CHAPTER 5
SHEAR AND DIAGONAL TENSION IN BEAMS

5.1 BASIC CONCEPTS

5.1.1 Shear versus Flexural Failures

Due to the following points, shear, or diagonal tension, failure may be more dangerous than flexural failure:

- It has greater uncertainty in predicting,
- It is not yet fully understood, in spite of many decades of experimental research and the use of highly sophisticated analytical tools,
- If a beam without properly designed shear reinforcement is overloaded to failure, shear collapse is likely to occur suddenly, with no advance warning of distress, see ***Figure 5.1-1*** below.

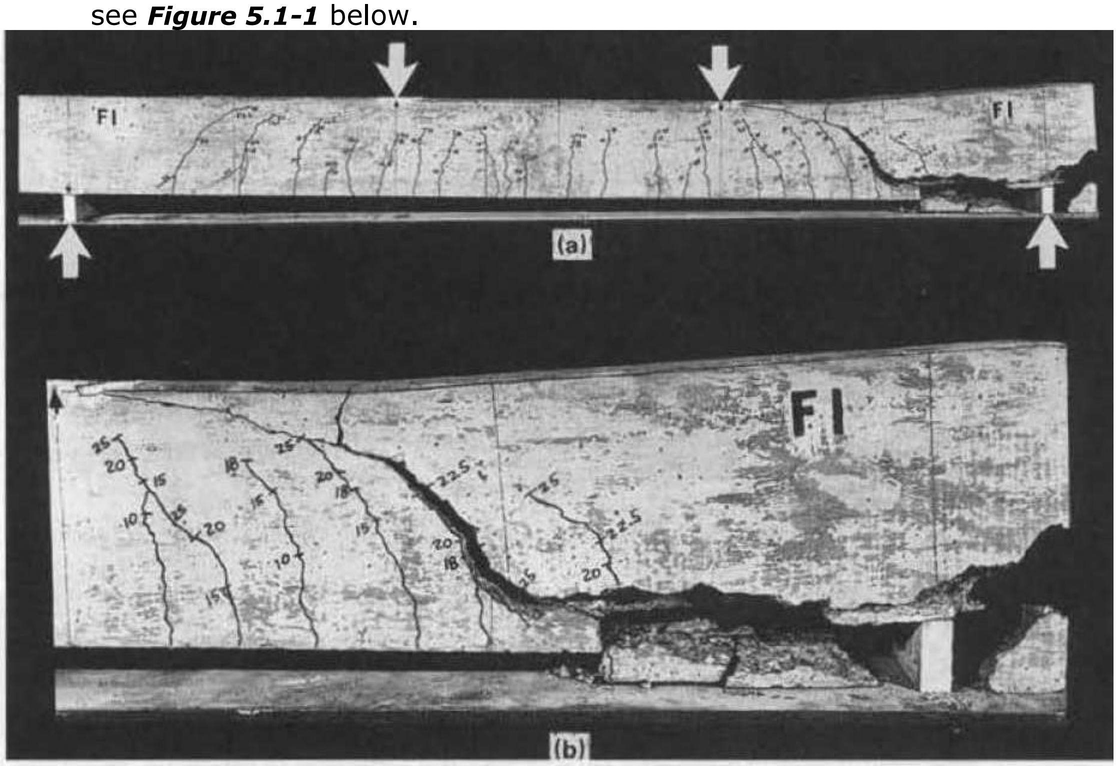


Figure 5.1-1: Shear failure of reinforced concrete beam: (a) overall view, (b) detail near right support.

5.1.2 Direct Shear versus Diagonal Tension

- It is important to realize that shear analysis and design in reinforced concrete structure are not really concerned with shear as such.
- The shear stresses in most beams are far below the direct shear strength of the concrete.
- The real concern is with diagonal tension stress, resulting from the combination of shear stress and longitudinal flexural stress.
- Difference between direct shear and diagonal tension is presented in sub article below.

5.1.2.1 Vertical and Horizontal Shears

- The simplest form of shear is the ***Vertical Shear Stress*** indicated in ***Figure 5.1-2*** below.

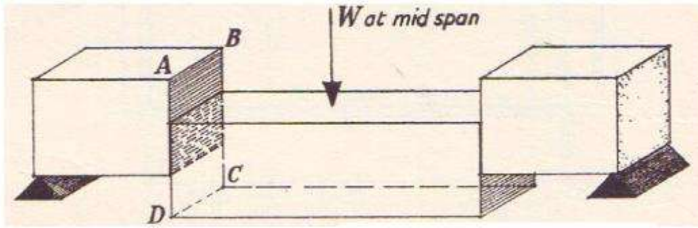


Figure 5.1-2: Vertical shear stresses.

- For homogenous beams and plain concrete beams before cracking, vertical shear stresses can be estimated from the following relation:

$$v = \frac{V.Q}{Ib}$$ **Eq. 5.1-1**

where:
V is total shear at section,
Q is statical moment about the neutral axis of that portion of cross section lying between a line through the point in question parallel to the neutral axis and nearest face (upper or lower) of the beam,
I is the moment of inertia of cross section about neutral axis,
b is width of beam at a given point.

- Distribution of vertical shear stress along beam depth is presented in ***Figure 5.1-3*** below:

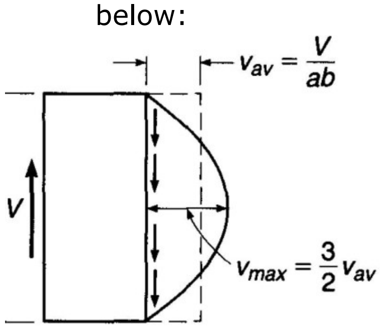


Figure 5.1-3: Shear stress distribution in homogeneous rectangular beams.

5.1.2.2 Horizontal Shear Stresses

- Referring to ***Figure 5.1-4*** below, imagine that a ball is placed between the two cut sections at X, because of the vertical shear action, the ball will turn in a clockwise direction.

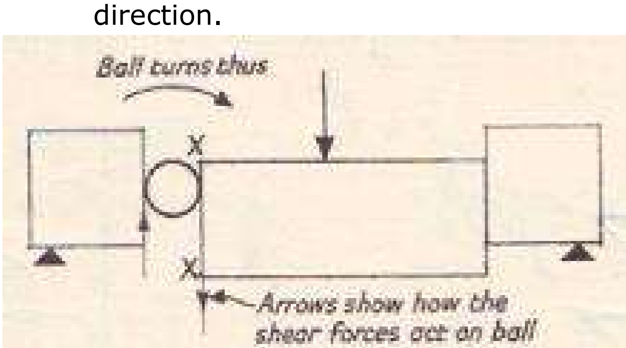


Figure 5.1-4: Conceptual view to imagine role of horizontal shear in resisting possible elemental rotation.

- Then in order to prevent turning, the cube shown below must be acted upon by horizontal forces shown in ***Figure 5.1-5*** below (Morgan, 1958):

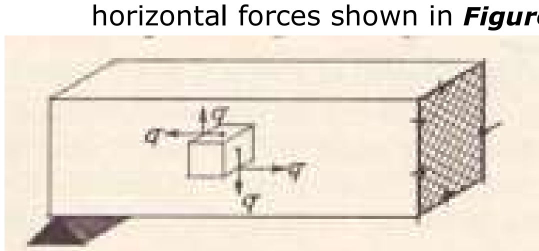


Figure 5.1-5: Horizontal shear stresses.

- These horizontal forces produce another type of shear stress called as ***Horizontal Shear Stress***.
- Thus, one can conclude that the ***vertical shear stress is accompanied by horizontal shear stress of equal intensity*** (Morgan, 1958).

5.1.2.3 Diagonal Tension and Compression

- Force (1) in ***Figure 5.1-6*** below can be combined with force (3) to produce a resultant force of $q\sqrt{2}$. Similarly force (2) and (4) produce a resultant force of $q\sqrt{2}$.

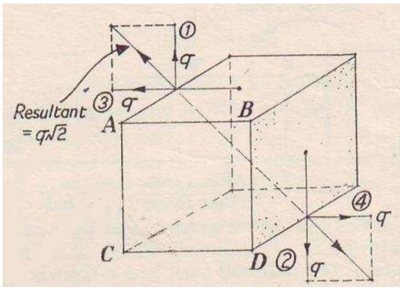


Figure 5.1-6: Diagonal tensile resultant of horizontal and vertical shear stresses.

- Thus, resultant of the vertical and horizontal shear stresses is a pull that exerted along the diagonal plane of the cube tending to cause the diagonal tension failure indicated in ***Figure 5.1-7*** below.

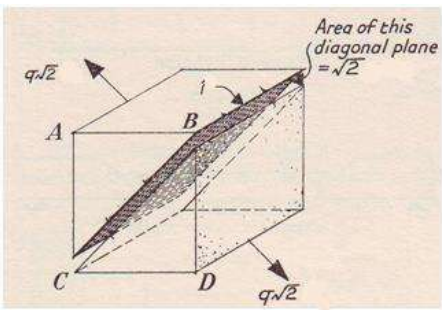


Figure 5.1-7: Diagonal tensile resultant of horizontal and vertical shear stresses, 2.

- Similarly, the vertical and horizontal shear stresses produce a compression force by combining force (2) with force (3) and force (1) with force (4) (see ***Figure 5.1-8*** below).

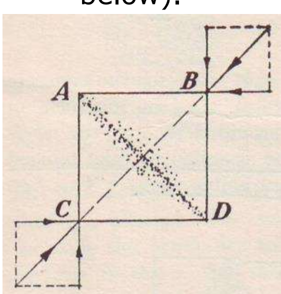


Figure 5.1-8: Diagonal compression resultant of horizontal and vertical shear stresses.

- Therefore***, whenever pure shear stress is acting on an element, it may be thought of as causing tension along one of the diagonals and compression along the other*** (Popov, 1976).

5.1.2.4 Stress Trajectories

Based on the above discussion for the relation between shear stresses and corresponding diagonal stresses, ***stress trajectories*** in a homogeneous simply supported beam with a rectangular section are presented in ***Figure 5.1-9*** below.

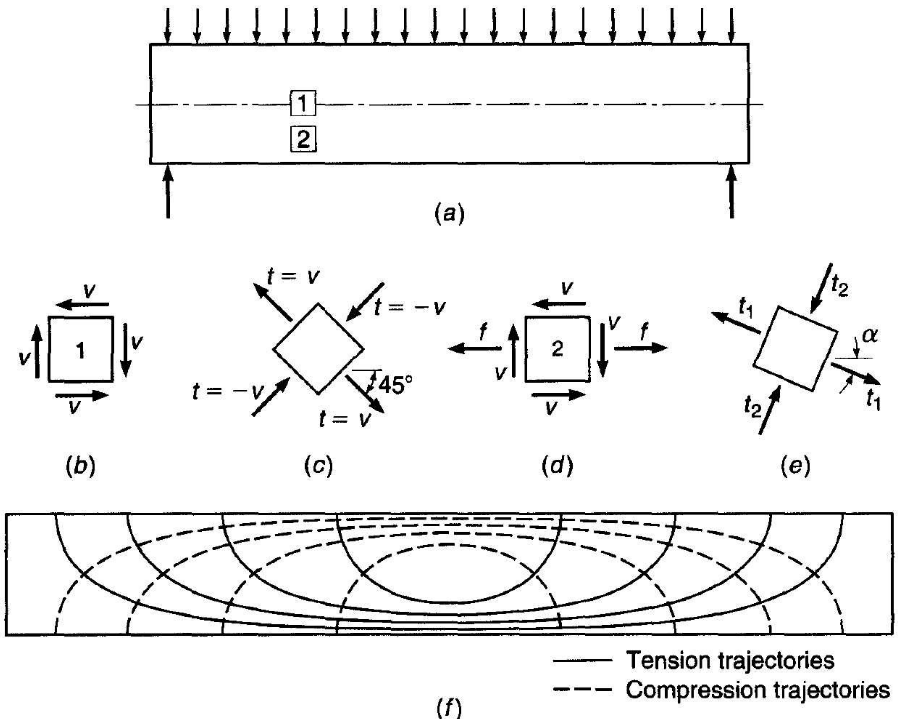


Figure 5.1-9: Stress trajectories in homogeneous rectangular beam.

5.1.2.5 Modes of Failure due to Shear or Diagonal Stresses

- Failure due to vertical shear stress is as shown in ***Figure 5.1-10*** below.

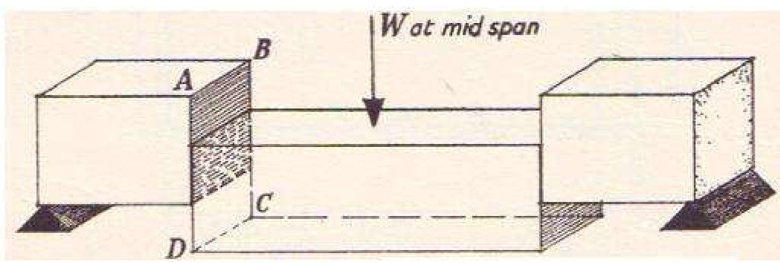


Figure 5.1-10: Mode of failure due to vertical shear stresses.

- Failure due to horizontal shear stress is as shown in ***Figure 5.1-11*** below.

Figure 5.1-11: Mode of failure due to horizontal shear stresses.

- Failure due to diagonal compression stress is as shown in ***Figure 5.1-12*** below.

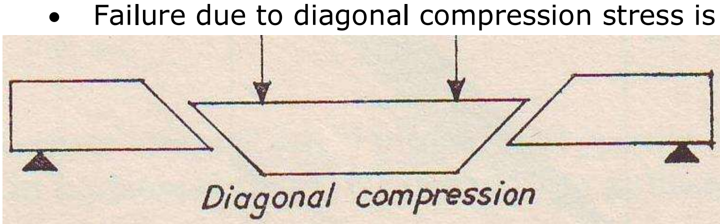


Figure 5.1-12: Mode of failure due to diagonal compression stresses.

- Failure due to diagonal tension stress is as shown in ***Figure 5.1-13*** below.

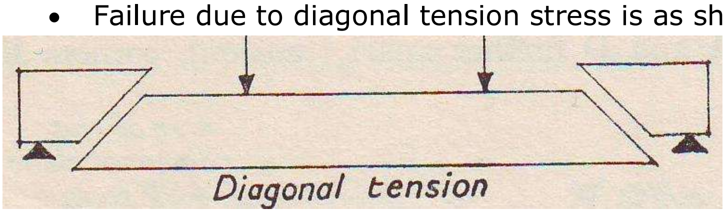


Figure 5.1-13: Mode of failure due to diagonal tensile stresses.

- It is known that concrete is at least ten times as strong in compression as it is in tension, so the typical shear failure in reinforced concrete beams is actually a diagonal tension failure.

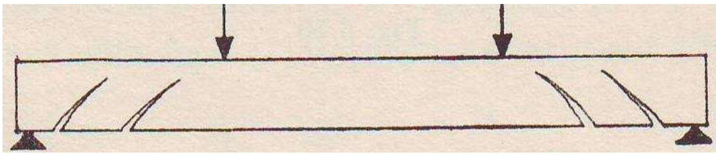


Figure 5.1-14: Cracks in concrete beams due to diagonal tension.

- When the shear stress is higher than the safe value of the concrete, steel in the form of vertical stirrups or inclined bars must be provided to take the exceed shear force.

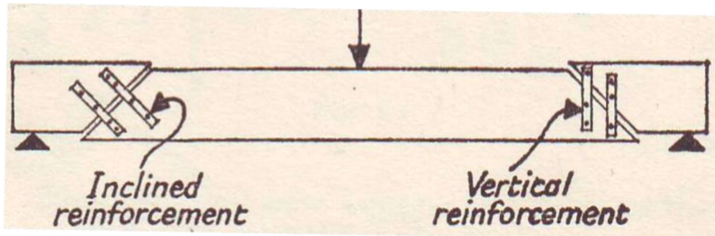


Figure 5.1-15: Conceptual view of vertical and inclined shear reinforcement.

5.1.3 Direct Shear

There are some circumstances in which consideration of direct shear is appropriate:

- The design of composite members combining precast beams with a cast-in-place top slab.

Figure 5.1-16: Composite members combining precast beams with a cast-in-place top slab.

- The horizontal shear stresses on the interface between components are important. The ***shear-friction theory*** is useful in this and other cases. This theory is out of our scope.

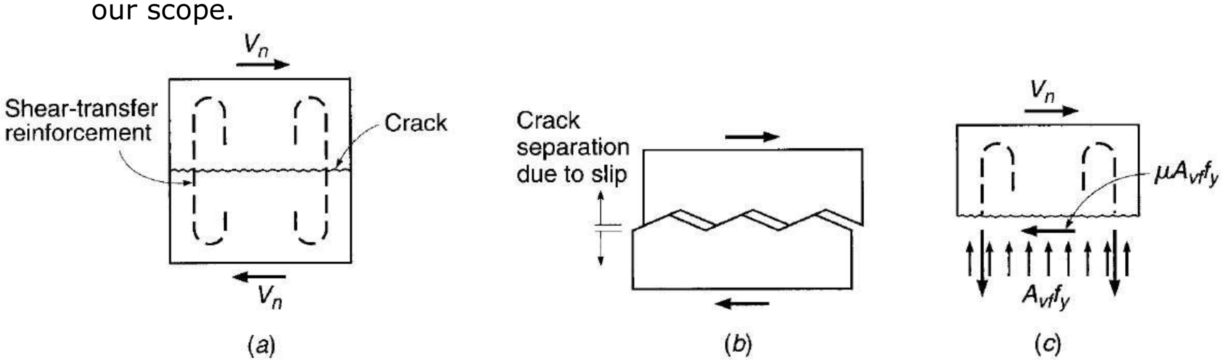


Figure 5.1-17: Basis of shear-friction design method: (a) applied shear; (b) enlarged representation of crack surface; (c) free-body sketch of concrete above crack.

5.1.4 ACI Code Provisions for Shear Design

- It is clear from the previous discussion; the problem under consideration is a problem of diagonal tension stresses.
- As the ACI Code uses the shear forces as an indication of the diagonal tension, then all design equations according to ACI Code are presented regarding shear forces.
- According to ACI Code (***9.5.1.1***), the design of beams for shear is to be based on the relation:

$$V_u \leq \emptyset V_n \qquad \textbf{Eq. 5.1-2}$$

- According to the ACI Code (21.2.1), the strength reduction factor, ϕ, for shear is 0.75.
- According to article ***22.5.1.1***, nominal shear strength, V_n, can be computed based on the following relation:

$$V_n = V_c + V_s \qquad \textbf{Eq. 5.1-3}$$

or,

$$V_u \leq V_n = \emptyset(V_c + V_s) \qquad \textbf{Eq. 5.1-4}$$

where:

V_u is the total shear force applied at a given section of the beam due to factored loads,

V_n is the nominal shear strength, equal to the sum of the contributions of the concrete (V_c) and the steel (V_s) if present.

