of i^{th} forward path

Δ: 1 − (sum of all individual loops) + (sum of the gain product of all possible combination of two nontouching loops) − (sum of gain product of all possible combination of three nontouching loops) + (....) − (....) + ...

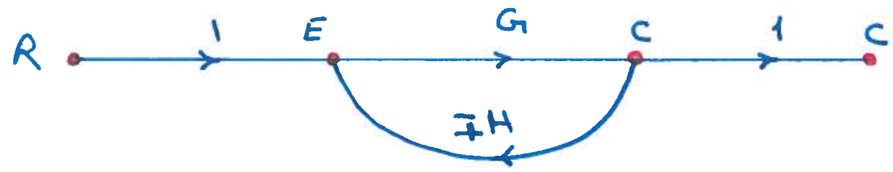
$$\rightarrow \Delta = 1 - \sum_j P_{j1} + \sum_j P_{j2} - \sum_j P_{j3} + \cdots$$

P_{jK} = j^{th} possible product of K nontouching loops

Δ_i: Same as Δ but formed by loops not touching the i^{th} forward path.

Δ is called the signal flow graph determinant or characteristic function, and if $\Delta = 0$ it is called the characteristic equation.

example: Consider the below signal flow graph,



There is only one forward path,

$$P_1 = G \quad ; \quad P_2 = P_3 = \cdots = 0$$

There is only one feedback loop, hence

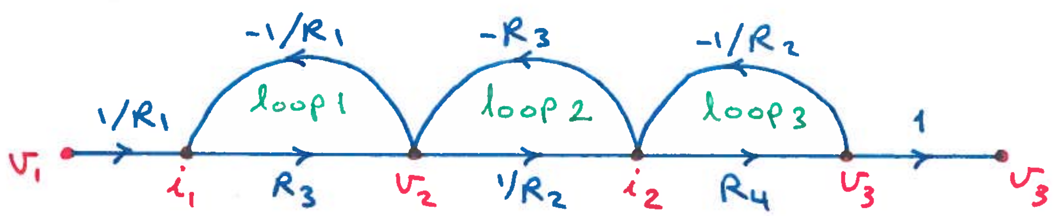
$$P_{11} = \mp GH \quad ; \quad P_{jk} = 0 \quad j \neq 1 \text{ and } k \neq 1$$

$$\rightarrow \Delta = 1 - P_{11} = 1 \pm GH$$

$$\Delta_1 = 1 - 0 = 1$$

$$\rightarrow T = \frac{C}{R} = \frac{P_1 \Delta_1}{\Delta} = \frac{G}{1 \pm GH}$$

example: Find the transfer function from the below signal flow graph,



sol There is one Forward path,

$$P_1 = \frac{1}{R_1} * R_3 * \frac{1}{R_2} * R_4 * 1 = \frac{R_3 R_4}{R_1 R_2}$$

There are three feedback loops, loop gains are,

$$P_{11} = -\frac{1}{R_1} R_3 \quad ; \quad P_{21} = -\frac{1}{R_2} R_3 \quad ; \quad P_{31} = -\frac{1}{R_2} R_4$$

There are two non touching loops, loop 1 and loop 3,

P_{12} = gain product of the only two nontouching loops $= P_{11} \cdot P_{31}$

$$\rightarrow P_{12} = \frac{R_3 R_4}{R_1 R_2}$$

There are no three loops that do not touch, therefore

$$\Delta = 1 - (P_{11} + P_{21} + P_{31}) + P_{12} = 1 + \frac{R_3}{R_1} + \frac{R_3}{R_2} + \frac{R_4}{R_2} + \frac{R_3 R_4}{R_1 R_2}$$

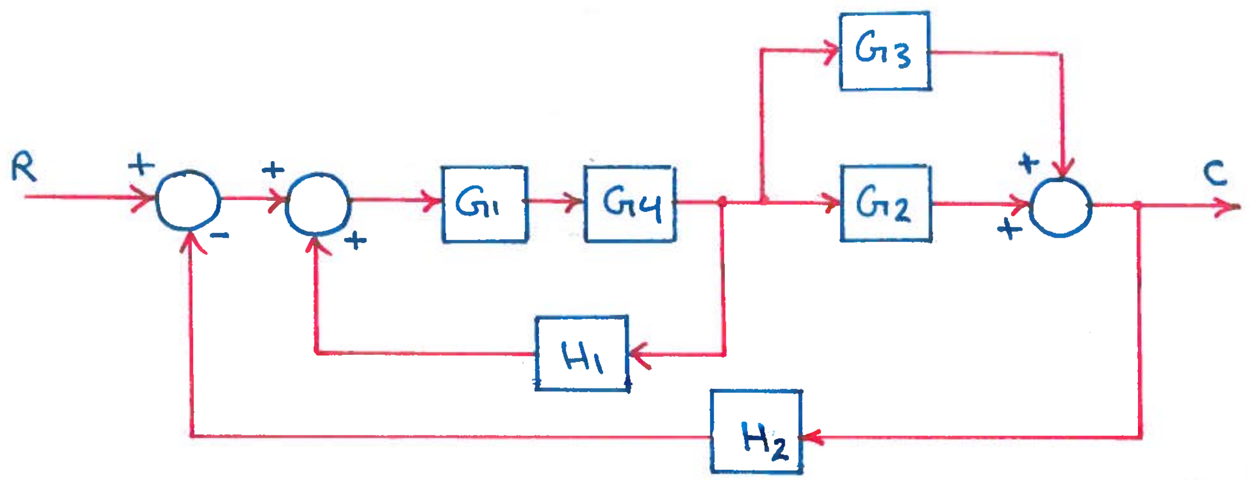
$$= \frac{R_1 R_2 + R_1 R_3 + R_1 R_4 + R_2 R_3 + R_3 R_4}{R_1 R_2}$$

Since all loops touch the forward path,

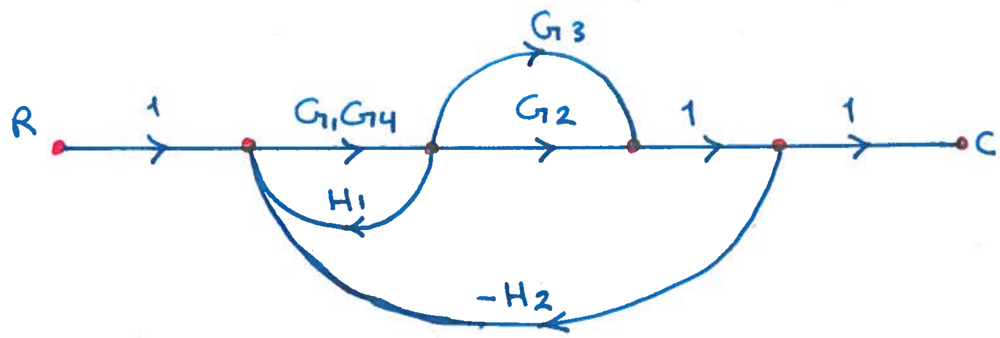
$$\rightarrow \Delta_1 = 1$$

finally $$\frac{v_3}{v_1} = \frac{P_1 \Delta_1}{\Delta} = \frac{R_3 R_4}{R_1 R_2 + R_1 R_3 + R_1 R_4 + R_2 R_3 + R_3 R_4}$$

example: Determine the control ratio C/R using Mason's rules for the below Block diagram,



