CHAPTER 4

FLEXURE ANALYSIS AND DESIGN OF BEAMS

4.1 BENDING OF HOMOGENOUS BEAMS

- For the homogenous beams (i.e., the beams made from single homogenous material like steel or wood) and in the elastic range, the bending stresses can be computed based on the following relation:

  $$f = \frac{M.y}{I}$$

  where
  f is the bending stress at distance y from the neutral axis.
  M is the bending moment at the section,
  I is the moment of inertia of cross section about neutral axis.

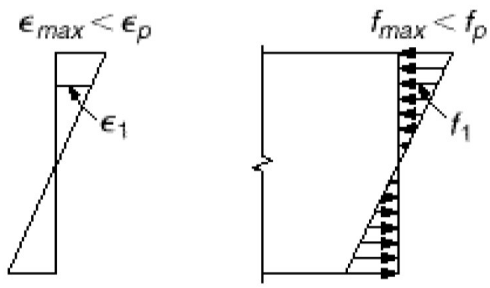


Figure 4.1-1: Stress distribution according to conventional flexural formula.

- For this homogenous elastic beam, the neutral axis passes through the center of gravity of the section.
- In general, the conventional flexure formula, $M.c/I$, is not applicable for RC beams as it has been derived for homogenous materials with linear elastic behavior.
- In Article 4.2 below a more fundamental flexural formula has been derived to take into account the nonlinear and composite nature of RC beams.

4.2 CONCRETE BEAM BEHAVIOR

4.2.1 Behavior of Plain Concrete

Plain concrete beams are inefficient flexure members because the tension strength in bending is a small fraction of the compression strength. Then we will focus on the analysis of reinforced concrete beams only.

4.2.2 Reinforce Concrete Beam Behavior

4.2.2.1 Suitability of Conventional Bending Formula for Analysis of RC Beams

As the reinforced concrete beam

- Is made from two materials
- Cracks in the concrete,
- Behaves no-linear in concrete and steel

above conventional bending relation of $(M.c)/I$ for the homogenous beam cannot be applied to analysis of RC beams.

4.2.2.2 More Rigorous Relations

4.2.2.2.1 Model Beam and Experimental Works

- Experiment works pertained to flexural behavior of RC beams are usually conducted through model beam in below:

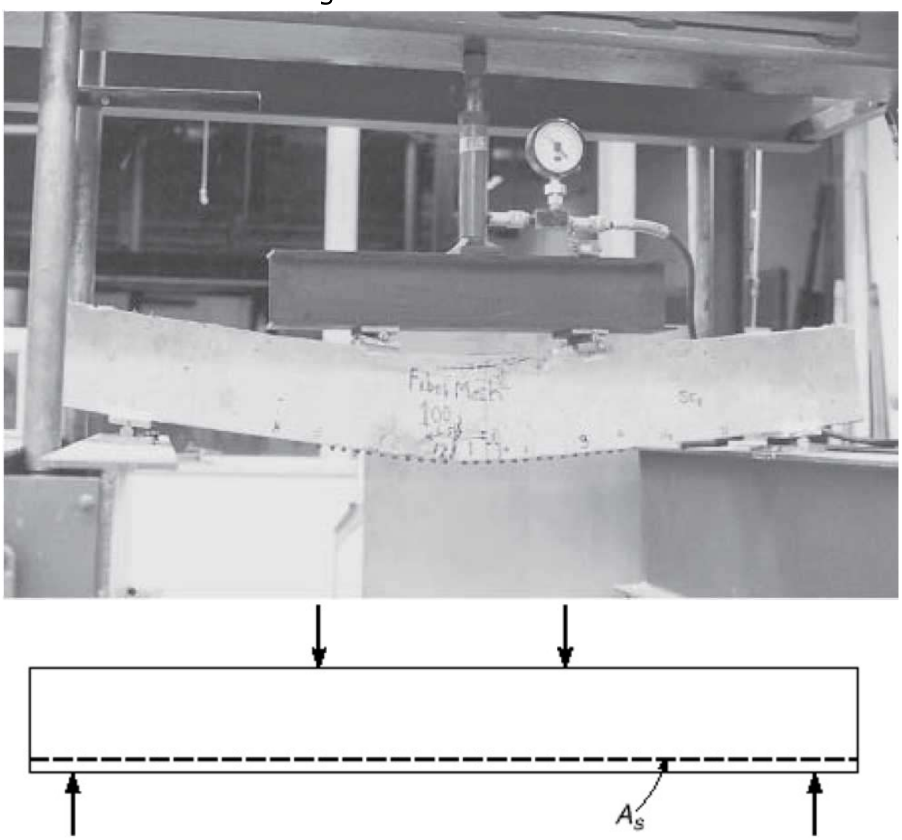


Figure 4.2-1: Model beam for experimental works of RC beams behavior in flexure.

- A beam loaded at third points mainly due to the fact that the mid region is under ***pure bending***, then ***the analysis can exclude the effect of shear stresses and focusing on flexure stresses only***.
- When the load on above beam is gradually increased from zero to the magnitude that will cause the beam to fail ***following three different stages of behavior can be clearly distinguished***.

4.2.2.2.2 Stresses Elastic and Section Uncracked

- At low loads, as long as the maximum tensile stress in the concrete is smaller than the tensile strength of concrete, the entire concrete is effective in resisting stress, in compression on one side and in tension on the other side of the neutral axis.
- At this stage, all stresses in the concrete are of small magnitude and are proportional to strains (i.e. the stresses are varied linearly with the depth). The distribution of strains and stresses in concrete and steel over the depth of the section is as shown in Figure 4.2-2 below.

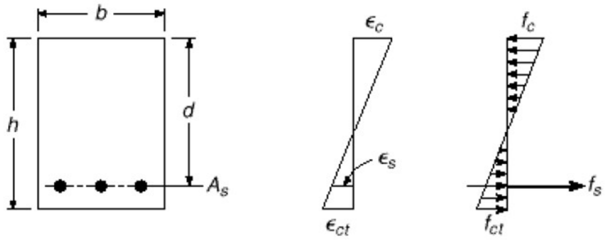


Figure 4.2-2: Strain and stress distribution during elastic uncracked stage.

- Then the only difference from the homogenous beam is in the presence of the steel reinforcement.
- It can be shown (see any text on strength of materials) that one can take account of the presence of the steel reinforcement by replacing the actual steel-and-concrete cross section with a *fictitious section* thought of as consisting of concrete only. In this "***Transformed Section***," the actual area of the reinforcement is replaced with an equivalent concrete area equal to $(n-1)A_s$, located at the level of the steel, as shown in Figure 4.2-3 below:

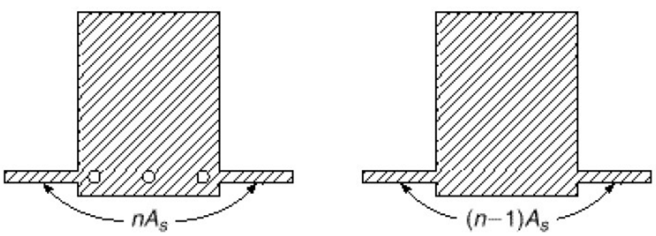


Figure 4.2-3: Transformed section for elastic uncracked RC beam.

where

$$n = \frac{Es}{Ec}$$

is the modular ratio.

- Once the transformed section has been obtained, the usual methods

$$f = \frac{M.c}{I}$$

of analysis of elastic homogeneous beams apply.

- **Computing of E_s and E_c:**
  As discussed in Chapter 2, according to (ACI318M, 2014), article 19.2.2, modulus of elasticity, E_c, for concrete can be estimated based on following correlation:
  - For values of w_c between 1440 and 2560 kg/m^3
    $E_c = w_c^{1.5} 0.043\sqrt{f_c'}\ (in\ MPa)$
  - For normalweight concrete
    $E_c = 4700\sqrt{f_c'}\ \ (in\ MPa)$

  According to the (ACI318M, 2014) (**20.2.2.2**), modulus of elasticity, E_s, for nonprestressed bars and wires shall be permitted to be taken as:
  $E_{Steel} = E_s = \ 200\,000\text{MPa}$
- This stage ends when tensile stress in concrete reaching a limit state. As discussed in Chapter 2, concrete tensile strength can be predicated based on
  - Direct Tensile Strength f_t'.
  - Split-Cylinder Strength f_{ct}.
  - Modulus of Rupture f_r

Example 4.2-1

For the beam shown in Figure 4.2-4 below, find the maximum magnitude of the load "P" such that the section stays in the uncracked elastic state.

Given:

- fc' = 25 MPa
- fy = 400 MPa
- Neglect the beam selfweight.

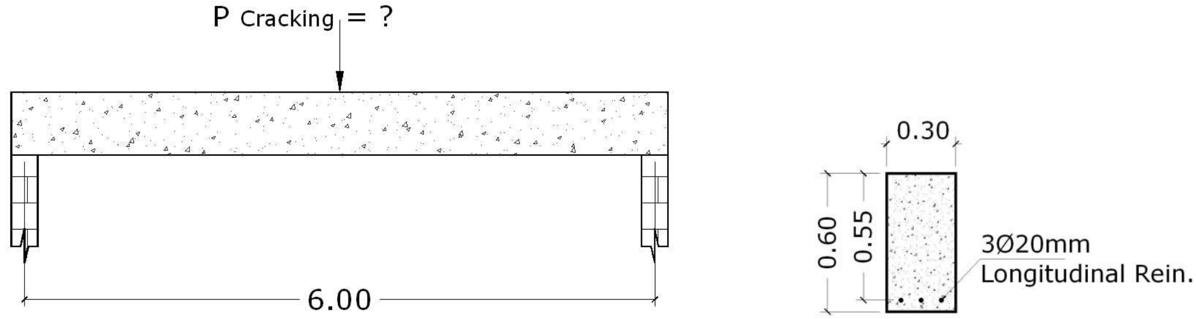
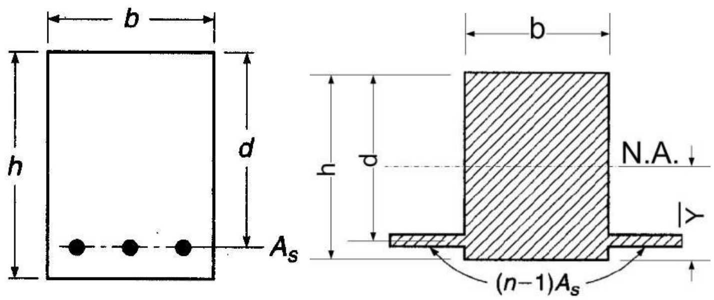


Figure 4.2-4: Beam of elastic uncracked section for Example 4.2-1.

Solution

- As the flexure formula is derived for the ***homogenous section***, then the steel must be transformed for the equivalent concrete to obtain a homogenous section that formed from a single material:



$$\because E_S = 200\,000\ MPa, and\ E_C = 4700\sqrt{f_c'} = 4700\sqrt{25} = 23\,500\ MPa, \therefore n \approx 8.5$$

$$A_S = \left(\pi \frac{20^2}{4}\right) \times 3 = 942\ mm^2$$

$$\therefore (n-1)A_s = 7\,065\ mm^2$$

$$\sum M_{of\ Area\ about\ lower\ face} = \bar{y}.A$$

$$\bar{y}.(300 \times 600 + 7\,065) = (300 \times 600) \times 300 + (7\,065) \times 50$$

$$\Rightarrow \bar{y} = 290mm\ < 300\ ok.$$

- Compute the moment of inertia for the transformed section:

$$I_{N.A.} = \left[\left(300 \times \frac{600^3}{12}\right) + (300 \times 600 \times 10^2)\right] + 7\,065\ \times (290 - 50)^2$$

$$I_{N.A.} = 5.82 \times 10^9\ mm^4$$

- Use the flexure formula to compute the cracking moment:

$$M_{Crack} = \frac{f_r \times I_{N.A.}}{c}$$

$$f_r = 0.62\sqrt{f_c'} = 0.62\sqrt{25} = 3.1\ MPa$$

$$M_{Crack} = \frac{3.1 \frac{N}{mm^2} \times (5.82 \times 10^9 mm^4)}{290mm}\ 62.2 \times 10^6\ N.mm = 62.2\ kN.m$$

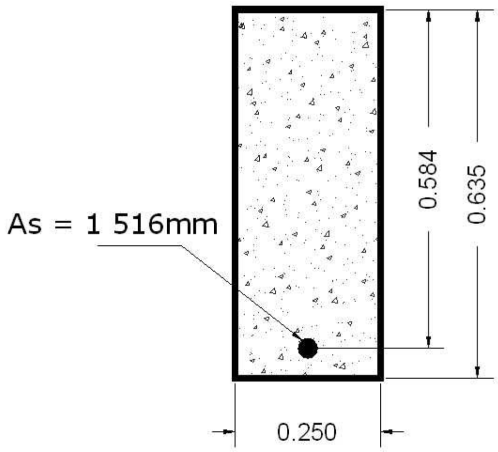
- Compute the P_{Crack}:

$$M_{Crack} = \frac{P_{Crack} \times L}{4} \Rightarrow P_{Crack} = \frac{62.2\ kN.m\ \times 4}{6m} = 41.5\ kN \blacksquare$$

4.2.2.2.3 Home Work for Article 4.2.2.2.2: Analysis of Uncracked Elastic Section

Problem 4.2-1

A rectangular beam with dimensions of b= 250mm, h = 635mm, and d = 584mm. The f_c' = 28 MPa, f_y = 400 MPa, and E_{Steel} = 200,00 MPa. Check the state of section and determine the stresses caused by a bending moment of M = 61 kN.m.



Hint:

Start your solution with assumption that the section under a moment of 61 kN.m stills within the 1st Stage. This assumption should be checked later.

Answers:

$f_{c\,Ten.} = 3.04\ MPa < f_r$

Then, the section is uncracked elastic section.

$f_{c\,Comp.} = 3.37\ MPa$

$f_{SteeL} = 20.2\ MPa$

4.2.2.2.4 Second Stage: Elastic Cracked Section

- When the load is further increased, the tension strength of the concrete is reached.
- Tension cracks develop and propagate quickly upward to or closed to level of the neutral plane, which in turn shift upward with progressive cracking.
- In ***well-designed beams the width of these cracks is so small*** (***hairline cracks***) that they are not harmful from the view point of the either corrosion protection or appearance ***(i.e. in current design philosophy, the design is based on permitting of hairline cracks***).
- In cracked section, the concrete does not transmit any tension stresses. Hence the steel is called upon to resist the entire tension. If the concrete stress do not exceeded approximately $f_c'/2$ and the steel stress has not reached the yield point, stresses and strains continue to be closely proportional. Then the distribution of strains and stresses are as shown below.

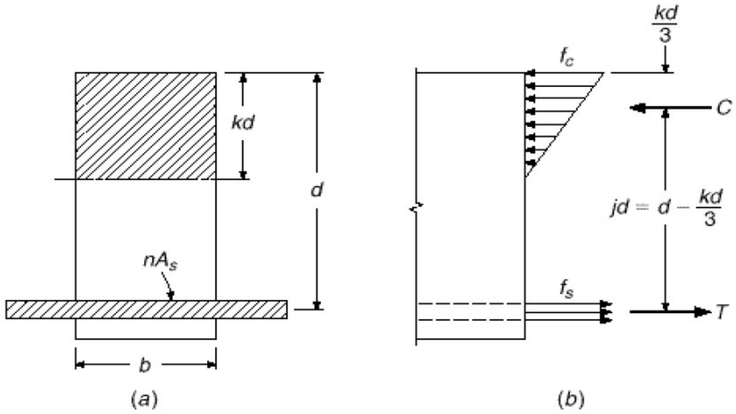


Figure 4.2-5: Strain and stress distribution during elastic cracked stage.

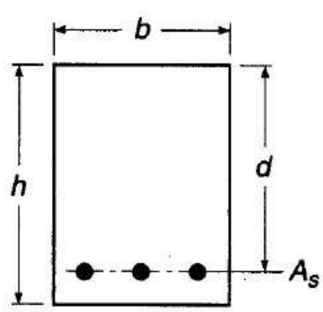
- This situation (elastic cracked section) ***generally occurs in structures under normal service loads (unfactored loads)***.
- The stresses and strains in the elastic cracked section can be computed based on transform the steel to an equivalent concrete, and then use the conventional flexure formula $f = M.y/I$.

Example 4.2-2

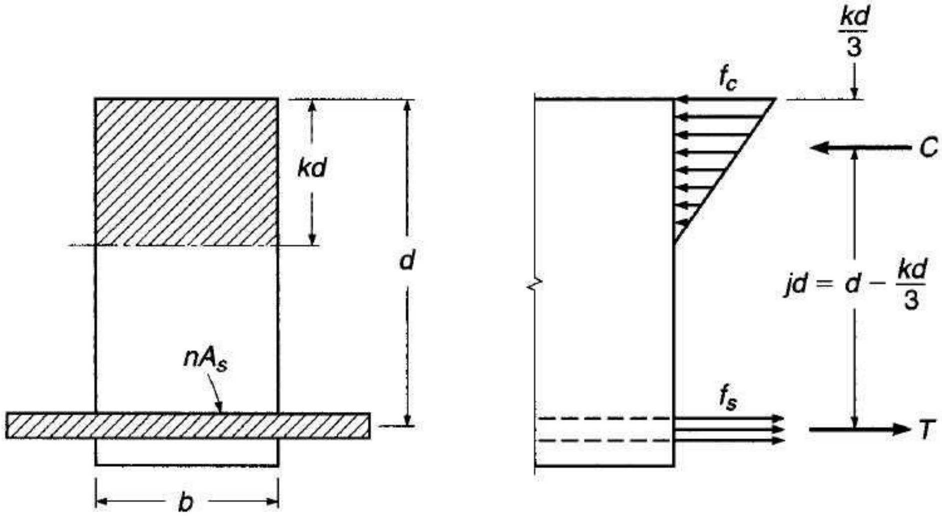
Resolve Example 4.2-1, to compute the maximum magnitude of the load P for the cracked elastic section (assuming that the elastic limit of concrete is equal to $f_c'/2$).

Solution

- As was discussed in the mechanics of materials the application of the conventional flexure formula ($f = M.c/I$) is based on the assumption of homogenous and linear section.
- Linearity of section is assured for concrete and will be assumed for the steel (and should be checked later). While homogeneity of section will be assured through the transformed section concept. That based on transformation of original nonhomogeneous original section shown



- In to the equivalent homogenous section shown below:



Then

$A_S = \left(\pi \frac{20^2}{4}\right) \times 3 = 942\ \text{mm}^2$

$\because E_S = 200\,000\ \text{MPa, and } E_C = 4700\sqrt{f_c'} = 4700\sqrt{25} = 23\,500\ \text{MPa},$

$\therefore n \approx 8.5$

$\therefore nA_s = 8\,007\ \text{mm}^2$

- Application of flexure formula:
  As the section is transformed to a homogenous one, then the flexure formula can now be applied:
  - Compute kd
    As the N.A. passes through the section centroid:

    $\sum_{i=1}^{2} \text{M of Area about N.A.} = 0 \Rightarrow (300 \times kd) \times kd/2 = 8007\text{mm}^2 \times d(1-K)$

    $k^2 + 0.0979k - 0.0979 = 0$

    $k = \frac{-0.0979 \mp \sqrt{(0.0979^2 + 4 \times 0.0979}}{2 \times 1} = 0.267,\ \ kd = 147\text{mm}$
  - Compute IN.A.

    $I_{N.A.} = \frac{kd^3 \times b}{3} + nAs \times (d - kd)^2$

    $I_{N.A.} = \frac{147^3 \times 300}{3} + 8007 \times (550 - 147)^2 = 1.62 \times 10^9 \text{mm}^4$
  - Compute M

    $\because f_c = \frac{(f_c')}{2} = \frac{M.c}{I_{N.A.}}$

    $\therefore M = \frac{25}{2} \times \frac{1.62 \times 10^9}{147\text{mm}} = 137.7\ \text{kN.m}$

    $\because M = \frac{PL}{4}, \qquad \therefore P = M.\frac{4}{L} = \frac{137.7 \times 4}{6} = 91.8\ \text{kN}$
- Check the assumption of $f_s \leq f_y$ as assumed:

  $f_s = \frac{M.c}{I} \times n = \frac{137.7 \times 10^6 \times (550 - 147)}{1.62 \times 10^9} \times 8.5 = 291\text{MPa}$

  $\because f_s = 291\ \text{MPa} < 400\ MPa\ \therefore ok.$

  Then the assumption of $f_s \leq f_y$ is correct and the solution that based on it is a final solution.