- Thus, according to ACI Code, the design problem for shear can be reduced to provisions for computing of V_u, V_c, and V_s if present. Each one of these quantities is discussed in some details in the articles below.

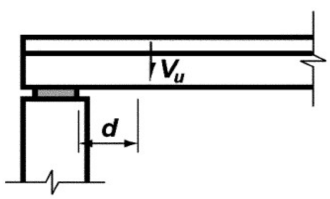
5.2 COMPUTING OF APPLIED FACTORED SHEAR FORCE V_u

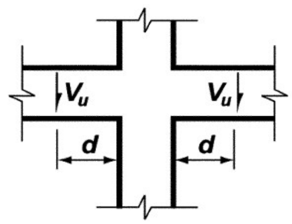
5.2.1 Basic Concepts

- The applied shear force can be computed based on given loads and spans.
- Generally, the applied factored shear force V_u is computed at the face of supports.
- According to ACI Code (**9.4.3.2**), ***sections between the face of support and a critical a section located "d" from the face of support for nonprestressed shall be permitted to be designed for*** V_u ***at that critical section*** if following conditions are satisfied: Discussion similar to that of classroom is preferable to add here to explain physical aspects of the three conditions below.
  - Support reaction, in the direction of applied shear, introduces compression into the end regions of the member.
  - Loads are applied at or near the top of the member.
  - No concentrated load occurs between the face of support and location of critical.

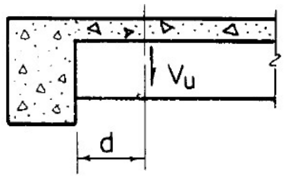
5.2.2 Examples on Computing of V_u

- For the figures below, critical section for computing of V_u will be taken at a distance "d" from the face of support as all above conditions are satisfied (Nilson, Design of Concrete Structures, 14th Edition, 2010). It is preferable to put these cases in groups, for example, floor beam supported on a deeper girder and a girder with same depth can be put in the same group. Besides, it is preferable that each group and corresponding figures have subtitle and caption.

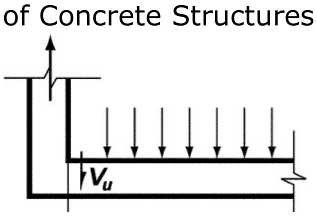




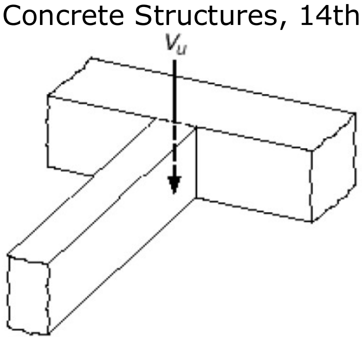
- For the figure below, the critical section for computing of V_u is at distance "d" from the face of support for a floor beam supported by a deeper main girder as all above conditions are satisfied (Kamara, 2005) (Page 12-3).



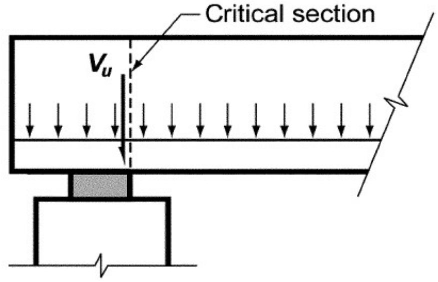
- For the figure below, the critical section for computing of V_u is at the face of support as member framing into a supporting member in tension (Nilson, Design of Concrete Structures, 14th Edition, 2010) (Page 131).



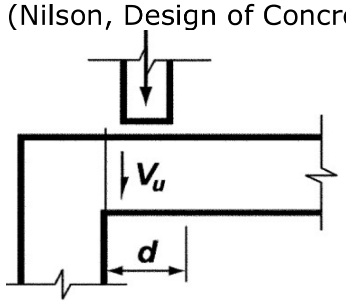
- For the figure below, the critical section for computing of V_u is at the face of support if the beam is supported by a girder of similar depth (Nilson, Design of Concrete Structures, 14th Edition, 2010).



- For the figure below, the critical section for computing of V_u is at the face of support as loads are not applied at or near the top of the member (Nilson, Design of Concrete Structures, 14th Edition, 2010).



- For the figure below, the critical section for computing of V_u is at the face of support as concentrated load occurs within a distance "d" from the face of support (Nilson, Design of Concrete Structures, 14th Edition, 2010).



5.3 SHEAR STRENGTH PROVIDED BY CONCRETE V_c

5.3.1 Upper Bound of Concrete Compressive Strength, f_c', in Estimating V_c

- According to ***article 22.5.3.1*** of ACI code, except for ***article 22.5.3.2***, related to prestressed beams and joist construction, ***the value of*** $\sqrt{f_c'}$ ***used to calculate*** V_c ***shall not exceed 8.3 MPa***.
- The above statement is because of a lack of test data and practical experience with concretes having compressive strengths greater than 70 MPa.

5.3.2 Plain Concrete Beams

- As the load increases in such a beam, a tension crack will form where the tensile stresses are largest, and it will immediately cause the beam to fail.
- Except for beams of very unusual proportions, the largest tensile stresses are those caused at the outer fiber by bending alone, at the section of maximum bending moment. In this case, shear has little, if any, influence on the strength of a beam.

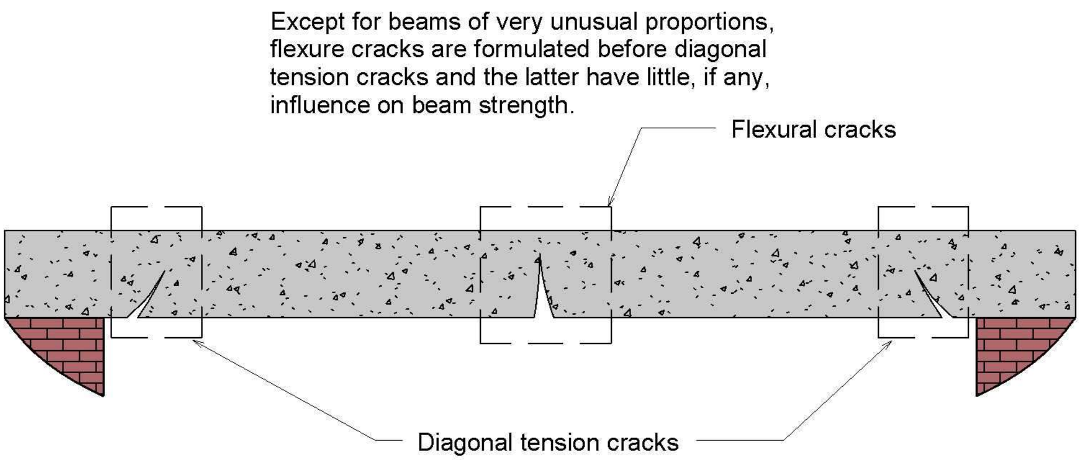


Figure 5.3-1: Behavior of plain concrete beams.

5.3.3 Reinforced Concrete Beams without Shear Reinforcement

- For beams designed properly for flexure, diagonal cracks may propagate faster than flexural cracks, and shear aspects may govern the beam failure.

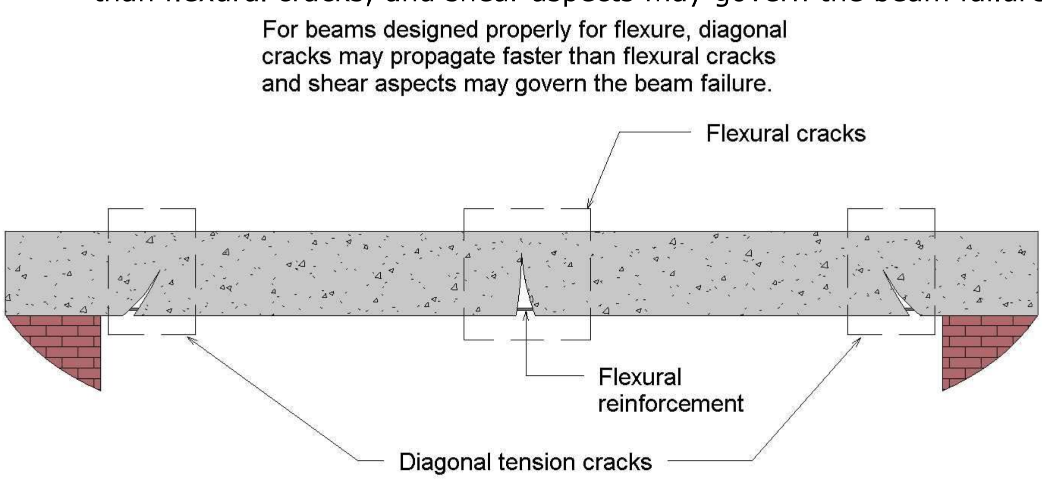


Figure 5.3-2: Behavior of a beam reinforced for flexure only.

- For concrete beams ***reinforced for flexure only***, shear force required ***to initiates diagonal cracks in web-shear cracks region***, or ***to propagate cracks in a flexure-shear region*** can be estimated from relation below, ***Article 22.5.5.1*** of (ACI318M, 2014):

$$V_c = 0.17\lambda\sqrt{f_c'}\,b_w d \qquad \textbf{Eq. 5.3-1}$$

where:

λ is the lightweight modification factor that taken from ***Table 5.3-1*** below, Table 19.2.4.2 of (ACI318M, 2014).

Table 5.3-1: Modification factor λ, Table 19.2.4.2 of (ACI318M, 2014).

Concrete	Composition of aggregates	λ
All-lightweight	Fine: ASTM C330M Coarse: ASTM C330M	0.75
Lightweight, fine blend	Fine: Combination of ASTM C330M and C33M Coarse: ASTM C330M	0.75 to 0.85[1]
Sand-lightweight	Fine: ASTM C33M Coarse: ASTM C330M	0.85
Sand-lightweight, coarse blend	Fine: ASTM C33M Coarse: Combination of ASTM C330M and C33M	0.85 to 1[2]
Normalweight	Fine: ASTM C33M Coarse: ASTM C33M	1

[1]Linear interpolation from 0.75 to 0.85 is permitted based on the absolute volume of normalweight fine aggregate as a fraction of the total absolute volume of fine aggregate.

[2]Linear interpolation from 0.85 to 1 is permitted based on the absolute volume of normalweight coarse aggregate as a fraction of the total absolute volume of coarse aggregate.

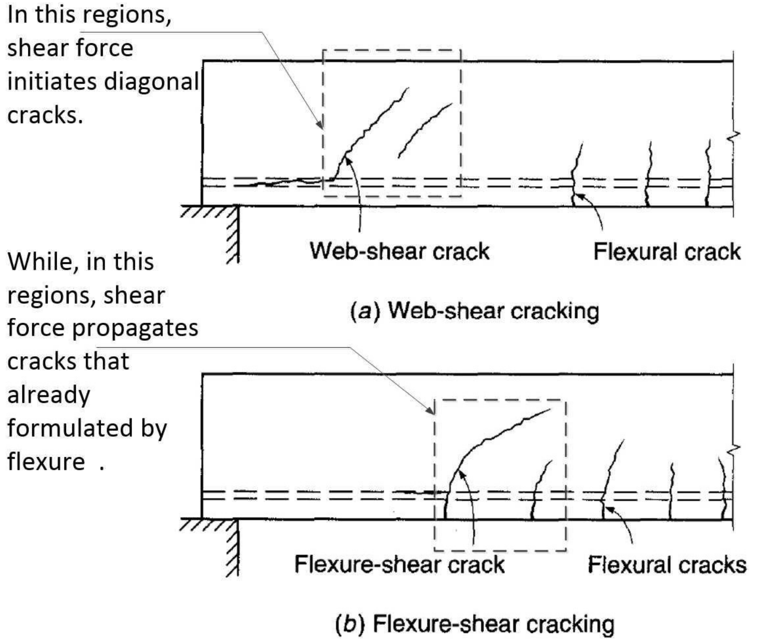


Figure 5.3-3: Diagonal tension cracking in reinforced concrete beams.

- With referring to ***Figure 5.3-3*** above, it is useful to note that the Eq. 5.3-1 is ***more suitable flexure-shear crack*** and ***relatively conservative for web-shear cracks***. A more accurate relation has been presented in ***Article 5.8*** of this chapter.
- In spite of its conservative nature in the web-shear crack region, in practice, most of the beams are usually designed based on Eq. 5.3-1.
- For solid circular members, the area used to compute V_c shall be taken as shown in ***Figure 5.3-4*** (***Article 22.5.2.2*** of ACI Code).

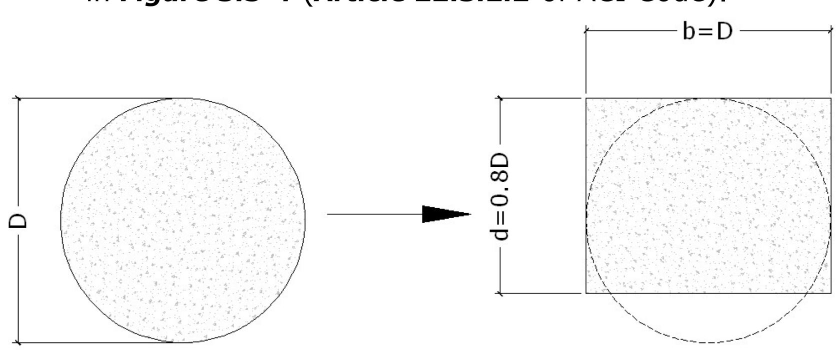


Figure 5.3-4: Effective area for shear in solid circular sections.

5.3.4 Beams Reinforced for Shear

- As for flexural behavior, current ACI code permits formation of web-shear cracks and flexure-shear cracks when beams are reinforced for shear and diagonal tension.
- With shear reinforcements, that resist propagation of web-shear cracks, the free body diagram for one side of crack at failure stage would be as shown ***Figure 5.3-5*** below.

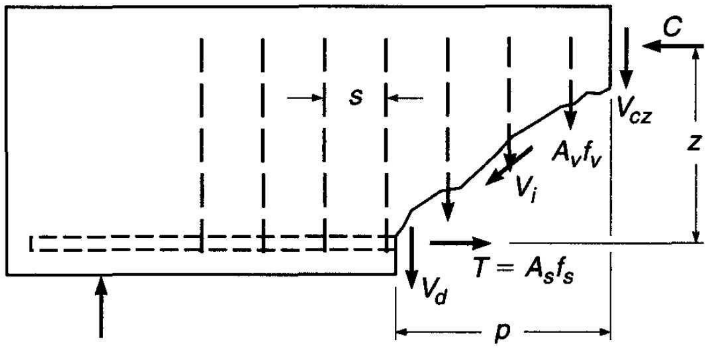


Figure 5.3-5: Forces at a diagonal crack in a beam with vertical stirrups.

where
$A_v f_v$ is shear force resisted by each stirrup, will be discussed in detail in ***Article 5.4.2*** of this chapter,
V_{cz} shear force resisted by uncracked concrete portion,
V_i shear force resisted by the interlocking of concrete on two sides of the crack,
V_d shear force resisted by longitudinal rebars, dowel action,

- From equilibrium in vertical direction,

$$V_{ext} = V_{cz} + V_d + V_{iy} + V_s \quad \textbf{Eq. 5.3-2}$$

- ***Empirically*** and ***conservatively*** current ACI code assumes that:

$$V_{cz} + V_d + V_{iy} \approx V_c = 0.17\lambda\sqrt{f'_c}\, b_w d \quad \textbf{Eq. 5.3-3}$$

- Therefore, in the current ACI code, the relation:

$$V_c = 0.17\lambda\sqrt{f'_c}\, b_w d \quad \textbf{Eq. 5.3-4}$$

has two roles:
  - It is used ***rationally*** to estimate the shear force that either initiates web-shear cracks or propagate flexure-shear cracks.
  - It is used ***empirically*** to estimate the order for summation of V_{cz}, V_d, and V_{iy}.

5.4 SHEAR STRENGTH PROVIDED BY SHEAR REINFORCEMENT V_s

5.4.1 Type of Shear Reinforcement

- Several types and arrangements of shear reinforcement permitted by ACI are illustrated in ***Figure 5.4-1*** (Kamara, 2005) (Page 12-6).

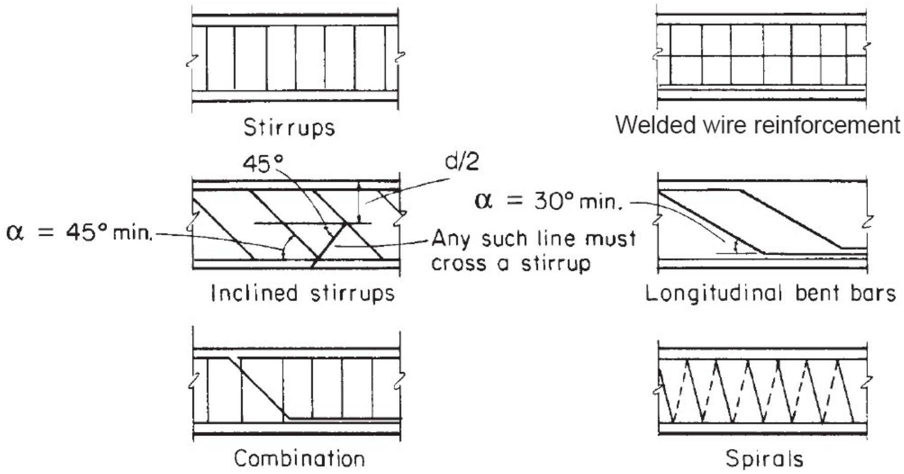


Figure 5.4-1: Types of shear reinforcement.

- Spirals, circular ties, or hoops are explicitly recognized as types of shear reinforcement starting with the 1999 code (Kamara, 2005) (Page 12-6).
- Vertical stirrups are the most common type of shear reinforcement.
- Inclined stirrups and longitudinal bent bars are rarely used as they require a special care during placement in the field.
- U-shaped bars similar to those presented in ***Figure 5.4-2*** below are the most common, although multiple-leg stirrups such as shown are sometimes necessary.

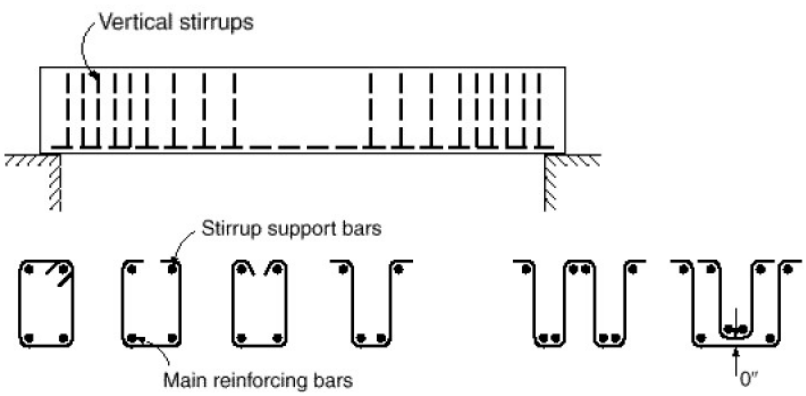


Figure 5.4-2: U stirrups shear reinforcement.

