BJT Transistor Modeling

One of our first concerns in the sinusoidal ac analysis of transistor networks is the magnitude of the input signal. It will determine whether small-signal or large-signal techniques should be applied. There are two models commonly used in the small-signal ac analysis of transistor networks: the r_e model and the hybrid equivalent model.

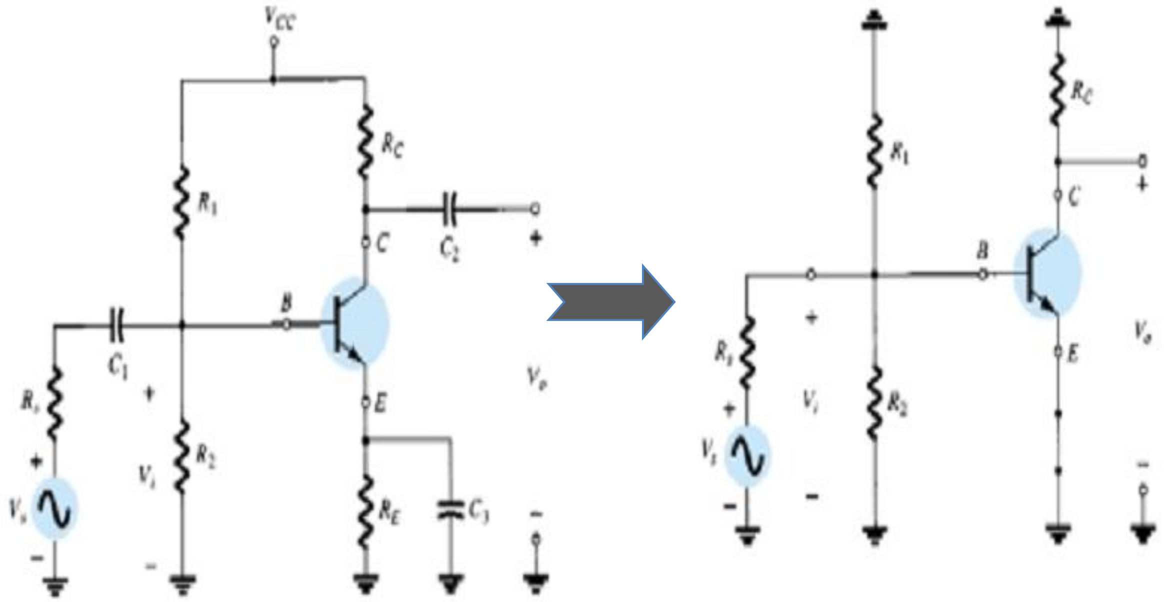
1- Amplification in The AC Domain:

That is, the output sinusoidal signal is greater than the input signal or, stated another way, the output ac power is greater than the input ac power. The question then arises as to how the ac power output can be greater than the input ac power? Conservation of energy dictates that over time the total power output, Po, of a system cannot be greater than its power input, Pi, and that the efficiency defined by $\eta = P_o/P_i$ cannot be greater than 1. The factor missing from the discussion above that permits an ac power output greater than the input ac power is the applied dc power. In fact, a conversion efficiency is defined by $\eta = P_{o(ac)}/P_{i(dc)}$.