  $\therefore P = 91.8\ \text{kN}$ ■

Example 4.2-3

Show that the neutral axis of cracked elastic reinforced concrete section with rectangular shape under flexure stress can be located based on the following relation:

$$k = \sqrt{(\rho n)^2 + 2\rho n} \; - \rho n$$

where

ρ is reinforcement ratio that defined as follows:

$$\rho = \frac{A_s}{bd}$$

Solution

As the neutral axis for an elastic beam passes through the centroid of its cross sectional area, then:

$$\sum_{i=1}^{2} \text{M of Area about N.A.} = 0 \;\Rightarrow (b \times kd) \times kd/2 \;= nA_S \times d(1 - K)$$

$$\left[\left(\frac{bd^2}{2}\right)k^2 + (nA_S d)k - nA_S d = 0\right] \div d$$

$$\left[\left(\frac{bd}{2}\right)k^2 + (nA_S)k - nA_S = 0\right] \div bd$$

$$\left(\frac{1}{2}\right)k^2 + (n\rho)k - n\rho = 0$$

Quadratic formula can be used to solve the above quadratic equation[1]:

$$k = \frac{\left(-n\rho \;\pm\; \sqrt{(n\rho)^2 + 4 \times \frac{1}{2} \times n\rho}\right)}{2 \times \frac{1}{2}}$$

As the negative distance has no meaning in our case, then the answer will be in terms of positive root:

$$k = \left(-n\rho \;\pm\; \sqrt{(n\rho)^2 + 2n\rho}\right)$$

As the dimension factor k cannot be a negative value, then the actual root for above equation will be:

$$k = \sqrt{(n\rho)^2 + 2n\rho} \; - n\rho \quad \blacksquare$$

Example 4.2-4

Relocate the neutral axis of Example 4.2-2, based on the general relation that has been derived in Example 3.

Solution

$$n = 8.5$$

$$A_S = 942 \text{ mm}^2$$

$$\rho = \frac{A_s}{bd} = \frac{942 \text{ mm}^2}{550 \times 300 \text{ mm}^2} = 5.71 \times 10^{-3}$$

$$n\rho = 0.0485$$

$$k = \sqrt{(0.0485)^2 + 2 \times 0.0485} \; - 0.0485 \; = 0.267 \quad \blacksquare$$

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[1] Quadratic equation is a equation that has the following general form:

$$ax^2 + bx + c = \quad \textit{where } a \neq 0$$

This equation can be solved based on the *Quadratic Formula*:

$$x = \frac{\left(-b \;\pm\; \sqrt{b^2 - 4ac}\right)}{2a}$$

Example 4.2-5 Analysis of Working Stresses in a Reinforced Concrete Beam with General Shape

The simply supported beam shown in Figure 4.2-6 below has the following data: $f_{c\ allowable} = 7\ MPa$, $f_{s\ allowable} = 124\ MPa$, and $n = 12$.

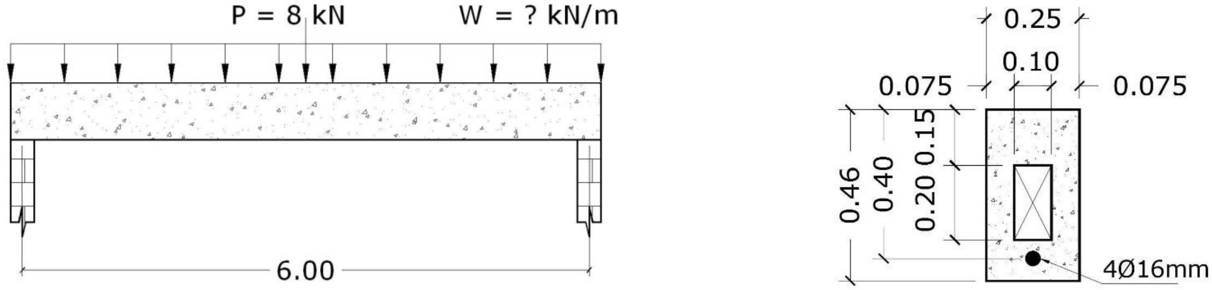


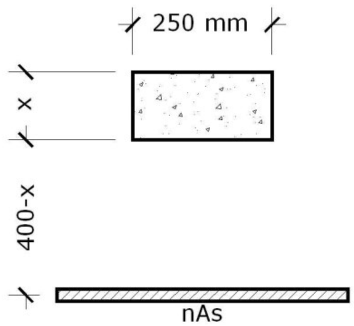
Figure 4.2-6: Simply supported beam for Example 4.2-5.

Compute the following:

- Section allowable bending moment.
- Total uniform load (including beam selfweight) that could the beam carry in addition to a concentrated load of 8 kN at mid-span.

Solution

- Compute the section moment of inertia of transformed section:
  - Assume that the neutral axis to be above the hollow section.



$$A_s = 4 \times \frac{\pi 16^2}{4} = 804\ mm^2 \quad \Longrightarrow nA_s = 12 \times 804mm^2 = 9\ 648mm^2$$

$$\sum \text{Area Moment about N.A.} = 0$$

$$250mm \times x \times \frac{x}{2} = 9\ 648mm^2 \times (400 - x)$$

$$x^2 + 77.2x - 30\ 874 = 0$$

$$x = \frac{-77.2 \pm \sqrt{77.2^2 + 4 \times 1 \times 30\ 874}}{2 \times 1} = 141mm$$

$$I_{N.A.} = \frac{141^3 \times 250}{3} + 9\ 648 \times (400 - 141)^2 = 881 \times 10^6 mm^4$$

  - Compute the allowable bending moment based on concrete allowable stresses:

$$F_{c\ allowable} = \frac{M_{allowable} \cdot c_{Top}}{I_{N.A.}} \Longrightarrow M_{allowable} = \frac{7MPa \times 881 \times 10^6 mm^4}{141mm} = 43.7\ kN.m$$

  - Compute the allowable bending moment based on steel allowable stresses:

$$F_{s\ allowable} = n\frac{M_{allowable} \cdot c_{Bottom}}{I_{N.A.}} \Longrightarrow M_{allowable} = \frac{124MPa \times 881 \times 10^6 mm^4}{12 \times (400 - 141)mm} = 35.1\ kN.m$$

  - Compute the allowable bending moment:

$$M_{allowable} = \text{Minimum of } 43.7\ kN.m \text{ and } 35.1\ kN.m = 35.1\ kN.m \blacksquare$$

- Compute the allowable total load:

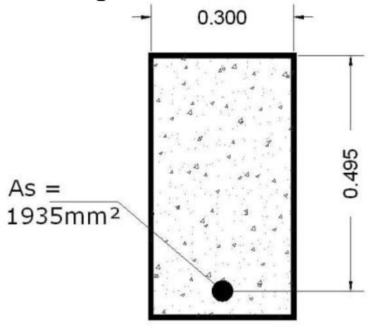
$$M = \frac{WL^2}{8} + \frac{PL}{4} \Longrightarrow 35.1\ kN.m = \frac{W\frac{kN}{m} \times 6^2 m^2}{8} + \frac{8kN \times 6m}{4}$$

$$W = 5.13\frac{kN}{m} \blacksquare$$

4.2.2.2.5 Home Work of Article 4.2.2.2.4: Analysis of Working Stresses in Beams with Rectangular Sections

Problem 4.2-2

For the beam shown below if the $E_s = 200\,000\ MPa$, $E_C = 20\,000\ MPa$, $f_c' = 21MPa$, and $f_y = 400\ MPa$, determine the maximum stresses in the steel and concrete if the applied bending moment is 115kN.m.



Answers

$k = 0.396\ kd = 196\ mm\ I_{N.A.} = 2.48 \times 10^9 mm^4$

$f_c = 9.09\ MPa < \frac{f_c'}{2}\ Ok.$ ■ $f_s = 139\ MPa < f_y\ Ok.$ ■

Problem 4.2-3

What is the maximum allowable bending moment for the beam of Problem 1, if the maximum allowable stresses are $f_s = 152MPa$ and $f_C = 8.33\ MPa$.

Answers

$M_{Allowable\ Based\ on\ Steel\ Allowable\ Stresses} = 126\ kN.m$
$M_{Allowable\ Based\ on\ Concrete\ Allowable\ Stresses} = 105\ kN.m$
$M_{Allowable} = Minimum\ (126, 105) = 105\ kN.m$ ■

Problem 4.2-4

What is the maximum allowable bending moment for the beam of Problem 1, if the maximum allowable stresses are $f_s = 132\ MPa$ and $f_C = 9.33\ MPa$.

Answers

$M_{Allowable\ Based\ on\ Steel\ Allowable\ Stresses} = 109\ kN.m$
$M_{Allowable\ Based\ on\ Concrete\ Allowable\ Stresses} = 118\ kN.m$
$M_{Allowable} = Minimum\ (109, 118) = 109\ kN.m$ ■

Problem 4.2-5

A concrete beam shown below has a simple span of 5m. It has $f_c = 9.00MPa$, $f_s = 124MPa$ and n = 10.

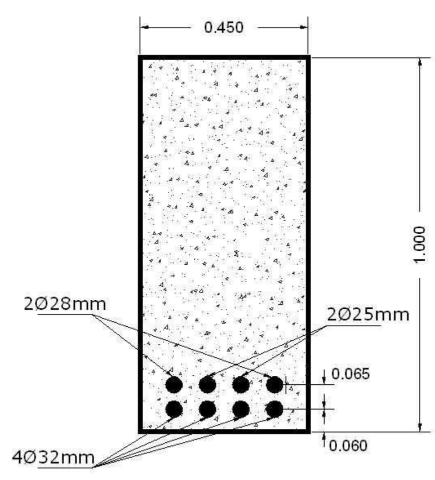
For this beam compute the following:

- Section allowable bending moment.
- Value of concentrated force "P" that the beam could carry at it midspan.

Notes on Problem 4.2-4

Sometimes, beam width is not sufficient to put the required reinforcement in a single layer, and then the reinforcement is put in two or more layers.

For analysis purposes, theses layers are usually replaced with a single layer that has an area equal to area of all layers and located at centroid of steel layers.



Answers

- Section allowable bending moment.

  $A_{of\ Rebar\ 25mm} = 490\ mm^2$
  $A_{of\ Rebar\ 28mm} = 615\ mm^2$
  $A_{of\ Rebar\ 32mm} = 804\ mm^2$
  $\bar{y}_{Measured\ from\ reinforcement\ center\ to\ beam\ lower\ face} = 86.5\ mm < \ 92.5\ mm\ Ok.$
  $d\ =\ 913\ mm$
  $As\ =\ 5\ 426\ mm^2$
  $\rho =\ 13.2 \times 10^{-3}$
  $n\rho = 0.132$
  $k =\ 0.398$
  $kd = 364\ mm$
  $nA_s =\ 54\ 260\ mm^2$
  $I_{N.A.}\ = 23.6 \times 10^9 mm^4$
  $M_{Allowable\ Based\ on\ Steel\ Allowable\ Stresses} = 533\ kN.m$
  $M_{Allowable\ Based\ on\ Concrete\ Allowable\ Stresses} = 584\ kN.m$
  $M_{Allowable}\ =\ 533\ kN.m$ ■
- Value of concentrated force "P" that the beam could carry at it mid-span.

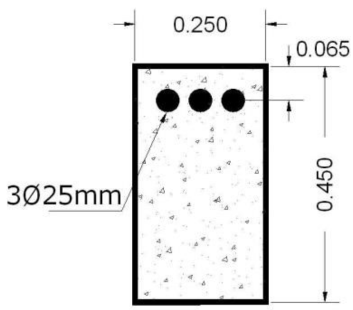
  $W_{Selfweight} =\ 10.8\frac{kN}{m}\ \ P =\ \ 399\ kN$ ■

Problem 4.2-6

The figure shown below is a cross-section of a cantilever beam supporting a uniform load of $2.5\frac{kN}{m}$ including its own weight and a concentrated load of $30\ kN$ at its free end. It has $f_c =\ 8.00\ MPa$, $f_s =\ 124\ MPa$, and $n\ =\ 10$.

For this beam compute the following:

- The safe resisting moment of the beam.
- The maximum beam span.



Answers

- The safe resisting moment of the beam:

  $d\ =\ 385\ mm\ \ \ As\ =\ 1\ 470\ mm^2\ \ \rho =\ 15.1 \times 10^{-3}\ \ n\rho = 0.151$
  $k =\ 0.419\ \ kd = 163\ mm\ nA_s =\ 14\ 700\ mm^2\ I_{N.A.}\ = 1.12 \times 10^9 mm^4$
  $M_{Allowable\ Based\ on\ Steel\ Allowable\ Stresses} = 61.2\ kN.m$
  $M_{Allowable\ Based\ on\ Concrete\ Allowable\ Stresses} = 55.0\ \ kN.m$
  $M_{Allowable}\ = Minimum\ (61.2, 55.0) =\ 55.0\ kN.m$ ■
- The maximum beam span.

  $L = 1.71\ m$ ■

4.2.2.2.6 Third Stage: Flexure Strength

- When the load is still further increased, flexure strength of the beam is reached. Failure can be caused in one of the following two ways:
  - ***SECONDARY COMPRESSION FAILURE (TENSION-CONTROLLED SECTION):***
    - When relatively ***moderate amount of reinforcement are employed, at some value of the load the steel will reach its yield point***.
    - At that stress the reinforcement yields suddenly and stretched a large amount, and the tension cracks in the concrete widen visibly and propagate upwards, with simultaneous significant deflection of the beam. When this happens, the strains in the remaining compression zone of the concrete increase to such degree that crushing of the concrete. Then the stresses history will be as shown in the figures below:

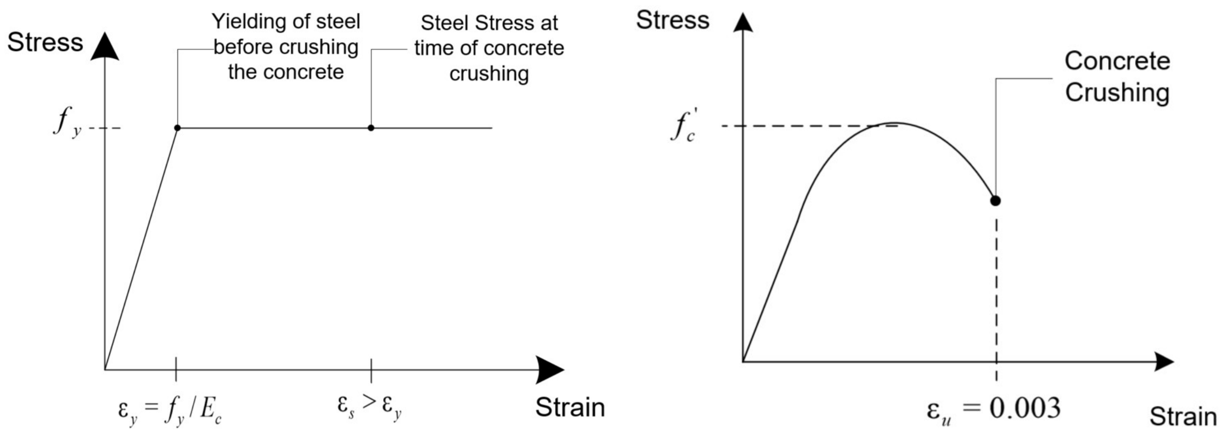


Figure 4.2-7: Stress –strain state for secondary compressive failure.

  - ***COMPRESSION FAILURE (COMPRESSION CONTROLLED SECTION)*:**
    - On the other hand, ***if large amount of reinforcement or normal amount of steel with very high strength are employed, the compression strength of the concrete may be exhausted before the steel start yielding***.
    - Then the stresses history during the compression failure will be as shown below:

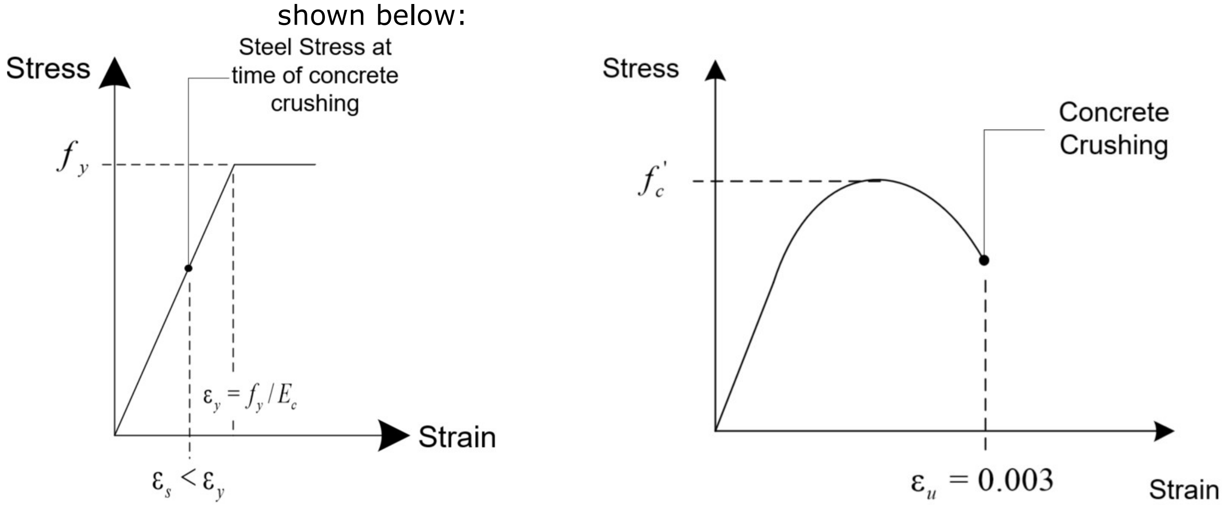


Figure 4.2-8: Stress –strain state for compressive failure.

    - Compression failure is sudden, of an almost explosive nature, and occurs without warning.
    - For this reason, (ACI318M, 2014), ***Article 21.2.1***, required to dimension beams in such a manner that should they be overloaded, failure would be initiated by yielding of the steel rather than by crushing of concrete (*Secondary Compression Failure*).

4.2.2.2.7 Nominal Flexure Strength M_n of a Rectangular Section with Secondary Compression Failure.

- It is clear that at or near ultimate loads, stresses are no longer proportional to the strain, then the conventional flexure formula ($f = \frac{M.c}{I}$) cannot be applied for the analysis and design of the section.
- And the analysis and design of the section must be based on the direct application of the basic principles (compatibility relation, stress-strain relation, and the equilibrium conditions) and as follow:
    - **COMPATIBILITY CONDITIONS**
      Based on
        - The ***kinematic assumption*** of the ***plane section before loading remain plane after loading***. This assumption is adopted by (ACI318M, 2014) in ***article 22.2.1.2***.
        - The ***assumption of the secondary compression failure*** (i.e., the failure starting with the yielding of the steel and the crushing of concrete). According to (ACI318M, 2014), ***article 22.2.2.1***, concrete crushing occurs when maximum strain at the extreme concrete compression fiber reaches a value of $\epsilon_u = 0.003$.

      the strain distribution will be as shown below:

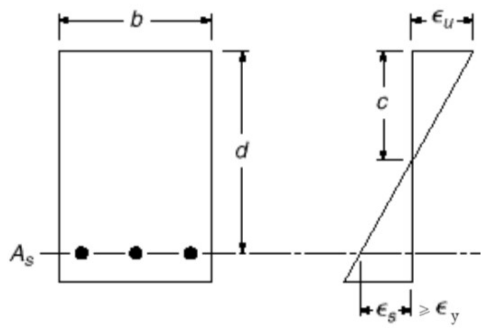


Figure 4.2-9: Strain distribution.

        - The kinematic assumption remains applicable even when materials behave inelastically, (Popov, 1968).
    - **STRESS-STRAIN RELATION:**
        - Based on the actual stress-strain relations of the concrete and reinforcing steel, the stress distribution will be as shown below:

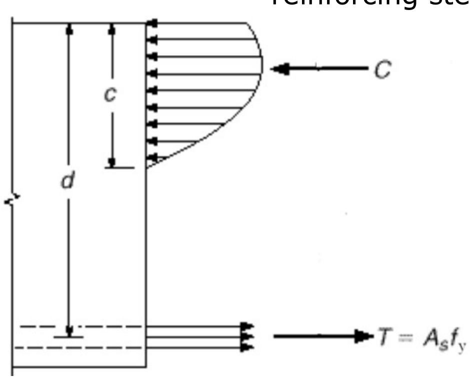


Figure 4.2-10: Stress distribution for a beam with secondary compression failure.

        - It is not really necessary to know the shape of the concrete stress distribution. What is necessary to know is, (ACI318M, 2014) ***article 22.2.2.3***:
            - The total resultant compression force "*C*" in the concrete.
            - Its vertical location.

              Evidently, then, one can think of the actual complex stress distribution as replaced by a fictitious one of some simple geometric shape, provided that this fictitious distribution results in the same total compression force "C" applied at the same location as in the actual member when it is on the point of failure.

- Historically, a number of simplified, fictitious equivalent stress distributions have been proposed by investigators in various countries. The one generally accepted was first proposed by ***C. S. Whitney*** and was subsequently elaborated and checked experimentally by others. The actual stress distribution immediately before failure and the fictitious equivalent distribution are shown in Figure below, (ACI318M, 2014), ***article 22.2.2.4***,

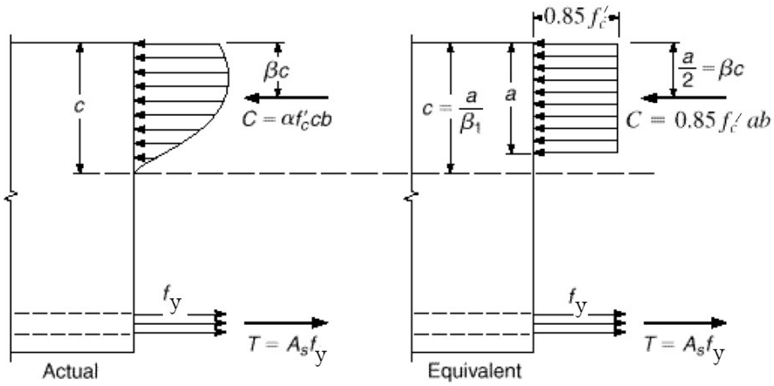


Figure 4.2-11: Whitney simplified equivalent stress distributions.