5.4.2 Theoretical Spacing between Vertical Stirrups

- Theoretical spacing for vertical stirrups can be related to other design parameters based on following relations:

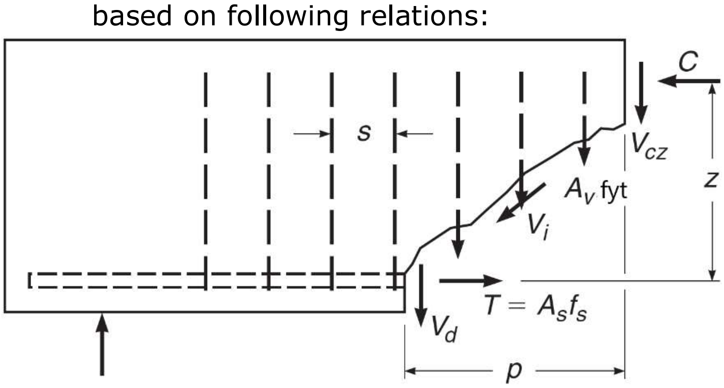


Figure 5.4-3: Forces at a diagonal crack in a beam with vertical stirrups, reproduced for convenience.

$$V_s = \text{Fore per each stirrup} \times \text{No. of stirrups through the inclined crack}$$

$$V_s = (A_v \times f_{yt})_{\text{Fore per each stirrup}} \times \left(\frac{p}{s}\right)_{\text{No.of stirrups through the inclined crack}}$$

where:

$A_v = $ area of shear reinforcement $= \frac{\pi \varnothing_{Stirrups}^2}{4} \times$ No. of Legs

- If the crack is assumed to have an angle of 45 degree with the horizon, then p can be computed approximately based on following relation:
  $p \approx d$
  Then:
  $$V_s = \frac{A_v f_{yt} d}{s}$$ **Eq. 5.4-1**
  Above relation that suitable for analysis purpose, can be solved for s to be more suitable for design purpose:
  $$s = \frac{A_v f_{yt} d}{V_s} \blacksquare$$ **Eq. 5.4-2**
- In addition to this theoretical spacing for shear reinforcement, ACI Code also includes many other nominal requirements that related to shear reinforcement. ACI practical procedure for shear design has been summarized in article below.

5.5 SUMMARY OF PRACTICAL PROCEDURE FOR SHEAR DESIGN

5.5.1 Essence of the Problem

- Generally, beam dimensions (b and h) are determined based on considerations other than shear and diagonal tension requirements.
- Then, in a shear problem, the designer deals with a beam that has pre-specified dimensions and main unknowns in the design problem are the shear reinforcement (if needed) and its details that can be summarized as follows:
  - The diameter of shear reinforcement.
  - Spacing (for economic aspect, a beam may be divided to sub-regions with different shear reinforcements) for shear reinforcements.
  - Anchorage requirements for shear reinforcements.
- The detailed procedure for each one of the above three unknowns will be discussed below.

5.5.2 Bar Diameter for Stirrups and Stirrups Support Bars

- As was previously discussed in Chapter 4, bar diameters that used for shear reinforcements usually include 10mm, or 13mm.
- A Bar diameter of 16mm rarely used as shear reinforcement.
- Where no top bars are required for flexure, stirrups support bars must be used. These are usually about the same diameter as the stirrups themselves (Nilson, Design of Concrete Structures, 14th Edition, 2010).

5.5.3 Spacing for Shear Reinforcements

Computing of required spacing can be summarized as follows:

- Draw the shear force diagram based on factored load and span length, and divide the diagram into the three distinct regions shown in ***Figure 5.5-1*** (Kamara, 2005) (Page 12-9):

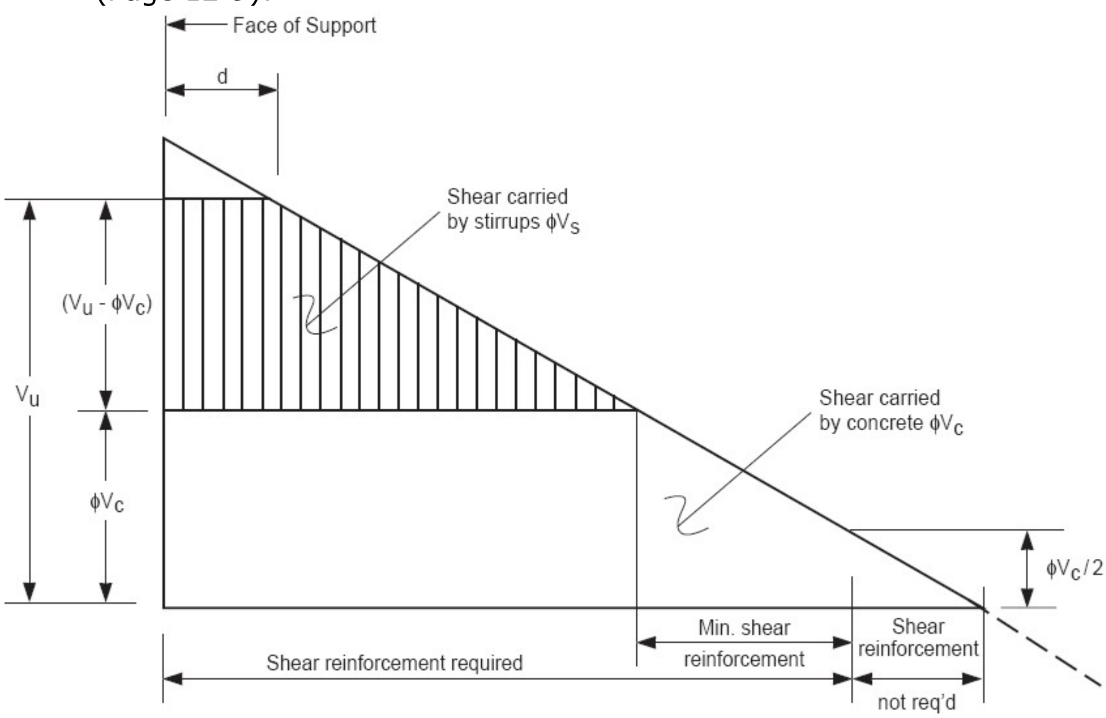


Figure 5.5-1: Three distinguish regions of shear force diagram.

- Based on ***Table 5.5-1***, compute the required spacing for each one of the regions shown above (if shear reinforcement is required for this region) (Kamara, 2005) (Page 12-8):

Table 5.5-1: ***ACI provisions for shear design.***

Region	$V_u \le \emptyset \frac{V_c}{2}$	$\emptyset \frac{V_c}{2} < V_u \le \emptyset V_c$	$\emptyset V_c \le V_u$
V_s	None	None	$= \frac{V_u - \emptyset V_c}{\emptyset} \le 0.66\sqrt{f_c'} b_w d$ Else, change beam dimensions.
$S_{Theoretical}$	None	None	$= \frac{A_v f_{yt} d}{V_s}$
$S_{for\ Av\ minimum}$ (9.6.3.3)	None	$\text{minimum}\ (\frac{A_v f_{yt}}{0.062\sqrt{f_c'} b_w}\ \text{or}\ \frac{A_v f_{yt}}{0.35 b_w})$	$\text{minimum}\ (\frac{A_v f_{yt}}{0.062\sqrt{f_c'} b_w}\ \text{or}\ \frac{A_v f_{yt}}{0.35 b_w})$
$S_{maximum}$ (9.7.6.2.2)	None	$\text{Minimum}\ \left[\frac{d}{2}\ \text{or}\ 600\text{mm}\right]$	$V_s \le 0.33\sqrt{f_c'} b_w d$: $\text{Minimum}\ \left[\frac{d}{2}\ \text{or}\ 600\text{mm}\right]$; $V_s > 0.33\sqrt{f_c'} b_w d$: $\text{Minimum}\ \left[\frac{d}{4}\ \text{or}\ 300\text{mm}\right]$
$S_{Required}$	None	$\text{Minimum}\ \left[S_{for\ Av\ minimum}, S_{maximum}\right]$	$\text{Minimum}\ \left[S_{Theoretical}, S_{for\ Av\ minimum}, S_{maximum}\right]$

- Notes on A_{vmin}:
  According to ***Article 9.6.3.1***, for cases presented in Table below, $A_{v\ minimum}$ is not required even with $\emptyset \frac{V_c}{2} < V_u \le \emptyset V_c$:

Table 5.5-2: Cases where $A_{v\,mi}$ is not required if $0.5\phi Vc < Vu \le \phi V$, Table 9.6.3.1 of (ACI318M, 2014).

Beam type	Conditions
Shallow depth	$h \le 250$ mm
Integral with slab	$h \le$ greater of $2.5t_f$ or $0.5b_w$ and $h \le 600$ mm
Constructed with steel fiber-reinforced normalweight concrete conforming to 26.4.1.5.1(a), 26.4.2.2(d), and 26.12.5.1(a) and with $f_c' \le 40$ MPa	$h \le 600$ mm and $V_u \le \phi 0.17\sqrt{f_c'} b_w d$
One-way joist system	In accordance with 9.8

5.5.4 Anchorage Requirement for Shear Reinforcements

5.5.4.1 Design Assumptions Regarding to Anchorage

Above design is based on assumption that the stirrups will yield at ultimate load. This will be true only if the stirrups are well anchored.

5.5.4.2 General Anchor Requirements

- Generally, the upper end of the inclined crack approach very closed to the compression face of the beam. Thus, the portion of the stirrups shown shaded in ***Figure 5.5-2*** must be able to anchor the stirrups.

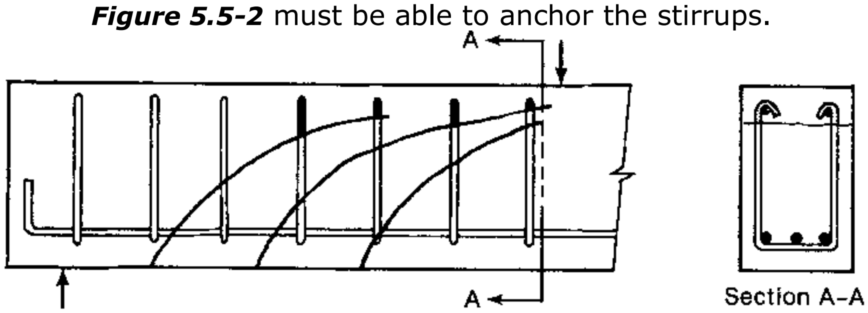


Figure 5.5-2: General requirements for anchorage of stirrups.

- ACI general anchor requirement can be summarized in ***Figure 5.5-3***.

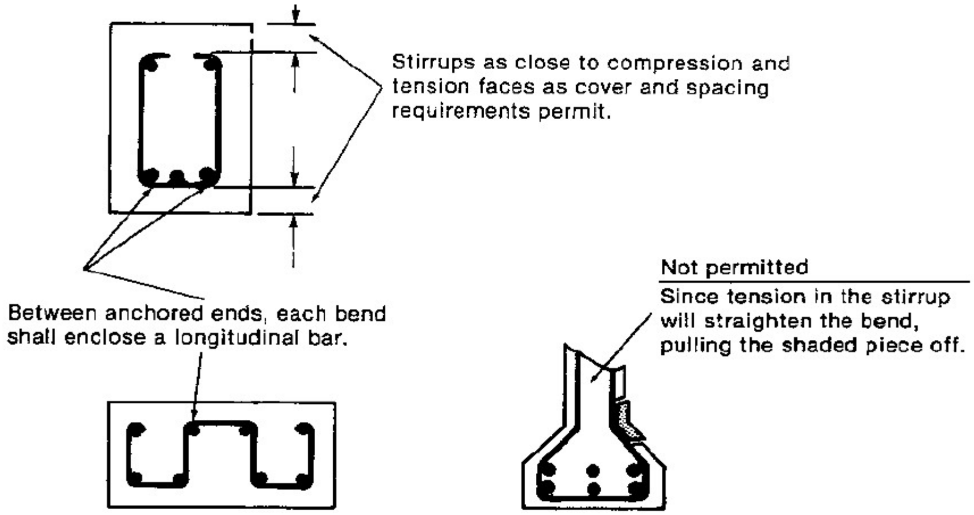


Figure 5.5-3: General requirements for anchorage of stirrups, continued.

- According to anchorage requirements, stirrups may be classified into the following two types.

5.5.4.3 Open Stirrups

- They may take any one of the shapes indicated in ***Figure 5.5-4***.

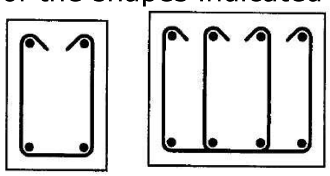


Figure 5.5-4: Open stirrups.

- As shown in ***Figure 5.5-5***, anchorage of an open stirrup depends on using standard hooks at the corners of the stirrups supporting rebars.

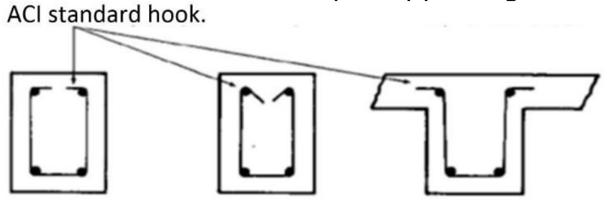
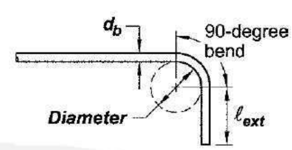
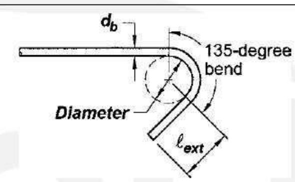
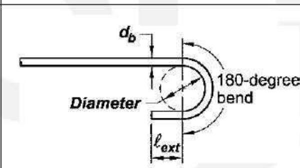


Figure 5.5-5: Standard hook anchorage for open stirrups.

- Minimum inside bend diameters and standard hook geometry ***for stirrups, ties, and hoops*** are presented in ***Table 5.5-3***.

Table 5.5-3: Minimum inside bend diameters and standard hook geometry for stirrups, ties, and hoops, Table 25.3.2 of (ACI318M, 2014).

Type of standard hook	Bar size	Minimum inside bend diameter, mm	Straight extension[1] ℓ_{ext}, mm	Type of standard hook
90-degree hook	No. 10 through No. 16	$4d_b$	Greater of $6d_b$ and 75 mm	
	No. 19 through No. 25	$6d_b$	$12d_b$	
135-degree hook	No. 10 through No. 16	$4d_b$	Greater of $6d_b$ and 75 mm	
	No. 19 through No. 25	$6d_b$		
180-degree hook	No. 10 through No. 16	$4d_b$	Greater of $4d_b$ and 65 mm	
	No. 19 through No. 25	$6d_b$		

[1]A standard hook for stirrups, ties, and hoops includes the specific inside bend diameter and straight extension length. It shall be permitted to use a longer straight extension at the end of a hook. A longer extension shall not be considered to increase the anchorage capacity of the hook.

- According to ACI (25.7.1.3b), for No. 19, through No. 25 stirrups with f_{yt} greater than 280 MPa, a standard stirrup hook around a longitudinal bar plus an embedment between mid-height of the member and the outside end of the hook equal to or greater than $0.17 d_b\, f_{yt}\, / (\lambda \sqrt{f_c'})$.

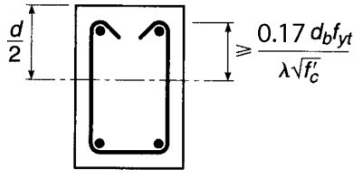


Figure 5.5-6: Embedment length for open stirrups with for No. 19, through No. 25 stirrups with f_{yt} greater than 280 MPa.

- This requirement has been included as it is not possible to bend a No. 19, No. 22, or No. 25 stirrup tightly around a longitudinal.

5.5.4.4 Closed Stirrups

- Its typical shapes are shown ***Figure 5.5-7***.

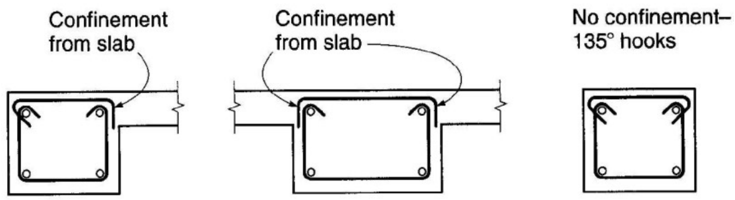


Figure 5.5-7: Typical closed stirrups.

- It may be taking the form of closed tie shown in ***Figure 5.5-8***.

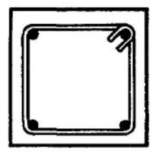


Figure 5.5-8: Tie reinforcement.

- Closed stirrups or closed ties should be used for:
  - For beams with compression reinforcements.
  - For members subjected to torsion.

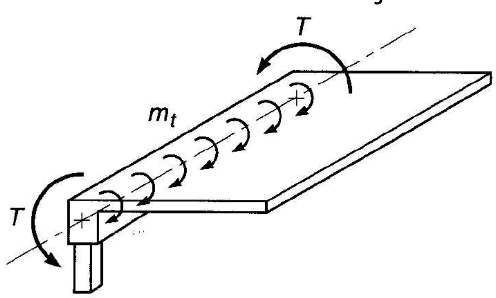


Figure 5.5-9: Beam subjected to torsion where closed stirrups should be adopted.

  - For reversals stresses.

5.5.4.5 Spliced Stirrup

- According to article ***25.7.1.7*** of (ACI318M, 2014), ***except where used for torsion*** or ***integrity reinforcement***, closed stirrups are permitted to be made using pairs of U-stirrups spliced to form a closed unit where lap lengths are at least $1.3 l_d$.
- In members with a total depth of at least 450 mm, such splices with $A_b f_{yt} \leq 40\ kN$ per leg shall be considered adequate if stirrup legs extend the full available depth of member.

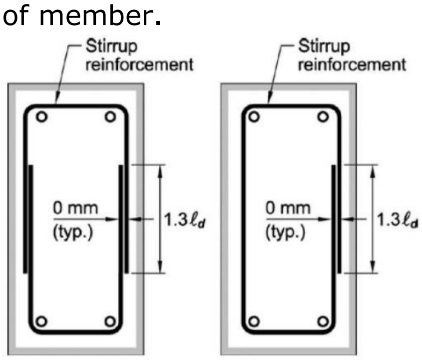


Figure 5.5-10: Closed stirrup configurations.