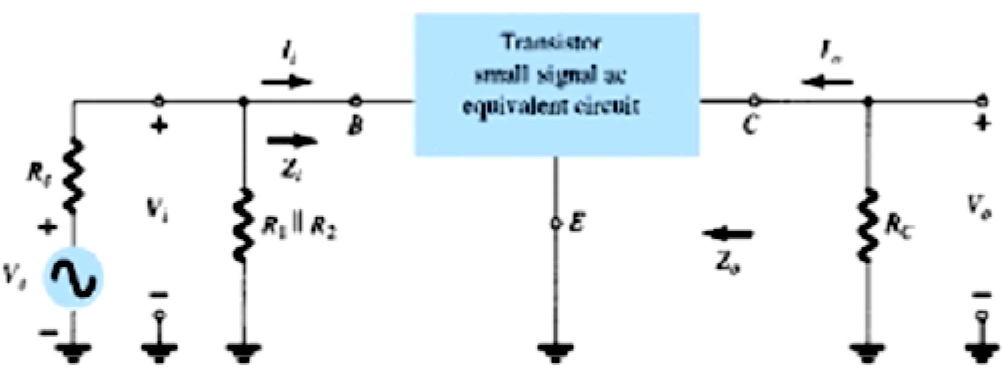
2- BJT Transistor Modeling

The ac equivalent of a network is obtained by:

a- Setting all dc sources to zero and replacing them by a short-circuit equivalent
b- Replacing all capacitors by a short-circuit equivalent
c- Removing all elements bypassed by the short-circuit equivalents introduced by steps a and b.
d- Redrawing the network in a more convenient and logical form.



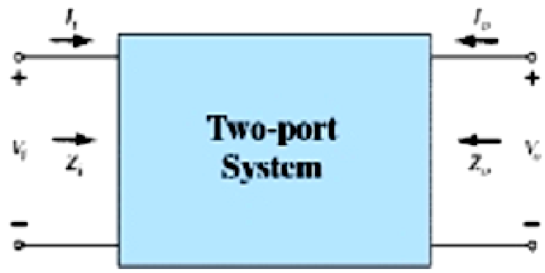
If we establish a common ground and rearrange the elements, R_1 and R_2 will be in parallel and R_C will appear from collector to emitter as shown in Fig. below.



For the two-port (two pairs of terminals) system, the input side (the side to which the signal is normally applied) is to the left and the output side (where the load is connected) is to the right. In fact, for most electrical and electronic systems, the general flow is usually from the left to the right.

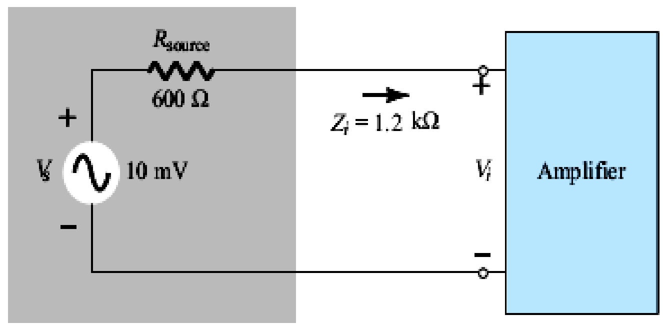
Input Impedance, Z_i: For the input side, the input impedance Z_i is defined by Ohm's law as: $Z_i = V_i / I_i$

It is particularly noteworthy that for frequencies in the low to mid-range (typically ≤100 kHz): The input impedance of a BJT transistor amplifier is purely resistive in nature and, depending on the manner in which the transistor is employed, can vary from a few ohms to megohms.



The importance of the input impedance of a system can best be demonstrated by:

The input voltage must be determined using the voltage-divider rule as follows:



$$V_i = \frac{Z_i V_s}{Z_i + R_{source}} = \frac{(1.2\ \text{k}\Omega)(10\ \text{mV})}{1.2\ \text{k}\Omega + 0.6\ \text{k}\Omega} = 6.67\ \text{mV}$$

Thus, only 66.7% of the full-input signal is available at the input.