- According to ACI code (**22.2.2.4.3**) β_1 can be computed based on Table below:

Table 4.2-1: Values of β1 for equivalent rectangular concrete stress distribution, Table 22.2.2.4.3 of (ACI318M, 2014).

f_c', MPa	β_1	
$17 \le f_c' \le 28$	0.85	(a)
$28 < f_c' < 55$	$0.85 - \dfrac{0.05(f_c' - 28)}{7}$	(b)
$f_c' \ge 55$	0.65	(c)

- **EQUILIBRIUM CONDITIONS:**

  According to (ACI318M, 2014), ***article 22.2.1.1***, equilibrium shall be satisfied at each section:

  $$\because \sum F_x = 0 \quad \Rightarrow 0.85 f_c' ba = A_s f_y$$

  $$\therefore a = \frac{A_s f_y}{0.85 f_c' b}$$

  $$\because \sum M_{\text{About Centroid of Compressive Force C}} = 0$$

  $$\therefore M_n = A_s f_y \left(d - \frac{a}{2}\right)$$

  Substitute the value of "a" into above equation:

  $$M_n = A_s f_y \left(d - \frac{1}{2}\ \frac{A_s f_y}{0.85 f_c' b}\right) \quad \text{or } M_n = A_s f_y d \left(1 - \frac{1}{2}\ \frac{A_s f_y}{0.85 f_c' bd}\right)$$

  Let $\rho = \frac{A_s}{bd}$

  $$M_n = \rho f_y b d^2 \left(1 - 0.59\ \frac{\rho f_y}{f_c'}\right) \quad \blacksquare$$

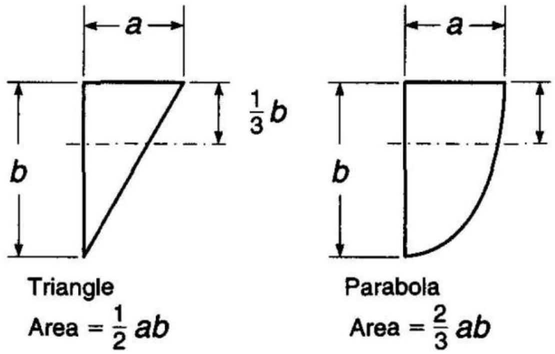
4.2.2.2.8 Home Work of Article 4.2.2.2.7: Behavior of Singly Reinforced Rectangular Concrete Beams

Problem 4.2-7

A rectangular beam made using concrete with $f_c' = 35\ MPa$ and steel with $f_y = 420\ MPa$ has a width b = 450mm, an effective depth d = 540mm, and a total depth h = 600mm. The beam is reinforced with four No. 29 bars. Compute the nominal moment capacity, assuming, see Figure below

a. an equivalent rectangular stress block,
b. a triangular stress block with a peak value of f_c',
c. a parabolic stress block with a peak value of f_c'. (see Fig. P3.13).

Compare and comment on your results, knowing that the rectangular stress block correlates within 4 percent with test results.



Aim of Problem

*This problem aims to highlight code regulations of 22.2.2.3 which states "**The relationship between concrete compressive stress and strain shall be represented by a rectangular, trapezoidal, parabolic, or other shape that results in prediction of strength in substantial agreement with results of comprehensive tests**".*

Answers

a. $a_{Rectangular} = 67.5\ mm$ $M_{n\ Rectangular} = 549\ kN.m$
b. $a_{Triangular} = 115\ mm$ $M_{n\ Triangular} = 544\ kN.m$
c. $a_{Parabolic} = 103\ mm$ $M_{n\ Parabolic} = 543\ kN.m$

$$\frac{M_{n\ Triangular}}{M_{n\ Rectangular}} = \frac{544}{549} = 0.991$$

$$\frac{M_{n\ Parabolic}}{M_{n\ Rectangular}} = \frac{543}{549} = 0.989$$

Comment:

In both cases the results are within a 4% margin or error and the rectangular stress block gives the higher value for the nominal moment.

Problem 4.2-8

A rectangular beam made using concrete with $f_c' = 42\ MPa$ and steel with $f_y = 420\ MPa$ has a width 500mm., an effective depth of d=440mm., and a total depth of h = 500mm. The concrete modulus of rupture $f_r = 3.6\ MPa$. The elastic moduli of the concrete and steel are, respectively, $E_c = 28000\ MPa$ and $E_s = 200000\ MPa$. The tensile steel consists of four No. 36. bars.

a. Find the maximum service load moment that can be resisted without stressing the concrete above $0.45f_c'$ or the steel above $0.4f_y$.
b. Determine whether the beam will crack before reaching the service load.
c. Compute the nominal flexural strength of the beam.
d. Compute the ratio of the nominal flexural strength of the beam to the maximum service load moment, and compare your findings to the ACI load factors and strength reduction factor.

Answers

a. $M_{sc} = 219\ kN.m\ M_{ss} = 179\ kN.m\ M_{ss} = 179\ kN.m$

b. $M_{cr} = \frac{bh^2}{6} f_r = 75\ kN.m$, therefore section cracks.

c. $a = 94.7\ mm\ M_n = 664\ kN.m$

d. $Ratio = \frac{M_n}{M_s} = \frac{664}{179} = 3.7 > \frac{\gamma}{\phi} = \frac{\frac{1.2+1.6}{2}}{0.9} = 1.56$

Comments:

The value of this ratio is greater than the ACI factors for strength, γ, divided by the ϕ factor, thus suggesting that the ***working stress design approach*** is more conservative than the ***strength design, or Load Resisting Factored Design LRFD, approach***.

4.2.2.2.9 Balanced Strain Condition (ACI 10.3.2)

- The secondary compression failure can be assured by keeping the reinforcement ratio ρ below a certain limiting value that called Balanced Steel Ratio ρ_b.
- It represents a limit amount of reinforcement necessary to make the beam fail by crushing of concrete at the same load that causes the steel yield.
- Computing the "***Balanced Steel Ratio***" is also can be written in terms of application of basic principles (Compatibility, Stress-Strain Relation, and Equilibrium):
  - Compatibility Conditions:
    Based on strain conditions shown below:

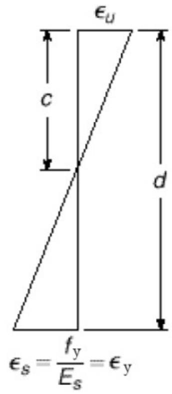


Figure 4.2-12: Strain distribution for balanced condition.

$$\frac{c_b}{\epsilon_u} = \frac{d}{\epsilon_u + \epsilon_y}$$

$$c_b = \frac{\epsilon_u}{\epsilon_u + \epsilon_y} d$$

$$[c_b = \frac{0.003}{0.003 + f_y/E_s} d] \times \frac{E_s}{E_s}$$

$$c_b = \frac{600}{600 + f_y} d$$

Above relation is a general relation and correct not only for rectangular section.

  - Stress-Strain Relation:
    Stress distribution for balanced condition can be derived from strain condition and as shown in Figure below:

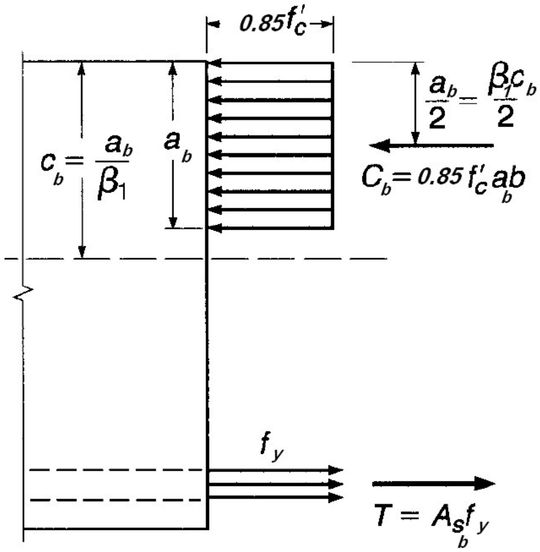


Figure 4.2-13: Stress distribution and forces for balanced condition.

  - Equilibrium Conditions:

$$\because \sum F_x = 0 \quad \Rightarrow 0.85 f'_c b a_b = A_{sb}\, f_y$$

$$A_{sb}\, f_y = 0.85 f'_c b \beta_1 \left(\frac{600}{600 + f_y} d\right) \quad \div f_y b d$$

$$\rho_b = 0.85\beta_1 \frac{f'_c}{f_y}\left(\frac{600}{600 + f_y}\right) \blacksquare$$

- It useful to notes that the ***Balanced Steel Ratio*** is a function of material strengths (f_c', and f_y) only and it is independent on beam dimensions.

Example 4.2-6

Compute the Balanced Steel Ratio for concretes that have compressive strength of $f'_c = 21\text{MPa}, 28\text{MPa}, \text{and } 35\text{MPa}$ when reinforced with reinforcing steel have grades of Grade 40, 50, and 60.

Solution

Microsoft Excel is so effective in prepare calculations table has cells that related to each other by algebraic or logical relations.

For our problem, calculation table will take the following form:

f_c' $MPa\,(psi)$	β_1	***Steel Grade***		
		40	***50***	***60***
		f_y MPa		
		280	***350***	***420***
21 (3000)	0.850	36.9 x 10^{-3}	27.4 x 10^{-3}	21.3 x 10^{-3}
28 (4000)	0.850	49.3 x 10^{-3}	36.5 x 10^{-3}	28.3 x 10^{-3}
35 (5000)	0.800	51.9 x 10^{-3}	42.9 x 10^{-3}	33.3 x 10^{-3}

4.2.2.2.10 ACI Maximum Steel Ratio ρ_{max}, (ACI318M, 2014), article 9.3.3.1

- In actual practice, the upper limit on ρ should be below ρ_b, for the following reasons:
  - For a beam with ρ exactly equal to ρ_b, the compressive strain limit of the concrete would be reached, theoretically, at precisely the same moment that the steel reaches its yield stress, without significant yielding before failure.
  - Material properties are never known precisely.
  - The actual steel area provided will always be equal to or larger than required, based on selected reinforcement ratio ρ, tending toward overreinforcement.
- Then to ensure under-reinforced behavior (ACI318M, 2014) (**9.3.3.1**) establishes a minimum net tensile strain ϵ_t, at the nominal member strength of **0.004**.
- By way of comparison, ϵ_y, the steel strain at the balanced condition, is 0.002 for Grade 420 (See Figure below).

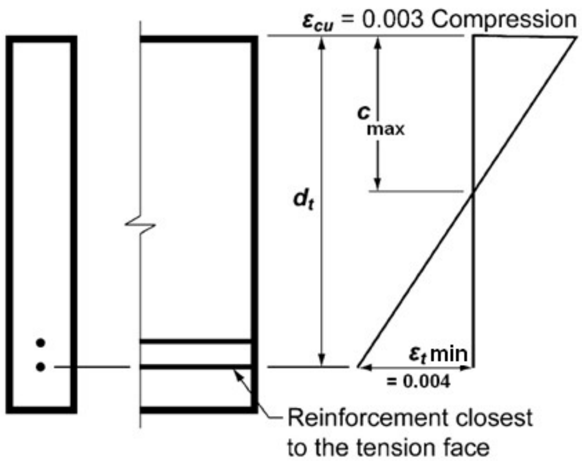


Figure 4.2-14: Strain limits for nonprestressed beams.

where d_t is the distance from extreme compression fiber to centroid of extreme layer of longitudinal tension steel.

- Based on strain distribution (compatibility conditions) c_{max} will be:

$$c_{max} = \frac{\epsilon_u}{\epsilon_u + 0.004} d_t$$
$$c_{max} = \frac{0.003}{0.003 + 0.004} d_t$$
$$c_{max} = 0.429\, d_t \ \blacksquare$$

Above relation is a general relation (i.e., it is applicable for rectangular and non-rectangular section).

- Above relation can be read as follows:
  According to ACI, the lowest permitted location of N.A. is located at 42.9% of d_t measured from compressive face.
- Thickness of equivalent rectangular stress distribution will be:

$$a_{max} = \beta_1 \frac{\epsilon_u}{\epsilon_u + 0.004} d_t$$

- Based on stress-strain relation and equilibrium conditions, ***Maximum Steel Area*** $A_{s\,max}$rmitted by the ACI will be:

$$\because \sum F_x = 0 \ \Rightarrow 0.85 f_c' b a_{max} = A_{s\,max}\, f_y$$
$$A_{s\ max}\, f_y = 0.85 f_c' b \left(\beta_1 \frac{\epsilon_u}{\epsilon_u + 0.004} d_t\right)$$

If d_t is conservatively taken equal to d, then:

$$A_{s\ max}\, f_y = 0.85 f_c' b \left(\beta_1 \frac{\epsilon_u}{\epsilon_u + 0.004} d\right) \quad \div f_y b d$$
$$\rho_{max} = 0.85 \beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} \ \blacksquare$$

Example 4.2-7

Compute the ACI ***Maximum Steel Ratio*** ρ_{max} for concretes that have compressive strength of $f_c' = 21MPa, 28MPa, and\ 35MPa$ when reinforced with reinforcing steel have grades of Grade 40, 50, and 60.

Solution

For our problem, calculations table will take the following form:

f_c' $MPa\,(psi)$	β_1	***Steel Grade***		
		40	***50***	***60***
		f_y MPa		
		280	***350***	***420***
21 (3000)	0.850	23.2×10^{-3}	18.6×10^{-3}	15.5×10^{-3}
28 (4000)	0.850	31.0×10^{-3}	24.8×10^{-3}	20.6×10^{-3}
35 (5000)	0.800	36.4×10^{-3}	29.1×10^{-3}	24.3×10^{-3}

4.2.2.2.11 ACI Flexure Strength Reduction Factor ∅ ((ACI318M, 2014) 21.2.2)

- The ACI Code encourages the use of lower reinforcement ratios by allowing higher strength reduction factors (∅) in such beams.
- To do that, ACI Code classified the concrete sections into:
  - Tension Controlled Section:
    - Is the member with a net tensile strain greater than or equal to 0.005. The corresponding strength reduction factor is 0.9.
    - Term member in above definition including beams and columns.
    - The selection of a net tensile strain of (0.005) is included to encompass the yield strain of all reinforcing steel including high-strength rebars.
  - Compression-Controlled Section:
    - Is the member that having a net tensile strain of less than 0.002.
    - Based on comparison with required strain of 0.004 for maximum steel ratio in beam, one can conclude that the term "member" in above definition including columns only, i.e. it must be clear that, it is not permitted by ACI Code to design concrete beams as compression-controlled members.
    - The strength reduction factor for compression-controlled members (columns) is 0.65. A value of 0.75 may be used if the members are spirally reinforced (see Figure below).

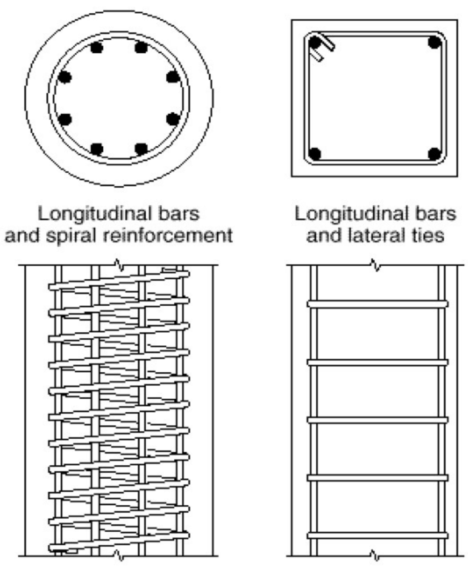


Figure 4.2-15: Spiral and tied columns.

    - Difference between Spiral Columns and Tied Columns will be discussed in more detail later.
    - A value of 0.002 corresponds approximately to the yield strain for steel with Grade 60.
  - Transition Zone Section:
    - Between net tensile strains of 0.002 and 0.005, the strength reduction factor varies lineally, and The ACI Code allows a linear interpolation of ∅ based on ϵ_t as shown in Figure below.

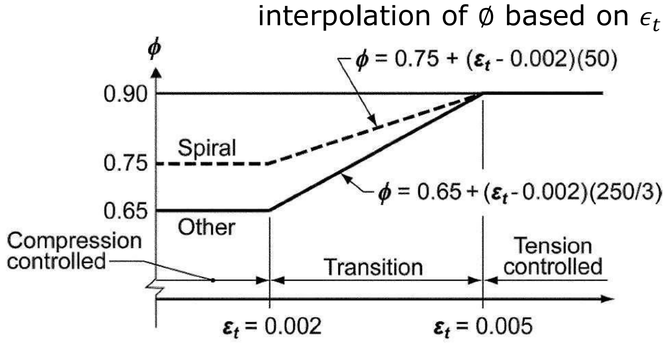


Figure 4.2-16: Strength reduction, ϕ, for transition factor.

- For beams, transition zone reduce to a range of 0.004 to 0.005 instead of range of 0.002 to 0.005 and as shown in Figure below.

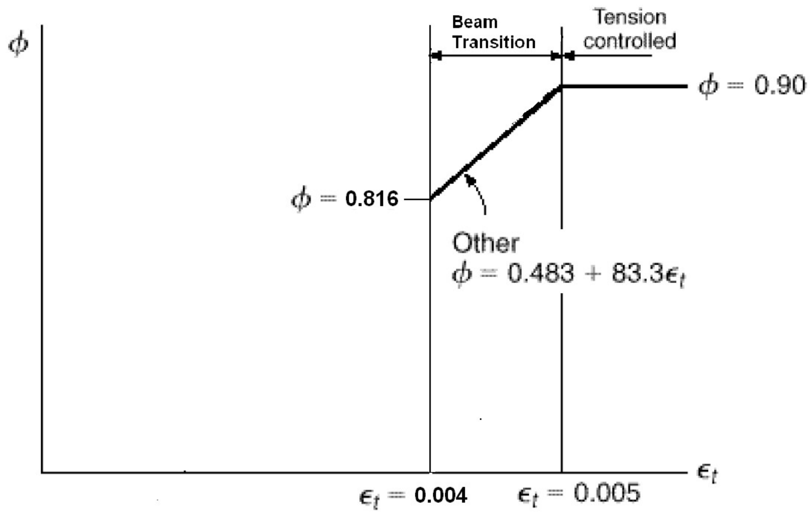


Figure 4.2-17: Transition zone for beams.

- Calculation of the nominal moment capacity frequently involves determination of the depth of the equivalent rectangular stress block "a" that related to "c" by the relation of $a = c/\beta_1$. Then it is some times more convenient to compute c/d ratios in terms the net tensile strain and as shown in Figure below.

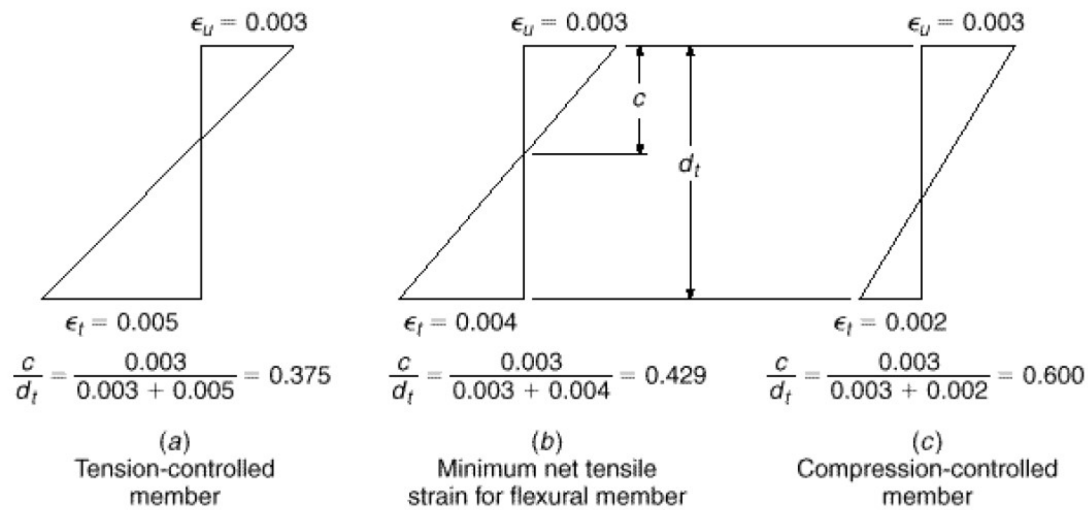


Figure 4.2-18: Section definition in terms of c/d_1 ratio.

Example 4.2-8

According to current design philosophy and for a beam with state of strains shown in Figure 4.2-19 below:

- Is the beam classified as failed or not?
- Is beam ratio, ρ, less than or greater than the maximum steel ratio, $\rho_{maximum}$?
- What is the flexural strength reduction factor, ϕ, for the beam?