- The development length may be defined as ***the length of embedment necessary to develop the full tensile strength of the bar***. It will be discussed in details in ***Chapter 6***.
- Its approximate value can be computed from Table below:

Table 5.5-4: Simplified tension development length in bar diameters l_d/d_b for uncoated bars and normalweight concrete

		No. 6 (No. 19) and Smaller[a]			No. 7 (No. 22) and Larger		
		f'_c, psi			f'_c, psi		
	f_y, ksi	4000	5000	6000	4000	5000	6000
(1) Bottom bars							
Spacing, cover and ties as per Case *a* or *b*	40	25	23	21	32	28	26
	50	32	28	26	40	35	32
	60	38	34	31	47	42	39
Other cases	40	38	34	31	47	42	39
	50	47	42	39	59	53	48
	60	57	51	46	71	64	58
(2) Top bars							
Spacing, cover and ties as per Case *a* or *b*	40	33	29	27	41	37	34
	50	41	37	34	51	46	42
	60	49	44	40	62	55	50
Other cases	40	49	44	40	62	55	50
	50	62	55	50	77	69	63
	60	74	66	60	92	83	76

Case *a*: Clear spacing of bars being developed or spliced $\geq d_b$, clear cover $\geq d_b$, and stirrups or ties throughout l_d not less than the Code minimum.

Case *b*: Clear spacing of bars being developed or spliced $\geq 2d_b$, and clear cover not less than d_b.

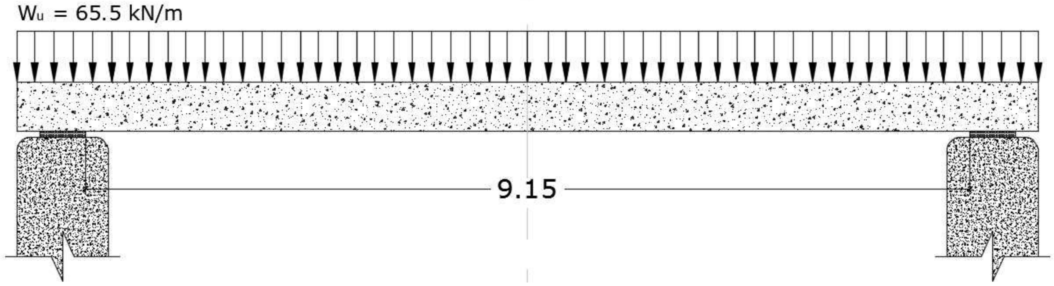
[a] ACI Committee 408 recommends that the values indicated for bar sizes No. 7 (No. 22) and larger be used for all bar sizes.

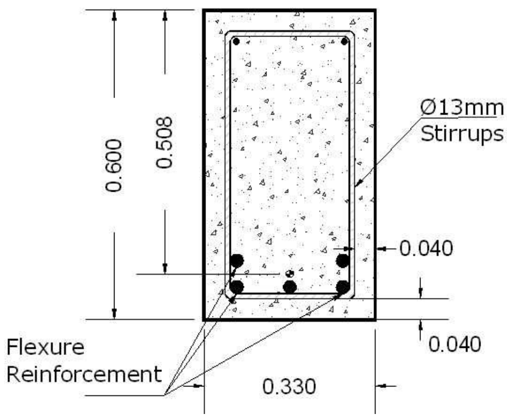
5.6 Basic Design Examples

Example 5.6-1

Check adequacy of proposed size and determine required spacing of vertical stirrups for a 9.15m span simply supported beam with following data:

$b_w = 330\text{mm},\ d = 508\text{mm},\ f_c' = 21\ \text{MPa},\ f_{yt} = 275\ \text{MPa},\ W_u = 65.5\frac{\text{kN}}{\text{m}}$



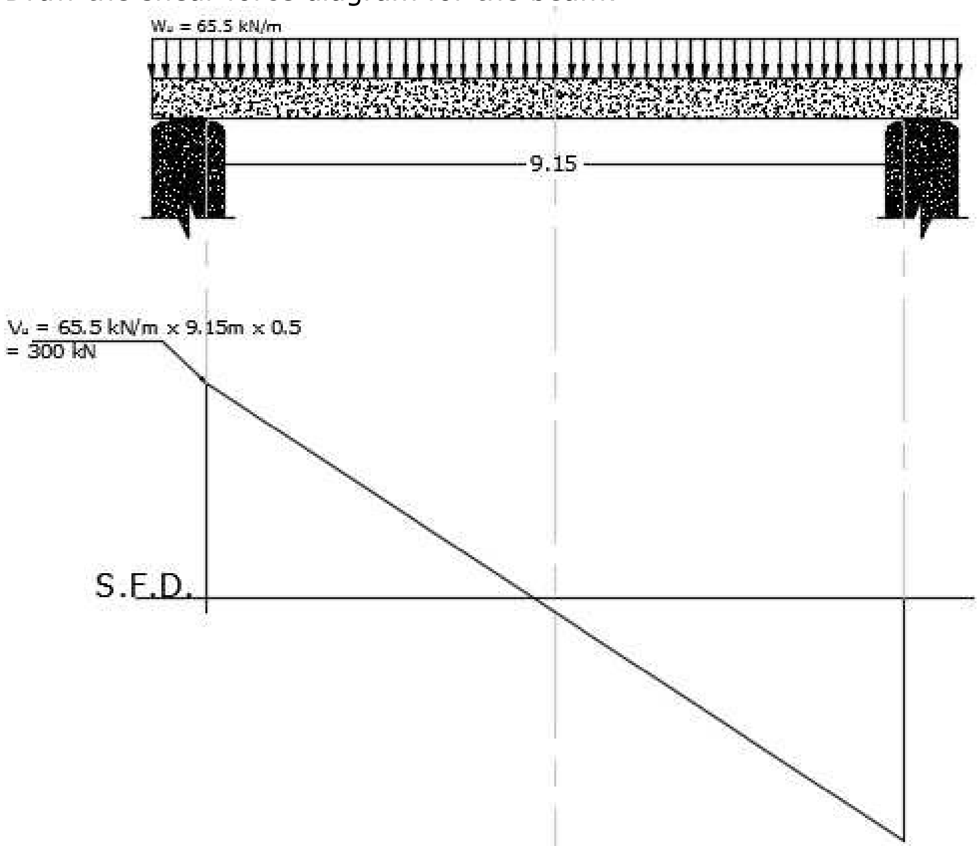


Proposed beam section.

Figure 5.6-1: Simply supported beam for Example 5.6-1.

Solution

- Regarding to bar diameter for stirrups, the proposed diameter of 13mm is common and accepted one.
- Draw the shear force diagram for the beam:



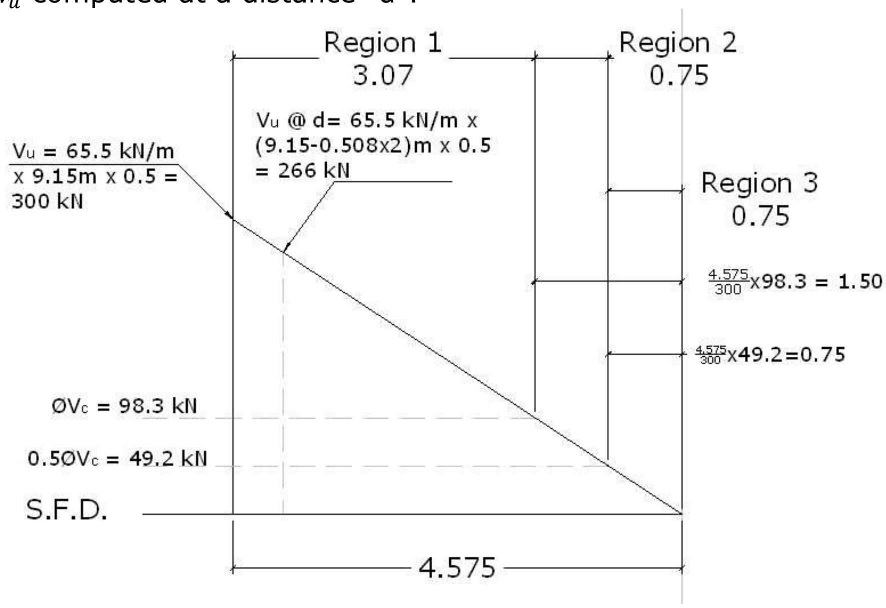
- Compute of Shear Strength Provided by Concrete V_c:

  $V_c = 0.17\lambda\sqrt{f_c'}\, b_w d$

  As $\lambda = 1.0$ for normal weight concrete, then:

  $V_c = 0.17\sqrt{f_c'}\, b_w d = 0.17 \times \sqrt{21}\dfrac{N}{mm^2} \times 330mm \times 508mm = 131\,000\ N = 131\ kN$

  $\emptyset V_c = 0.75 \times 131\ kN = 98.3\ kN$
- Based on value of $\emptyset V_c$ divide the shear force diagram into three regions indicated below. As all limitations of article (**9.4.3.2**) are satisfied, then sections located less than a distance "d" from face of support shall be permitted to be designed for V_u computed at a distance "d".



To compute shear force at distance "d" from face of supported any one of the following two approaches can be adopted:

- Based on differential equations of equilibrium:

From mechanics of materials, to satisfy equilibrium of an infinitesimal element, following differential equations should be satisfied:

$$w = \frac{dV}{dx}$$

$$V = \frac{dM}{dx}$$

The first equation indicates that the load value, w, represents the slope for shear diagram while the second equation indicates that the value of shear force represents the slope of the bending moment diagram. It is useful to note that both equations are consistent in units.

From the first equation:

$dV = wdx$

Integrate to obtain

$$V_2 - V_1 = \int_1^2 w\, dx$$

Or

$$V_{u\,@\,distance\ d} = \int_{From\ face\ of\ support}^{To\ a\ distance\ d\ from\ face\ of\ support} w\, dx \ + V_{u\,@\,face\ of\ support}$$

It is worthwhile to note that the above ***finite integral is equal to area under load diagram from face of support to a distance "d" from face of support***.

$V_{u\,@\,distance\ d} = (-65.5 \times 0.508 \ + 300) \approx 266\ kN$

- Based on Symmetry

From problems that have symmetry, shear force at distance "d" can be determined based on following relation:

$$V_{u\ at\ distance\ d\ from\ face\ of\ support}=\frac{1}{2}\times W_u(l_n-2d)$$

where l_n is the clear span measured from face to face of supports.

$$V_{u\ at\ distance\ d\ from\ face\ of\ support}=\frac{1}{2}\times 65.5\times(9.15-2\times 0.508)=266\ kN$$

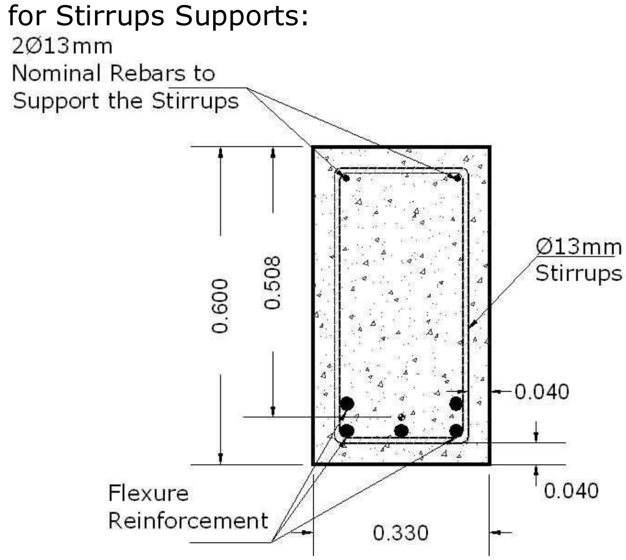
- Compute stirrups spacing for each region based on the table presented below:
  Try U Shape stirrups of 13mm diameter, then $\mathrm{A_v}$ will be:

$$\mathrm{A_v}=\frac{\pi\times 13^2}{4}\times 2=265\ \mathrm{mm}^2$$

Stirrups Spacing for Example 5.6-1

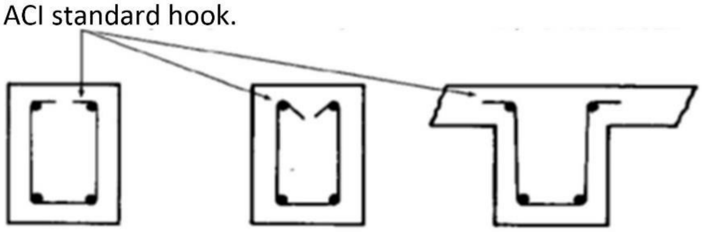
Region	$V_u\leq\emptyset\frac{V_c}{2}$	$\emptyset\frac{V_c}{2}<V_u\leq\emptyset V_c$	$\emptyset V_c\leq V_u$
V_s	None	None	$=\frac{V_u-\emptyset V_c}{\emptyset}=0.66\sqrt{f_c'}b_w d$ $\frac{266-98.3}{0.75}\ ?\ 0.66\times\sqrt{21}\times 330\times 508$ $224kN<\ 507\ kN\ Ok$ Beam dimensions are adequate.
$S_{Theoretical}$	None	None	$=\frac{A_v f_{yt} d}{V_s}=\frac{265\times 275\times 508}{224\ 000}=165\ mm$
$S_{for\ Av\ minimum}$	None	$minimum\ (\frac{A_v f_{yt}}{0.062\sqrt{f_c'}b_w}\ or\ \frac{A_v f_{yt}}{0.35 b_w})$ $=630\ mm$	$minimum\ (\frac{A_v f_{yt}}{0.062\sqrt{f_c'}b_w}\ or\ \frac{A_v f_{yt}}{0.35 b_w})$ $minimum\ (\frac{265\times 275}{0.062\sqrt{21}\times 330}\ or\ \frac{265\times 275}{0.35\times 330})$ $minimum\ (\ 777\ or\ 630)$ $=630\ mm$
$S_{maximum}$	None	$Minimum\ \left[\frac{d}{2}\ or\ 600mm\right]$ $=254\ mm$	$V_s\leq 0.33\sqrt{f_c'}b_w d$ $224kN\leq 0.33\sqrt{21}\times 330\times 508$ $224\ kN\leq 254\ kN$ $Minimum\ \left[\frac{d}{2}\ or\ 600mm\right]$ $Minimum\ \left[\frac{508}{2}\ or\ 600mm\right]=254\ mm$ $V_s>0.33\sqrt{f_c'}b_w d$ $Minimum\ \left[\frac{d}{4}\ or\ 300mm\right]$
$S_{Required}$	None	$Minimum\ [S_{for\ Av\ minimum}, S_{maximum}]$ $Minimum\ [630\ \ mm, 254\ mm]$ $=254$ mm Use $\emptyset 13mm$ @ $250mm$	$Minimum\ \left[\begin{matrix}S_{Theoretical},\\ S_{for\ Av\ minimum}, S_{maximum}\end{matrix}\right]$ $Minimum\ \left[\begin{matrix}165\ mm,\\ 630\ \ mm, 254\ mm\end{matrix}\right]$ $=165\ mm$ Use $\emptyset 13mm$ @ $150mm$

- Selecting of Nominal Reinforcement for Stirrups Supports:
  As no top bars are required for flexure, stirrups support bars must be used. These are usually about the same diameter as the stirrups themselves (Nilson, Design of Concrete Structures, 3th Edition , 2003) (Page 180).

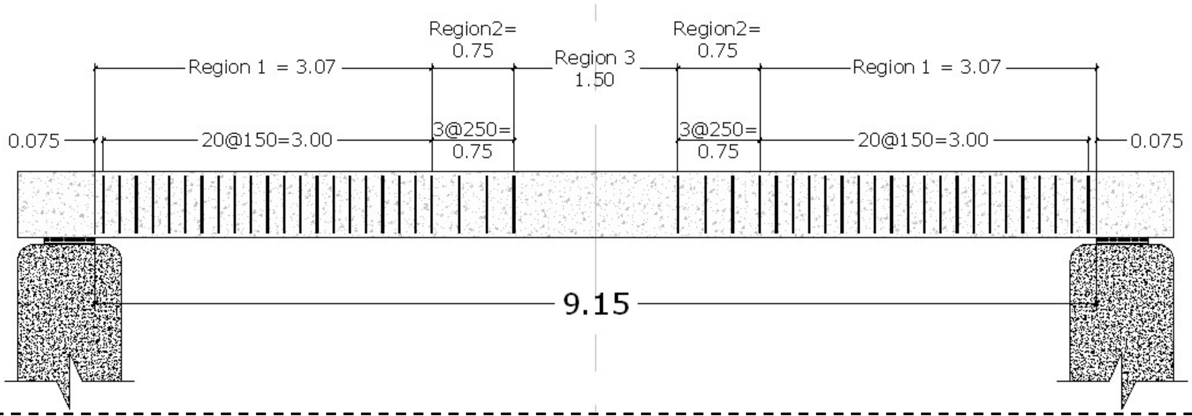


- Anchorage Requirement for Shear Reinforcements:
  If one assumes that no compression reinforcement is required for this beam, any one of following anchorage can be used:

ACI standard hook.



- Final stirrup spacing would be as indicated in below:



Example 5.6-2

Re-design ***Example 5.6-1*** but with using same spacing along beam span. Then compare the two designs.

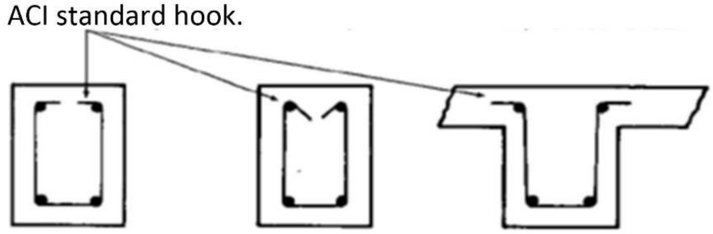
Solution

It practices, structural designers may use the same spacing along beam span. This spacing should be computed based on maximum shear force and can be used in other regions where shear forces are less than the force that used in design.

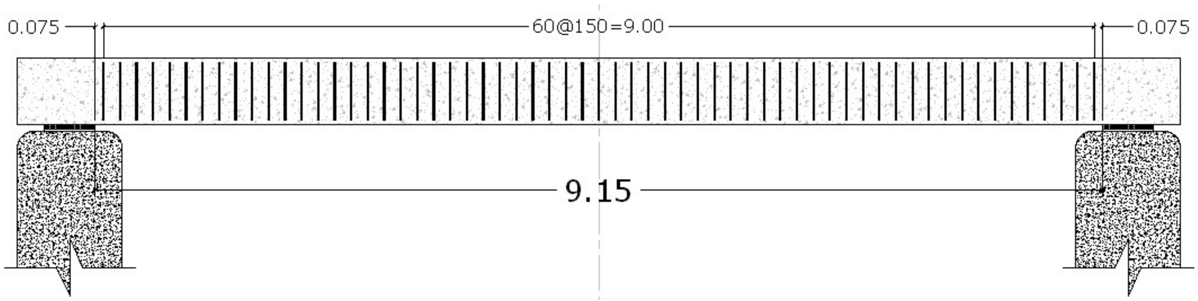
- Compute V_u:
  As all limitations of article (**9.4.3.2**) are satisfied, then sections located less than a distance "d" from face of support shall be designed for V_u computed at a distance "d".
  $$V_u = \frac{\left[65.5 \frac{kN}{m} \times (9.15 - 2 \times 0.508) m\right]}{2} = 266\ kN$$
- Compute Concrete Shear Strength V_c:
  $$V_c = 0.17\lambda\sqrt{f_c'}\, b_w d$$
  With $\lambda = 1.0$ for normal weight concrete:
  $$V_c = 0.17\sqrt{f_c'}\, b_w d = 0.17 \times \sqrt{21} \frac{N}{mm^2} \times 330mm \times 508mm = 131\ 000\ N = 131\ kN$$
  $$\emptyset V_c = 0.75 \times 131\ kN = 98.3\ kN$$
  $$\because V_u = 266\ kN > \emptyset V_c = 98.3\ kN$$
  Then the beam will be designed based of provisions of $V_u > \phi V_c$.