Sol signal flow graph is as below:



There are three feedback loops:

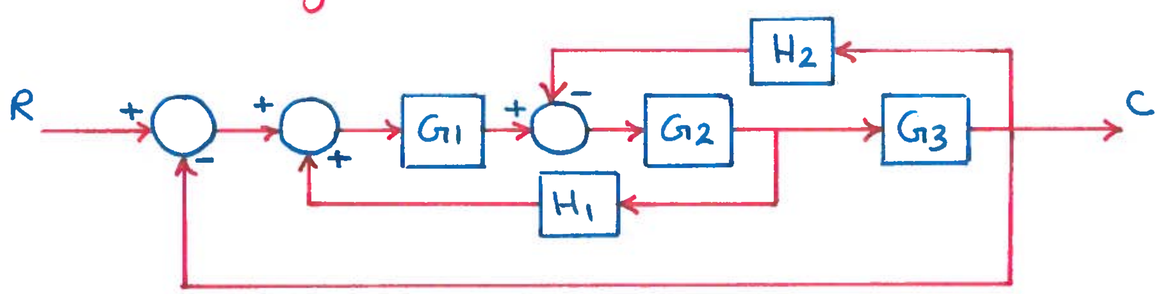
$$P_{11} = G_1G_4H_1 \ ; \ P_{21} = -G_1G_2G_4H_2 \ ; \ P_{31} = -G_1G_3G_4H_2$$

There are no nontouching loops and all loops touch both forward paths, then:

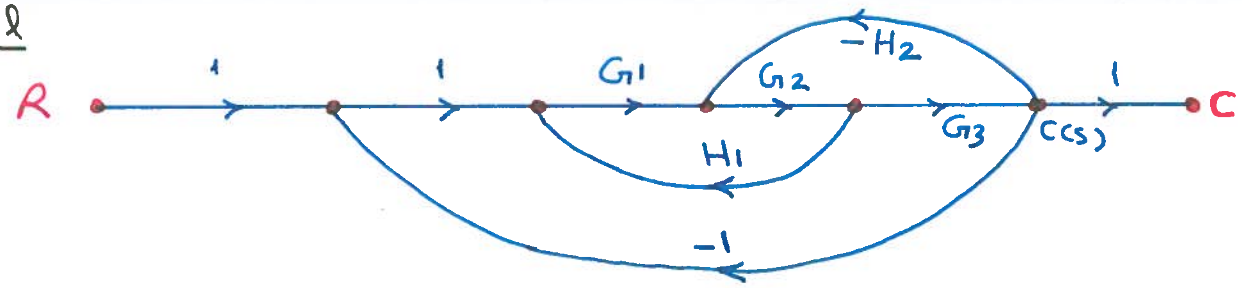
$$\Delta_1 = 1 \ , \ \Delta_2 = 1$$

$$\rightarrow T = \frac{C}{R} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{G_1G_2G_4 + G_1G_3G_4}{1 - G_1G_4H_1 + G_1G_2G_4H_2 + G_1G_3G_4H_2}$$

example: obtain the closed loop T.F, C/R, using Mason's rules for the block diagram shown,



sol



Forward path gain is,

$$P_1 = G_1 G_2 G_3$$

There are three individual loops,

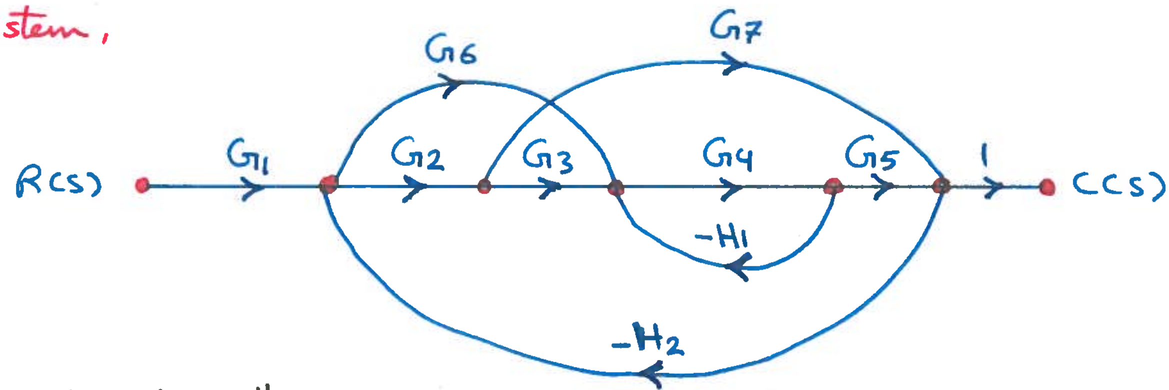
$P_{11} = G_1 G_2 H_1$; $P_{21} = -G_2 G_3 H_2$; $P_{31} = -G_1 G_2 G_3$

there are no nontouching loops , $\Delta_1 = 1$

$\rightarrow \Delta = 1 - (P_{11} + P_{21} + P_{31}) = 1 - G_1 G_2 H_1 + G_2 G_3 H_2 + G_1 G_2 G_3$

$$\rightarrow \frac{C}{R} = \frac{P_1 \Delta_1}{\Delta} = \frac{G_1 G_2 G_3}{1 - G_1 G_2 H_1 + G_2 G_3 H_2 + G_1 G_2 G_3}$$

example: obtain the closed-loop transfer function for the below system,



Forward gain paths are :

$P_1 = G_1 G_2 G_3 G_4 G_5$; $P_2 = G_1 G_6 G_4 G_5$; $P_3 = G_1 G_2 G_7$

There are four loops, these are :

$P_{11} = -G_4 H_1$; $P_{21} = -G_2 G_7 H_2$; $P_{31} = -G_6 G_4 G_5 H_2$;

$P_{41} = -G_2 G_3 G_4 G_5 H_2$

There are two non touching loops , P_{11} and P_{21}

$\rightarrow \Delta = 1 - (P_{11} + P_{21} + P_{31} + P_{41}) + P_{11} P_{21} + 0$

$$\rightarrow \Delta = 1 + G_4 H_1 + G_2 G_7 H_2 + G_6 G_4 G_5 H_2 + G_2 G_3 G_4 G_5 H_2 + G_2 G_4 G_7 H_1 H_2$$

now, we have all the loops touching P_1 , $\rightarrow \Delta_1 = 1$

for P_2, all loops are touching it as well;
therefore $\Delta_2 = 1$

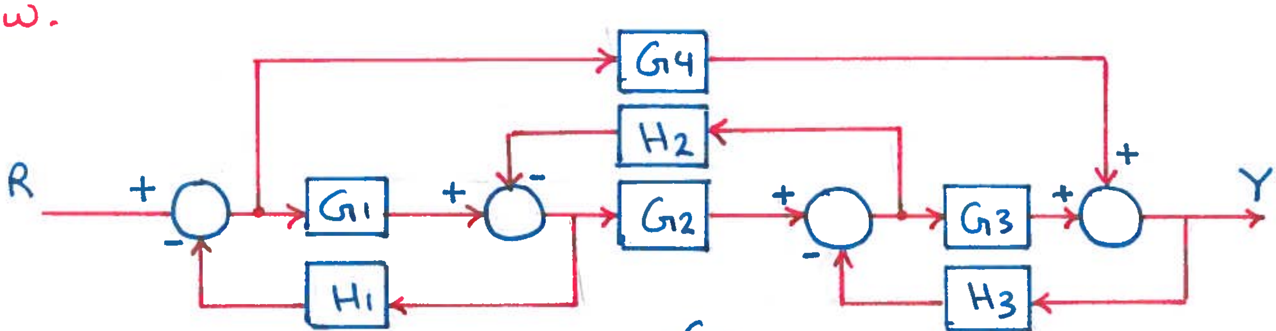
for P_3, we have nontouching loop for P_3 which is P_{11},