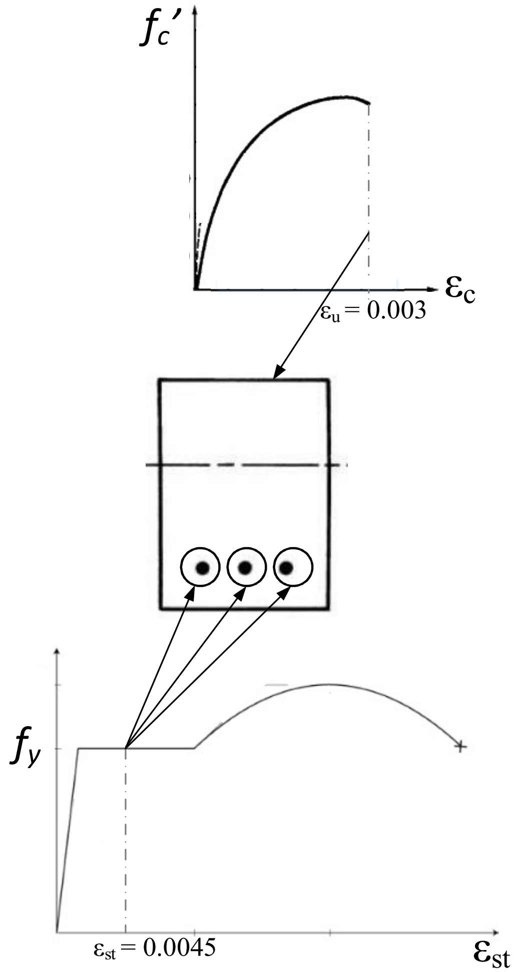


Figure 4.2-19: State of strains for Example 4.2-8.

Solution

- As concrete strain reaches 0.003, then the beam is at failure stage according to current ACI design philosophy.
- As steel strain of 0.0045 is greater than strain of 0.004 for $\rho_{Maximum}$, and as steel strain is inversely proportional to steel ratio, then provided steel ratio is lower than maximum ratio.
- The section is within the transition zone, $0.004 < \epsilon_t < 0.005$, then strength factored should be calculated based on following relation
  $\phi = 0.483 + 83.3\epsilon_t = 0.483 + 83.3 \times 0.0045 = 0.858$

4.2.2.2.12 ACI Minimum Reinforcement ((ACI318M, 2014), Article 9.6.1)

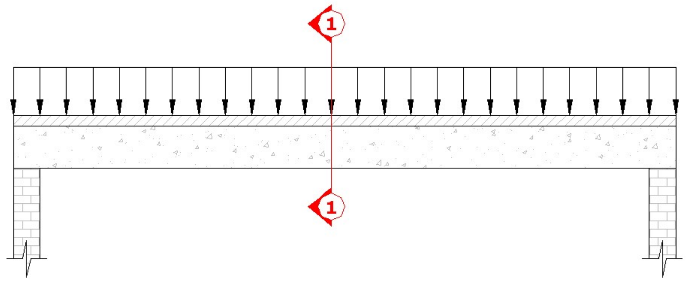
- Another mode of failure may occur in very lightly reinforced beams. If the flexural strength of the cracked section is less than the moment that produced cracking of the previously uncracked section the beam will fail immediately and without warning of distress upon formation of the first flexural crack.
- To ensure against this type of failure, a lower limit has been established for the reinforcement ratio by equating the cracking moment computed from the concrete modulus of rupture to the strength of the cracked section.
  $\because\ M_n = M_{Cracking} \quad \rightrightarrows \quad A_{s\ minimum}$
- Based on above concept, (ACI318M, 2014) (article 9.6.1) gives the following provisions for minimum steel Area:
  - At every section of a flexural member where tensile reinforcement is required by analysis. As provided shall not be less than that given by:
    $$A_{s\ minimum} = \frac{0.25\sqrt{f_c'}}{f_y} b_w d \geq \frac{1.4}{f_y} b_w d$$
    The relation $\frac{1.4}{f_y} b_w d$ had been derived based on substituting $f_c' = 31.4\ MPa$ into more accurate relation $\frac{0.25\sqrt{f_c'}}{f_y} b_w d$.
    For many years the relation $\frac{1.4}{f_y} b_w d$ was used as $A_{s\ min}$. For concrete with high strength, it is not sufficient and $A_{s\ min}$ must be computed based on the more accurate relation $\frac{0.25\sqrt{f_c'}}{f_y} b_w d$.
  - For members that have following properties:
    - Statically determinate.
    - With a flange in tension.

    Asmin shall be computed based on the following equation:
    $$A_{s\ min} = minimum\ \left(\frac{0.25\sqrt{f_c'}}{f_y} b_f d, \frac{0.50\sqrt{f_c'}}{f_y} b_w d\right)$$
    It's useful to note that above two conditions usually satisfy in the cantilever spans.
    When the flange of a section is in tension, the amount of tensile reinforcement needed to make the strength of the reinforced section equal that of the unreinforced section is about twice that for a rectangular section or that of a flanged section with the flange in compression. A higher amount of minimum tensile reinforcement is particularly necessary in cantilevers and other statically determinate members where there is no possibility for redistribution of moments.
  - The requirements of $A_{s\ min}$ need not be applied if, at every section, $A_{s\ Provided}$ is at least one-third greater than that required by analysis, i.e.:
    $$A_{S\ Provided} = 1\frac{1}{3} A_{S\ Require}$$
    This exception is intended to solve the problem of $A_{s\ min}$ for members that have large cross sectional areas.

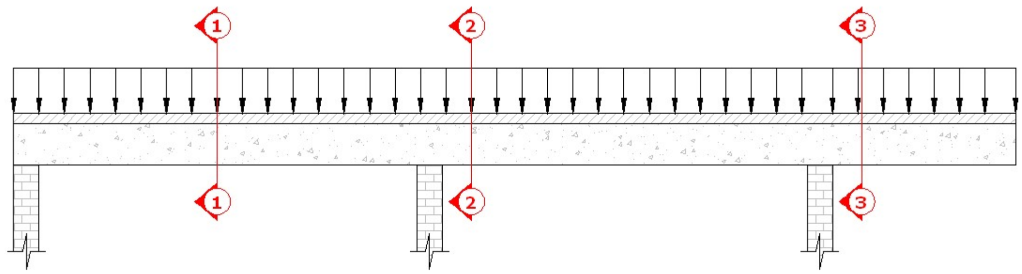
Example 4.2-9

State the relation that must be used for computing $A_{s\,min}$ for beams shown in Figure 4.2-20 below.

- Beam 1:



- Beam 2:



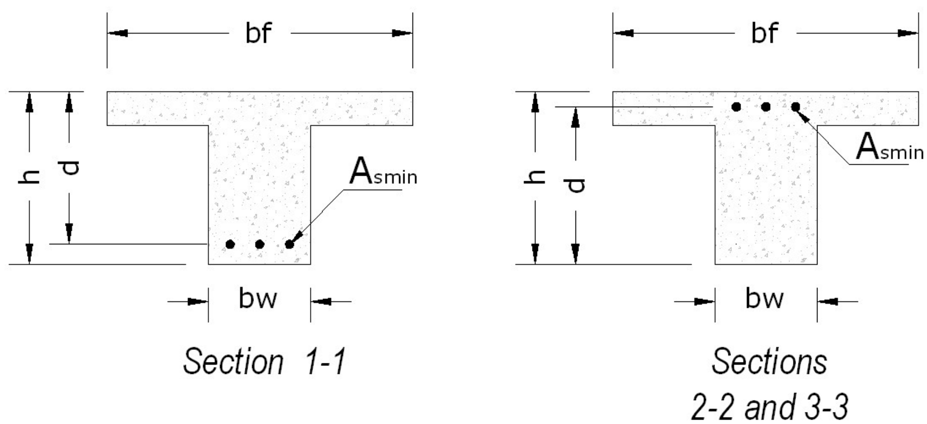


Figure 4.2-20: Beam for Example 4.2-9.

Solution

- For Beam 1:
  Section 1-1:
  As the section flange is under compression stress, then $A_{s\,min}$ is computed based on the following relation:

$$A_{s\,minimum} = \frac{0.25\sqrt{f_c'}}{f_y} b_w d \geq \frac{1.4}{f_y} b_w d$$

- For Beam 2:
  Section 1-1
  As the section flange is under compression stress, then $A_{s\,min}$ is computed based on the following relation:

$$A_{s\,minimum} = \frac{0.25\sqrt{f_c'}}{f_y} b_w d \geq \frac{1.4}{f_y} b_w d$$

  Section 2-2
  In spite of the section flange is under tensile stress, but as the span is a statically indeterminate span then $A_{s\,min}$ is computed based on the following relation:

$$A_{s\,minimum} = \frac{0.25\sqrt{f_c'}}{f_y} b_w d \geq \frac{1.4}{f_y} b_w d$$

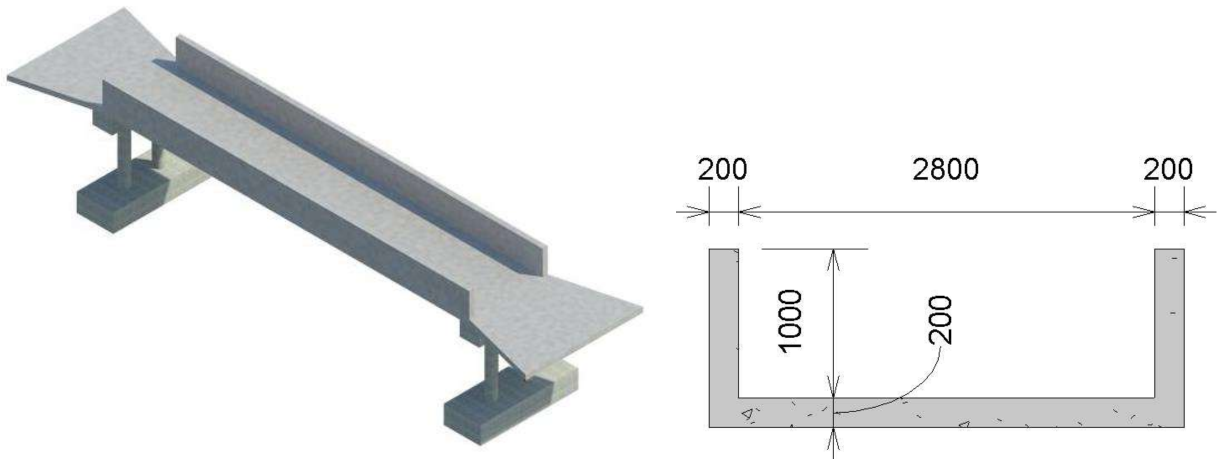
Section 3-3

As the section flange is under tensile stress, and the span is a statically determinate span then $A_{s\,min}$ is computed based on the following relation:

$$A_{s\,min} = minimum\left(\frac{0.25\sqrt{f_c'}}{f_y}b_f d, \frac{0.50\sqrt{f_c'}}{f_y}b_w d\right)$$

Example 4.2-10

For a simply supported pedestrian bridge shown in Figure 4.2-21 below, compute the minimum reinforcement area according to ACI requirements.



3D View **Cross Section**

Figure 4.2-21: Pedestrian bridge for Example 4.2-10.

Solution

For this statically determinate pedestrian bridge with a flange in tension, minimum flexure reinforcement should computed based on:

$$A_{s\,min} = minimum\left(\frac{0.25\sqrt{f_c'}}{f_y}b_f d, \frac{0.50\sqrt{f_c'}}{f_y}b_w d\right)$$

Assuming two layers of $\phi 20mm$ longitudinal rebars and $\phi 12mm$ stirrups:

$$d = 1200 - 40 - 12 - 20 - \frac{25}{2} = 1115mm$$

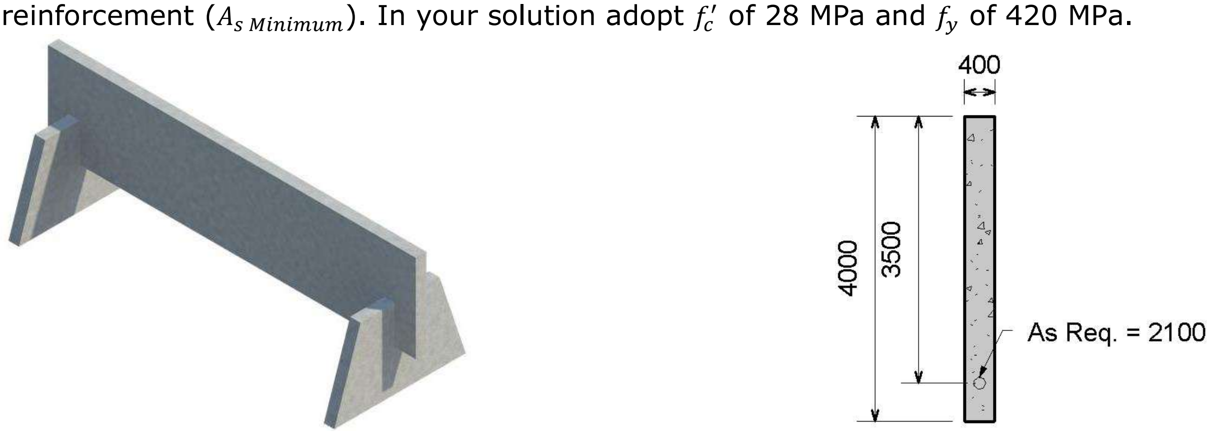
$$A_{s\,\min f\quad two\,legs} = minimum\left(\frac{0.25 \times \sqrt{21}}{420} \times 3200 \times 1115, \frac{0.50 \times \sqrt{21}}{420} \times (2 \times 200 \times 1115)\right)$$

$$A_{s\,\min for\,two\,legs} = minimum\ (9733, \quad 2809)$$

$$A_{s\,\min f\quad two\,legs} = 2809\,mm$$

Example 4.2-11

For monument that shown in Figure 4.2-22 below, required steel reinforcement ($A_{S\,Required}$) has been found equal to $2100mm^2$. Compare this area with ACI minimum reinforcement ($A_{s\,Minimum}$). In your solution adopt f_c' of 28 MPa and f_y of 420 MPa.



3D View **Cross Section**

Figure 4.2-22: Monument for **Example 4.2-11**.

Solution

According to ACI 9.6.1.3, the requirements of ACI 9.6.1.2 (traditional $A_{s\,Minimum}$ requirements), and ($A_{s\,Minimum}$ requirements for a statically determinate section with flange in tension) need not be applied if, at every section, $A_{s\,provided}$ is at least one-third greater than that required by analysis.

$$A_{s\,Minimum} = \frac{1.4}{f_y} b_w d = \frac{1.4}{420} \times 400 \times 3500 = 4667\ mm^2$$

$$A_{s\,Minimum} = 4667\ mm^2 > 1\frac{1}{3} \times 2100 = 2799\ mm^2$$

Then, use:

$A_{Provided} = 2799\ mm^2$

4.3 PROCEDURE AND EXAMPLES FOR FLEXURE ANALYSIS OF RECTANGULAR BEAMS WITH TENSION REINFORCEMENT

4.3.1 Procedures

- Generally, in an analysis problem the following information are knowns:
  - Beam dimensions and reinforcement (b, h, d, and A_s).
  - Materials strength (f_y and $f_c^{'}$).

  and the following information are required:
  - To check if the section is adequate to general requirements of ACI code to see if the provided steel reinforcement agrees with ACI limits on $A_{s\,max}$ and $A_{s\,min}$.
  - To compute the design flexural strength of section ($\emptyset M_n$).
  - To compute the maximum live or dead or other loads that can be supported by the considered beam.
- Based on above knowns and requirements, the procedure for analysis of a rectangular beam with tension reinforcement can be summarized as follows:
  - Check if the provided steel reinforcement agrees with ACI limits on $A_{s\,max}$ and $A_{s\,min}$:

    $\rho \leq \rho_{max} = 0.85\beta_1 \dfrac{f_c'}{f_y} \dfrac{\epsilon_u}{\epsilon_u + 0.004}$ $\quad A_s \geq A_{s\,minimum} = \dfrac{0.25\sqrt{f_c'}}{f_y} b_w d \geq \dfrac{1.4}{f_y} b_w d$
  - Compute the nominal strength M_n of the section:

    $$M_n = \rho f_y b d^2 \left(1 - 0.59 \frac{\rho f_y}{f_c'}\right)$$
  - Compute the strength reduction factor $\emptyset$:
    - Compute steel stain based on the following relations:

      $$a = \frac{A_s f_y}{0.85 f_c' b}$$

      $$c = \frac{a}{\beta_1}$$

      $$\epsilon_t = \frac{d - c}{c} \epsilon_u$$
    - If $\epsilon_t \geq 0.005$, then $\emptyset = 0.9$
    - If $\epsilon_t < 0.005$, then $\emptyset = 0.483 + 83.3\epsilon_t$
  - Compute the design strength $\emptyset M_n$ of the section:

    $\emptyset M_n = \emptyset \times M_n$
  - If maximum live, or dead, or other loads are required, then factored moment must be computed based on the following relation

    $M_u = \emptyset M_n$

    and the required loads can be computed based on the bending moment diagram of the problem under consideration.

4.3.2 Examples

Example 4.3-1

Check the adequacy of the beam of ***Example 4.2-1*** according to ACI Code (318M-14) and determine the maximum factored load P_u that can be supported by this beam. In your checking and computation assume that $f_c' = 25\,MPa$, $f_y = 400\,MPa$ and that beam selfweight can be neglected.

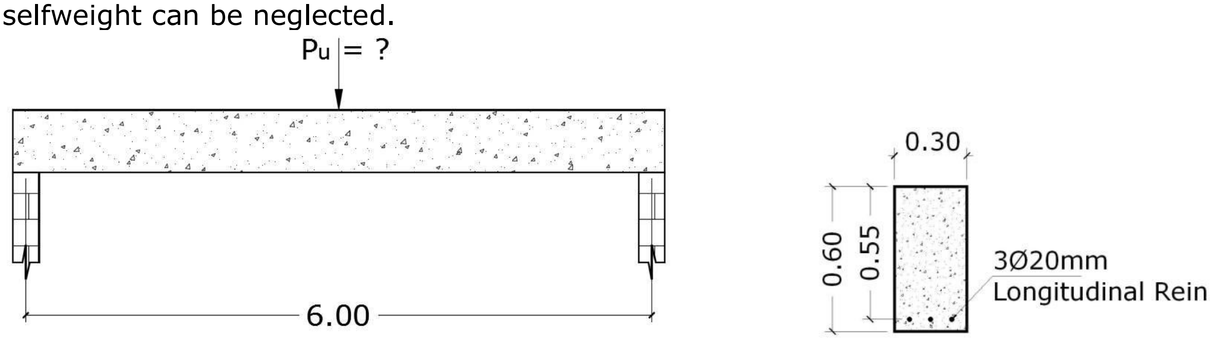


Figure 4.3-1: Simply supported beam for Example 4.3-1.

Solution

- Check if the provided steel reinforcement agrees with ACI limits on $A_{s\,max}$ and $A_{s\,min}$:

$$A_{bar} = \frac{\pi \times 20^2}{4} = 314\, mm^2$$
$$A_s = 3 \times 314 = 942\, mm^2$$
$$\rho = \frac{942}{300 \times 550} = 5.71 \times 10^{-3}$$
$$\rho_{max} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004}$$
$$\because f_c' < 28\, MPa \quad \therefore \beta_1 = 0.85$$
$$\rho_{max} = 0.85 \times 0.85 \times \frac{25}{400} \frac{0.003}{0.003 + 0.004} = 19.4 \times 10^{-3}$$
$$\therefore \rho < \rho_{max} \quad Ok. \blacksquare$$
$$A_{s\,minimum} = \frac{0.25\sqrt{f_c'}}{f_y} b_w d \geq \frac{1.4}{f_y} b_w d$$
$$\because f_c' < 31\, MPa$$
$$\therefore A_{s\,minimum} = \frac{1.4}{f_y} b_w d = \frac{1.4}{400} \times 300 \times 550 = 525\, mm^2 < A_s \quad Ok. \blacksquare$$

- Compute the nominal strength M_n of the section:

$$M_n = \rho f_y b d^2 \left(1 - 0.59\, \frac{\rho f_y}{f_c'}\right)$$
$$M_n = 5.71 \times 10^{-3} \times 400 \times 300 \times 550^2 \left(1 - 0.59\, \frac{5.71 \times 10^{-3} \times 400}{25}\right) = 196\, kN.m$$

- Compute strength reduction factor $\emptyset$:
  Compute steel stain based on the following relations:

$$a = \frac{A_s f_y}{0.85 f_c' b} \Rightarrow a = \frac{942\, mm^2 \times 400\, MPa}{0.85 \times 25\, MPa\, \times 300mm} = 59.1\, mm$$
$$c = \frac{a}{\beta_1} \Rightarrow c = \frac{59.1\, mm}{0.85} = 69.5\, mm$$
$$\epsilon_t = \frac{d - c}{c} \epsilon_u \Rightarrow \epsilon_t = \frac{550 - 69.5}{69.5} \times 0.003 = 20.7 \times 10^{-3}$$

  As $\epsilon_t > 0.005$, then $\emptyset = 0.9$.