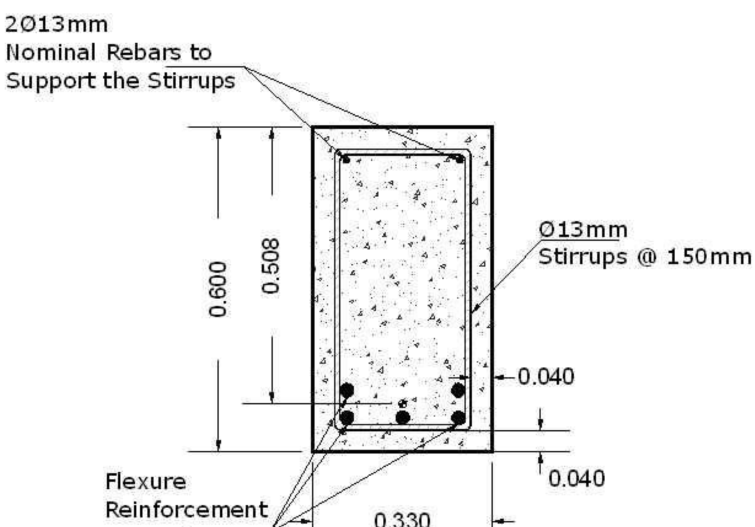
Shear Spacing Design of Example 5.6-2	
Region	$\emptyset V_c \le V_u$
V_s	$= \frac{V_u - \emptyset V_c}{\emptyset} = 0.66\sqrt{f_c'}b_w d$ $\frac{266 - 98.3}{0.75}\ ?\ 0.66 \times \sqrt{21} \times 330 \times 508 \Longrightarrow 224kN < 507\ kN\ Ok$ Beam dimensions are adequate.
$S_{Theoretical}$	$= \frac{A_v f_{yt} d}{V_s} = \frac{265 \times 275 \times 508}{224\ 000} = 165\ mm$
S for Av minimum	$minimum\ (\frac{A_v f_{yt}}{0.062\sqrt{f_c'}b_w}\ or\ \frac{A_v f_{yt}}{0.35 b_w})$ $minimum\ \left(\frac{265 \times 275}{0.062\sqrt{21} \times 330}\ or\ \frac{265 \times 275}{0.35 \times 330}\right) = minimum\ (777\ or\ 630)$ $= 630\ mm$
$S_{maximum}$	$V_s \le 0.33\sqrt{f_c'}b_w d$ $224kN \le 0.33\sqrt{21} \times 330 \times 508 \Longrightarrow 224\ kN \le 254\ kN$ $Minimum\ \left[\frac{d}{2}\ or\ 600mm\right] \Longrightarrow Minimum\ \left[\frac{508}{2}\ or\ 600mm\right] = 254\ mm$
	$V_s > 0.33\sqrt{f_c'}b_w d \Longrightarrow Minimum\ \left[\frac{d}{4}\ or\ 300mm\right]$
$S_{Required}$	$Minimum\ [S_{Theoretical}, S_{for\ Av\ minimum}, S_{maximum}]$ $Minimum\ [165\ mm, 630\ mm, 254\ mm] = 165\ mm$ **Use $\emptyset 13mm\ @\ 150mm$**

- Anchorage Requirement for Shear Reinforcements:
  As for previous example, if one assumes that no compression reinforcement is required for this beam, any one of following anchorage can be used:



- Comparison between two designs:
  Required Number of Stirrups for the more accurate design of Example 5.6-1 is:
  $$\text{No. of Stirrups} = \left[\left(\frac{3.0}{0.150} + 1\right) + \frac{0.75}{0.250}\right] \times 2 = 48 \text{ U Stirrups}$$
  Required Number of Stirrups for the simplified design of Example 5.6-2 is:
  $$\text{No. of Stirrups} = \left(\frac{9.0}{0.150} + 1\right) = 61 \text{ U Stirrups}$$
  Then dividing the beam into three regions and design of each region for its shear force can save 13 stirrups.





Example 5.6-3

Design Region 1 and Region 2 of floor beam indicated in ***Figure 5.6-2*** for shear. The beam has a width of 375mm and an effective depth of 775mm. Assume that the designer intends to use:

- $f_c' = 27.5\ \text{MPa}$.
- $f_{yt} = 414\ \text{MPa}$.
- Stirrups of 10mm diameter ($A_{Bar} = 71\text{mm}^2$).

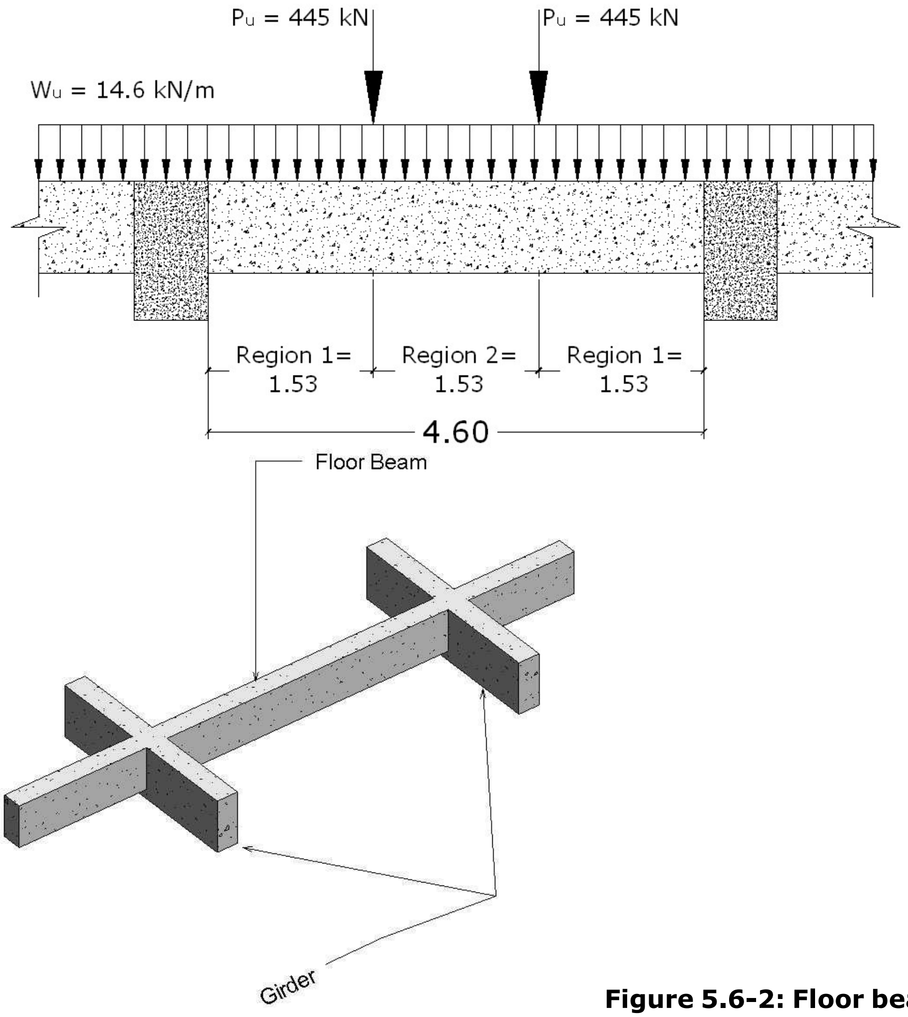


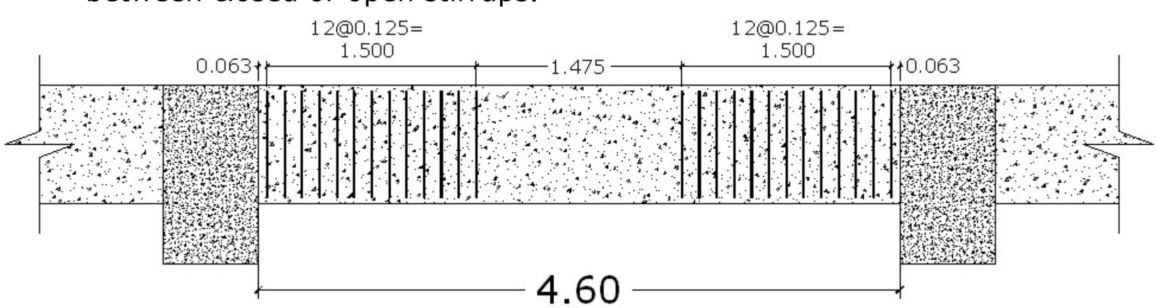
Figure 5.6-2: Floor beam for Example 5.6-3.

Solution

- Shear Reinforcement for Region 1:
  - Compute factored shear force V_u:
    As girder is deeper than floor beam, then all ACI limitations are satisfied and the shear force for Region 1 can be determined at distance "d" from face of support.
    $$V_u = 14.6\frac{\text{kN}}{\text{m}} \times (4.6\text{m} - 2 \times 0.775\text{m}) \times \frac{1}{2} + 445\ \text{kN} = 467\ \text{kN}$$
  - Shear strength of concrete V_c:
    $$V_c = 0.17\lambda\sqrt{f_c'}\,b_w d$$
    with $\lambda = 1.0$ for normal weight concrete:
    $$V_c = 0.17\sqrt{f_c'}\,b_w d = 0.17 \times \sqrt{27.5}\frac{\text{N}}{\text{mm}^2} \times 375\text{mm} \times 775\text{mm} = 259\ \text{kN}$$
  - Stirrups spacing:
    $\emptyset V_c = 0.75 \times 259\ \text{kN} = 194\ \text{kN}$
    $\because V_u = 467\ \text{kN} > \emptyset V_c = 194\ \text{kN}$
    Then, shear reinforcement must be used and its spacing can be computed from Table below:
    $A_v = 71 \times 2 = 142\ \text{mm}^2$

Shear Spacing Design of Example 5.6-3 for Region 1	
Region	$\emptyset V_c \leq V_u$
V_s	$= \frac{V_u - \emptyset V_c}{\emptyset} = 0.66\sqrt{f_c'}b_w d$ $\frac{467 - 194}{0.75}$? $0.66 \times \sqrt{27.5} \times 375 \times 775$ $364 < 1\,006\ kN\ Ok$ Beam dimensions are adequate.
$S_{Theoretical}$	$= \frac{A_v f_{yt} d}{V_s} = \frac{142 \times 414 \times 775}{364\,000} = 125\ mm$
$S_{for\ Av\ minimum}$	$minimum\ (\frac{A_v f_{yt}}{0.062\sqrt{f_c'}b_w}\ or\ \frac{A_v f_{yt}}{0.35 b_w})$ $minimum\ (\frac{142 \times 414}{0.062\sqrt{27.5} \times 375}\ or\ \frac{142 \times 414}{0.35 \times 375})$ $minimum\ (482\ or\ 448)$ $= 448\ mm$
$S_{maximum}$	$V_s \leq 0.33\sqrt{f_c'}b_w d$ $364kN \leq 0.33\sqrt{27.5} \times 375 \times 775$ $364kN\ kN \leq 503\ kN$ $Minimum\ \left[\frac{d}{2}\ or\ 600mm\right]$ $Minimum\ \left[\frac{775}{2}\ or\ 600mm\right] = 387\ mm$
	$V_s > 0.33\sqrt{f_c'}b_w d$ $Minimum\ \left[\frac{d}{4}\ or\ 300mm\right]$
$S_{Required}$	$Minimum\ [S_{Theoretical}, S_{for\ Av\ minimum}, S_{maximum}]$ $Minimum\ [125\ mm, 448\ mm, 387\ mm]$ $= 125\ mm$ ***Use*** $\boldsymbol{\emptyset 10mm\ @\ 125mm}$

- Shear Reinforcement for Region 2:
  - Factored shear force V_u:
    Due to symmetry
    $$V_u = \left(14.6\frac{kN}{m} \times 1.53m\right) \times \frac{1}{2} = 11.1\ kN$$
  - Shear strength of concrete V_c:
    According to simplified equation of the code, concrete shear force is constant along span of prismatic beam. Therefore, concrete shear strength of Region 2 would be equal to that of Region 1.
    $$V_c = 259\ kN$$
    $$\emptyset V_c = 0.75 \times 259\ kN = 194\ kN$$
    $$\because \frac{\emptyset V_c}{2} = \frac{194\ kN}{2} = 97\ kN > V_u$$
    Then, no shear reinforcement is required for Region 2.
- Anchorage
  As nothing is mentioned about longitudinal reinforcement, then one cannot select between closed or open stirrups.

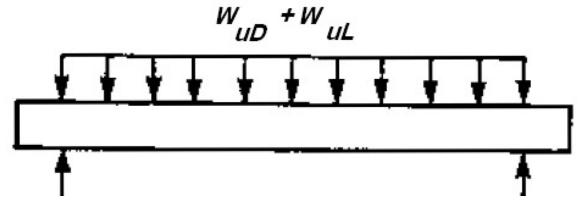


Example 5.6-4

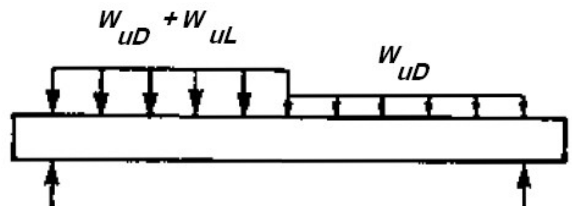
For a simply supported beam, that has a clear span of 6m, design 10mm U stirrup at a mid-span section. In your design, assume that load pattern must be included and assume:

- $f_c' = 21\ MPa, f_{yt} = 420\ MPa$
- $h = 500, d = 450\ mm, b_w = 300mm$
- $W_{ud} = 60 \frac{kN}{m}$ (Including Beam Selfweight) and $W_{ul} = 200 \frac{kN}{m}$

Solution



- Compute V_u
  Although the dead load is always present over the full span, the live load may act over the full span as shown or over a part of span as shown in below.
  Based on influence line for shear at mid-span of simply supported beam, the maximum effect of live load occurs when this load acting on one half of beam span as indicated in above. Therefore, for design case when load pattern is important, shear force must be computed based on partial loading of one half of beam span:



$$V_{u\ @\ mid\ span} = 0.0_{\text{Shear due to } W_D} + \frac{W_{uL} L}{8}_{\text{Shear Due to LL on half of Beam Span}} = \frac{W_{uL} L}{8}$$

$$= \frac{200 \frac{kN}{m} \times 6m}{8} = 150\ kN$$

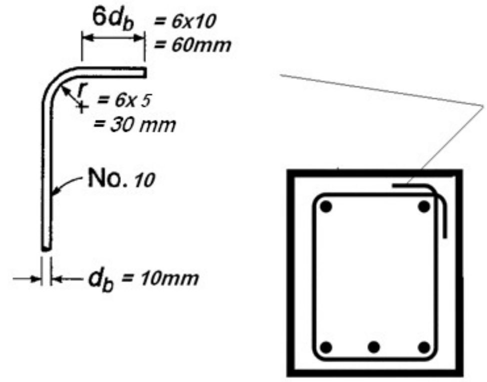
- Compute V_c

$$V_c = 0.17\sqrt{f_c'}\, b_w d = 0.17 \times \sqrt{21} \frac{N}{mm^2} \times 300mm \times 450mm = 105\ kN \Rightarrow \emptyset V_c = 0.75 \times 105\ kN = 78.8\ kN$$

- Stirrups Design
  As
  $V_u > \emptyset V_c$
  then shear stirrups is designed as presented in Table below.

Shear Spacing Design of Example 5.6-4	
Region	$\emptyset V_c \leq V_u$
V_s	$= \frac{V_u - \emptyset V_c}{\emptyset} = 0.66\sqrt{f_c'} b_w d$ $\frac{150 - 78.8}{0.75}$? $0.66 \times \sqrt{21} \times 300 \times 450 \Rightarrow 94.9\ kN < 408\ kN\ Ok$ Beam dimensions are adequate.
$S_{Theoretical}$	$= \frac{A_v f_{yt} d}{V_s} = \frac{157 \times 420 \times 450}{94.9 \times 10^3} = 313\ mm$
$S_{for\ Av\ minimum}$	$minimum\ (\frac{A_v f_{yt}}{0.062\sqrt{f_c'} b_w}\ or\ \frac{A_v f_{yt}}{0.35 b_w})$ $minimum\ (\frac{157 \times 420}{0.062\sqrt{21} \times 300}\ or\ \frac{157 \times 420}{0.35 \times 300}) \Rightarrow minimum\ (774\ or\ 628) = 628\ mm$
$S_{maximum}$	$V_s \leq 0.33\sqrt{f_c'} b_w d$ $94.9\ kN \leq 0.33\sqrt{21} \times 300 \times 450$ $94.9\ kN < 204\ kN$ $Minimum\ [\frac{d}{2}\ or\ 600mm] \Rightarrow Minimum\ [\frac{450}{2}\ or\ 600mm] = 225\ mm$
	$V_s > 0.33\sqrt{f_c'} b_w d \Rightarrow Minimum\ [\frac{d}{4}\ or\ 300mm]$
$S_{Required}$	$Minimum\ [S_{Theoretical}, S_{for\ Av\ minimum}, S_{maximum}]$ $Minimum\ [313\ mm, 628\ mm, 225\ mm] = 225\ mm$ ***Use*** $\emptyset 10mm\ @\ 225mm$

- Stirrups Details
  As movable live load is a reversal load, then closed stirrup must be used here as shown in the figure below.



Example 5.6-5

For the roof system shown in ***Figure 5.6-3*** below, design shear reinforcement for a typical floor beam and a typical girder.