Output Impedance, Z_o: In particular, for frequencies in the low to mid-range (typically≤100 kHz): The output impedance of a BJT transistor amplifier is resistive in nature and, depending on the configuration and the placement of the resistive elements, Z_o, can vary from a few ohms to a level that can exceed 2 MΩ.

$$Z_o = V_o/I_o$$

Voltage Gain, A_v: One of the most important characteristics of an amplifier is the small-signal ac volt-age gain as determined by: $A_V = V_o/V_i$

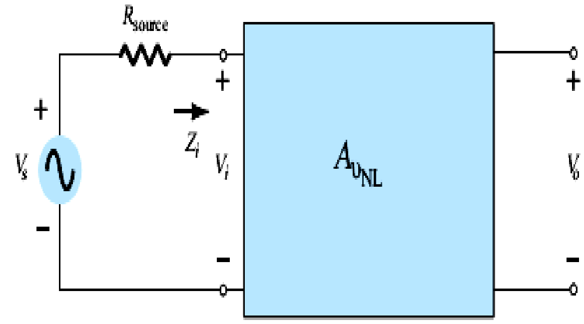
For the system of Fig. below having a source resistance R_s, the level of V_i would first have to be determined using the voltage-divider rule before the gain V_o/V_s could be calculated. That is,

$$V_i = \frac{Z_i V_s}{Z_i + R_s}$$

with $$\frac{V_i}{V_s} = \frac{Z_i}{Z_i + R_s}$$

and $$A_{v_s} = \frac{V_o}{V_s} = \frac{V_i}{V_s} \cdot \frac{V_o}{V_i}$$

so that $$A_{v_s} = \frac{V_o}{V_s} = \frac{Z_i}{Z_i + R_s} A_{v_{NL}}$$



Current Gain, A_i: The current gain defined by:

$$A_i = I_o / I_i$$

For the loaded situation

$$I_i = V_i / Z_i \quad and \quad I_o = -V_o / R_L$$

with $$A_i = \frac{I_o}{I_i} = -\frac{V_o/R_L}{V_i/Z_i} = -\frac{V_o Z_i}{V_i R_L}$$

and $$A_i = -A_v \frac{Z_i}{R_L}$$

Phase Relationship: For the typical transistor amplifier at frequencies that permit ignoring the effects of the reactive elements, the input and output signals are either 180° out of phase or in phase.

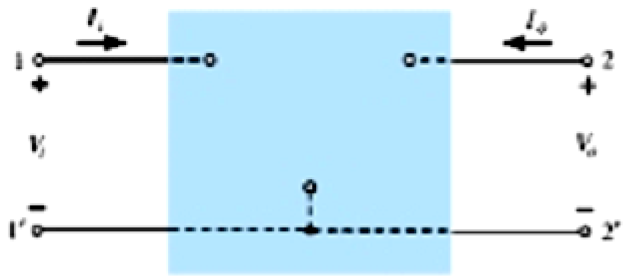
3- The Hybrid Equivalent Model:

Our description of the hybrid equivalent model will begin with the general two-port system of Fig. below. It is the most frequently employed in transistor circuit analysis.

$$V_i = h_{11} I_i + h_{12} V_o$$

$$I_o = h_{21} I_i + h_{22} V_o$$

If we arbitrarily set V_o=0 (short circuit the output terminals) and solve for h_{11}



$$h_{11} = \left.\frac{V_i}{I_i}\right|_{V_o = 0}$$

Ohms

If I_i is set equal to zero by opening the input leads, the following will result for h_{12}

$$h_{12} = \left.\frac{V_i}{V_o}\right|_{I_i = 0}$$

unitless

It has no units since it is a ratio of voltage levels and is called the open-circuit reverse transfer voltage ratio parameter.

If V_o is equal to zero by again shorting the output terminals, the following will result for h_{21}

$$h_{21} = \left.\frac{I_o}{I_i}\right|_{V_o=0}$$

unitless

Note that we now have the ratio of an output quantity to an input quantity. The term forward will now be used rather than reverse as indicated for h12

$$h_{22} = \left.\frac{I_o}{V_o}\right|_{I_i=0}$$

siemens

It is called the open-circuit out-put admittance parameter.