$\rightarrow \Delta_3 = 1 - P_{11} = 1 + G_4 H_1$

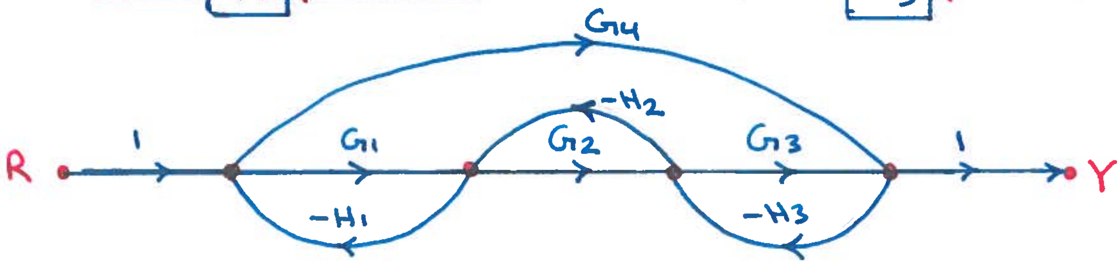
$\rightarrow T.F = \frac{C}{R} = \frac{1}{\Delta}(P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3)$

$$= \frac{G_1G_2G_3G_4G_5 + G_1G_6G_4G_5 + G_1G_2G_7(1+G_4H_1)}{1+G_4H_1+G_2G_7H_2+G_6G_4G_5H_2+G_2G_3G_4G_5H_2+G_2G_4G_7H_1H_2}$$

example: Consider the block diagram shown below, draw the equivalent signal flow graph and find the transfer function using Mason's law.



sol



There are two forward Paths, path gains are:

$P_1 = G_1G_2G_3$; $P_2 = G_4$

There are Four i ividual loops with loop gains:

$P_{11} = -G_1H_1$; $P_{21} = -G_2H_2$; $P_{31} = -G_3H_3$; $P_{41} = -G_4H_1H_2H_3$

There is one combination of two nontouching loops, their loop gain product is : $P_{11} \cdot P_{31} = G_1G_3H_1H_3$

$\rightarrow \Delta = 1 - (-G_1H_1 - G_2H_2 - G_3H_3 - G_4H_1H_2H_3) + G_1G_3H_1H_3 + 0$

now, we have all loops touching $P_1 \rightarrow \Delta_1 = 1$

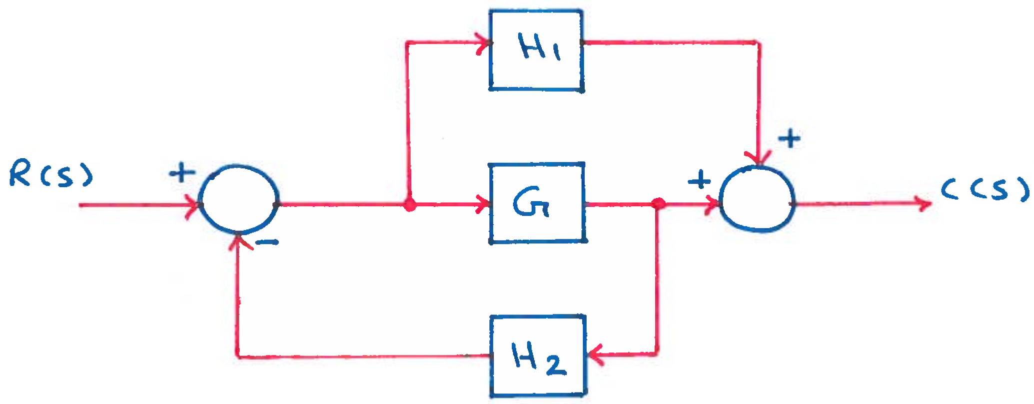
We have one loop (P_{21}) nontouching P_2

$\rightarrow \Delta_2 = 1 - (P_{21}) = 1 - (-G_2H_2) = 1 + G_2H_2$

$$\rightarrow T = \frac{Y}{R} = \frac{1}{\Delta}(P_1\Delta_1 + P_2\Delta_2)$$

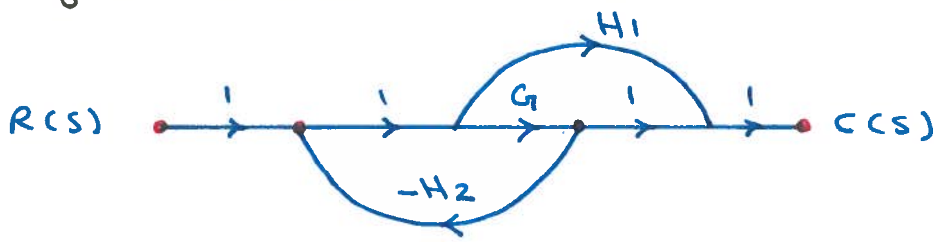
$$= \frac{G_1G_2G_3 + G_4(1 + G_2H_2)}{1 + G_1H_1 + G_2H_2 + G_3H_3 + G_4H_1H_2H_3 + G_1G_3H_1H_3}$$

example: use Mason's rules to obtain the transfer function for the below block diagram,



<u>sol</u>

signal flow graph as below,



we have two forward paths with gains:

$P_1 = G$; $P_2 = H_1$

We have one loop:

$$P_{11} = -GH_2$$

this loop is touching P_1 and P_2

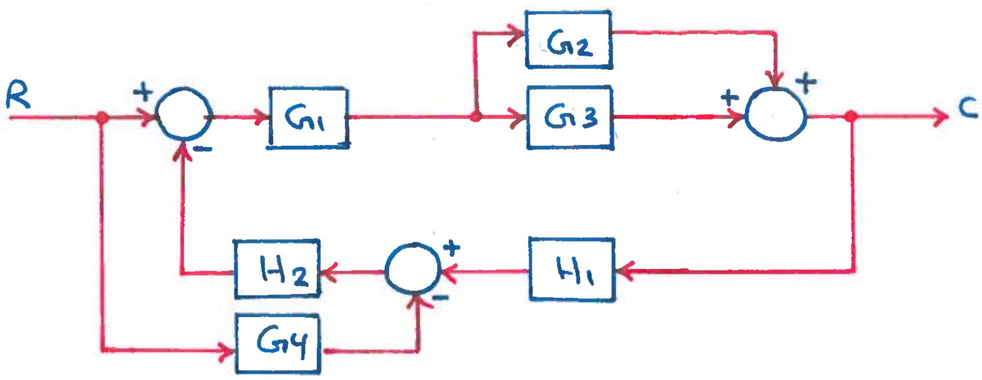
hence $\Delta_1 = \Delta_2 = 1$

$$\Delta = 1 - (P_{11}) + 0 = 1 + GH_2$$

$$\rightarrow T.F = \frac{C}{R} = \frac{1}{\Delta}(P_1\Delta_1 + P_2\Delta_2)$$

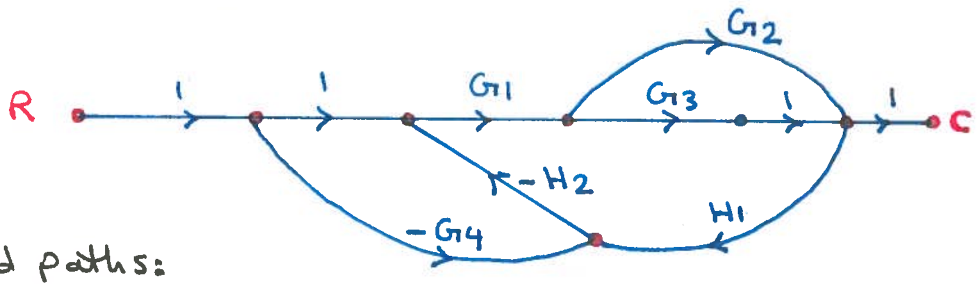
$$= \frac{G + H_1}{1 + GH_2}$$

example: Find C/R for the system whose block diagram representation is shown, by signal flow graph technique.



Sol

The signal flow graph will be as shown below:



Forward paths:

$P_1 = G_1G_3$; $P_2 = G_1G_2$; $P_3 = G_1G_3G_4H_2$; $P_4 = G_1G_2G_4H_2$

Loops: $P_{11} = -G_1G_3H_1H_2$; $P_{21} = -G_1G_2H_1H_2$

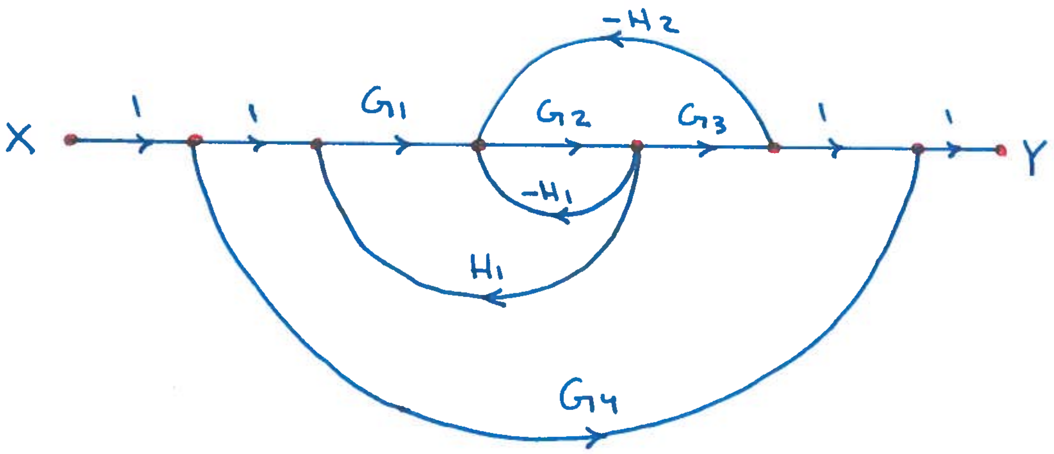
loops are touching all paths ; $\rightarrow \Delta_1 = \Delta_2 = \Delta_3 = \Delta_4 = 1$

$\rightarrow \Delta = 1-(P_{11}+P_{21}) = 1 + G_1G_2H_1H_2 + G_1G_3H_1H_2$

$$\rightarrow \frac{C}{R} = \frac{1}{\Delta}(P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3 + P_4\Delta_4)$$

$$= \frac{G_1G_3 + G_1G_2 + G_1G_3G_4H_2 + G_1G_2G_4H_2}{1 + G_1G_2H_1H_2 + G_1G_3H_1H_2}$$

example: obtain the closed loop transfer function for the system shown,



Sol

We have two forward paths:

$P_1 = G_1 G_2 G_3$

$P_2 = G_4$

We have three feedback loops :

$P_{11} = G_1 G_2 H_1$

$P_{21} = -G_2 H_1$

$P_{31} = -G_2 G_3 H_2$

$$\Delta = 1 - (P_{11} + P_{21} + P_{31})$$

$$= 1 - G_1 G_2 H_1 + G_2 G_3 H_2 + G_2 H_1$$

for P_1, there are no nontouching loops for it,

$\rightarrow \Delta_1 = 1$

for P_2, all feedback loops are non touching and since Δ_i is same as Δ but it is formed by nontouching loops

$$\rightarrow \Delta_2 = \Delta = 1 - G_1 G_2 H_1 + G_2 G_3 H_2 + G_2 H_1$$

$$\rightarrow \text{Transfer function} = \frac{Y}{X} = \frac{1}{\Delta}(P_1 \Delta_1 + P_2 \Delta_2)$$

$$\rightarrow \frac{Y}{X} = \frac{G_1 G_2 G_3 + G_4 (1 + G_2 H_1 - G_1 G_2 H_1 + G_2 G_3 H_2}{1 - G_1 G_2 H_1 + G_2 G_3 H_2 + G_2 H_1}$$

example: Find $\mathcal{L}(3e^{-t} - e^{-2t})$

sol $\mathcal{L}(3e^{-t} - e^{-2t}) = 3\mathcal{L}(e^{-t}) - \mathcal{L}(e^{-2t})$

$$= \frac{3}{s+1} - \frac{1}{s+2} = \frac{2s+5}{s^2+3s+2}$$

example: find $\mathcal{L}^{-1}\left(\frac{2}{s+1} - \frac{4}{s+3}\right)$

sol

$$\mathcal{L}^{-1}\left(\frac{2}{s+1} - \frac{4}{s+3}\right) = 2\mathcal{L}^{-1}\left(\frac{1}{s+1}\right) - 4\mathcal{L}^{-1}\left(\frac{1}{s+3}\right)$$
$$= 2e^{-t} - 4e^{-3t}$$

example: Find $\mathcal{L}\left[\frac{d}{dt}(e^{-t})\right]$ if $\lim_{t\to 0} e^{-t} = 1$

sol

$$\mathcal{L}\left[\frac{d}{dt}(e^{-t})\right] = s\left(\frac{1}{s+1}\right) - 1 = \frac{-1}{s+1}$$