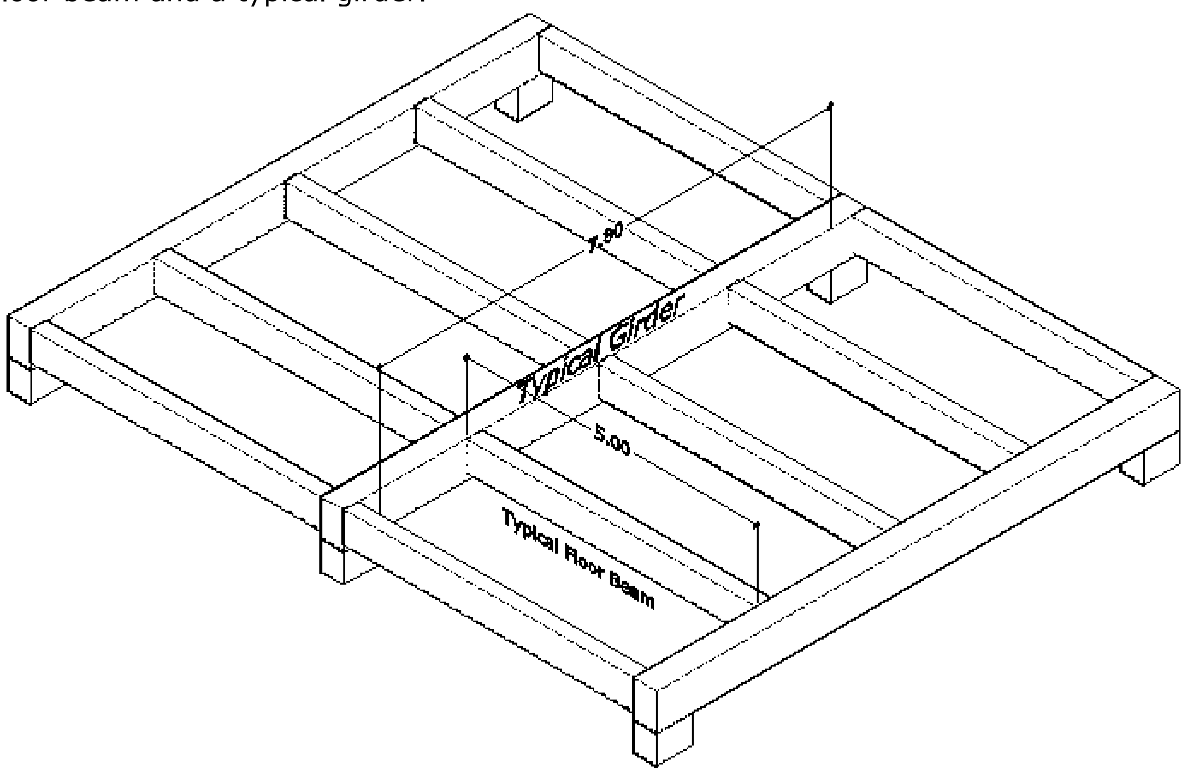
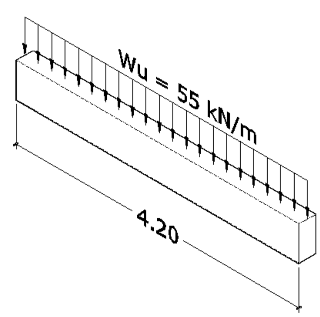


Figure 5.6-3: Roof system for Example 5.6-5.

In your design, assume that:

- $f_c' = 21\ MPa$ and $f_{yt} = 420\ MPa$.
- Floor beams have b = 250mm, h = 450mm, and d = 400mm and subjected to a uniformly distributed factored load of W_u = 55 kN/m transferred from the supported slab.
- Girders have b = 400mm, h = 600mm, and d = 520mm.
- Selfweight of floor beams and girders should be included in your design.
- Try 10mm U stirrups for the floor beam and 12mm U stirrups for the girder.

Solution



- Design Shear Reinforcement for Floor Beam:
  - Computing of Vu:
    As the girder is deeper than the floor beam, then critical section for the floor beam can be taken at distance "d" from face of support (girder in this case).

$$W_u = 55\frac{kN}{m} + \left((0.45 \times 0.25\ m^2) \times 24\frac{kN}{m^3}\right) \times 1.2 = 58\frac{kN}{m}$$

$$V_{u\ @\ d\ from\ face\ of\ support} = \left(58\frac{kN}{m} \times (5.0 - 0.4 \times 2)m\right) \times \frac{1}{2} = 122\ kN$$

- Compute V_c:

$$V_c = 0.17\sqrt{f_c'}\ b_w d = 0.17 \times \sqrt{21}\frac{N}{mm^2} \times 250mm \times 400mm = 77.9\ kN$$

$$\emptyset V_c = 0.75 \times 77.9\ kN = 58.4\ kN$$

- Design of Shear Reinforcement:

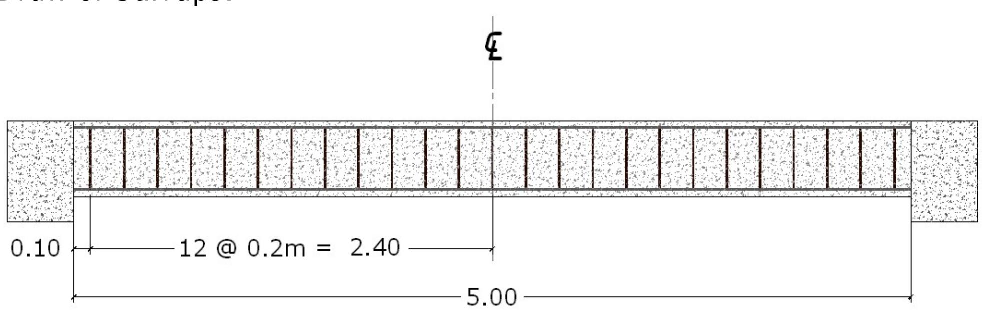
  As

  $V_u > \emptyset V_c$

  Then shear reinforcement must be designed based on zone 1 (see the table below).

Stirrups Design of Example 5.6-5 (Floor Beam)	
Region	$\emptyset V_c \leq V_u$
V_s	$= \frac{V_u - \emptyset V_c}{\emptyset} = 0.66\sqrt{f_c'}b_w d$ $\frac{122 - 58.4}{0.75}\ ?\ 0.66 \times \sqrt{21} \times 250 \times 400 \Rightarrow 84.8\ kN < 302\ kN\ Ok$ Beam dimensions are adequate.
$S_{Theoretical}$	$= \frac{A_v f_{yt} d}{V_s} = \frac{157 \times 420 \times 400}{84.8 \times 10^3} = 311\ mm$
$S_{for\ Av\ minimum}$	$minimum\ (\frac{A_v f_{yt}}{0.062\sqrt{f_c'}b_w}\ or\ \frac{A_v f_{yt}}{0.35 b_w})$ $minimum\ (\frac{157 \times 420}{0.062\sqrt{21} \times 250}\ or\ \frac{157 \times 420}{0.35 \times 250}) \Rightarrow minimum\ (928\ or\ 754)$ $= 754\ mm$
$S_{maximum}$	$V_s \leq 0.33\sqrt{f_c'}b_w d$ $84.8\ kN \leq 0.33\sqrt{21} \times 250 \times 400 \Rightarrow 84.8\ kN \leq 151\ kN$ $Minimum\ \left[\frac{d}{2}\ or\ 600mm\right] \Rightarrow Minimum\ \left[\frac{400}{2}\ or\ 600mm\right] = 200\ mm$
	$V_s > 0.33\sqrt{f_c'}b_w d$ $Minimum\ \left[\frac{d}{4}\ or\ 300mm\right]$
$S_{Required}$	$Minimum\ [S_{Theoretical}, S_{for\ Av\ minimum}, S_{maximum}]$ $Minimum\ [311\ mm,\ 754\ mm,\ 200\ mm] = 200\ mm$ **Use** $\emptyset 10mm\ @\ 200mm$

- Draw of Stirrups:



- Design of Shear Reinforcement for Girder:
  - Compute of Vu:

    Forces acting on the girder are summarized in the figure below. Shear force, R_u, transfers from floor beams to the supporting girder can be computed as follows:

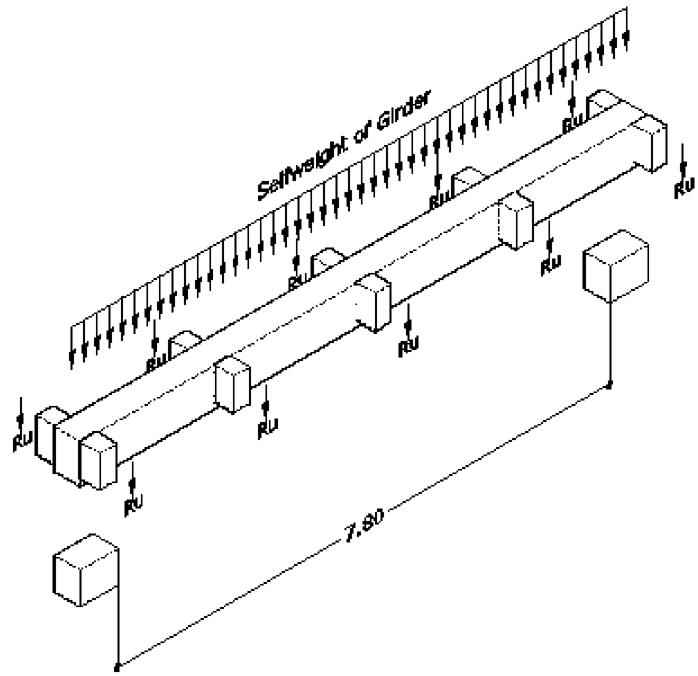
$$R_u = 58\frac{kN}{m} \times \frac{5m}{2} = 145\ kN$$

Shear force due to girder selfweight is

$$V_{u\ Due\ to\ Girder\ Selfweight} = \left((0.6 \times 0.4)m^2 \times 24\frac{kN}{m^3} \times (7.8 - 0.52 \times 2)m \times \frac{1}{2}\right)1.2 = 23.4\ kN$$

Therefore, the total factored shear force would be:

$$V_u = \left((3 \times 145\ kN_{Reactions\ from\ 3\ floor\ beam}) \times 2_{Two\ faces}\right)\frac{1}{2} + 23.4\ kN = 458\ kN$$



- Compute V_c:

  $V_c = 0.17\sqrt{f'_c}\, b_w d = 0.17 \times \sqrt{21}\frac{N}{mm^2} \times 400mm \times 520mm = 162\ kN$

  $\emptyset V_c = 0.75 \times 162\ kN = 121\ kN$
- Design of Shear Reinforcement:

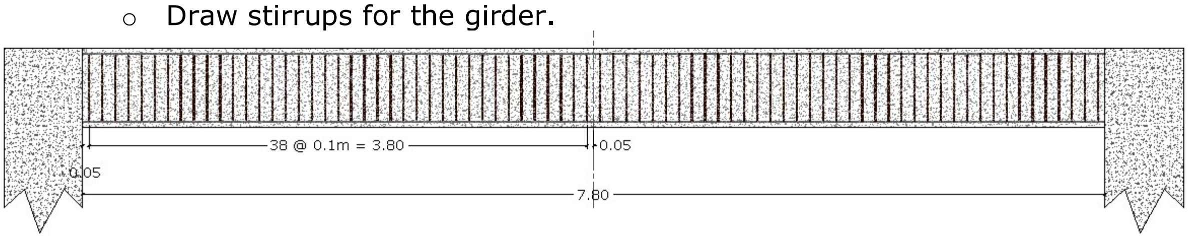
  As

  $V_u > \emptyset V_c$

  then, shear reinforcement is designed as indicated in the table below.

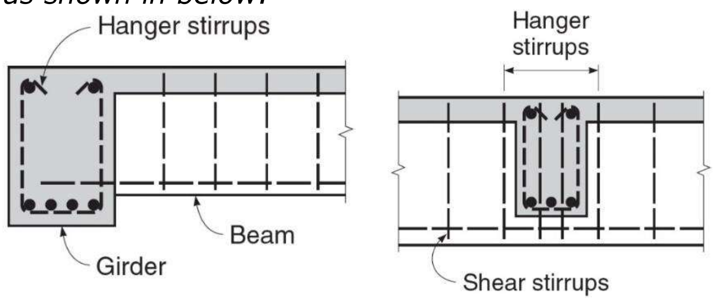
Stirrups Design of Example 5.6-5 (Girder Design)	
Region	$\emptyset V_c \le V_u$
V_s	$= \frac{V_u - \emptyset V_c}{\emptyset} = 0.66\sqrt{f_c'}b_w d \Rightarrow \frac{458 - 121}{0.75}\ ?\ 0.66 \times \sqrt{21} \times 400 \times 520$ $\Rightarrow 449\ kN < 629\ kN\ Ok$ Beam dimensions are adequate.
$S_{Theoretical}$	$= \frac{A_v f_{yt} d}{V_s} = \frac{226 \times 420 \times 520}{449 \times 10^3} = 110\ mm$
$S_{for\ Av\ minimum}$	$minimum\ (\frac{A_v f_{yt}}{0.062\sqrt{f_c'}b_w}\ or\ \frac{A_v f_{yt}}{0.35 b_w})$ $minimum\ (\frac{226 \times 420}{0.062\sqrt{21} \times 400}\ or\ \frac{226 \times 420}{0.35 \times 400}) \Rightarrow minimum\ (835\ or\ 678)$ $= 678\ mm$
$S_{maximum}$	$V_s \le 0.33\sqrt{f_c'}b_w d$
	$V_s > 0.33\sqrt{f_c'}b_w d$ $449\ kN > 0.33\sqrt{21} \times 400 \times 520 \Rightarrow 449\ kN > 314\ kN$ $Minimum\ \left[\frac{d}{4}\ or\ 300mm\right] \Rightarrow Minimum\ \left[\frac{520}{4}\ or\ 300mm\right] = 130\ mm$
$S_{Required}$	$Minimum\ [S_{Theoretical}, S_{for\ Av\ minimum}, S_{maximum}]$ $\Rightarrow Minimum\ [110\ mm, 678mm, 130\ mm]$ $= 110\ mm$ ***Use*** $\emptyset \mathbf{12mm} @ \mathbf{100mm}$

- Draw stirrups for the girder.

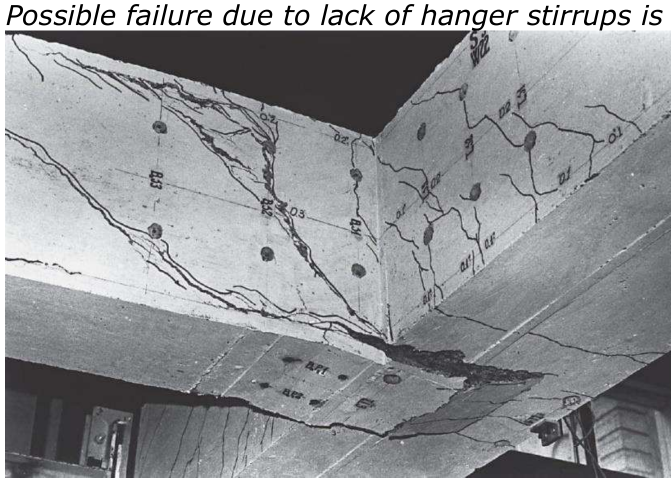


Important Notes

- *It is useful to note that shear forces in this example have been determined based on assumption of equal shear at beam-ends. More accurate assumption will be discussed later when we study the analysis and design of slabs and continuous beams.*
- *Hanger Stirrups:*
  - *Proper detailing of steel in the region of beam-to-girder connection such a joint requires the use of well-anchored "hanger" stirrups in the girder, as shown in below:*



  - *The hanger stirrups are required in addition to the normal girder stirrups.*
  - *Possible failure due to lack of hanger stirrups is presented in below:*

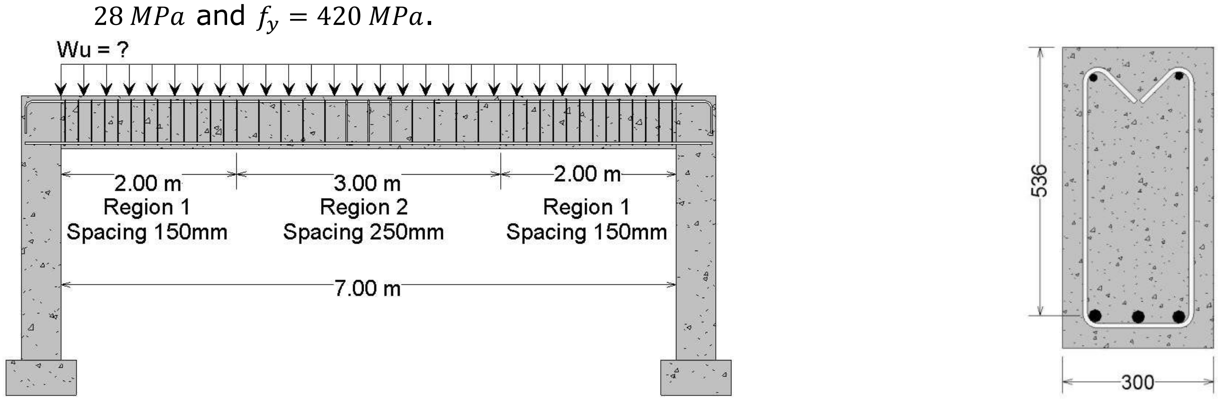


  - *Design of hanger stirrups is out of our scope, for more information about their design see* (Darwin, Dolan, & Nilson, 2016)*, page 557.*

Example 5.6-6

For the singly reinforced beam of the portal frame shown in ***Figure 5.6-4*** below, a designer has proposed to use open U stirrups with diameter of 10mm and with indicated spacing for shear reinforcement of the beam.

- Is using of open U stirrups justified according to ACI requirements? Explain your answer.
- Based on proposed spacing and beam shear strength, what is the maximum uniformly factored load W_u that could be applied? In your solution assume $f_c' = 28\ MPa$ and $f_y = 420\ MPa$.



Longitudinal Sectional View. **Beam Cross Section**

Figure 5.6-4: Frame for Example 5.6-6.

Solution

- Using of Open U Stirrups:
  As the beam is singly reinforced and with assuming that it is not subjected to torsion nor to reversal loads, then using of open U stirrups is justified according to ACI code.
- Maximum Uniformly Distributed Load $\boldsymbol{W_u}$:
  Based on Shear Strength of Region 1:

$$V_c = 0.17 \times 1.0 \times \sqrt{28} \times 300 \times 536 = 145\ kN$$

$$A_v = \frac{\pi \times 10^2}{4} \times 2 = 157\ mm^2$$

$$V_s = \frac{A_v f_y d}{s} = \frac{157 \times 420 \times 536}{150} = 236\ kN < 0.33 \times 1.0 \times \sqrt{28} \times 300 \times 536 = 281\ kN$$

$$S_{maximum} = minimum\left(\frac{536}{2}, 600\right) = 268\ mm > 150mm \therefore Ok.$$

$$\phi V_n = 0.75 \times (145 + 236) = 286\ kN$$

$$V_u = \frac{W_u \times (7.0 - 2 \times 0.536)}{2} = 286 \Longrightarrow W_u = 96.5\frac{kN}{m}$$

Based on Shear Strength of Region 2:

$$V_s = \frac{A_v f_y d}{s} = \frac{157 \times 420 \times 536}{250} = 141\ kN$$

$$S_{maximum} = 268\ mm > 250mm \therefore Ok.$$

$$\phi V_n = 0.75 \times (145 + 141) = 214\ kN$$

$$V_u = \frac{W_u \times 3}{2} = 214$$

$$W_u = 143\frac{kN}{m}$$

Finally,

$$W_u = minimum\ (96.5, 143) = 96.5\frac{kN}{m} \blacksquare$$

Example 5.6-7

For a frame shown in ***Figure 5.6-5*** below, based on shear capacity of Girder 300x600, what are maximum values for point load "P_u", and distributed load "W_u" that can be supported by the beam?