- Compute section design strength $\emptyset M_n$:

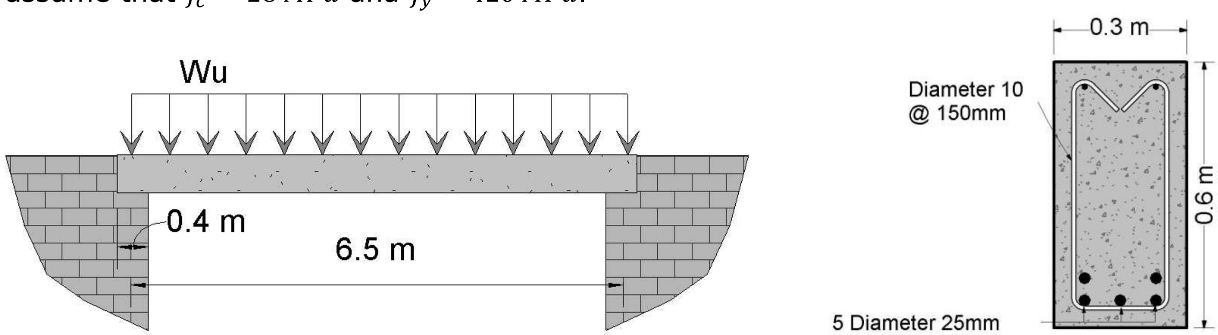
$$\emptyset M_n = \emptyset \times M_n \Rightarrow \emptyset M_n = 0.9 \times 196\, kN.m = 176\, kN.m \blacksquare$$

- Compute Maximum Factored Load P_u:
  As the selfweight can be neglected as stated in the example statement, then P_u can be computed based on the following relation:

$$\because M_u = \frac{P_u L}{4} = \emptyset M_n = 176\, kN.m \Rightarrow P_u = \frac{4 \times 176\, kN.m}{6.0\, m} = 117\, kN \quad \blacksquare$$

Example 4.3-2

Check flexure adequacy of a simply supported beam shown in Figure 4.3-2 below when subjected to a factored load of $W_u = 70 \frac{kN}{m}$ (Including beam selfweight). In your solution assume that $f_c' = 28\, MPa$ and $f_y = 420\, MPa$.



Elevation view. **A mid-span section**

Figure 4.3-2: Simply supported beam for Example 4.2-2.

Solution

- Check the proposed beam for general limits of the ACI code:

$$d = 600 - 40 - 10 - 25 - \frac{25}{2} = 512\ mm$$

$$A_s = \frac{\pi \times 25^2}{4} \times 5 = 2454\ mm^2$$

$$\rho_{Provided} = \frac{2454}{512 \times 300} = 16.0 \times 10^{-3}$$

$$\rho_{Maximum} = 0.85^2 \times \frac{28}{420} \times \frac{0.003}{0.007} = 20.6 \times 10^{-3} > \rho_{Provided} \therefore Ok.$$

$$A_{s\ Minimum} = \frac{1.4}{420} \times 300 \times 512 = 512\ mm^2 < A_{s\ Provided} \therefore Ok.$$

- Compute its nominal strength, M_n:

Instead of using the relation of

$$\mathrm{M_n} = \rho \mathrm{f_y} \mathrm{bd}^2 \left(1 - 0.59\ \frac{\rho \mathrm{f_y}}{\mathrm{f_c'}}\right)$$

the nominal flexural strength, M_n, can be determined based on simple statics with referring to forces diagram in Figure 4.3-3:

$$\Sigma F_x = 0$$

$$C = T$$

$$0.85 f_c' ba = A_s f_y$$

Solve for a

$$a = \frac{(A_s f_y)}{0.85 f_c' b} = \frac{420 \times 2454}{0.85 \times 28 \times 300} = 144\ mm$$

$$M_n = \Sigma M_{about\ C} = T \times Arm = (A_s f_y) \times \left(d - \frac{a}{2}\right)$$

$$= (2454 \times 420) \times \left(512 - \frac{144}{2}\right)$$

$$= 453\ kN.m$$

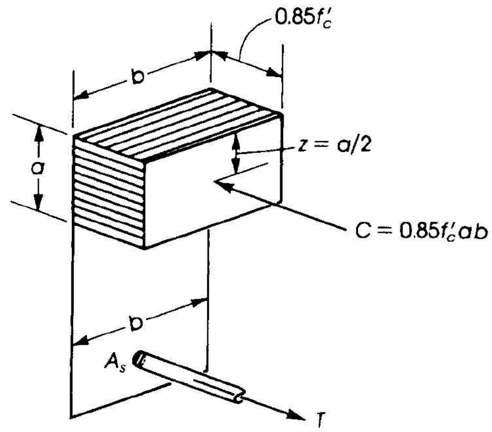


Figure 4.3-3: Forces diagram for a rectangular beam with tension reinforcement only.

This second approach is important as:

- It can be applied to sections other than rectangular sections.
- It focuses on basic principles of the applied mechanics and can be used without the need to remember of a ready relation.

- The strength reduction factor, ϕ, can be determined as follows:

$$c = \frac{144}{0.85} = 169\ mm$$

$$\epsilon_t = \frac{d - c}{c} \epsilon_u \Longrightarrow \epsilon_t = \frac{512 - 169}{169} \times 0.003 = 0.00609 > 0.005$$

Then:

$$\phi = 0.9$$

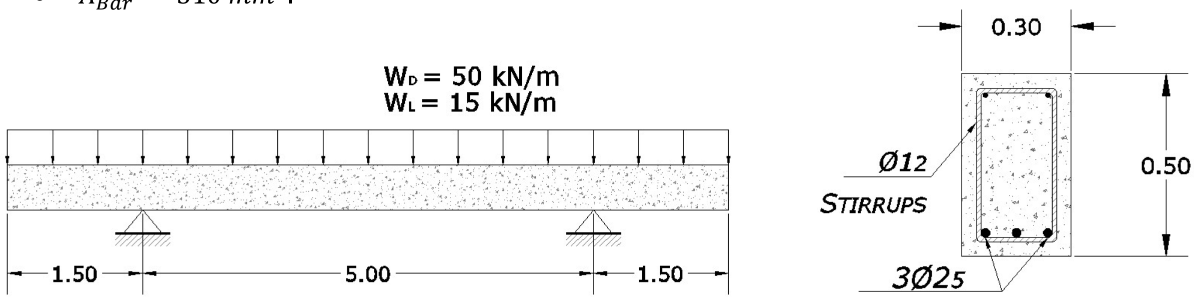
$$\phi M_n = 0.9 \times 453 = 408\ kN.m$$

$$M_u = \frac{W_u l^2}{8} = \frac{70 \times 6.5^2}{8} = 370\ kN.m < \phi M_n \therefore Ok.$$

Example 4.3-3

Check the adequacy of the proposed section when used at mid span of the beam shown in Figure 4.3-4. Assume that

- $f_c' = 21 MPa$ and f_y = 420MPa.
- Selfweight should be included.
- $A_{Bar} = 510\ mm^2$.



Elevation view. **Cross section**

Figure 4.3-4: Overhang beam for Example 4.2-3.

Solution

- Check if the provided reinforcement is accepted according to ACI requirements:

$$A_{bar} = 510\ mm^2$$
$$A_s = 3 \times 510 = 1530\ mm^2$$
$$d = 500 - 40 - 12 - \frac{25}{2} = 435\ mm$$
$$\rho_{Provided} = \frac{1530\ mm^2}{435 \times 300\ mm^2} = 11.7 \times 10^{-3}$$
$$\rho_{Maximum} = 0.85^2 \times \frac{21}{420} \times \frac{0.003}{0.007} = 15.5 \times 10^{-3}$$
$$\rho_{Provided} < \rho_{Maximum} \therefore Ok.$$
$$\because f_c' < 31\ MPa$$
$$A_{s\ minimum} = \frac{1.4}{420} \times 300 \times 435 = 435 mm^2$$
$$A_s > A_{s\ minimum}\ Ok.$$

- Compute Nominal Flexure Strength:

$$M_n = 11.7 \times 10^{-3} \times 420 \times 300 \times 435^2 \times (1 - 0.59 \times \frac{11.7 \times 10^{-3} \times 420}{21})$$
$$M_n = 240\ kN.m$$

- Compute flexure strength reduction factor:

$$a = 120\ mm$$
$$c = 141\ mm$$
$$\epsilon_t = \frac{435 - 141}{141} \times 0.003 = 6.26 \times 10^{-3} > 0.005$$
$$\therefore \phi = 0.9$$

- Design Moment:

$$\phi M_n = 0.9 \times 240 = 216\ kN.m$$

- Check section adequacy when used at mid-span:

$$W_{self} = 0.3 \times 0.5 \times 24 = 3.6 \frac{kN}{m}$$
$$W_D = 53.6 \frac{kN}{m}$$
$$W_u = maximum(1.4 \times 53.6\ or\ 1.2 \times 53.6 + 1.6 \times 15)$$
$$W_u = maximum(75.0\ or\ 88.3) = 88.3 \frac{kN}{m}$$
$$M_{u\ support} = \frac{88.3 \times 1.5^2}{2} = 99.8\ kN.m$$
$$M_{u\ @\ mid-span} = \frac{88.3 \times 5^2}{8} - 99.3 = 177\ kN.m < \phi M_n\ Ok$$

4.3.3 Homework Problems

In addition to practice on concepts, *Problem* 4.3-1, *Problem 4.3-2*, and *Problem 4.3-3* aim to show how M_n can be affected by changing in material properties (f_y and f_c').

Problem 4.3-1

A rectangular beam has a width 250 mm, and an effective depth 505 mm. It is reinforced with 3 Φ 25 (assume A_{bar} = 510 mm²). If f_y = 420 MPa and f_c' = 20 MPa. Check the beam adequacy and compute its design flexural strength according to the ACI Code.

Answers

- Check if the provided steel reinforcement agrees with ACI limits on A_{smax} and A_{smin}:
  $A_s = 1\,530\ \text{mm}^2$
  $\rho = 12.1 \times 10^{-3}$
  $\rho_{max} = 14.7 \times 10^{-3}$
  $\therefore\ \rho < \rho_{max}$ Ok. ■
  $A_{s\,minimum} = 421\ \text{mm}^2 < A_s$ Ok. ■
- Compute section nominal strength M_n :
  $M_n = 275\ \text{kN.m}$
- Compute strength reduction factor ∅:
  a. Compute steel stain:
     $a = 151\ \text{mm}$
     $c = 178\ \text{mm}$
     $\epsilon_t = 0.00551$
  b. $\epsilon_t > 0.005$, then $\emptyset = 0.9$
- Compute section design strength $\emptyset M_n$:
  $\emptyset M_n = 247\ \text{kN.m}$■

Problem 4.3-2

Same as Problem 4.3-1 except that f_c'=40 MPa. Compare the flexure strength for this problem with that of Problem 4.3-1.

Answers

- Check if the provided steel reinforcement agrees with ACI limitss on A_{smax} and A_{smin}:
  $A_s = 1\,530\ \text{mm}^2$
  $\rho = 12.1 \times 10^{-3}$
  $\beta_1 = 0.76 \ \geq 0.65$ Ok.
  $\rho_{max} = 26.4 \times 10^{-3}$
  $\therefore\ \rho < \rho_{max}$ Ok. ■
  $A_{s\,minimum} = 475\ \text{mm}^2 < A_s$ Ok. ■
- Compute section nominal strength Mn:
  $M_n = 300\ \text{kN.m}$
- Compute strength reduction factor ∅:
  - Compute steel stain:
    $a = 75.6\ \text{mm}$
    $c = 99.5\ \text{mm}$
    $\epsilon_t = 0.0122$
  - $\epsilon_t > 0.005$, then $\emptyset = 0.9$
- Compute section design strength $\emptyset M_n$:
  $\emptyset M_n = 270\ \text{kN.m}$■
- Comparison with the design strength of Problem 4.3-1:
  Increasing percentage in design strength due to increasing f_c' from 20 MPa to 40 MPa can be computed as follows:
  $$\text{Increasing Percent} = \frac{270 - 247}{247} \times 100\% = 9.31\%$$
  Note that doubling the concrete strength increased $\emptyset M_n$ by only 9.31%■.

Problem 4.3-3

Same as Problem 4.3-1 except that f_y = 300 MPa. Compare the flexure strength for this problem with that of Problem 4.3-1.

Answers

- Check if the provided steel reinforcement agrees with ACI limits on A_{smax} and A_{smin}:
  $A_s = 1\,530\ \text{mm}^2$
  $\rho = 12.1 \times 10^{-3}$
  $\rho_{max} = 20.6 \times 10^{-3}$
  $\therefore\ \rho < \rho_{max}$ Ok. ■
  $A_{s\ minimum} = 589\ \text{mm}^2 < A_s$ Ok. ■
- Compute section nominal strength M_n :
  $M_n = 207\ \text{kN.m}$
- Compute strength reduction factor $\emptyset$:
  - Compute steel stain:
    $a = 108\ \text{mm}$
    $c = 127\ \text{mm}$
    $\epsilon_t = 0.00893$
  - $\epsilon_t > 0.005$, then $\emptyset = 0.9$
- Compute section design strength $\emptyset M_n$:
  $\emptyset M_n = 186\ \text{kN.m}$■
- Comparison with the design strength of Problem 4.3-1:
  $$\text{Increasing Percent} = \frac{|186 - 247|}{247} \times 100\% = 24.7\%$$
  Note that reducing f_y by ($\frac{420-300}{420} = 28.6\%$) reduces $\emptyset M_n$ by 24.7%■.

Based on results of Problem 4.3-1*,* Problem 4.3-2*, and* Problem 4.3-3 *one concludes that the effect of changing in steel yield stress (f_y) is more significant than the effect of changing in concrete compressive strength (f_c').*

Problem 4.3-4

A rectangular beam has a width of 305 mm, and an effective depth of 444 mm. It is reinforced with $4\emptyset 29mm$ (assume $A_{bar} = 645mm^2$). If $f_y = 414MPa$ and $f_c' = 27.5\ MPa$. Check the beam adequacy and compute its design flexural strength according to the ACI Code.

Aim of the Problem:

This problem aims to show solution procedures for a section in the transition zone.

Answers

- Check if the provided steel reinforcement is in agreement with ACI requirements on A_{smax} and A_{smin}:
  $A_s = 2\,580\ \text{mm}^2$
  $\rho = 19.1 \times 10^{-3}$
  $\rho_{max} = 20.6 \times 10^{-3}$
  $\therefore\ \rho < \rho_{max}$ Ok. ■
  $A_{s\ minimum} = 458\ \text{mm}^2 < A_s$ Ok. ■
- Compute section nominal strength M_n :
  $M_n = 395\ \text{kN.m}$
- Compute strength reduction factor $\emptyset$:
  - Compute steel stain:
    $a = 150\ \text{mm}$
    $c = 176\ \text{mm}$
    $\epsilon_t = 0.00457$
  - $\epsilon_t < 0.005$, then $\emptyset = 0.864$
- Compute section design strength $\emptyset M_n$:
  $\emptyset M_n = \emptyset \times M_n$
  $\emptyset M_n = 0.864 \times 395\ \text{kN.m} = 341\ \text{kN.m}$■

Problem 4.3-5

Determine if the beam shown in Figure 4.3-5 is adequate as governed by ACI Code (ACI 318M-14). If $f_y = 414\text{MPa}$ and $f_c' = 27.5\ \text{MPa}$. Assume that $A_{bar} = 510\text{mm}^2$.

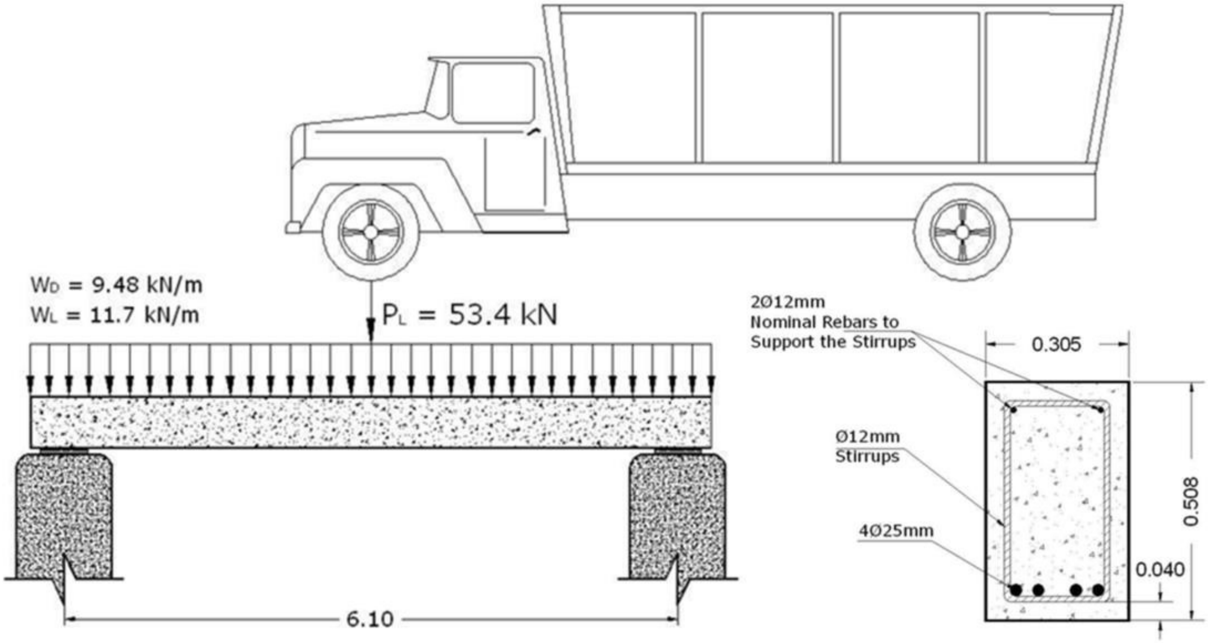


Figure 4.3-5: A simple bridge for the Problem 4.3-5.

Answers

- Check if the provided steel reinforcement agrees with ACI limits on $A_{s\,max}$ and $A_{s\,min}$:
  $A_s = 2\,040\ mm^2$
  $d = 508 - 40_{Cover} - 12_{Stirrups} - \frac{25}{2}_{\text{Half the Bar Diameter}} = 444\ mm$
  $\rho = 15.1 \times 10^{-3}$
  $\rho_{max} = 20.6 \times 10^{-3} \Rightarrow \therefore\ \rho < \rho_{max}$ Ok. ■
  $A_{s\,minimum} = 458\ mm^2 < A_s$ Ok. ■
- Compute the nominal strength M_n:
  $M_n = 325\ kN.m$
- Compute the strength reduction factor $\emptyset$:
  - Compute steel stain:
    $a = 118\ mm$
    $c = 139\ mm$
    $\epsilon_t = 0.00658$
  - $\epsilon_t > 0.005$, then $\emptyset = 0.9$
- Compute the design strength $\emptyset M_n$:
  $\emptyset M_n = 293\ kN.m$ ■
- Compute the Factored Moment:
  - Moment due to the Dead Loads:
    $W_{Selfweight} = 3.72 \frac{kN}{m} \Rightarrow W_{Dead} = 13.2 \frac{kN}{m}$
    $M_{Dead} = 61.4\ kN.m$
  - Moment due to the Live Load:
    $M_{Live} = 136\ kN.m$
  - Factored Moment M_u:
    $M_u =$ Maximum of $(1.4M_D$ or $1.2M_D + 1.6M_L)$
    $M_u =$ Maximum of $[86$ or $291] = 291\ kN.m$ ■
  - Check Section Adequacy:
    $\because\ \emptyset M_n = 293\ kN.m > M_u = 291\ kN.m \quad \therefore$ Ok. ■

Problem 4.3-6

Check adequacy of the beam shown in Figure 4.3-6 for bending according to the requirements of ACI 318M-14. Assume that $f'_c = 28\ MPa$, $f_y = 420MPa$ and $A_{Bar} = 510mm^2$.

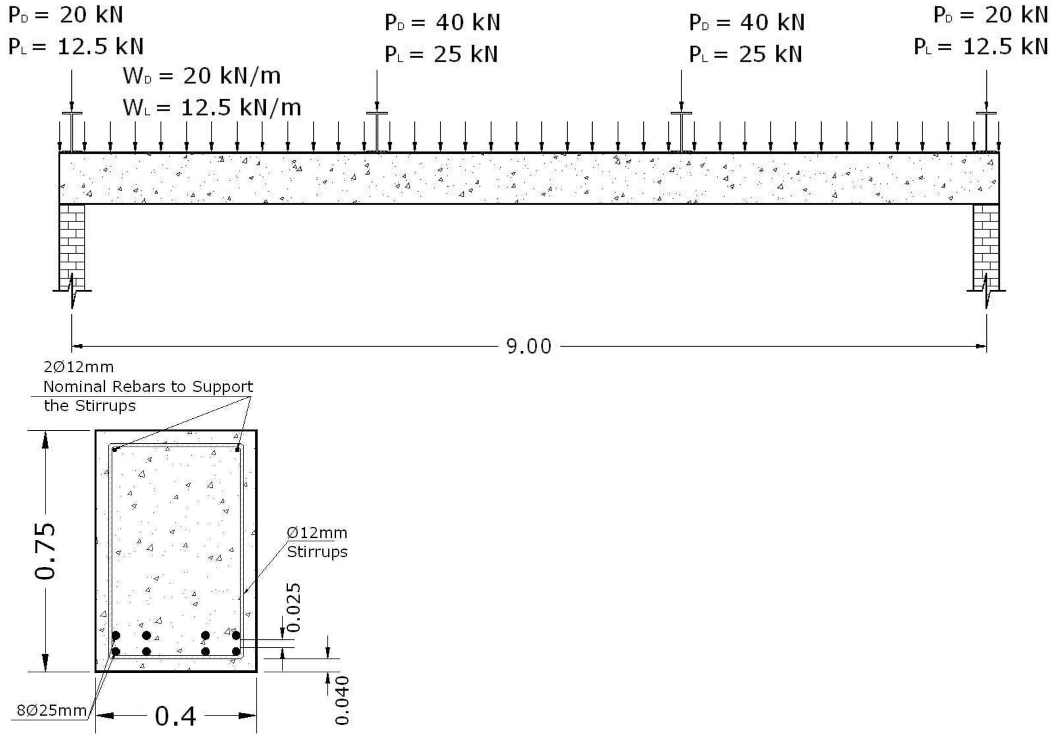


Figure 4.3-6: A simple beam for the Problem 4.3-5.