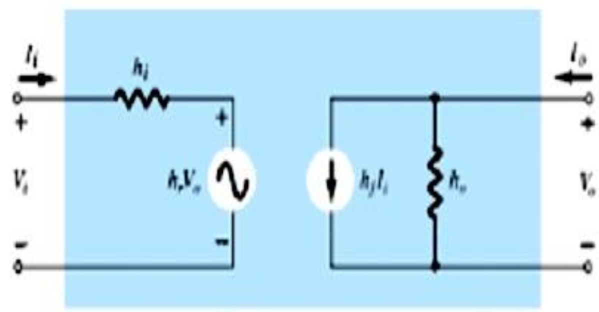
The complete "ac" equivalent circuit for the basic three-terminal linear device is indicated in Fig. below with a new set of subscripts for the h-parameters. The choice of letters is obvious from the following listing:

h_{11} →input resistance →h_i

h_{12} →reverse transfer voltage ratio → h_r

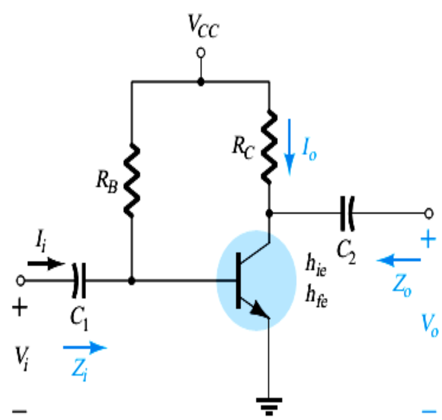
h_{21} →*forward transfer current ratio*→ h_f

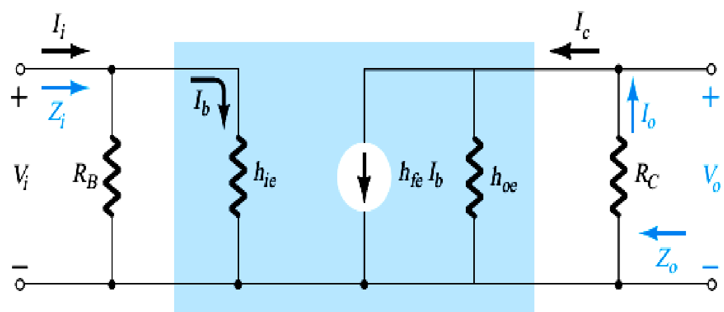
h_{22} →*output conductance* →h_o



4- Approximate Hybrid Equivalent Circuit

a- Fixed-Bias Configuration





$$V_o = -I_o R' = -I_C R'$$
$$= -h_{fe} I_b R'$$

Using $R' = 1/h_{oe} \| R_C$,

and $$I_b = \frac{V_i}{h_{ie}}$$

$$Z_i = R_B \| h_{ie}$$

with $$V_o = -h_{fe}\frac{V_i}{h_{ie}} R'$$

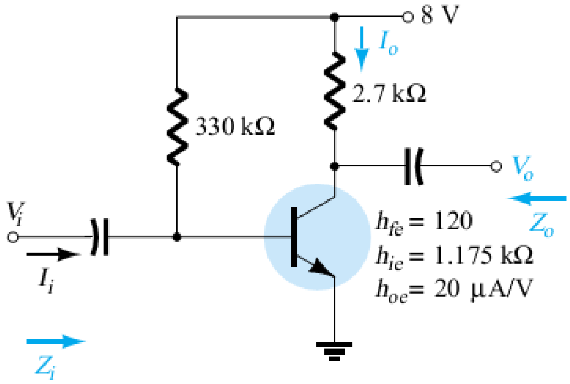
$$Z_o = R_C \| 1/h_{oe}$$

so that $$A_v = \frac{V_o}{V_i} = -\frac{h_{fe}(R_C \| 1/h_{oe})}{h_{ie}}$$

A_i: Assuming that $R_B \gg h_{ie}$ and $1/h_{oe} \geq 10R_C$, then $I_b \cong I_i$ and $I_o = I_c = h_{fe}I_b = h_{fe}\, I_i$ with