example: Find $\mathcal{L}\left[\int_0^t e^{-\tau} d\tau\right]$

sol

$$\mathcal{L}\left[\int_0^t e^{-\tau} d\tau\right] = \frac{1}{s}\left(\frac{1}{s+1}\right) = \frac{1}{s(s+1)}$$

example: find the initial value of $f(t) = e^{-3t}$

sol $F(s) = \mathcal{L}f(t) = \frac{1}{s+3}$

$$\rightarrow \lim_{t\to 0} e^{-3t} = \lim_{s\to\infty} s\,\frac{1}{s+3} = \lim_{s\to\infty} \cancel{s}\,\frac{1}{\cancel{s}(1+\frac{3}{s})} = 1$$

example: if $f(t) = 1 - e^{-t}$, find the final value of this function,

sol $F(s) = \mathcal{L}(1 - e^{-t}) = \frac{1}{s} - \frac{1}{s+1} = \frac{1}{s(s+1)}$

$$\rightarrow \lim_{t\to\infty} (1 - e^{-t}) = \lim_{s\to 0} \cancel{s}\,\frac{1}{\cancel{s}(s+1)} = 1$$

example: Find $\mathcal{L}(\cos t)$

sol $F(s) = \mathcal{L}[\cos t] = \frac{s}{s^2+1}$

example: Find $\mathcal{L}[e^{-2t}\cos t]$

sol $\mathcal{L}[e^{-2t}\cos t] = \dfrac{s+2}{(s+2)^2+1} = \dfrac{s+2}{s^2+4s+5}$

example: $F(s) = \dfrac{s}{(s+1)(s^2+1)}$; find $f(t)$,

sol

$$F_1(s) = \frac{1}{s+1} \rightarrow f_1(t) = e^{-t}$$

$$\rightarrow f_1(t-\tau) = e^{-(t-\tau)}$$

$$F_2(s) = \frac{s}{s^2+1} \rightarrow f_2(t) = \cos t$$

$$\rightarrow f_2(\tau) = \cos\tau$$

$$\rightarrow \mathcal{L}^{-1}\left[\frac{1}{s+1}\cdot\frac{s}{s^2+1}\right] = \int_0^t e^{-(t-\tau)}\cos\tau \, d\tau$$

$$= e^{-t}\int_0^t e^{\tau}\cos\tau \, d\tau$$

$$= \frac{1}{2}(\cos t + \sin t - e^{-t})$$

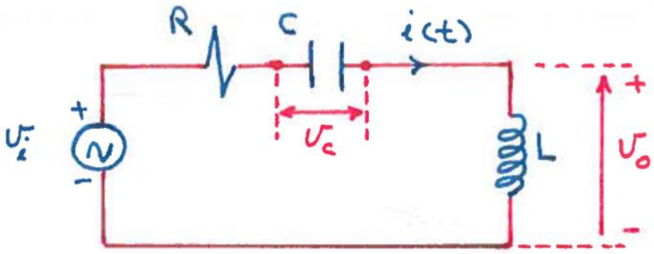
Transfer Function:

The transfer function of a linear time-invariant differential equation system is defined as the ratio of the Laplace transform of the output (response function) to the Laplace transform of the input (driving function) under assumption that all initial conditions are zero.

$$G(s) = \text{Transfer function} = \left.\frac{\mathcal{L}\,y(t)}{\mathcal{L}\,r(t)}\right|_{\text{zero initial conditions}}$$

$$\rightarrow G(s) = \frac{Y(s)}{R(s)}$$

example: Consider the below electrical network, find the transfer function $\frac{V_o}{V_i}$,



sol

$$v_i = iR + v_c + L\frac{di}{dt}$$

taking Laplace trasform

$$\rightarrow V_i = IR + V_c + LSI \ldots (1)$$, assuming zero initial conditions

now, $i = C\frac{dv_c}{dt}$

$$\rightarrow I = CSV_c \rightarrow V_c = \frac{I}{CS} \ldots (2)$$

sub. in (1)

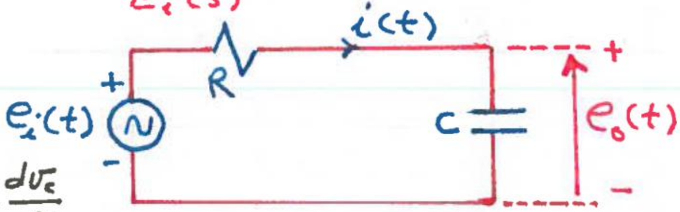
$$\rightarrow V_i = IR + \frac{I}{CS} + LSI$$

$$v_o = L\frac{di}{dt} \rightarrow V_o = LSI$$

$$\rightarrow \text{Transfer function} = \frac{V_o}{V_i} = \frac{LSI(s)}{I(s)\left[R + LS + \frac{1}{CS}\right]}$$

$$\rightarrow \frac{V_o}{V_i} = \frac{S^2}{S^2 + \frac{R}{L}S + \frac{1}{LC}}$$

example: Find the transfer function $\frac{E_o(s)}{E_i(s)}$ for the circuit shown,



sol

$e_i(t) = i(t)R + v_c(t)$; and $i(t) = C\frac{dv_c}{dt}$

$$\rightarrow E_i(s) = I(s)R + \frac{I(s)}{CS} \ldots (1)$$

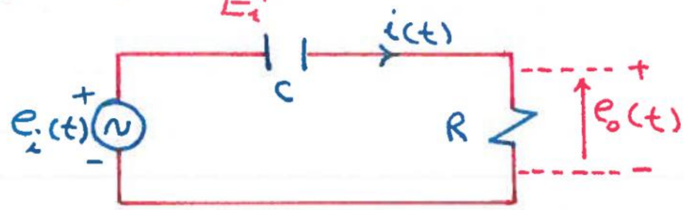
$$\text{and } E_o = V_c(s) = \frac{I(s)}{CS} \ldots (2)$$

$$\rightarrow \text{Transfer function} = \frac{E_o(S)}{E_i(S)} = \frac{\frac{I(s)}{CS}}{I(s)R + \frac{I(s)}{CS}}$$

$$= \frac{1}{RCS + 1} = \frac{1}{TS + 1}$$

where $T = RC$

example: Find the transfer function $\frac{E_o}{E_i}$ for the circuit shown,



sol

$e_i(t) = V_c(t) + i(t)R$

$i(t) = C\frac{dV_c}{dt} \rightarrow I(s) = CSV_c(s)$, Zero initial conditions

$\rightarrow V_c(s) = I(s)/CS$

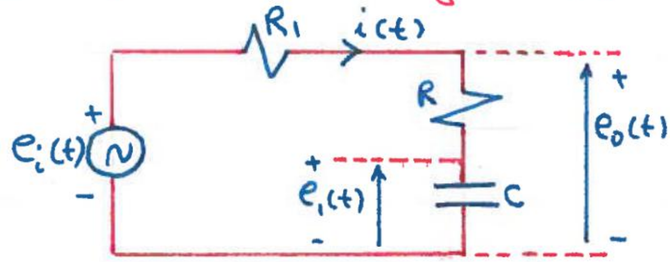
$\rightarrow E_i(s) = \frac{I(s)}{CS} + I(s)R = I(s)\left[R + \frac{1}{CS}\right]$ --- (1)