Answers

- Check if the provided steel reinforcement is in agreement with ACI requirements on $A_{s\,max}$ and $A_{s\,min}$:
  $A_s = 4\,080\ \text{mm}^2$
  $d = 750 - 40_{Cover} - 12_{Stirrups} - 25_{\text{ the Bar Diameter}} - \left(\frac{25}{2}\right)_{\text{Half the Spacing between Layers}} = 660\ \text{mm}$
  $\rho = 15.5 \times 10^{-3}$
  $\rho_{max} = 20.6 \times 10^{-3}\ \Rightarrow \therefore\ \rho < \rho_{max}$ Ok. ■
  $A_{s\,minimum} = 880\ \text{mm}^2 < A_s$ Ok. ■
- Compute the nominal strength M_n:
  $M_n = 979\ \text{kN.m}$
- Compute the strength reduction factor $\emptyset$:
  - Compute steel stain:
    $a = 180\ \text{mm}$
    $c = 212\ \text{mm}$
    $\epsilon_t = 0.00634$
  - $\epsilon_t > 0.005$, then $\emptyset = 0.9$.
- Compute the design strength $\emptyset M_n$:
  $\emptyset M_n = 881\ \text{kN.m}$■
- Compute the Factored Moment:
  - Moment due to the dead loads:
    $W_{Selfweight} = 7.2\frac{kN}{m} \Rightarrow W_{Dead} = 27.2\frac{kN}{m} \Rightarrow M_{Dead} = 395\ \text{kN.m}$
  - Moment due to the live load:
    $M_{Live} = 202\ \text{kN.m}$
  - Factored Moment M_u:
    $M_u = \text{Maximum of } (1.4M_D \text{ or } 1.2M_D + 1.6M_L)$
    $M_u = \text{Maximum of } [553 \text{ or } 797] = 797\ \text{kN.m}$ ■
  - Check Section Adequacy:
    $\because\ \emptyset M_n = 881\ \text{kN.m} > M_u = 797\ \text{kN.m}\ \ \therefore$ Ok. ■

4.4 PRACTICAL FLEXURE DESIGN OF A RECTANGULAR BEAM WITH TENSION REINFORCEMENT ONLY AND PRE-SPECIFIED DIMENSIONS (b AND h)

4.4.1 Essence of Problem

- In the design problem, usually the beam span, beam dimensions (b, and h), dead, live, and other loads are defined based on functional and/or architectural requirements.
- Materials strength ($f_c^{'}$ and f_y) are generally selected based on the available materials in the local market.
- Then, the main unknown in the design process is the reinforcement detail that can be summarized as follows:
  - Number and diameters of rebars.
  - Number of layers that required for these rebars.
  - Required concrete cover to protect the reinforcement against probable corrosion.
  - Points where bars are no longer needed for moments, i.e., points for bending or stopping of reinforcement, out the scope of this chapter and will be discussed thoroughly in Chapter 5.

4.4.2 Design Procedure

Based on above known and unknown quantities, design procedure can be summarized as follows:

1. Computed required factored applied moment (M_u) based on given loads and spans. Beam selfweight can be computed based on given dimensions (b, and h).
2. Computed the required nominal or theoretical flexure strength ($\mathrm{M_n}$) based on the following relation:
   $$\mathrm{M_n} = \frac{\mathrm{M_u}}{\emptyset}$$
   Strength reduction factored can be assumed 0.9, and checked later.
3. Compute the effective beam depth "d":
   Generally, in engineering practice the reinforcements are either put in one or two layers. Depend on number of layers, "d" can be computed based on one of the following relations[2]:
   $$\mathrm{d_{for\ One\ Layer}} = \mathrm{h - Cover - Stirrups} - \frac{\mathrm{Bar\ Diameter}}{2}$$
   $$d_{for\ Two\ Layer} = h - Cover - Stirrups - Bar\ Diameter - \frac{Spacing\ between\ Layers}{2}$$
   Based on above two relations, one can conclude that the designer must assume preliminary values for following items to be able to compute the effective depth "d":
   a. ***Number of Layers***
   Heavy loads required a large reinforcement area that cannot be put in one layer and vice versa.
   Diagnosis between heavy loads and light loads is generally depends on designer experience. For examination purposes, number of layers may be mentioned in question statement.
   b. ***Concrete Cover***
   To provide the steel with adequate concrete protection against corrosion, the designer must maintain a certain minimum thickness of concrete cover outside of the outermost steel:
   The thickness required will vary, depending upon:
      i. The type of member.

[2] It is useful to note that, the equation of effective depth for a beam with two reinforcement layers is based on the assumption that centroid of two layers lies at mid distance between two layers. This assumption is correct for two identical layers. For other conditions, it gives conservative results that accepted in the engineering practice.

ii. Conditions of exposure.
iii. Bar diameter.

According to article 20.6.1.3.1 of the (ACI318M, 2014), concrete cover can be determined based on Table 4.4-1 below.

Table 4.4-1: Specified concrete cover for cast-in-place nonprestressed concrete members, Table 20.6.1.3.1 of the (ACI318M, 2014).

Concrete exposure	Member	Reinforcement	Specified cover, mm
Cast against and permanently in contact with ground	All	All	75
Exposed to weather or in contact with ground	All	No. 19 through No. 57 bars	50
		No. 16 bar, MW200 or MD200 wire, and smaller	40
Not exposed to weather or in contact with ground	Slabs, joists, and walls	No. 43 and No. 57 bars	40
		No. 36 bar and smaller	20
	Beams, columns, pedestals, and tension ties	Primary reinforcement, stirrups, ties, spirals, and hoops	40

As a general case, (ACI318M, 2014) requirements for beams (that not exposed to weather) can be summarized in Figure 4.4-1.

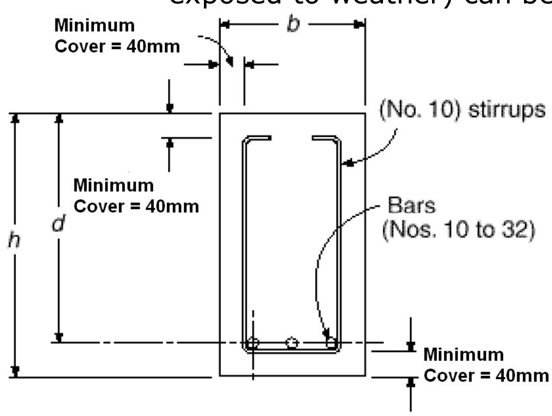


Figure 4.4-1: Cover requirements for beams not exposed to weather.

c. ***Bar Diameter***

As discussed in Chapter 2, data for metric rebars according to ASTM are summarized in Table 4.4-2 below.

No. 16 to No. 25 is usually used in for beam reinforcement. No. 13 rebars may be used in minor works like lintel beam reinforcement.

d. ***Stirrups Diameter***

Stirrups are the hoop reinforcement that used for shear reinforcement.

No. 10 and No. 13 are usually used as stirrups. No. 16 may be used in some large beams with heavy loads.

e. ***Spacing between Layers***

According to article 25.2.2 of (ACI318M, 2014), for parallel nonprestressed reinforcement placed in two or more horizontal layers,

i. reinforcement in the upper layers shall be placed directly above reinforcement in the bottom layer
ii. a clear spacing between layers of at least 25 mm.

Table 4.4-2ASTM standard metric reinforcing bars

Bar size, no.*	Nominal diameter, mm	Nominal area, mm^2	Nominal mass, kg/m
10	9.5	71	0.560
13	12.7	129	0.994
16	15.9	199	1.552
19	19.1	284	2.235
22	22.2	387	3.042
25	25.4	510	3.973
29	28.7	645	5.060
32	32.3	819	6.404
36	35.8	1006	7.907
43	43.0	1452	11.38
57	57.3	2581	20.24

*Bar numbers approximate the number of millimeters of the nominal diameter of the bar.

4. Compute the required steel ratio $\rho_{Required}$:
   The basic relation between variables for rectangular beam with tension reinforcement:
   $$M_n = \rho f_y bd^2\left(1 - 0.59\ \frac{\rho f_y}{f'_c}\right)$$
   can be solved to compute $\rho_{Required}$ from known f_y, f'_c, b, d, and M_n and as follows:
   $$\rho_{Required} = \frac{1 - \sqrt{1 - 2.36\frac{M_n}{f'_c bd^2}}}{1.18 \times \frac{f_y}{f'_c}}$$ ■
   Why only smaller one of two roots is adopted in the solution of above relation will be discussed thoroughly in Example 4.4-1 below.
   If the quantities under the square root $(1 - 2.36\frac{M_n}{f'_c bd^2})$ has a negative value, this gives an indication that the failure is a compression failure and that the section is rejected according to ACI Code requirements. Then the designer must increase one or both of beam dimensions (b and h) and resolve the problem from Step 3.
5. Check if the beam failure is secondary compression failure or compression failure:
   If:
   $$\rho_{Required} > \rho_{max} = 0.85\beta_1\ \frac{f'_c}{f_y}\ \frac{\epsilon_u}{\epsilon_u + 0.004}$$
   Then the designer must increase one or both of beam dimensions (b and h) and resolve the problem from Step 3.
6. Compute the required steel area:
   $$A_{S\ Required} = \rho_{Required} \times bd$$
7. Compute required rebars number:
   $$\text{No. of Rebars} = \frac{A_{S\ Required}}{A_{Bar}}$$
   Round the required rebars number to nearest larger integer number.
8. Check if the available width "b" is adequate to put the rebars in a single layer:
   $$b_{required} = 2 \times \text{Side Cover} + 2 \times \text{Stirrups Diamter} + \text{No. of Rebars} \times \text{Bar Diameter} + (\text{No. of Rebars} - 1) \times \text{Spacing between Rebars}$$
   If
   $$b_{required} > b_{available}$$
   Then reinforcement cannot be put in a single layer. If your calculations have be based on assumption of single layer, then you must retain to Step 3 and recalculate "d" based on two reinforcement layers.
   According to article 25.2.1 of (ACI318M, 2014), for parallel nonprestressed reinforcement in a horizontal layer, clear spacing shall be at least the greatest of:

$S_{Clear\ Minimum} = Maximum\left[25mm, d_b, \frac{4}{3}d_{agg}\right]$

As the maximum size of aggregate, d_{agg}, is usually selected to satisfy above relation, then it reduces into:

$S_{Clear\ Minimum} = Maximum\ [25mm, d_b]$

9. Checking cracks width or checking for $S_{Maximum}$:
    a. As was discussed in Chapter 1 (Design Criteria) and in Second Stage of beam behavior (Elastic Cracked Section), current design philosophy doesn't aim to design a concrete beam without cracks but aims to limit these cracks to be fine (called hairline cracks), invisible to a casual observer, permitting little if any corrosion of the reinforcement.
    b. Methods of cracks control:
        i. Previously, ACI 318 requirements were based on computing of actual cracks width (w in Figure below) and compared it with a maximum limits.

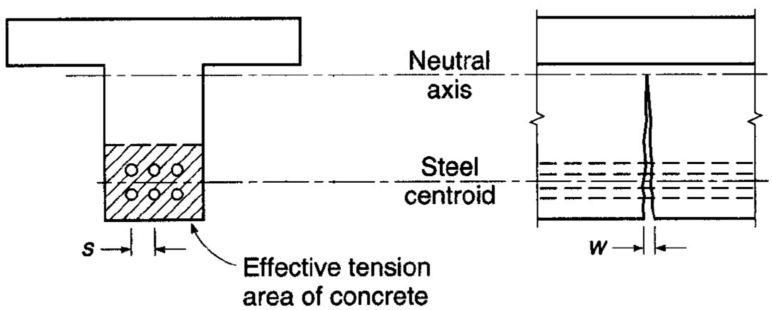


Figure 4.4-2: Crack width computations terminology of previous code editions.

        ii. Currently, ACI Code adopted a simpler approach that can be used for structures that ***not subjected to very aggressive exposure or designed to be watertight***.
        iii. This simplified approach based on following experimental and analytical fact:
        Generally, to control cracking, it is better to use a larger number of smaller-diameter bars to provide the required A_s than to use the minimum number of larger bars.
        iv. Then instead of working with crack width "w", we can work based on <u>center to center</u> spacing between bars "s in Figure above" which gives an indication on bars size as if we use a smaller number of bars with larger diameter instead of larger number of bars with smaller diameter spacing "s" will be larger and vice versa.
        v. According to ACI Code (24.3.2) maximum spacing (center to center) between bars for crack control purposes can be computed as follows:
        $s = 380\frac{280}{f_y} - 2.5c_c \leq 300\frac{280}{f_s}$
        where
            1. f_s is the stress in reinforcement closest to the tension face at service load shall be computed based on the unfactored moment. It shall be permitted to take fs as $2/3\,f_y$.
            2. cc is concrete clear cover.
        vi. For traditional reinforcement of Grade 60 and traditional cover, ACI Code commentary (R24.3.2) shows that s_{max} can be taken as 250 mm.

10. Check with $A_{s\ minimum}$ requirements:
If

$A_{S\ Provided} < A_{s\ minimum} = \frac{0.25\sqrt{f_c'}}{f_y} b_w d \geq \frac{1.4}{f_y} b_w d$

Then used:

$A_{S\ Provided} = A_{s\ minimum}$

And recalculate rebars number based on this area.

11. Check the assumption of $\emptyset = 0.9$:
    a. Compute steel stain based on the following relations:
    $$a = \frac{A_s f_y}{0.85 f_c' b}$$
    $$c = \frac{a}{\beta_1}$$
    $$\epsilon_t = \frac{d - c}{c} \epsilon_u$$
    b. If $\epsilon_t \geq 0.005$, then $\emptyset = 0.9$ Ok.
    c. If $\epsilon_t < 0.005$, then compute more accurate $\emptyset$:
    $\emptyset = 0.483 + 83.3\epsilon_t$
    and retain to Step 2.
12. Draw final detailed beam section.

4.4.3 Examples

Example 4.4-1

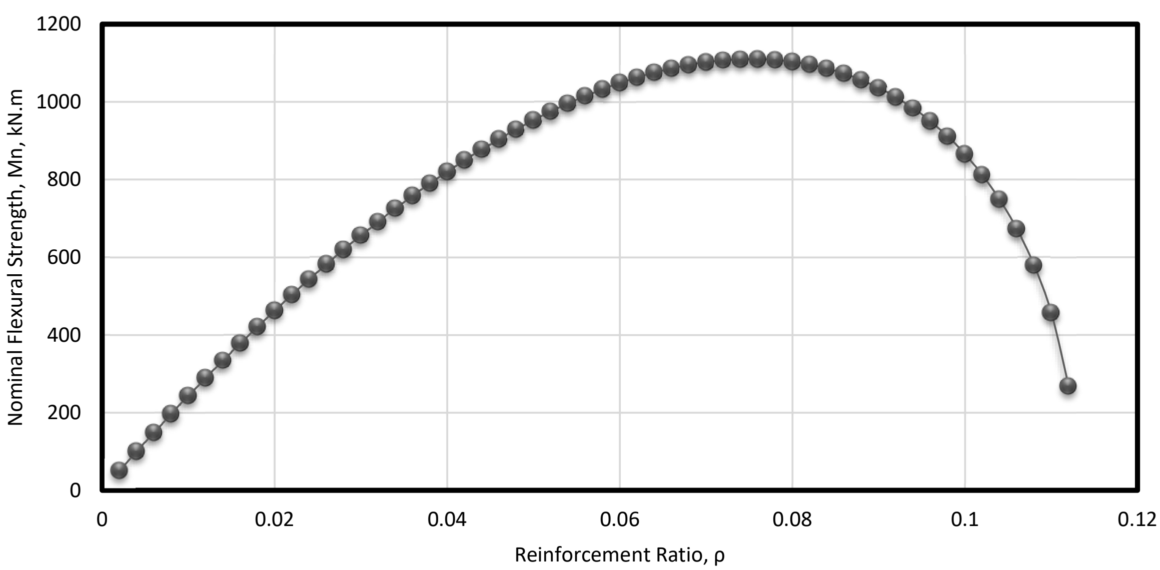
For a beam with an effective depth of 450mm and a width of 300mm, draw a relation between provided reinforcement ratio, $\rho_{Provided}$, and corresponding nominal flexural strength, M_n, and then discuss why only smaller root is adopted in the solution of relation.

$$\rho_{Required} = \frac{1 - \sqrt{1 - 2.36 \frac{M_n}{f_c' b d^2}}}{1.18 \times \frac{f_y}{f_c'}}$$

In your solution, assume $f_c' = 28\ MPa$ and $f_y = 420\ MPa$.

Solution

- For each value of ρ, compute corresponding value of M_n based on following relation:
  $$M_n = \rho f_y b d^2 \left(1 - 0.59\ \frac{\rho f_y}{f_c'}\right)$$
  and draw resulting values as shown in Figure below.



- For a specific flexural strength, the larger root of above relation either gives reinforcement ratio greater than $\rho_{maximum}$, a rejected design, or gives a larger value compared to first root, uneconomical design. Therefore, only smaller root should be considered in commuting required reinforcement ratio for a specific nominal strength.

Example 4.4-2

Design a simply supported rectangular reinforced concrete beam shown Figure 4.4-3 below. It is known that this beam is not exposed to weather and not in contact with ground.

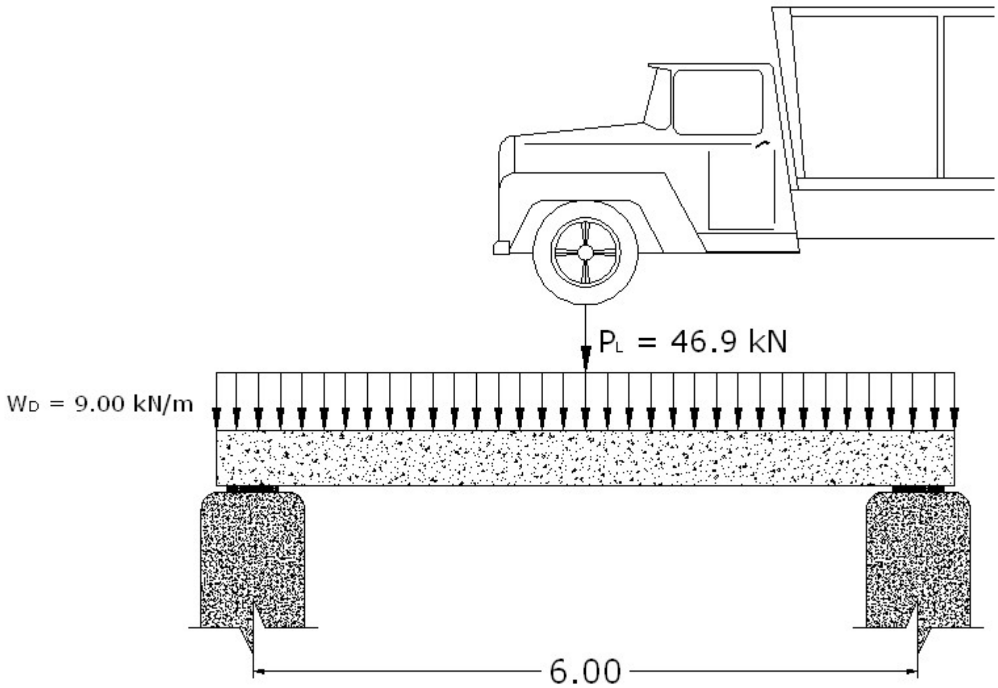


Figure 4.4-3: Simply supported bridge for Example 4.4-2.

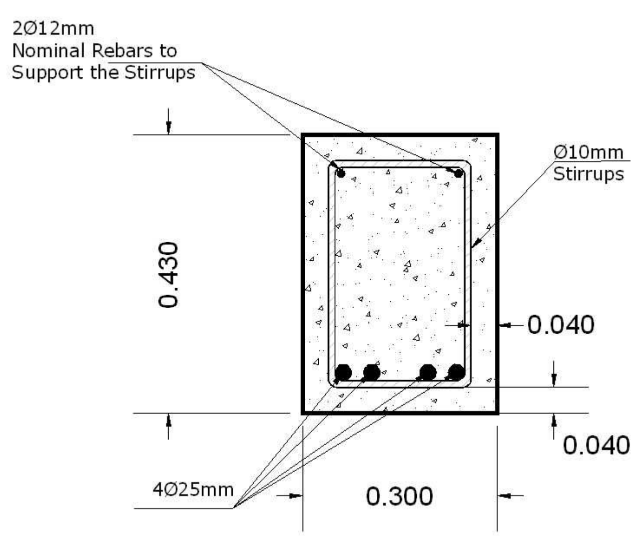
Assume that the designer intends to use:

- Concrete of $f_c{}' = 30\ MPa$.
- Steel of $f_y = 400\text{MPa}$.
- A width of 300mm and a height of 430mm (these dimensions have been determined based on deflection considerations).
- Rebar of No. 25 for longitudinal reinforcement.
- Rebar of No. 10 for stirrups.
- Single layer of reinforcement.