In your solution, assume that selfweight could be neglected, $f_c' = 28\ MPa$ and $f_y = 420\ MPa$.

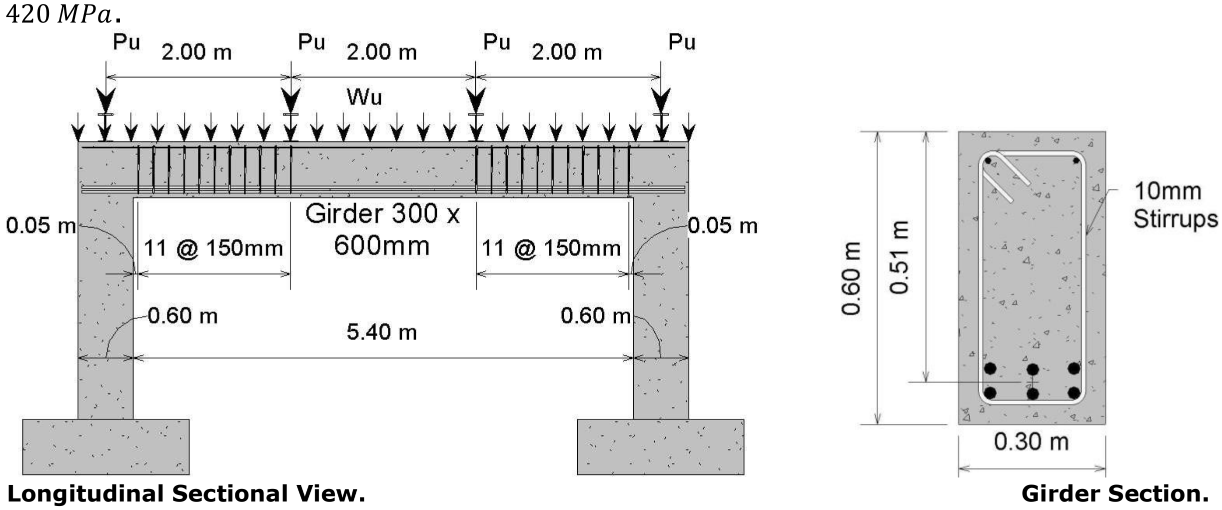


Figure 5.6-5: Frame for Example 5.6-7.

Solution

Distributed load "W_u" could be computed from middle region where no shear reinforcement are used:

$$V_u = \frac{\phi V_c}{2} = \frac{1}{2} \times \left(0.75 \times 0.17 \times \sqrt{28} \times 300 \times 510\right) \Longrightarrow V_u = \frac{\phi V_c}{2} = 51.6\ kN$$

$$V_u = \frac{W_u \times 2.00}{2} = 51.6\ kN \Longrightarrow W_u = 51.6\ kN \blacksquare$$

Point load "Pu" could be computed from support regions where stirrups of $\phi 10 @ 150mm$ are used.

$V_s = 2 \times \frac{\pi \times 10^2}{4} \times 420 \times \frac{510}{150} = 224\, kN \Rightarrow \mathrm{V_s} = 224 < \ 0.66\sqrt{28} \times 300 \times 510 = 534\, kN \therefore Ok.$

$\because \mathrm{V_s} = 224 < \ 0.33\sqrt{28} \times 300 \times 510 = 267\, kN \Rightarrow \mathrm{S} = 150\mathrm{mm} < \mathrm{Minimum}\ \left[\frac{510}{2} \mathrm{or}\ 600\right]$

$\mathrm{S} = 150\mathrm{mm} < 255\ \mathrm{mm} \therefore Ok.$

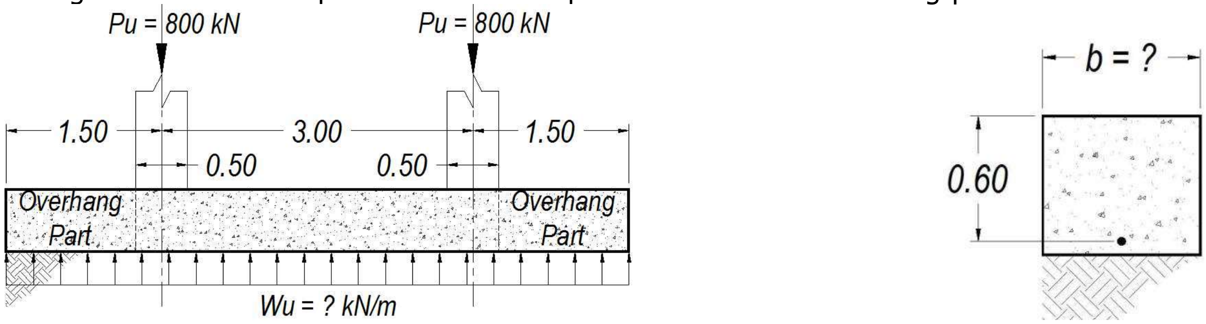
$V_c = \ 0.17 \times \sqrt{28} \times 300 \times 510 = 138\, kN \Rightarrow V_u = 0.75 \times (138 + 224) = 272\, kN$

$(W_u \times (5.4 - 0.51 \times 2) + 2P_u) = \ 2 \times 272 \Rightarrow (51.6\ \times (5.4 - 0.51 \times 2) + 2P_u) = \ 2 \times 272$

$\Rightarrow P_u = 159\, kN$ ■

Example 5.6-8

For beam shown in ***Figure 5.6-6*** below, select beam width such that concrete shear strength would be adequate for shear requirements in the overhang parts.



Logitudinal veiw **A Section in Overhang Region**

Figure 5.6-6: Foundation for Example 5.6-8.

In your solution, assume that:

- Beam selfweight can be neglected.
- $f_c' = 21\, MPa$

Solution

$W_u = \frac{800 \times 2}{6} = 267 \frac{kN}{m} \Rightarrow V_{u\, @\, d\, fram\, face\, of\, support} = 267 \frac{kN}{m} (1.5 - 0.25 - 0.6)m = 174\, kN$

$V_u = \frac{\phi V_c}{2} \Rightarrow 174000\ N = \frac{1}{2}\left(0.75\left(0.17\sqrt{21} \times b \times 600\right)\right) \Rightarrow b = 993\, mm$

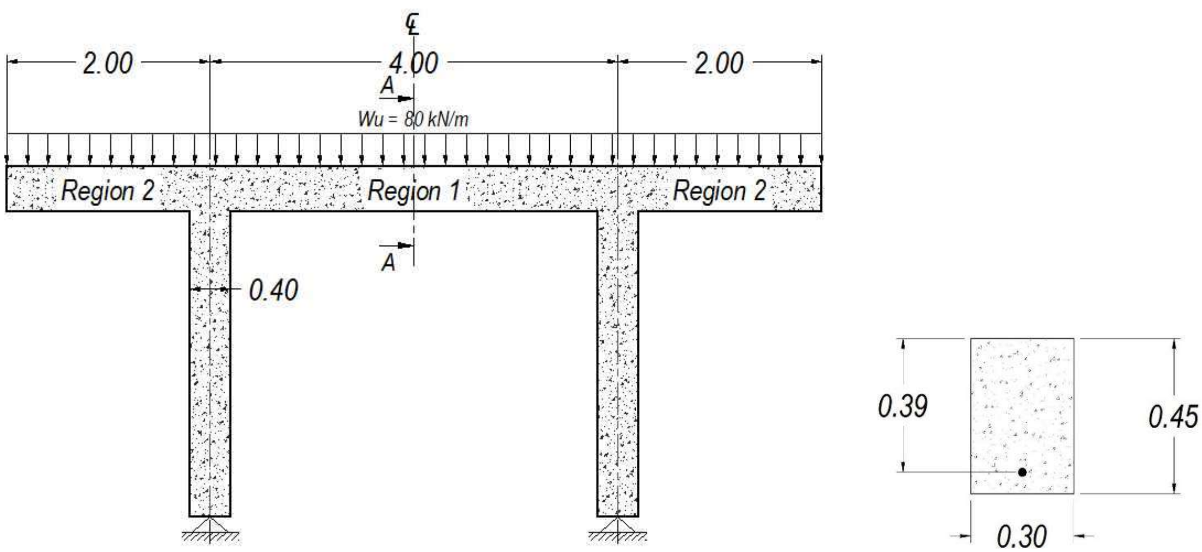
Say

$b = 1000\, mm$ ■

Example 5.6-9

For the frame shown in Figure 5.6-7 below,

- Design Region 1 for shear according ACI requirements.
- Is shear reinforcement for Region 1 adequate for Region 2?



Elevation view. **Section A-A**

Figure 5.6-7: Frame for Example 5.6-9.

In your solution, assume that:

- $f_c' = 21\,MPa,\ f_y = 420\,MPa$
- No.10 for stirrups.

Solution

Region 1:

$$V_{u\,for\,Region\,1} = 80\frac{kN}{m}(4 - 0.4 - 2\times 0.39)m \times \frac{1}{2} = 113\,kN$$

$$\phi V_c = 0.75 \times 0.17 \times \sqrt{21} \times 300 \times 390 = 68.4\,kN < V_u$$

Shear Spacing Design of Region 1	
Region	$\emptyset V_c \le V_u$
V_s	$= \frac{V_u - \emptyset V_c}{\emptyset} = 0.66\sqrt{f_c'}b_w d \Rightarrow \frac{113 - 68.4}{0.75}\ ?\ 0.66 \times \sqrt{21} \times 300 \times 390$ $59.5kN <\ 354\,kN\ Ok$ Beam dimensions are adequate.
$S_{Theoretical}$	$= \frac{A_v f_{yt} d}{V_s} = \frac{157 \times 420 \times 390}{59.5 \times 10^3} = 432\,mm$
$S_{for\ Av\ minimum}$	$minimum\ (\frac{A_v f_{yt}}{0.062\sqrt{f_c'}b_w}\ or\ \frac{A_v f_{yt}}{0.35 b_w}) \Rightarrow minimum\ (\frac{157 \times 420}{0.062\sqrt{21} \times 300}\ or\ \frac{157 \times 420}{0.35 \times 300})$ $minimum\ (773\ or\ 628) = 628\,mm$
$S_{maximum}$	$V_s \le 0.33\sqrt{f_c'}b_w d \Rightarrow 59.5kN \le 0.33\sqrt{21} \times 300 \times 390 \Rightarrow 59.5kN \le 177\,kN$ $Minimum\ \left[\frac{d}{2}\ or\ 600mm\right] \Rightarrow Minimum\ \left[\frac{390}{2}\ or\ 600mm\right] = 195\,mm$
	$V_s > 0.33\sqrt{f_c'}b_w d \Rightarrow Minimum\ \left[\frac{d}{4}\ or\ 300mm\right]$
$S_{Required}$	$Minimum\ [S_{Theoretical}, S_{for\ Av\ minimum}, S_{maximum}]$ $Minimum\ [432\,mm, 628\,mm, 195\,mm] = 195\,mm$ **Use** $\boldsymbol{\emptyset 10mm\ @\ 175mm}$

Region 2:

$$V_{u\,for\,Region\,2} = 80\frac{kN}{m}\left(2.0 - \frac{0.4}{2} - 0.39\right)m = 113\,kN$$

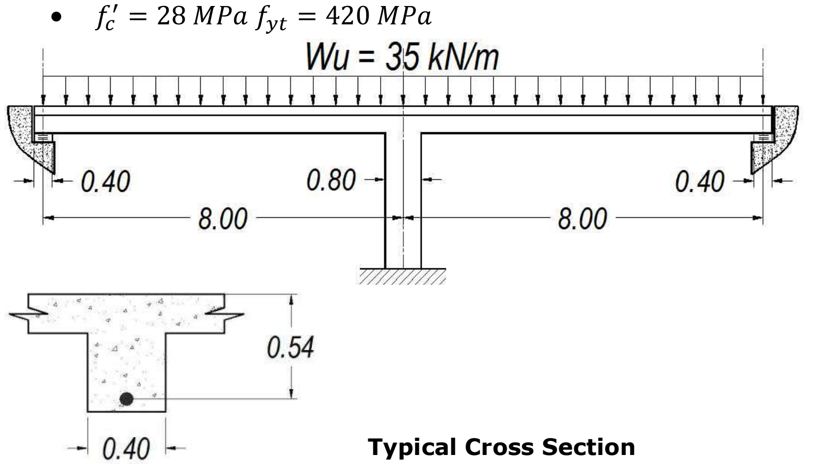
$$\because\ V_{u\,for\,Region\,2} = V_{u\,for\,Region\,1}$$

Therefore, the shear reinforcement for Region 1 is adequate for Region 2.

Example 5.6-10

Design for shear the most critical region of pedestrian bridge shown in Figure 5.6-8 below. In your solution, assume that:

- Shear force at interior support to be increased by 15%.
- Beam selfweight could be neglected.
- U stirrups with 10mm diameter.
- $f_c' = 28\,MPa\ f_{yt} = 420\,MPa$



Longitudinal Section

Typical Cross Section

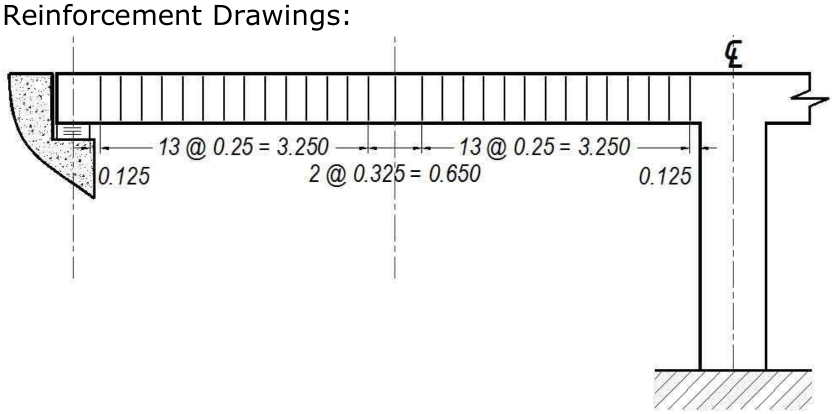
Figure 5.6-8: Pedestrian bridge for Example 5.6-10.

Solution

- Design Shear Force:
  As will be discussed in design of ***one-way slabs*** and ***continuous beams***, according to ACI code, the most critical shear for continuous beams occurs at the exterior face of first interior support with a shear force of 15% greater than average shear force for simple beams.
  $$V_{u\,@\,face\,of\,support} = 1.15\frac{W_u l_n}{2}, l_n = 8.0 - \frac{0.8}{2} - \frac{0.4}{2} = 7.4\,m$$
  $$V_{u\,@\,face\,of\,support} = 1.15\frac{\left(35\frac{kN}{m} \times 7.4m\right)}{2} = 149\,kN$$
  As all related conditions are satisfied, then shear at distance "d" could be used in beam design.
  $$V_{u\,@\,distance\,d\,from\,face\,of\,support} = 149\,kN - 35\frac{kN}{m} \times 0.54m = 130\,kN$$
- Concrete Shear Strength:
  $$\phi V_c = \phi(0.17\lambda\sqrt{f_c'}\,b_w d) = \phi(0.17 \times 1 \times \sqrt{28} \times 400 \times 540) = 146\,\text{kN}$$
  $$\because \emptyset\frac{V_c}{2} < V_u \leq \emptyset V_c$$
  then only nominal shear reinforcement is required.
- Required Shear Reinforcement:
  $$A_v = \frac{\pi \times 10^2}{4} \times 2 = 157\,mm^2$$

Shear reinforcement for Example 5.6-10	
Region	$\emptyset\frac{V_c}{2} < V_u \leq \emptyset V_c$
V_s	None
$S_{Theoretical}$	None
$S_{for\ Av\ minimum}$	$\text{minimum}\left(\frac{A_v f_{yt}}{0.062\sqrt{f_c'}b_w}\ \text{or}\ \frac{A_v f_{yt}}{0.35 b_w}\right)$ $\text{minimum}\left(\frac{157 \times 420}{0.062 \times \sqrt{28} \times 400}, \frac{157 \times 420}{0.35 \times 400}\right) \Rightarrow \text{minimum}(502, 471) = 471\,mm$
$S_{maximum}$	$\text{Minimum}\left[\frac{d}{2}\ \text{or}\ 600\text{mm}\right] = 270$
$S_{Required}$	$\text{Minimum}\left[\begin{matrix}471,\\270\end{matrix}\right] = 270\,mm$ ***Use U Stirrups ϕ10mm @ 250mm***

- Reinforcement Drawings:



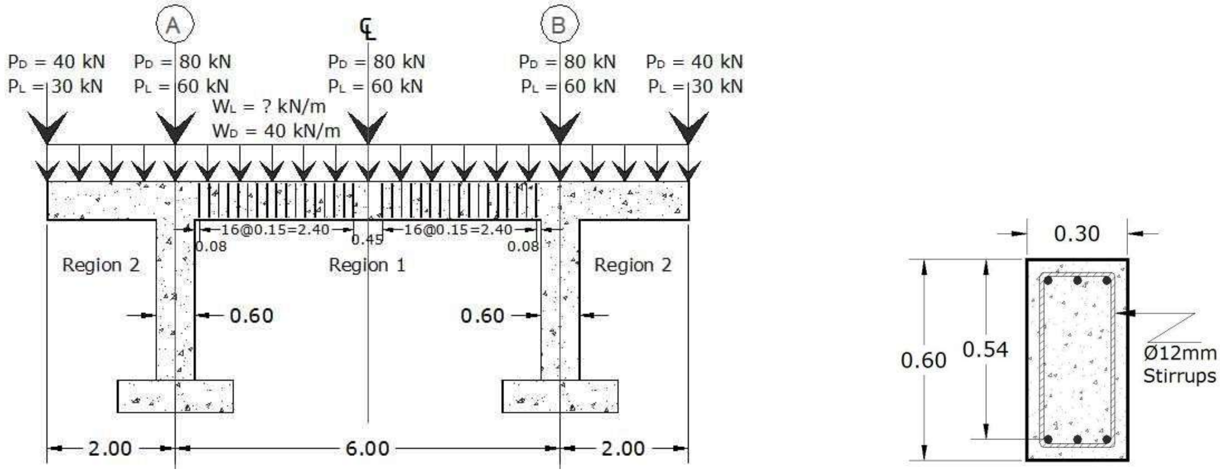
Example 5.6-11

For the frame that shown in Figure 5.6-9 below.

- Based on shear reinforcement that proposed for Region 1, what is maximum uniform distributed live load "W_L" that could be supported?
- Is shear reinforcement that proposed for Region 1 adequate when used in Region 2?

In your solution, assume that:

- U stirrups with 12mm diameter.
- $f_c' = 28\ MPa\ f_{yt} = 420\ MPa$
- $W_u = 1.2D + 1.6L$



Longitudinal Section **Typical Cross Section.**

Figure 5.6-9: Frame for Example 5.6-11.

Solution

- Based on shear reinforcement that proposed for Region 1, the maximum uniform distributed live load "W_L" that could be supported would be:

$$A_v = \frac{\pi \times 12^2}{4} \times 2 = 226\ mm^2 \Longrightarrow V_s = \frac{A_v f_{yt} d}{s} = \frac{226 \times 420 \times 540}{150} = 342\ kN$$

$$\mathrm{V_c} = \left(0.17\lambda\sqrt{\mathrm{f_c'}}\,\mathrm{b_w d}\right) = 0.17 \times \sqrt{28} \times 300 \times 540 = 146\ kN \Longrightarrow \phi V_n = \phi(V_c + V_s)$$

$$= 0.75 \times (146 + 342) = 366\ kN$$

$$V_{u\ @\ face\ of\ support} = \phi V_n = 366\ kN$$

$$P_u = 1.2 \times 80 + 1.6 \times 60 = 192\ kN$$

$$(W_u \times (6.0 - 0.6 - 0.54 \times 2) + 192) \times \frac{1}{2} = 366 \Longrightarrow W_u = 125 \frac{kN}{m}$$

$$W_D = 40 + (0.6 \times 0.3 \times 24) = 44.3\ kN$$

$$W_u = 125 = 1.2 \times 44.3 + 1.6 \times W_L \Longrightarrow W_l = 44.9\ kN\ \blacksquare$$

- Check if the shear reinforcement that proposed for Region 1 is adequate when used in Region 2?

$$P_u = 1.2 \times 40 + 1.6 \times 30 = 96.0\ kN$$

$$V_{u\ at\ d} = 125 \times \left(2.0 - \frac{0.6}{2} - 0.54\right) + 96.0 \Longrightarrow V_{u\ at\ d} = 241\ kN < \phi V_n \ \therefore Ok.\ \blacksquare$$

5.7 PROBLEMS FOR SOLUTION ON BASIC SHEAR ASPECTS

Problem 5.7-1

A reinforced concrete beam with a rectangular cross section is reinforced for moment only and subjected to a shear V_u of 40.0 kN. Beam width b=300mm and effective depth d=184mm, $f_c' = 21\text{MPa}$ and $f_y = 414\text{MPa}$. Is beam satisfactory for shear?

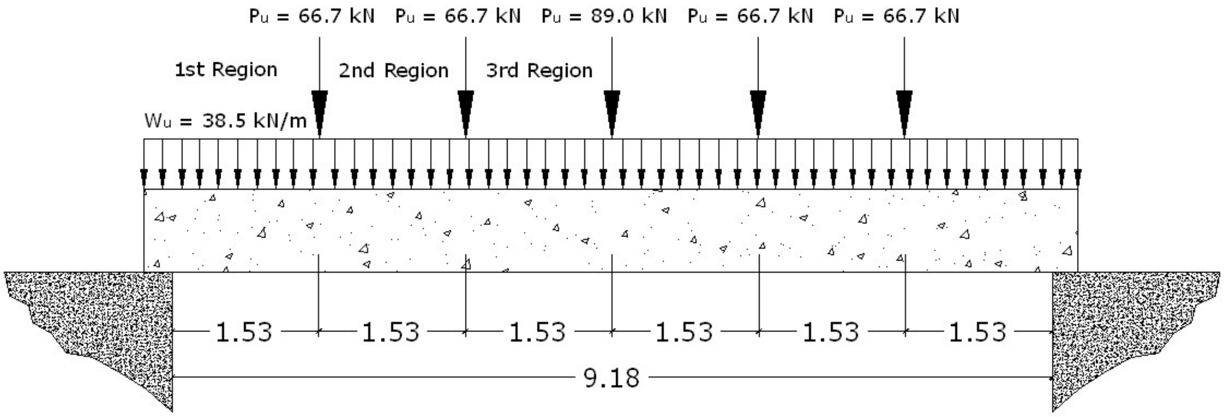
Answers

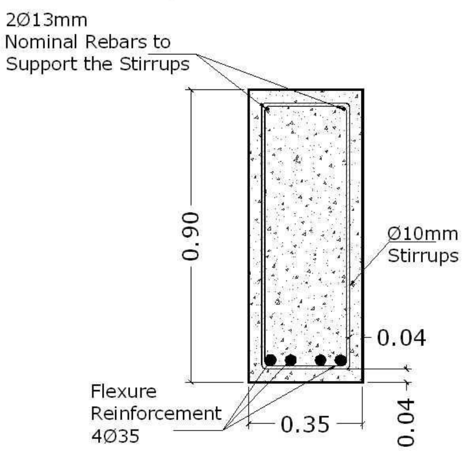
$V_c = 43.0, \frac{1}{2}\emptyset V_c \, ? \, V_u, \quad \because \frac{1}{2}\emptyset V_c < V_u$

Then shear reinforcement is required for this beam. As no shear reinforcement is provided, then the beam is inadequate for shear.

Problem 5.7-2

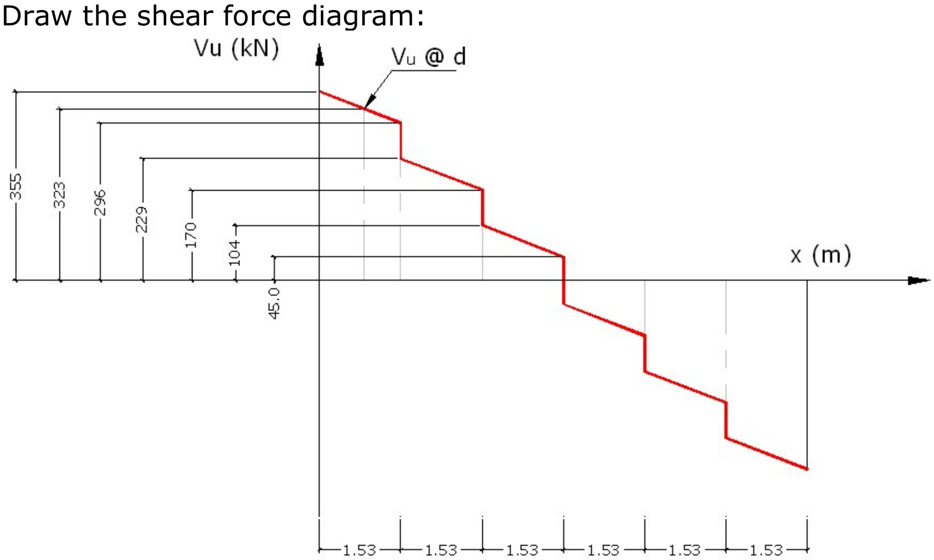
For beam shown below, design single-loop stirrups. The loads shown are factored loads. Use $f_c' = 21\text{MPa}$ and $f_y = 414\text{MPa}$. The uniformly load includes the beam selfweight.





Answers

Draw the shear force diagram:



$\mathrm{d} = 833\ \mathrm{mm}$

Shear Design for 1st Region:

$\mathrm{V_{u\ @\ d\ from\ Face\ of\ Support}} = 323\ \mathrm{kN}$

$\emptyset \mathrm{V_c} = 170\ \mathrm{kN}$

$\mathrm{A_v} = 157\ \mathrm{mm^2}$

Stirrups Design of Problem 5.7-2 (Region 1)	
Region	$\emptyset V_c \leq V_u$
V_s	$\frac{V_u - \emptyset V_c}{\emptyset} = 0.66\sqrt{f_c'}b_w d \Rightarrow 204kN < 882\ kN\ Ok$ Beam dimensions are adequate.
$S_{Theoretical}$	$= \frac{A_v f_{yt} d}{V_s} = 266\ mm$
$S_{for\ Av\ minimum}$	$minimum\ (\frac{A_v f_{yt}}{0.062\sqrt{f_c'}b_w}\ or\ \frac{A_v f_{yt}}{0.35 b_w}) \Rightarrow minimum\ (654\ or\ 531) = 531\ mm$
$S_{maximum}$	$V_s \leq 0.33\sqrt{f_c'}b_w d \Rightarrow 204\ kN \leq 441\ kN$ $Minimum\ \left[\frac{833}{2}\ or\ 600mm\right] = 416\ mm$
	$V_s > 0.33\sqrt{f_c'}b_w d \Rightarrow Minimum\ \left[\frac{d}{4}\ or\ 300mm\right]$
$S_{Required}$	$Minimum\ [S_{Theoretical}, S_{for\ Av\ minimum}, S_{maximum}] \Rightarrow Minimum\ [266mm, 531\ mm, 416\ mm]$ $=266\ mm$ Use $\emptyset 10mm\ @\ 250mm$

Shear Design for 2nd Region:

Stirrups Design of Problem 5.7-2 (Region 2)	
Region	$\emptyset V_c \leq V_u$
V_s	$= \frac{V_u - \emptyset V_c}{\emptyset} = 0.66\sqrt{f_c'}b_w d \Rightarrow 78.7kN < 882\ kN\ Ok$ Beam dimensions are adequate.
$S_{Theoretical}$	$= \frac{A_v f_{yt} d}{V_s} = 688\ mm$
$S_{for\ Av\ minimum}$	$minimum\ (\frac{A_v f_{yt}}{0.062\sqrt{f_c'}b_w}\ or\ \frac{A_v f_{yt}}{0.35 b_w}) \Rightarrow minimum\ (654\ or\ 531) = 531\ mm$
$S_{maximum}$	$V_s \leq 0.33\sqrt{f_c'}b_w d \Rightarrow 204\ kN \leq 441\ kN \Rightarrow Minimum\ \left[\frac{833}{2}\ or\ 600mm\right] = 416\ mm$
	$V_s > 0.33\sqrt{f_c'}b_w d \Rightarrow Minimum\ \left[\frac{d}{4}\ or\ 300mm\right]$
$S_{Required}$	$Minimum\ [S_{Theoretical}, S_{for\ Av\ minimum}, S_{maximum}]$ $Minimum\ [688\ mm, 531\ mm, 416\ mm] = 416mm$ Use $\emptyset 10mm\ @\ 400mm$

Shear Design for 3rd Region:

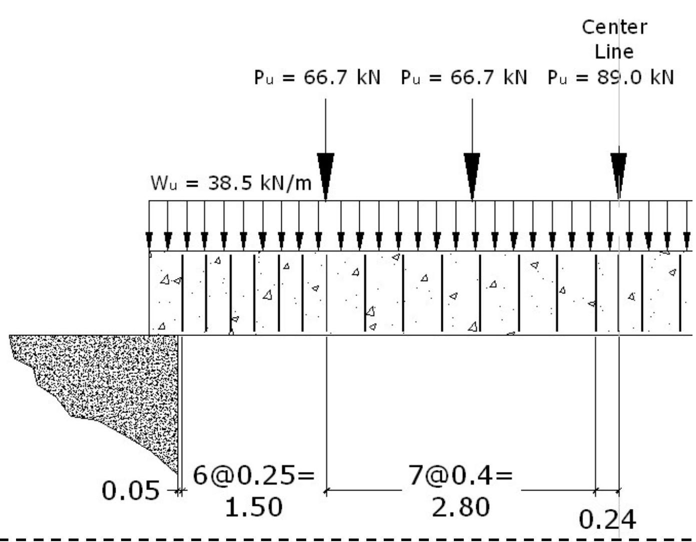
$\because \mathrm{V_u} = 104\ \mathrm{kN} < \emptyset \mathrm{V_c} = 170\ \mathrm{kN}$

Then, only nominal requirement is required for 2nd Region:

$\mathrm{S_{Required}} = \mathrm{Minimum}\ [531\ \mathrm{mm}, 416\ \mathrm{mm}]$

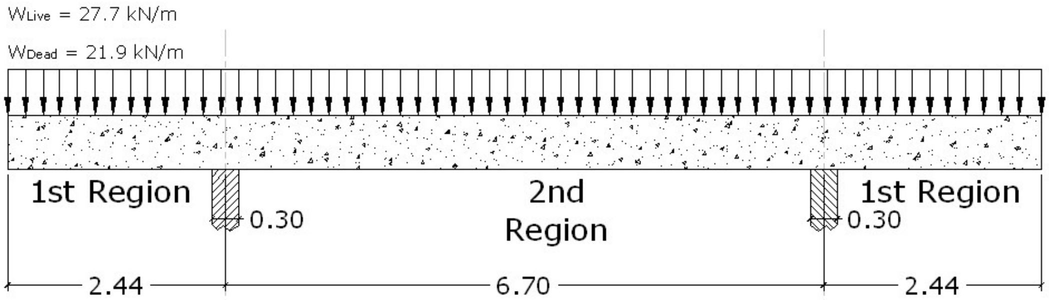
$\mathrm{S_{Required}} = 416\mathrm{mm}$

$\Rightarrow$ Use $\emptyset 10\mathrm{mm}\ @\ 400\mathrm{mm}$



Problem 5.7-3

Design stirrups for the beam shown. Service loads are 21.9 kN/m dead load (including beam selfweight) and 27.7 kN/m live load. Beam width "b" is 325mm and effective depth "d" is 600mm for both top and bottom reinforcement. Use $f_c' = 21\text{MPa}$ and $f_y = 414\text{MPa}$. Use 10mm U Stirrups.



Answers

Computed the factored load:

$W_u = \text{maximum of } [1.4 \text{ Dead or } 1.2 \text{ Dead } + 1.6 \text{ Live}]$

$W_u = \text{maximum of } \left[31.0\frac{\text{kN}}{\text{m}} \text{ or } 70.6\frac{\text{kN}}{\text{m}}\right] = 70.6\frac{\text{kN}}{\text{m}}$

Shear Design for Region 1:

$V_{u\,@\,d} = 70.6(2.44 - 0.15 - 0.6) = 119\,kN$

$\emptyset V_c = 0.75 \times 152 \text{ kN} = 114 \text{ kN}$

$A_v = 157 \text{ mm}^2$

Summary of stirrups design for this region is given in Table below.

Shear Design for Region 2:

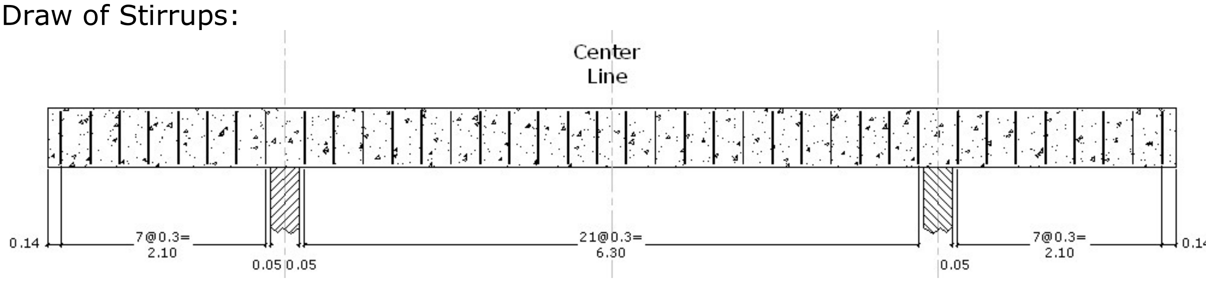
$V_{u\,@\,d} = 70.6\,(6.7 - 0.15 \times 2 - 0.6 \times 2) \times \frac{1}{2} = 184\,kN$

$\emptyset V_c = 114 \text{ kN}$

$A_v = 157 \text{ mm}^2$

Summary of stirrups design for this region is given in the table below.

Draw of Stirrups:



Stirrups Design of (Region 1)	
Region	$\emptyset V_c \leq V_u$
V_s	$= \frac{V_u - \emptyset V_c}{\emptyset} = 0.66\sqrt{f_c'}b_w d$ $6.67kN < 590\,kN\,Ok$ Beam dimensions are adequate.
$S_{Theoretical}$	$= \frac{A_v f_{yt} d}{V_s} = 5847\,mm$
$S_{for\,Av\,minimum}$	$minimum\,(\frac{A_v f_{yt}}{0.062\sqrt{f_c'}b_w}\,or\,\frac{A_v f_{yt}}{0.35 b_w}) \Rightarrow minimum\,(704\,or\,571) = 571\,mm$
$S_{maximum}$	$V_s \leq 0.33\sqrt{f_c'}b_w d \Rightarrow 6.67kN \leq 295\,kN \Rightarrow Minimum\,\left[\frac{d}{2}\,or\,600mm\right]$ $Minimum\,\left[\frac{600}{2}\,or\,600mm\right] = 300\,mm$
	$V_s > 0.33\sqrt{f_c'}b_w d \Rightarrow Minimum\,\left[\frac{d}{4}\,or\,300mm\right]$
$S_{Required}$	$Minimum\,[S_{Theoretical}, S_{for\,Av\,minimum}, S_{maximum}] \Rightarrow Minimum\,[5847\,mm, 571\,mm, 300\,mm]$ $= 300\,mm$ **Use** $\emptyset\mathbf{10}\boldsymbol{mm}\,@\,\mathbf{300}\,\boldsymbol{mm}$

Stirrups Design of (Region 2)	
Region	$\emptyset V_c \leq V_u$
V_s	$\frac{V_u - \emptyset V_c}{\emptyset} = 0.66\sqrt{f_c{}'}b_w d \Longrightarrow 93.3kN < 590\ kN\ Ok$ Beam dimensions are adequate.
$S_{Theoretical}$	$= \frac{A_v f_{yt} d}{V_s} = 418\ mm$
$S_{for\ Av\ minimum}$	$minimum\ (\frac{A_v f_{yt}}{0.062\sqrt{f_c{}'}b_w}\ or\ \frac{A_v f_{yt}}{0.35 b_w}) \Longrightarrow minimum\ (704\ or\ 571) = 571\ mm$
$S_{maximum}$	$V_s \leq 0.33\sqrt{f_c{}'}b_w d \Longrightarrow 93.3kN \leq 295\ kN$ $Minimum\ \left[\frac{d}{2}\ or\ 600mm\right] \Longrightarrow Minimum\ \left[\frac{600}{2}\ or\ 600mm\right] = 300\ mm$
	$V_s > 0.33\sqrt{f_c{}'}b_w d \Longrightarrow Minimum\ \left[\frac{d}{4}\ or\ 300mm\right]$
$S_{Required}$	$Minimum\ [S_{Theoretical}, S_{for\ Av\ minimum}, S_{maximum}]$ $Minimum\ [418\ mm, 571\ mm, 300\ mm] = 300mm$ Use $\emptyset 10mm\ @\ 300\ mm$