$e_o(t) = i(t)R \rightarrow E_o(s) = I(s)\cdot R$ --- (2)

$\rightarrow$ Transfer function $= \frac{E_o(S)}{E_i(S)} = \frac{I(s)\cdot R}{I(s)[R+\frac{1}{CS}]} = \frac{RCS}{RCS+1} = \frac{TS}{TS+1}$

where $T = RC$

example: For the circuit shown below, find the voltage transfer function,



sol

$e_i(t) = i(t)R_1 + i(t)R + e_1(t)$

$= i(t)(R_1+R) + \frac{1}{C}\int i(t)dt$

$E_i(s) = I(s)[R_1+R] + \frac{1}{CS}I(s)$, Zero initial Conditions

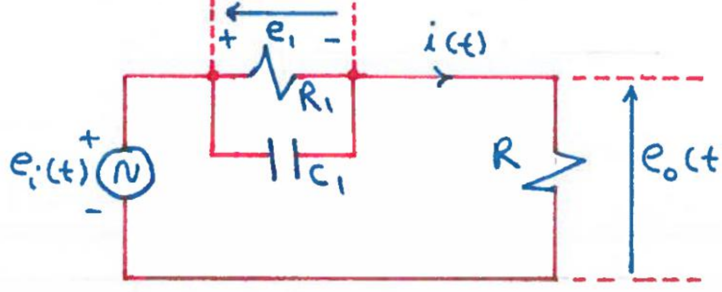
$E_i(s) = I(s)\left[R+R_1+\frac{1}{CS}\right]$ ---- (1)

$e_o(t) = i(t)R + e_1(t) = i(t)R + \frac{1}{C}\int i(t)dt$

$\rightarrow E_o(S) = I(S)R + \frac{1}{CS}I(S) = I(s)\left[R + \frac{1}{CS}\right]$ --- (2)

$\rightarrow \frac{E_o}{E_i} = \frac{I(s)[R+\frac{1}{CS}]}{I(s)[R+R_1+\frac{1}{CS}]} = \frac{RCS+1}{CS[R+R_1]+1}$

example: Find the transfer function for the below circuit,



sol $e_o(t) = i(t)R \rightarrow E_o(S) = I(S)R$ --- ①

$e_i(t) = e_1(t) + i(t)R$

$$e_1(t) = i(t) * Z_1 \rightarrow E_1(S) = I(S) * \frac{R_1 \cdot \frac{1}{C_1 S}}{R_1 + \frac{1}{C_1 S}}$$

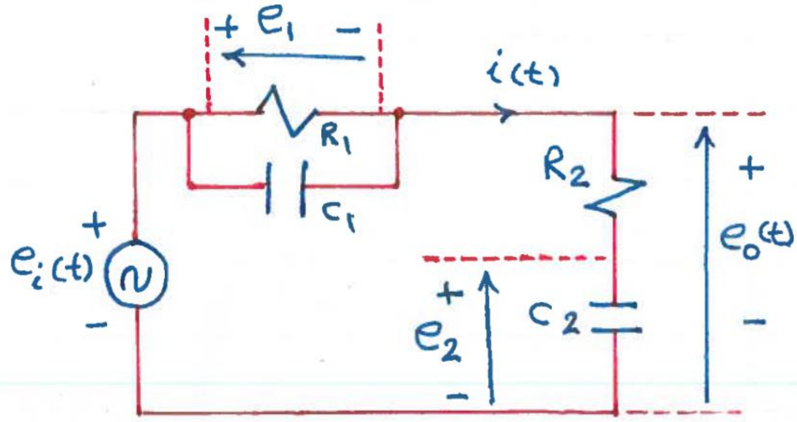
$$= I(S) \cdot \frac{R_1}{R_1 C_1 S + 1}$$

$$\rightarrow E_i(S) = I(S) \cdot \frac{R_1}{R_1 C_1 S + 1} + I(S)R \quad \text{--- ②}$$

$$\rightarrow \text{Transfer function} = \frac{E_o}{E_i} = \frac{I(S)\, R}{I(S) \cdot \left[\frac{R_1}{R_1 C_1 S + 1} + R \right]}$$

$$= \frac{R(R_1 C_1 S + 1)}{R(R_1 C_1 S + 1) + R_1}$$

example: Find the Transfor function for the below circuit,



sol

note that $e_o = e_i \dfrac{Z_2}{Z_1 + Z_2}$

$$\rightarrow E_o = E_i \frac{Z_2(S)}{Z_1(S) + Z_2(S)}$$

$$\frac{E_o}{E_i} = \frac{R_2 + \frac{1}{C_2 S}}{R_2 + \frac{1}{C_2 S} + \frac{R_1 / C_1 S}{R_1 + \frac{1}{C_1 S}}}$$

$$\rightarrow \frac{E_o}{E_i} = \frac{(R_1 R_2 C_1 C_2) S^2 + (R_1 C_1 + R_2 C_2) S + 1}{(R_1 R_2 C_1 C_2) S^2 + (R_1 C_1 + R_2 C_2 + R_1 R_2 C_2) S + 1}$$

Common Laplace Transform Pairs

Time Domain Function		Laplace Domain Function
Name	Definition*	
Unit Impulse	$\delta(t)$	1
Unit Step	$\gamma(t)$ †	$\frac{1}{s}$
Unit Ramp	t	$\frac{1}{s^2}$
Parabola	t^2	$\frac{2}{s^3}$
Exponential	e^{-at}	$\frac{1}{s+a}$
Asymptotic Exponential	$\frac{1}{a}(1-e^{-at})$	$\frac{1}{s(s+a)}$
Dual Exponential	$\frac{1}{b-a}\left(e^{-at}-e^{-bt}\right)$	$\frac{1}{(s+a)(s+b)}$
Asymptotic Dual Exponential	$\frac{1}{ab}\left[1+\frac{1}{a-b}\left(be^{-at}-ae^{-bt}\right)\right]$	$\frac{1}{s(s+a)(s+b)}$
Time multiplied Exponential	te^{-at}	$\frac{1}{(s+a)^2}$
Sine	$\sin(\omega_0 t)$	$\frac{\omega_0}{s^2+\omega_0^2}$
Cosine	$\cos(\omega_0 t)$	$\frac{s}{s^2+\omega_0^2}$
Decaying Sine	$e^{-at}\sin(\omega_d t)$	$\frac{\omega_d}{(s+a)^2+\omega_d^2}$
Decaying Cosine	$e^{-at}\cos(\omega_d t)$	$\frac{s+a}{(s+a)^2+\omega_d^2}$
Generic Oscillatory Decay	$e^{-at}\left[B\cos\left(\omega_d t\right)+\frac{C-aB}{\omega_d}\sin\left(\omega_d t\right)\right]$	$\frac{Bs+C}{\left(s+a\right)^2+\omega_d^2}$
Prototype Second Order Lowpass, underdamped	$\frac{\omega_0}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_0 t}\sin\left(\omega_0\sqrt{1-\zeta^2}\,t\right)$	$\frac{\omega_0^2}{s^2+2\zeta\omega_0 s+\omega_0^2}$
Prototype Second Order Lowpass, underdamped - Step Response	$1-\frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_0 t}\sin\left(\omega_0\sqrt{1-\zeta^2}\,t+\phi\right)$ $\phi=\tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)$	$\frac{\omega_0^2}{s(s^2+2\zeta\omega_0 s+\omega_0^2)}$

*All time domain functions are implicitly=0 for t<0 (i.e. they are multiplied by unit step, $\gamma(t)$).
†u(t) is more commonly used for the step, but is also used for other things. $\gamma(t)$ is chosen to avoid confusion (and because in the Laplace domain it looks a little like a step function, $\Gamma(s)$).