$$A_i = \frac{I_o}{I_i} \cong h_{fe}$$

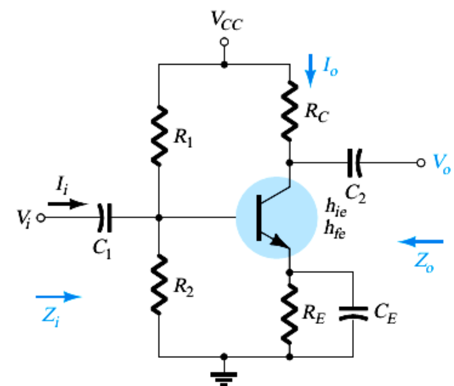
Example: For the circuit shown: Determine



(a) Z_i.
(b) Z_o.
(c) A_v.
(d) A_i.

b- Voltage-Divider Configuration (Bypassed Emitter-Biase):

For the voltage-divider bias configuration of Fig. below, the resulting small-signal ac equivalent network will have the same appearance as fixed bias configuration, with R_B replaced by $R' = R_1 \| R_2$.



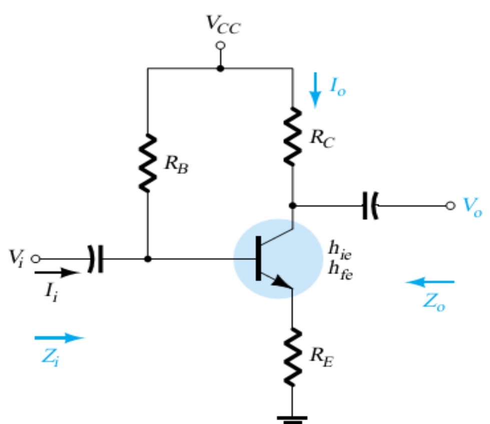
$$Z_i = R' \| h_{ie}$$

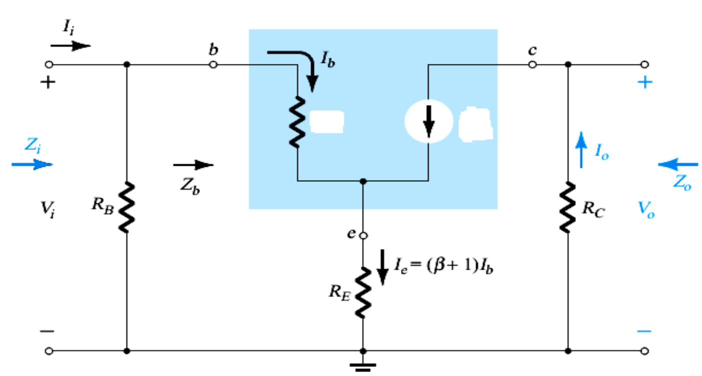
$$A_v = -\frac{h_{fe}(R_C \| 1/h_{oe})}{h_{ie}}$$

$$Z_o \cong R_C$$

$$A_i = -\frac{h_{fe} R'}{R' + h_{ie}}$$

c- Unbypassed Emitter-Bias Configuration:





***Zi*:**

$$Z_b \cong h_{fe}\, R_E$$

and

$$Z_i = R_B \| Z_b$$

***Zo*:**

$$Z_o = R_C$$

***Av*:**

$$A_v = -\frac{h_{fe}\, R_C}{Z_b} \cong -\frac{h_{fe} R_C}{h_{fe}\, R_E}$$

and

$$A_v \cong -\frac{R_C}{R_E}$$

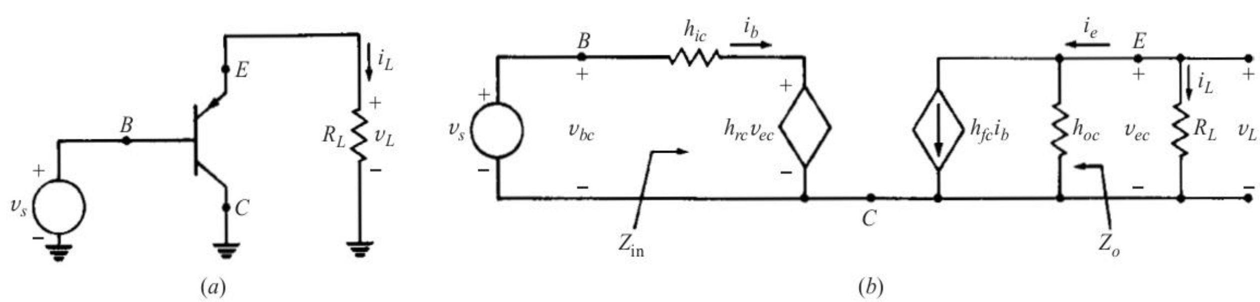
***Ai*:**

$$A_i = \frac{h_{fe} R_B}{R_B + Z_b}$$

or

$$A_i = -A_v \frac{Z_i}{R_C}$$

d- CC Amplifier Analysis:



Let $h_{ic} = 1k\Omega$, $h_{rc} = 1$, $h_{fc} = -10$, $h_{oc} = 12\mu S$, and $R_L = 2k\Omega$. Drawing direct analogies with the CE amplifier, find expressions for the (a) current-gain ratio Ai, (b) voltage-gain ratio Av,(c)input impedance Zin, and (d) output impedance Zo. (e) Evaluate this typical CC amplifier.

$$A_i = \frac{h_{fc}}{1 + h_{oc}R_L} = -\frac{-101}{1 + (12 \times 10^{-6})(2 \times 10^3)} = 98.6$$

Note that $A_i \approx -h_{fc}$, and that the input and output currents are in phase because $h_{fc} < 0$.

(b)

$$A_v = -\frac{h_{fc}R_L}{h_{ic} + R_L(h_{ic}h_{oc} - h_{fc}h_{rc})} = -\frac{(-101)(2 \times 10^3)}{1 \times 10^3 + (2 \times 10^3)[(1 \times 10^3)(12 \times 10^{-6}) - (-101)(1)]} = 0.995$$

Observe that $A_v \approx 1/(1 - h_{ic}h_{oc}/h_{fc}) \approx 1$. Since the gain is approximately 1 and the output voltage is in phase with the input voltage, this amplifier is commonly called a *unity follower*.

(c)

$$Z_{\text{in}} = h_{ic} - \frac{h_{rc}h_{fc}R_L}{1 + h_{oc}R_L} = 1 \times 10^3 - \frac{(1)(-101)(2 \times 10^3)}{1 + (12 \times 10^{-6})(2 \times 10^3)} = 8.41\,\text{M}\Omega$$

Note that $Z_{\text{in}} \approx -h_{fc}/h_{oc}$.

(d)

$$Z_o = \frac{1}{h_{oc} - h_{fc}h_{rc}/h_{ic}} = \frac{1}{12 \times 10^{-6} - (-101)(1)/(1 \times 10^3)} = 9.9\,\Omega$$

Note that $Z_o \approx -h_{ic}/h_{fc}$.

(e) Based on the typical values of this example, the characteristics of the CB amplifier can be summarized as follows:

1. High current gain

2. Voltage gain of approximately unity

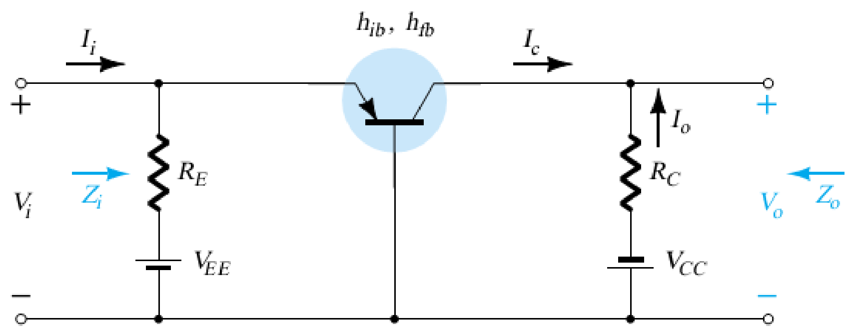
3. Power gain approximately equal to current gain

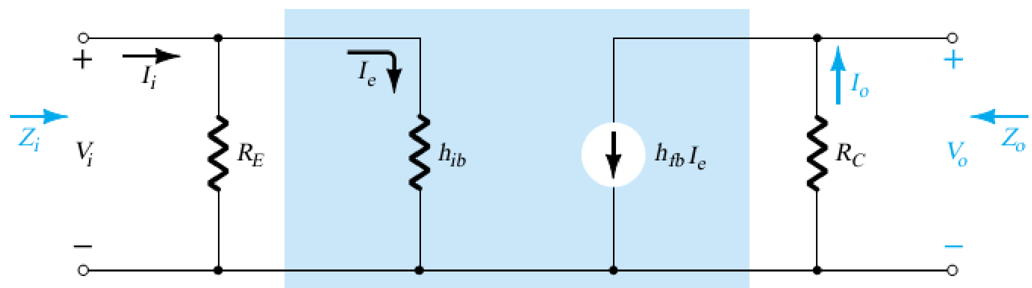
4. No current or voltage phase shift

5. Large input impedance

6. Small output impedance

e- Common-Base Configuration





***Z_i*:**

$$Z_i = R_E \| h_{ib}$$

***Z_o*:**

$$Z_o = R_C$$

***A_v*:**

$$V_o = -I_o R_C = -(h_{fb} I_e) R_C$$

with $I_e = \dfrac{V_i}{h_{ib}}$ and $V_o = -h_{fb}\dfrac{V_i}{h_{ib}} R_C$

so that

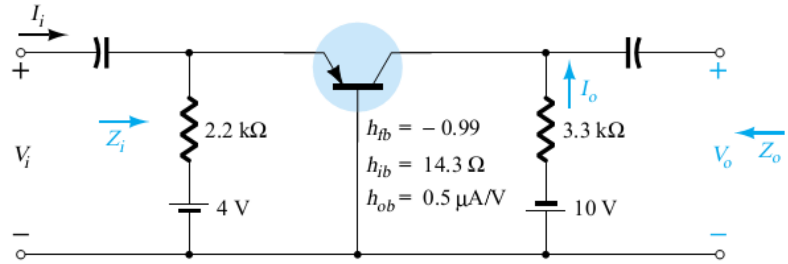
$$A_v = \frac{V_o}{V_i} = -\frac{h_{fb} R_C}{h_{ib}}$$

***A_i*:**

$$A_i = \frac{I_o}{I_i} = h_{fb} \cong -1$$

Example: For the network of Fig. below, determine:

(a) Z_i.
(b) Z_o.
(c) A_v.
(d) A_i.



Solution

(a) $Z_i = R_E \| h_{ib} = 2.2\ \text{k}\Omega \| 14.3\ \Omega = \mathbf{14.21\ \Omega} \cong h_{ib}$

(b) $r_o = \dfrac{1}{h_{ob}} = \dfrac{1}{0.5\ \mu\text{A/V}} = 2\ \text{M}\Omega$

$Z_o = \dfrac{1}{h_{ob}} \| R_C \cong R_C = \mathbf{3.3\ k\Omega}$

(c) $A_v = -\dfrac{h_{fb}\, R_C}{h_{ib}} = -\dfrac{(-0.99)(3.3\ \text{k}\Omega)}{14.21} = \mathbf{229.91}$

(d) $A_i \cong h_{fb} = \mathbf{-1}$