Solution

1. Computed required factored applied moment (M_u):
   a. Moment due to Dead Loads:
   $$W_{\text{Selfweight}} = 0.43\text{m} \times 0.3\text{m} \times 24\frac{\text{kN}}{\text{m}^3} = 3.1\frac{\text{kN}}{\text{m}}$$
   $$W_{\text{Dead}} = 9.00\frac{\text{kN}}{\text{m}} + 3.10\frac{\text{kN}}{\text{m}} = 12.1\ \frac{\text{kN}}{\text{m}}$$
   $$M_{\text{Dead}} = \frac{12.1\frac{\text{kN}}{\text{m}} \times 6.0^2\text{m}^2}{8} = 54.5\ \text{kN.m}$$
   b. Moment due to Live Load:
   $$M_{\text{Live}} = \frac{46.9\text{kN} \times 6.0\text{m}}{4} = 70.4\ \text{kN.m}$$
   c. Factored Moment M_u:
   $$M_u = \text{Maximum of } (1.4M_D \ \text{ or } 1.2M_D + 1.6M_L)$$
   $$M_u = \text{Maximum of } [1.4 \times 54.5 \ \text{ or } (1.2 \times 54.5 + 1.6 \times 70.4)] =$$
   $$M_u = \text{Maximum of } [76.3 \ \text{ or } 178] = 178\ \text{kN.m} \ \blacksquare$$
2. Computed the required nominal or theoretical flexure strength (M_n) based on the following relation:
   $$M_n = \frac{M_u}{\emptyset}$$
   Strength reduction factored can be assumed 0.9, and checked later.
   $$M_n = \frac{178\ \text{kN.m}}{0.9} = 198\ \text{kN.m}$$

3. Compute the effective beam depth "d":
   Assuming that reinforcement can be put in a single reinforcement, then:
   $$d_{\text{for One Layer}} = 430 - 40 - 10 - \frac{25}{2} = 368 \text{ mm}$$
4. Compute the Required Steel Ratio ρ_{Required}:
   $$\rho_{\text{Required}} = \frac{1 - \sqrt{1 - 2.36 \frac{M_n}{f'_c b d^2}}}{1.18 \times \frac{f_y}{f'_c}}$$
   $$\rho_{\text{Required}} = \frac{1 - \sqrt{1 - 2.36 \frac{198 \times 10^6 \text{N.mm}}{30 \times 300 \times 368^2}}}{1.18 \times \frac{400}{30}} = 13.6 \times 10^{-3}$$
5. Check if the beam failure is secondary compression failure or compression failure:
   $$\rho_{max} = 0.85\beta_1 \frac{f'_c}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004}$$
   $$\beta_1 = 0.85 - \frac{30 - 28}{7} \times 0.05 = 0.836 > 0.65\ Ok$$
   $$\rho_{max} = 0.85 \times 0.836\ \frac{30}{400} \frac{0.003}{0.003 + 0.004} = 22.8\ \times 10^{-3} > \rho_{\text{Required}}\ \text{Ok.}$$
6. Compute the required steel area:
   $$A_{S\ \text{Required}} = \rho_{\text{Required}} \times bd$$
   $$A_{S\ \text{Required}} = 13.6 \times 10^{-3} \times 300 \times 368 = 1501 \text{ mm}^2$$
7. Compute required rebars number:
   $$A_{\text{Bar}} = \frac{\pi \times 25^2}{4} = 490 \text{ mm}^2$$
   $$\text{No. of Rebars} = \frac{1\,501 \text{ mm}^2}{490 \text{ mm}^2} = 3.06$$
   Try 4Ø25mm.
   $$A_{S\ \text{Provided}} = 4 \times 490 \text{ mm}^2 = 1\,960 \text{ mm}^2$$
8. Check if the available width "b" is adequate to put the rebars in a single layer:
   $$b_{\text{required}} = 2 \times \text{Side Cover} + 2 \times \text{Stirrups} + \text{No. of Rebars} \times \text{Bar Diameter} + (\text{No. of Rebars} - 1) \times \text{Spacing between Rebars}$$
   $$b_{\text{required}} = 2 \times 40\text{mm} + 2 \times 10 + 4 \times 25\text{mm} + 3 \times 25\text{mm} = 275\text{mm} < 300\text{mm Ok.}$$
9. Checking for $S_{maximum}$ for Crack Control:
   $$s = \frac{300 - 40 \times 2 - 10 \times 2 - 25}{3} = 58.3\ mm < s_{max}\ \ Ok$$
10. Check with $A_{s\ \text{minimum}}$ requirements:
   $$A_{s\ \text{minimum}} = \frac{1.4}{f_y} b_w d = \frac{1.4}{400} \times 300 \times 368 = 386 \text{ mm}^2 < A_{S\ \text{Provided}} = 1960 \text{ mm}^2 \text{ Ok.}$$
11. Check the assumption of $\emptyset = 0.9$:
   a. Compute steel stain based on the following relations:
   $$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{1\,960\ \times\ 400}{0.85 \times 30 \times 300} = 102 \text{ mm} \Rightarrow c = \frac{102 \text{ mm}}{0.836} = 122 \text{ mm}$$
   $$\epsilon_t = \frac{368 \text{ mm} - 122 \text{ mm}}{122 \text{ mm}} \times 0.003 = 6.05 \times 10^{-3}$$
   b. As $\epsilon_t \geq 0.005$, then $\emptyset = 0.9$ Ok.
12. Final Reinforcement Details:



Example 4.4-3

Design lintel beam shown in Figure 4.4-4 below. In your solution, assume that the beam supports in addition to its own weight all brick works that lie directly on it and supports a dead load of 12 kN/m and live load of 8 kN/m transferred from supported slab. Seats of the lintel beam can be simulated as simple supports in your design.

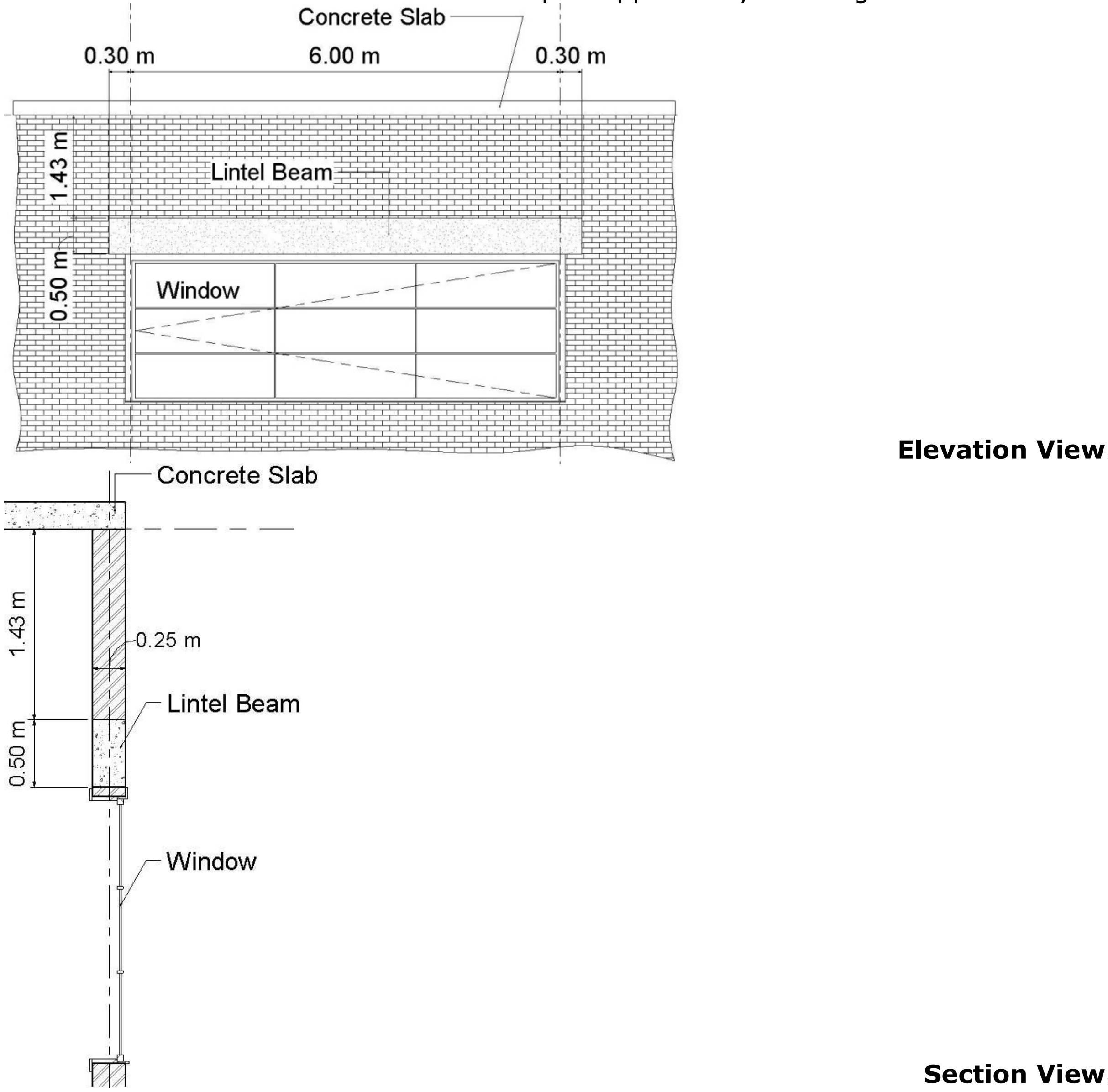


Figure 4.4-4: Lintel beam for Example 4.4-3.

Solution

$W_{self} = 0.5 \times 0.25 \times 24 = 3.0 \frac{kN}{m}, W_{Brick} = 1.43 \times 0.25 \times 19 = 6.79 \frac{kN}{m}$

$W_D = 3.0 + 6.79 + 12 = 21.8 \frac{kN}{m}$

$W_L = 8.0 \frac{kN}{m}$

$W_u = maximum\ (1.4 \times 21.8\ or\ 1.2 \times 21.8 + 1.6 \times 8.0) = maximum\ (30.5\ or\ 39) = 39 \frac{kN}{m}$ ■

$M_u = \frac{W_u l_n^2}{8} = \frac{39 \times 6.3^2}{8} = 193\ kN.m$

Try $\phi 20$ for longitudinal reinforcement in two layers and $\phi 12$ for stirrups.

$d = 500 - 40 - 12 - 20 - \frac{25}{2} = 415\ mm$

Computed the required nominal or theoretical flexure strength (M_n) based on the following relation:

$M_n = \frac{M_u}{\emptyset}$

Strength reduction factored can be assumed 0.9, and checked later.

$M_n = \frac{193}{0.9} = 214\ \text{kN.m}$

Compute the Required Steel Ratio $\rho_{Required}$:

$$\rho_{Required} = \frac{1 - \sqrt{1 - 2.36 \frac{M_n}{f_c' bd^2}}}{1.18 \times \frac{f_y}{f_c'}} = \frac{1 - \sqrt{1 - 2.36 \times \frac{214 \times 10^6}{28 \times 250 \times 415^2}}}{1.18 \times \frac{420}{28}} = 13.4 \times 10^{-3}$$

Check if the beam failure is secondary compression failure or compression failure:

$$\rho_{max} = 0.85\beta_1 \ \frac{f_c'}{f_y} \ \frac{\epsilon_u}{\epsilon_u + 0.004} = 0.85 \times 0.85 \ \frac{28}{420} \ \frac{0.003}{0.003 + 0.004} = 20.6 \ \times 10^{-3} > \rho_{Required} \ \text{Ok.}$$

Compute the required steel area:

$A_{S\ Required} = \ \rho_{Required} \times bd = \ 13.4 \times 10^{-3} \times 250 \times 415 = 1390\ mm^2$

Compute required rebars number:

$A_{Bar} = \frac{\pi \times 20^2}{4} \approx 314\ mm^2 \ \Rightarrow \text{No. of Rebars } = \frac{1390}{314} = 4.43$

Try $5\emptyset 20$mm.

$A_{S\ Provided} = 5 \times 314 = 1570\ mm^2$

Check if the available width "b" is adequate to put the rebars in a single layer:

$$b_{required} = 2 \times \text{Side Cover} + 2 \times \text{Stirrups } + \text{No. of Rebars} \times \text{Bar Diameter} + (\text{No. of Rebars} - 1) \times \text{Spacing between Rebars}$$

$b_{required} = 2 \times 40 + 2 \times 12 + \ 5 \times 20 + 4 \times 25 = 304mm > 250mm$

Therefore, reinforcement should be placed in two layers as assumed, $3\phi 20$ for first layer and $2\phi 20$ for the second one.

Checking for $S_{maximum}$ for Crack Control:

$$s \ = \frac{250 - 40 \times 2 - 12 \times 2 - 20}{2} = 63\ mm < s_{max} \ Ok$$

Check with $A_{s\ minimum}$ requirements:

$$A_{s\ minimum} = \frac{1.4}{f_y} b_w d = \frac{1.4}{420} \times 250 \times 415 = 349\ mm^2 < A_{S\ Provided} = 1570\ mm^2\ \text{Ok.}$$

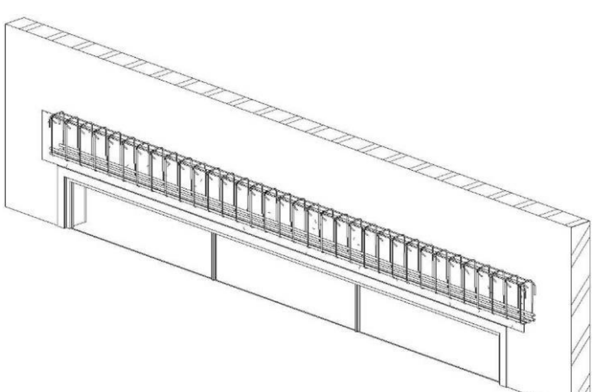
Check the assumption of $\emptyset = 0.9$:

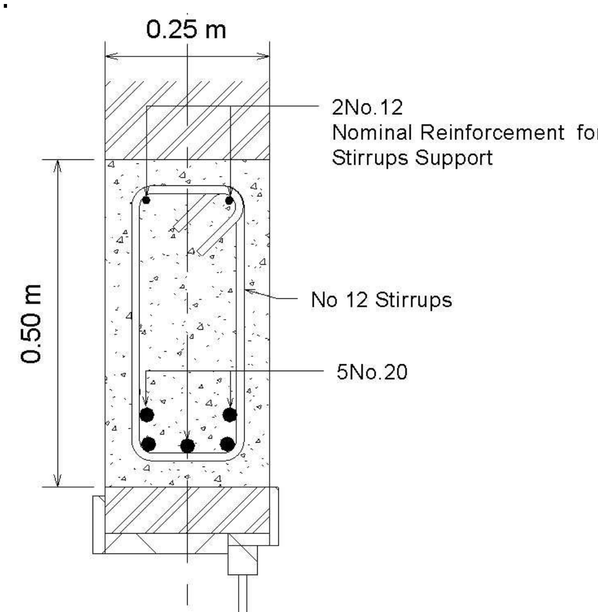
Compute steel stain based on the following relations:

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{1570 \ \times \ 420}{0.85 \times 28 \times 250} = 111\ mm \Rightarrow c = \frac{111}{0.85} = 131\ mm \Rightarrow \epsilon_t = \frac{415 - 131}{131} \times 0.003 = 6.50 \times 10^{-3}$$

As $\epsilon_t \ \geq 0.005$, then $\emptyset = 0.9$ Ok.

Finally, beam details are presented in below:





Example 4.4-4

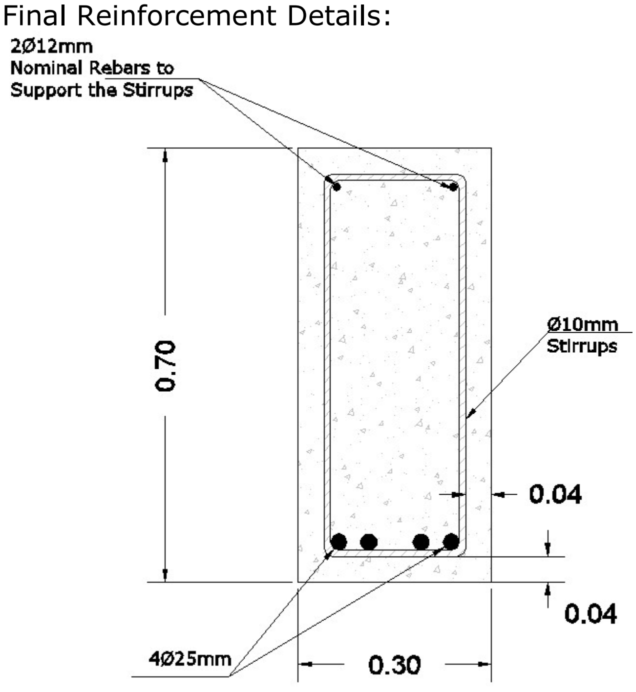
Design a rectangular beam to support a dead load of $35kN/m$ and a live load of $25kN/m$ acting on a simple span of $6m$. Assume that the designer intends to use:

- A width of 300mm and a depth of 700mm. These dimensions have been determined based on architectural considerations.
- $f_c' = 21MPa$ and $f_y = 420MPa$.
- Bar diameter of 25mm for longitudinal reinforcement.
- One layer of reinforcement.
- Bar diameter of 10mm for stirrups.

Solution

1. Computed required factored applied moment (M_u):
   a. Moment due to Dead Loads:
   $$W_{Selfweight} = 0.3m \times 0.7m \times 24\frac{kN}{m^3} = 5.04\frac{kN}{m}$$
   $$W_{Dead} = 35.0\frac{kN}{m} + 5.04\frac{kN}{m} = 40.0\frac{kN}{m}$$
   $$M_{Dead} = \frac{40.0\frac{kN}{m} \times 6.0^2m^2}{8} = 180 \; kN.m$$
   b. Moment due to Live Load:
   $$M_{Live} = \frac{25\;kN/m \times 6.0^2m^2}{8} = 113\;kN.m$$
   c. Factored Moment M_u:
   $$M_u = Maximum\;of\;(1.4M_D \;\; or\; 1.2M_D + 1.6M_L)$$
   $$M_u = Maximum\;of\;[1.4 \times 180 \;\; kN.m \;\; or\; (1.2 \times 180 \;\; kN.m + 1.6 \times 113\;kN.m)]$$
   $$M_u = Maximum\;of\;[252 \;\; or\; 397] = 397\;kN.m \;\blacksquare$$
2. Computed the required nominal or theoretical flexure strength (M_n) based on the following relation:
   $$M_n = \frac{M_u}{\emptyset}$$
   Strength reduction factored can be assumed 0.9, and checked later.
   $$M_n = \frac{397\;kN.m}{0.9} = 441\;kN.m$$
3. Compute the effective beam depth "d":
   Assume that, reinforcement can be put in single reinforcement, then:
   $$d_{for\;One\;Layer} = 700 - 40 - 10 - \frac{25}{2} = 637\;mm$$
4. Compute the Required Steel Ratio $\rho_{Required}$:
   $$\rho_{Required} = \frac{1 - \sqrt{1 - 2.36\frac{M_n}{f_c'bd^2}}}{1.18 \times \frac{f_y}{f_c'}}$$
   $$\rho_{Required} = \frac{1 - \sqrt{1 - 2.36\frac{441 \times 10^6 N.mm}{21 \times 300 \times 637^2}}}{1.18 \times \frac{420}{21}} = 9.75 \times 10^{-3}$$
5. Check if the beam failure is secondary compression failure or compression failure:
   $$\rho_{max} = 0.85\beta_1\;\frac{f_c'}{f_y}\;\frac{\epsilon_u}{\epsilon_u + 0.004}$$
   $$\beta_1 = 0.85$$
   $$\rho_{max} = 0.85 \times 0.85\;\frac{21}{420}\;\frac{0.003}{0.003 + 0.004} = 15.5\;\times 10^{-3} > \rho_{Required}\;\;Ok.$$

6. Compute the required steel area:
$A_{S\,Required} = \rho_{Required} \times bd$
$A_{S\,Required} = 9.75 \times 10^{-3} \times 300 \times 637\,mm = 1\,863\,mm^2$
7. Compute required rebars number:
$$A_{Bar} = \frac{\pi \times 25^2}{4} = 490\,mm^2$$
$$No.of\ Rebars = \frac{1\,863\,mm^2}{490\,mm^2} = 3.80$$
Try $4\emptyset 25mm$.
$A_{S\,Provided} = 4 \times 490\,mm^2 = 1\,960\,mm^2$
8. Check if the available width "b" is adequate to put the rebars in a single layer:
$$b_{required} = 2 \times Side\ Cover + 2 \times Stirrups + No.of\ Rebars \times Bar\ Diameter + (No.of\ Rebars - 1) \times Spacing\ between\ Rebars$$
$b_{required} = 2 \times 40mm + 2 \times 10 + 4 \times 25mm + 3 \times 25mm = 275mm < 300mm\ Ok.$
9. Check with $A_{s\,minimum}$ requirements:
$$A_{s\,minimum} = \frac{1.4}{f_y} b_w d = \frac{1.4}{420} \times 300 \times 637 = 637\,mm^2 < A_{S\,Provided} = 1\,960\,mm^2\ Ok.$$
10. Check $S_{maximum}$:
With four rebars, two stirrup legs, two covers, and a width of 300mm, the requirement of maximum spacing is necessary satisfied.
11. Check the assumption of $\emptyset = 0.9$:
    a. Compute steel stain based on the following relations:
$$a = \frac{A_s f_y}{0.85 f_c' b}$$
$$a = \frac{1\,960 \times 420}{0.85 \times 21 \times 300} = 154\,mm$$
$$c = \frac{154\,mm}{0.85} = 181\,mm$$
$$\epsilon_t = \frac{d - c}{c} \times \epsilon_u$$
$$\epsilon_t = \frac{637\,mm - 181\,mm}{181\,mm} \times 0.003 = 7.56 \times 10^{-3}$$
    b. As $\epsilon_t \geq 0.005$, then $\emptyset = 0.9$ Ok.
12. Final Reinforcement Details:



Example 4.4-5

What are the required longitdinal reinforcements for Scetion A and Section B of the beam shown in Figure 4.4-5 below?