Automatic Control systems

Introduction:

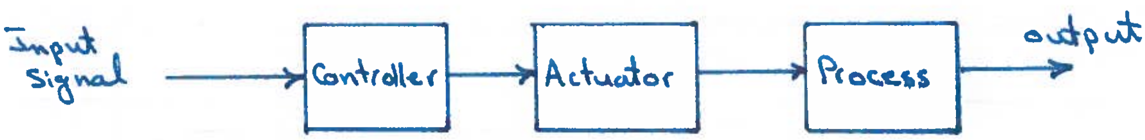
Engineering is concerned with understanding and controlling the materials and forces of nature for the benefit of humankind. Control system engineers are concerned with understanding the controlling segments of their environment, ofen called systems, to provide useful economic products for society. The present challenge to control engineers is the modeling and control of modern, complex, interrelated systems such as traffic control systems, chemical processes, and robotic systems as well many useful and interesting industrial automation systems.

Control engineering integrates the concepts of network theory and communication theory, therefore it is not limited to any engineering discipline but equally applicable to chemical, mechanical, environmental, civil, and electrical engineering.

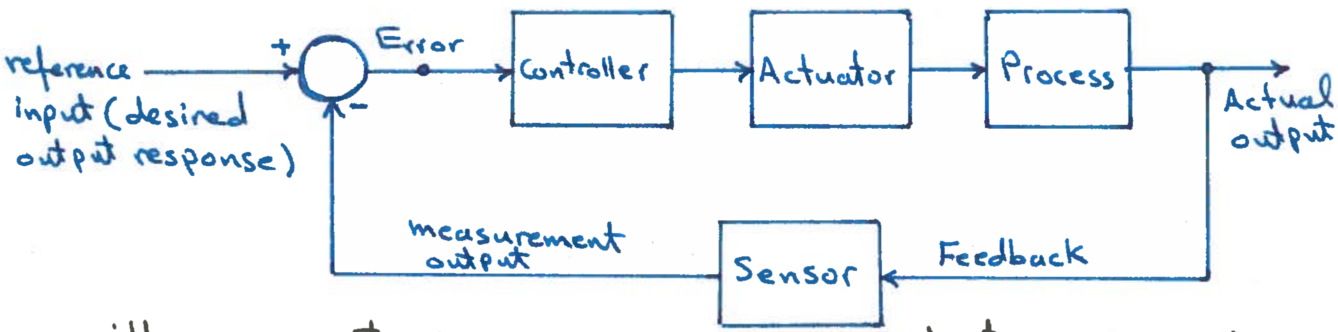
A control system is an interconnection of components forming a system configuration that will provide a desired system response. A component or process to be controlled can be represented by a block.

The input-output relationship represents the cause-and-effect relationship of the process, which in turn represents a processing of the input signal to provide an output signal variable, often with power amplification.

An Open-loop Control system uses a controller and an actuator to obtain the desired response. It is a system without feedback.

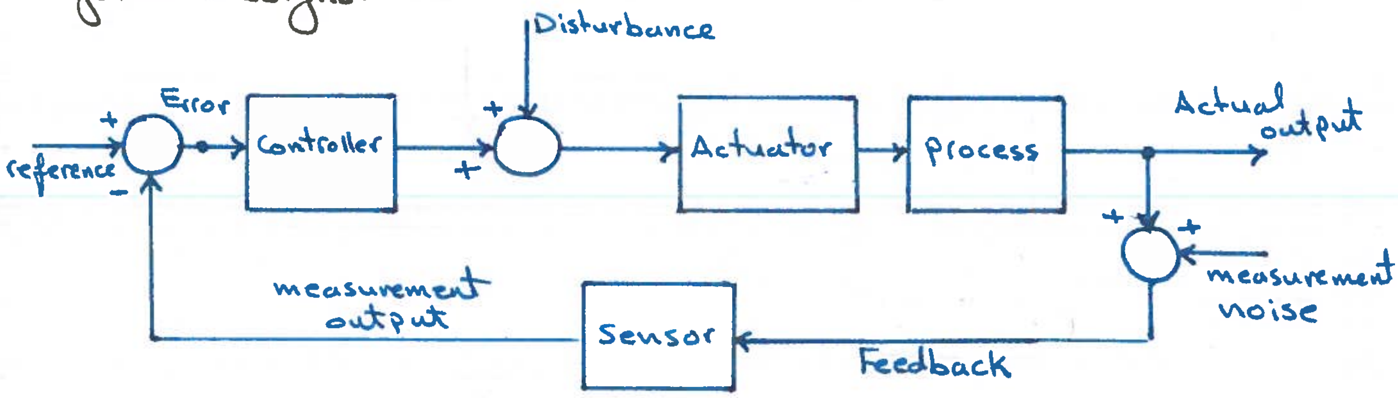


A closed-loop feedback control system utilizes an additional measure of the actual output to compare the actual output with the desired output response (input signal).



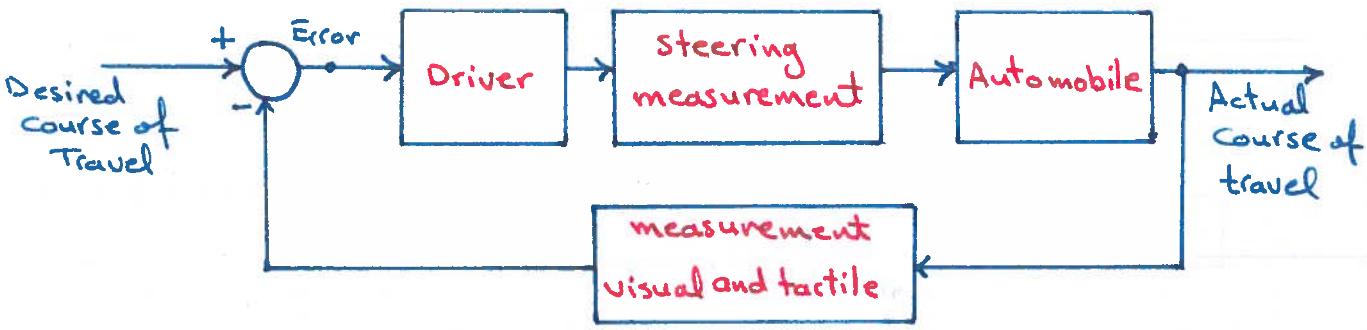
with an acurate sensor, the measured output is a good approximation of the actual output of the system.

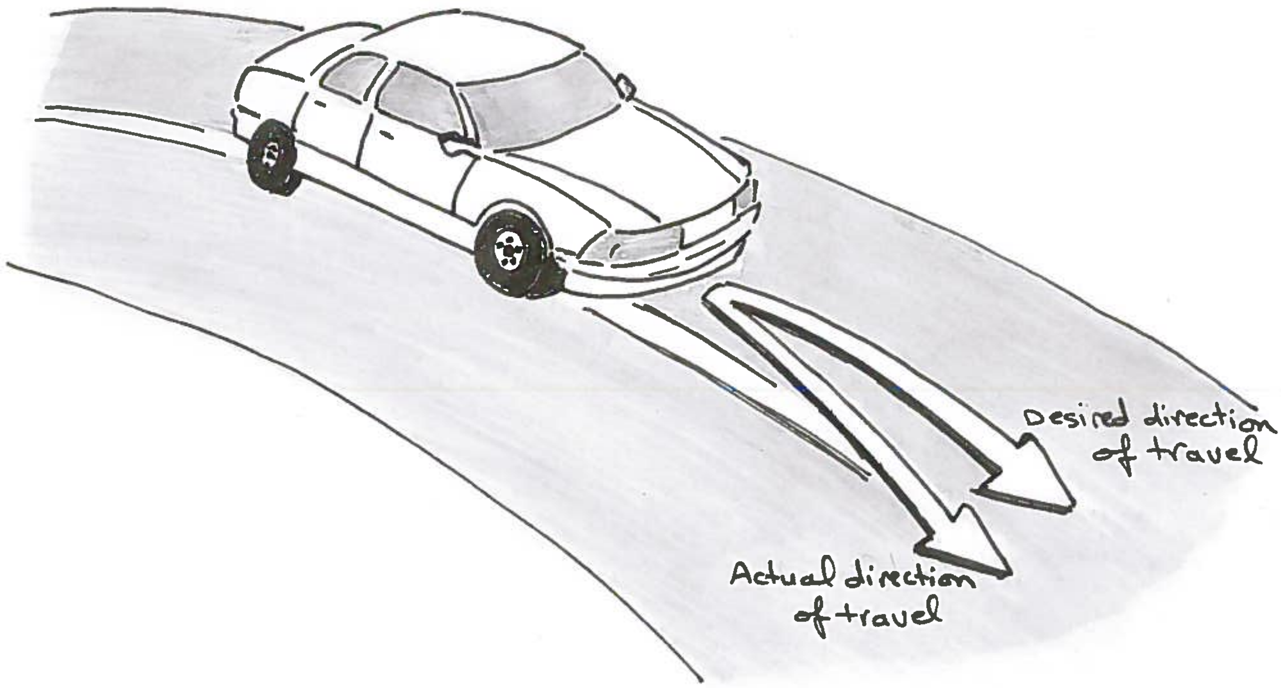
Closed loop control has many advantages over open loop control including the ability to reject external disturbances and improve measurement noise attenuation. External disturbances and measurement noise in real world must be addressed in practical system designs.

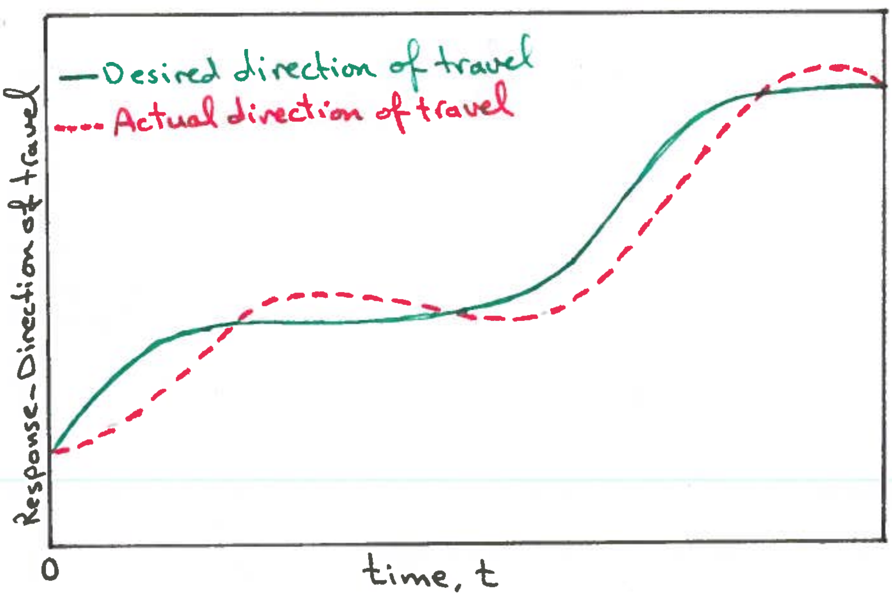


Example of A control system:

A simple block diagram of an automobile steering control system is shown below:







The desired course is compared with measurement of the actual course in Order to generate a measure of the error. This measurement is obtained by visual and tactile (body movement) feedback, as provided by the feel of the steering wheel by the hand (Sensor).

Definitions :

Controlled Variable : is the quantity or condition that is measured and controlled.

Control signal / manipulated variable : is the quantity or Condition that is varied by the Controller so as to affect the value of the controlled Variable.

Control: measuring the value of the Controlled Variable of the system and applying the control signal to the system to correct or limit deviation of the measured value from a desired value.

Plants: Any physical object to be controlled, such as mechanical device, heating furnace, chemical reactor, or a spacecraft.

Process: A continuing operation marked by a series of gradual changes and lead to a particular result. it consists of a series of controlled actions.

Systems: A system is a combination of components that act together and perform a certain task.

Disturbance: A disturbance is a signal that tends to affect the value of the output of the system.

Feedback control: It refers to an operation that tends to reduce the difference between the output of a system and same reference input.

Open-loop Control system: is one in which the control action is independent of the output.

closed-loop control system: is one in which the control action is somehow dependent on the output.

Bandwidth: The bandwidth of a system is a frequency response measure of how well the system responds to (or filters) variations (or frequencies) in the input signal.

Stable system: The definition of a stable system can be based upon the response of the system to bounded inputs, that is, inputs whose magnitudes are less than some finite value for all time.

A continuous or discrete-time system is said to be stable if every bounded input produces a bounded output.

Major advantages of Open-loop Control system:

- Simple construction and ease of maintenance.
- less expensive than closed loop system.
- no stability problems.
- Convenient when output is hard to measure.

Major disadvantages of open-loop system:

- Disturbances cause errors and output may differ from the desired.
- recalibration is necessary from time to time.

Advantages of Closed-loop Control system:

- Ability to minimize the effect of external disturbance & noise.
- the use of feedback makes the system response insensitive to external disturbance.
- Can use inacurate components to obtain the accurate control of a given plant.

Disadvantages of closed-loop System:

- Stability is a major problem.
- more expensive than open loop system.

The LAPLACE TRANSFORM:

The laplace transform method substitutes relatively easily solved algebric equations for the more difficult differential equations. The time-response solution is obtained by the following operations:

1- obtain the linearized differential equations.

2- obtain the laplace transformation of the differential equations.