In your solution assume that:

1. $f_c' = 28\ MPa$ and $f_y = 420\ MPa$.
2. Rebar of 25mm is used for longtudinal reinforcement.
3. Single layer of reinforcement.
4. Rebars of 12mm is used for shear reinforcement.
5. Beam slefweight can be neglcted.
6. Uniform subgarde reaction.

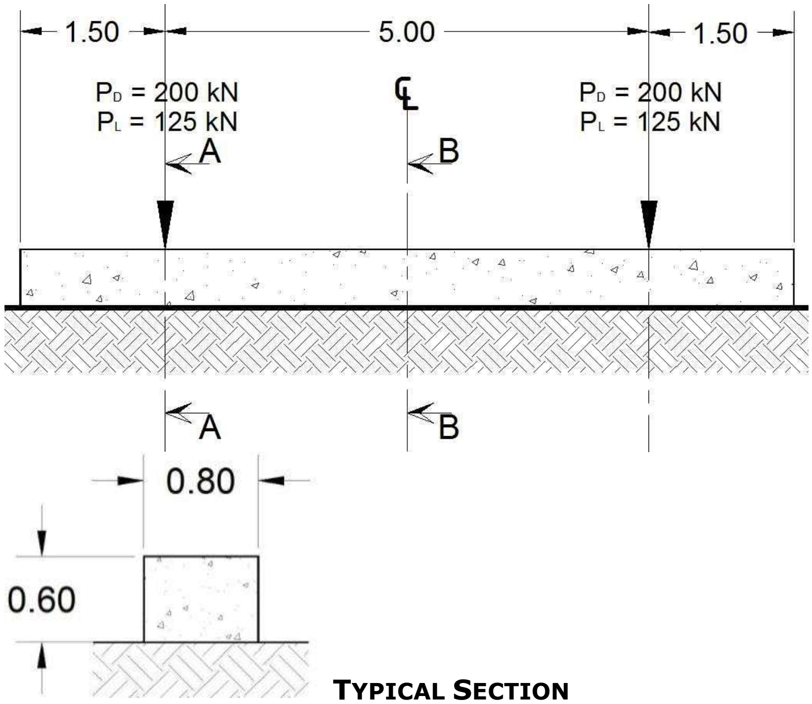


Figure 4.4-5: Foundation beam for Example 4.4-5.

Solution

Design Forces

$$P_u = maximum\ (1.4P_{Dead}\ Or\ 1.2P_{Dead} + 1.6P_{Live})$$
$$P_u = maximum\ (1.4 \times 200\ Or\ 1.2 \times 200 + 1.6 \times 125)$$
$$P_u = maximum\ (280\ Or\ 440)$$
$$P_u = 440\ kN$$
$$W_u = (440\ kN \times 2) \times \frac{1}{8m} = 110\frac{kN}{m}$$
$$M_{u\ @\ Section\ A-A} = 110\frac{kN}{m} \times 1.5m \times \frac{1.5m}{2} = 124\ kN.m$$
$$M_{u\ @\ Section\ B-B} = -\left(\frac{110\frac{kN}{m} \times 5^2 m^2}{8}\right) + 124$$
$$M_{u\ @\ Section\ B-B} = -220\ kN.m$$

Design of Section A-A

1. Computed the required nominal or theoretical flexure strength (M_n) based on the following relation:

$$\mathrm{M_n} = \frac{\mathrm{M_u}}{\emptyset}$$

Strength reduction factored can be assumed 0.9, and checked later.

$$\mathrm{M_n} = \frac{124\ \mathrm{kN.m}}{0.9} = 138\ \mathrm{kN.m}$$

2. Compute the effective beam depth "d":
   Assume that, reinforcement can be put in single reinforcement, then:
   $$d_{\text{for One Layer}} = 600 - 75_{\text{Casted and exposed to soil}} - 12 - \frac{25}{2} = 505 \text{ mm}$$
3. Compute the Required Steel Ratio ρ_{Required}:
   $$\rho_{\text{Required}} = \frac{1 - \sqrt{1 - 2.36 \frac{M_n}{f_c' b d^2}}}{1.18 \times \frac{f_y}{f_c'}}$$
   $$\rho_{\text{Required}} = \frac{1 - \sqrt{1 - 2.36 \frac{138 \times 10^6 \text{N.mm}}{28 \times 800 \times 505^2}}}{1.18 \times \frac{420}{28}}$$
   $$\rho_{\text{Required}} = 1.63 \times 10^{-3}$$
4. Check if the beam failure is secondary compression failure or compression failure:
   $$\rho_{\max} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004}$$
   $$\rho_{\max} = 0.85^2 \frac{28}{420} \frac{0.003}{0.003 + 0.004} = 20.6 \times 10^{-3} > \rho_{\text{Required}} \text{ Ok.}$$
5. Compute the required steel area:
   $$A_{S\text{ Required}} = \rho_{\text{Required}} \times bd$$
   $$A_{S\text{ Required}} = 1.63 \times 10^{-3} \times 800 \times 505 = 660 \text{ mm}^2$$
6. Check with $A_{s\text{ minimum}}$ requirements:
   $$A_{s\text{ minimum}} = \frac{1.4}{f_y} b_w d = \frac{1.4}{420} \times 800 \times 505 = 1347 \text{ mm}^2$$
   $$A_{s\text{ minimum}} = 1347 \text{ mm}^2 > 1\frac{1}{3} A_{S\text{Required}} = 878 \text{ mm}^2$$
   Then used
   $$A_s = 878\, mm^2$$
7. Compute required rebars number:
   $$A_{Bar} = \frac{\pi \times 25^2}{4} = 490\, mm^2$$
   $$No.of\ Rebars = \frac{878\, mm^2}{490\, mm^2} = 1.79$$
   Try $2\emptyset 25$mm.
   $$A_{S\text{ Provided}} = 2 \times 490 \text{ mm}^2 = 980 \text{ mm}^2$$
8. Check if the available width "b" is adequate to put the rebars in a single layer:
   $$b_{\text{required}} = 2 \times \text{Side Cover} + 2 \times \text{Stirrups} + \text{No. of Rebars} \times \text{Bar Diameter} + (\text{No. of Rebars} - 1) \times \text{Spacing between Rebars}$$
   $$b_{\text{required}} = 2 \times 50 + 2 \times 12 + 2 \times 25 + 25 = 199 \text{ mm} < 800\text{mm Ok.}$$
9. Checking for Smax for Crack Control:
   $$s = 800 - 50 \times 2 - 12 \times 2 - 25 = 651 \text{ mm} > s_{\max}\ \textit{Not Ok}$$
   Then use $\boldsymbol{5\phi 16mm}$ instead of $2\phi 25mm$.
   $$A_{S\text{ Provided}} = 5 \times 200 \text{ mm}^2 = 1000 \text{ mm}^2$$
10. Check the assumption of $\emptyset = 0.9$:
    a. Compute steel stain based on the following relations:
    $$a = \frac{A_s f_y}{0.85 f_c' b}$$
    $$a = \frac{10000 \times 420}{0.85 \times 28 \times 800} = 22.1 \text{ mm}$$
    $$c = \frac{22.1 \text{ mm}}{0.85} = 26 \text{ mm}$$
    $$\epsilon_t = \frac{505 \text{ mm} - 26 \text{ mm}}{26 \text{ mm}} \times 0.003 = 55.3 \times 10^{-3}$$
    b. As $\epsilon_t \geq 0.005$, then $\emptyset = 0.9$ Ok.

Design of Section B-B:

1. Computed the required nominal or theoretical flexure strength (M_n) based on the following relation:
$$M_n = \frac{M_u}{\emptyset}$$
Strength reduction factored can be assumed 0.9, and checked later.
$$M_n = \frac{220 \text{ kN.m}}{0.9} = 244 \text{ kN.m}$$
2. Compute the effective beam depth "d":
Assume that, reinforcement can be put in single reinforcement, then:
$$d_{\text{for One Layer}} = 600 - 50_{\text{Exposed to Soil}} - 12 - \frac{25}{2} = 525 \text{ mm}$$
3. Compute the Required Steel Ratio ρ_{Required}:
$$\rho_{\text{Required}} = \frac{1 - \sqrt{1 - 2.36 \frac{M_n}{f'_c bd^2}}}{1.18 \times \frac{f_y}{f'_c}}$$
$$\rho_{\text{Required}} = \frac{1 - \sqrt{1 - 2.36 \frac{244 \times 10^6 \text{N.mm}}{28 \times 800 \times 525^2}}}{1.18 \times \frac{420}{28}}$$
$$\rho_{\text{Required}} = 2.70 \times 10^{-3}$$
4. Check if the beam failure is secondary compression failure or compression failure:
$$\rho_{max} = 0.85\beta_1 \frac{f'_c}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004}$$
$$\rho_{max} = 0.85^2 \frac{28}{420} \frac{0.003}{0.003 + 0.004} = 20.6 \times 10^{-3} > \rho_{\text{Required}} \text{ Ok.}$$
5. Compute the required steel area:
$$A_{S \text{ Required}} = \rho_{\text{Required}} \times bd$$
$$A_{S \text{ Required}} = 2.70 \times 10^{-3} \times 800 \times 525 = 1134 \text{ mm}^2$$
6. Check with $A_{s \text{ minimum}}$ requirements:
$$A_{s \text{ minimum}} = \frac{1.4}{f_y} b_w d = \frac{1.4}{420} \times 800 \times 525 = 1400 \text{ mm}^2$$
$$A_{s \text{ minimum}} = 1400 \text{ mm}^2 < 1\frac{1}{3} A_{\text{SRequired}} = 1508 \text{ mm}^2 \; Ok.$$
Then used
$$A_s = A_{s \text{ minimum}} = 1400 \text{ mm}^2$$
7. Compute required rebars number:
$$A_{Bar} = \frac{\pi \times 25^2}{4} = 490 \; mm^2$$
$$No. of \; Rebars = \frac{1400 \text{ mm}^2}{490 \; mm^2} = 2.85$$
Try $3\emptyset 25$mm.
$$A_{S \text{ Provided}} = 3 \times 490 \text{ mm}^2 = 1470 \text{ mm}^2$$
8. Check if the available width "b" is adequate to put the rebars in a single layer:
$$b_{\text{required}} = 2 \times \text{Side Cover} + 2 \times \text{Stirrups} + \text{No. of Rebars} \times \text{Bar Diameter} + (\text{No. of Rebars} - 1) \times \text{Spacing between Rebars}$$
$$b_{\text{required}} = 2 \times 50 + 2 \times 12 + 3 \times 25 + 2 \times 25 = 249 \text{ mm} < 800\text{mm Ok.}$$
9. Checking for Smax for Crack Control:
$$s = \frac{800 - 50 \times 2 - 12 \times 2 - 25}{2} = 326 \text{ mm} > s_{max} \; Not \; Ok$$
Then use $\mathbf{5\phi 20mm}$ instead of $3\phi 25mm$.
$$A_{S \text{ Provided}} = 5 \times 314 \text{ mm}^2 = 1570 \text{ mm}^2$$

10. Check the assumption of $\emptyset = 0.9$:
    a. Compute steel stain based on the following relations:

$$a = \frac{A_s f_y}{0.85 f_c' b}$$

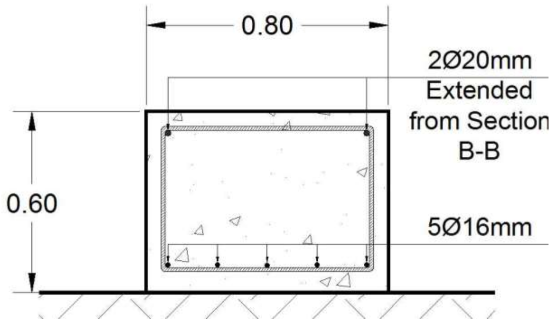
$$a = \frac{1570 \times 420}{0.85 \times 28 \times 800} = 34.6 \text{ mm}$$

$$c = \frac{34.6 \text{ mm}}{0.85} = 40.7 \text{ mm}$$

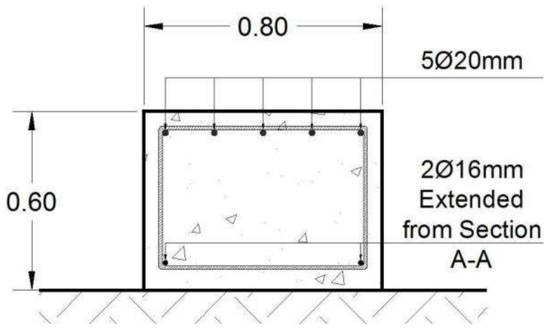
$$\epsilon_t = \frac{525 \text{ mm} - 34.6 \text{ mm}}{34.6 \text{ mm}} \times 0.003 = 42.5 \times 10^{-3}$$

    b. As $\epsilon_t \geq 0.005$, then $\emptyset = 0.9$ Ok.

Sections Details:



SECTION A-A



SECTION B-B

Example 4.4-6

Design Section A-A of beam shown in Figure 4.4-6 below for flexure requirements according to ACI 318M-14.

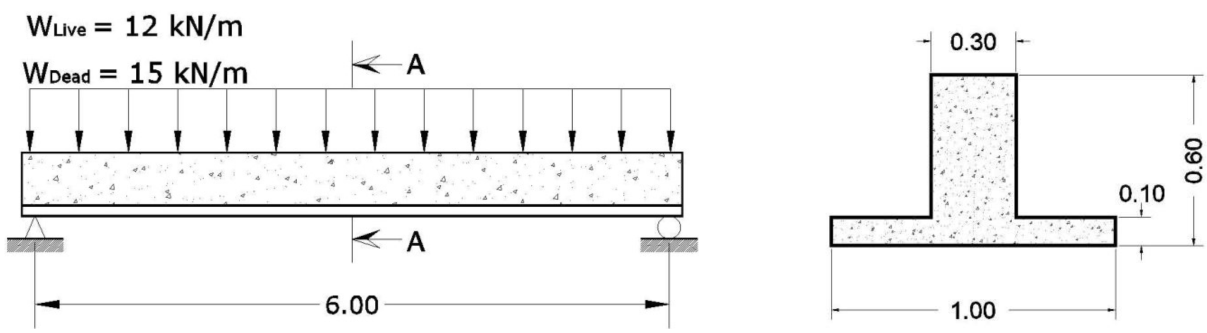


Figure 4.4-6: Inverted beam of Example 4.4-6.

In your solution, assume that:

1. f_c' = 28 MPa.
2. f_y = 420 MPa.
3. Rebar of No. 25 for longitudinal reinforcement.
4. Single layer of reinforcement.
5. Rebar of No. 10 for stirrups.
6. Rebars can be put in a single layer.

Solution

1. Design Moments:

$$W_{selfweight} = (0.1 \times 1.0 + 0.5 \times 0.3)m^2 \times 24\frac{kN}{m^2} = 6\frac{kN}{m}$$

$$W_{Dead} = 15 + 6 = 21\frac{kN}{m}$$

$$W_u = 1.4(21) \text{ or } [1.2 \times 21 + 1.6 \times 12]$$

$$W_u = 29.4\frac{kN}{m} \text{ or } 44.4\frac{kN}{m}$$

$$W_u = 44.4\frac{kN}{m}$$

$$M_u = \frac{W_u l^2}{8} = \frac{44.4\frac{kN}{m} \times 6.0^2 m^2}{8} = 200 \text{ kN.m}$$

2. Section Design:
   As the flange is on the tension side, section should be designed as a rectangular section.
3. Computed the required nominal or theoretical flexure strength (M_n) based on the following relation:

$$M_n = \frac{M_u}{\emptyset}$$

   Strength reduction factored can be assumed 0.9, and checked later.

$$M_n = \frac{200 \text{ kN.m}}{0.9} = 222 \text{ kN.m}$$

4. Compute the effective beam depth "d":
   Assume that, reinforcement can be put in single reinforcement, then:

$$d_{for\ One\ Layer} = 600 - 40 - 10 - \frac{25}{2} = 538 \text{ mm}$$

5. Compute the Required Steel Ratio $\rho_{Required}$:

$$\rho_{Required} = \frac{1 - \sqrt{1 - 2.36\frac{M_n}{f_c' b d^2}}}{1.18 \times \frac{f_y}{f_c'}}$$

$$\rho_{Required} = \frac{1 - \sqrt{1 - 2.36\frac{222 \times 10^6 \text{N.mm}}{28 \times 300 \times 538^2}}}{1.18 \times \frac{420}{28}}$$

$$\rho_{Required} = 6.46 \times 10^{-3}$$

6. Check if the beam failure is secondary compression failure or compression failure:

$$\rho_{max} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004}$$

$$\beta_1 = 0.85$$

$$\rho_{max} = 0.85^2 \frac{28}{420} \frac{0.003}{0.003 + 0.004} = 20.6 \times 10^{-3} > \rho_{Required} \text{ Ok.}$$

$$\rho_{max} = 20.6 \times 10^{-3} > \rho_{Required} \text{ Ok.}$$

7. Compute the required steel area:

$$A_{S\ Required} = \rho_{Required} \times bd$$

$$A_{S\ Required} = 6.46 \times 10^{-3} \times 300 \times 538 = 1042 \text{ mm}^2$$

8. Compute required rebars number:

$$A_{Bar} = \frac{\pi \times 25^2}{4} = 490 \text{ mm}^2$$

$$\text{No. of Rebars} = \frac{1042 \text{ mm}^2}{490 \text{ mm}^2} = 2.12$$

   Try $3\emptyset 25$mm.

$$A_{S\ Provided} = 3 \times 490 \text{ mm}^2 = 1\,470 \text{ mm}^2$$

9. Check if the available width "b" is adequate to put the rebars in a single layer:

$$b_{required} = 2 \times \text{Side Cover} + 2 \times \text{Stirrups} + \text{No. of Rebars} \times \text{Bar Diameter} + (\text{No. of Rebars} - 1) \times \text{Spacing between Rebars}$$

$b_{required} = 2 \times 40mm + 2 \times 10 + \ 3 \times 25mm + 2 \times 25mm = 225mm < 300mm$ Ok.

10. Checking for Smax for Crack Control:

$$S \ = (300 - 40 \times 2 - 10 \times 2 - 25) \times \frac{1}{2} = 87.5 \text{ mm} < S_{max} \text{ Ok}$$

Remember that S_{max} is c/c distance.

11. Check with $A_{s\,minimum}$ requirements:

As the span is statically determinate, and as flange is in tension side, As min will be computed based on following relation.

$$A_{s\,min} = \ minimum \ \left(\frac{0.25\sqrt{f_c'}}{f_y} b_f d, \frac{0.50\sqrt{f_c'}}{f_y} b_w d \right)$$

$$A_{s\,min} = minimum \left(\frac{0.25\sqrt{28}}{420} 1000 \times 538 \, , \frac{0.50\sqrt{28}}{420} 300 \times 538 \right)$$

$$A_{s\,min} = minimum(\, 1695 \, , \qquad 1017)$$

$$A_{s\,min} = 1017 \, mm^2 < A_{s\,provided} \ \therefore \ ok.$$

12. Check the assumption of $\emptyset = 0.9$:

i. Compute steel stain based on the following relations:

$$a = \frac{A_s f_y}{0.85 f_c' b}$$

$$a = \frac{1470 \ \times 420}{0.85 \times 28 \times 300} = 86.5 \text{ mm}$$

$$c = \frac{86.5 \text{ mm}}{0.85} = 102 \text{ mm}$$

$$\epsilon_t = \frac{538 \text{ mm} - 102 \text{ mm}}{102 \text{ mm}} \times 0.003 = 12.8 \times 10^{-3}$$

ii. As $\epsilon_t \ \geq 0.005$, then $\emptyset = 0.9$ Ok.

13. Final Reinforcement Details: