4.6 ANALYSIS OF A RECTANGULAR BEAM WITH TENSION AND COMPRESSION REINFORCEMENTS (A DOUBLY REINFORCED BEAM)

4.6.1 Basic Concepts

- Occasionally, beams are built with both tension reinforcement and compression reinforcement. These beams are called as beams with tension and compression reinforcement or doubly reinforced beams.
- Area and ratio of compression reinforcement have notations of A_s' and ρ' respectively (See Figure 4.6-1 below):

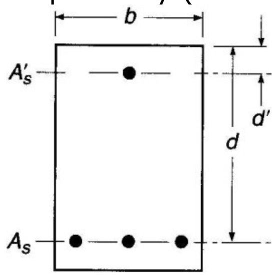


Figure 4.6-1: A doubly reinforced section.

- To be consistence with notations adopted in single reinforced beams, reinforcement ratio for tension reinforcement, ρ, is defined as:

$$\boldsymbol{\rho = \frac{A_s}{bd}}$$

- To simplify algebraic **operation through adopting same denominator, reinforcement notation for compression reinforcement, ρ', is defined as:**

$$\rho' = \frac{A_s'}{bd} \quad \textbf{Eq. 4.6-1}$$

- There are four reasons for using compression reinforcement in beams:
  - **Reduce Sustained-Load Deflection**
    First and most important, the addition of compression reinforcement reduces the long-term deflections of a beam subjected to sustained loads, see Figure 4.6-2 below.

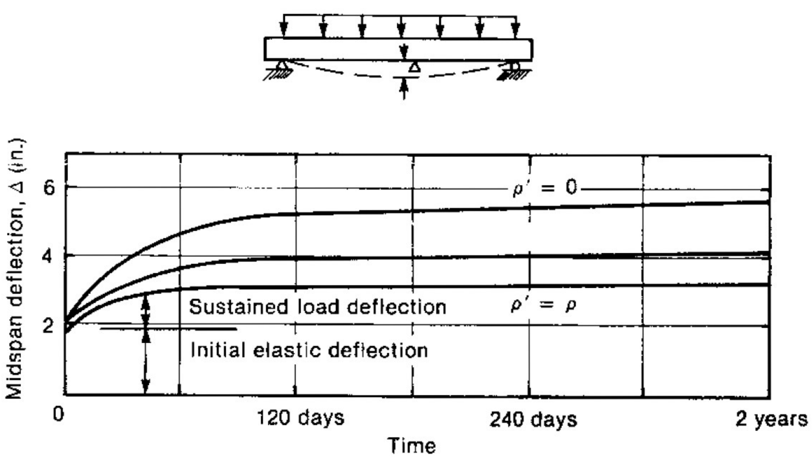


Figure 4.6-2: Compression reinforcement effects on deflection due to sustained loads.

  - **Fabrication Ease**
    When assembling the reinforcing cage for a beam, it is customary to provide bars in the corners of stirrups to hold stirrups in place in the form see Figure 4.6-3 below.

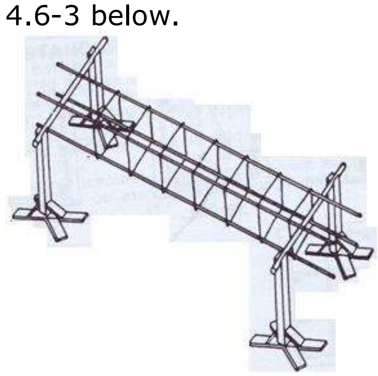


Figure 4.6-3: A reinforcement cage for fabrication ease.

- **Increase Ductility**
  It can be shown that the addition of compression reinforcement causes a reduction in the depth of the compression stress block "a". As "a" decreases the strain in the tension reinforcement at failure increases, resulting in more ductile behavior, see Figure 4.6-4 below.

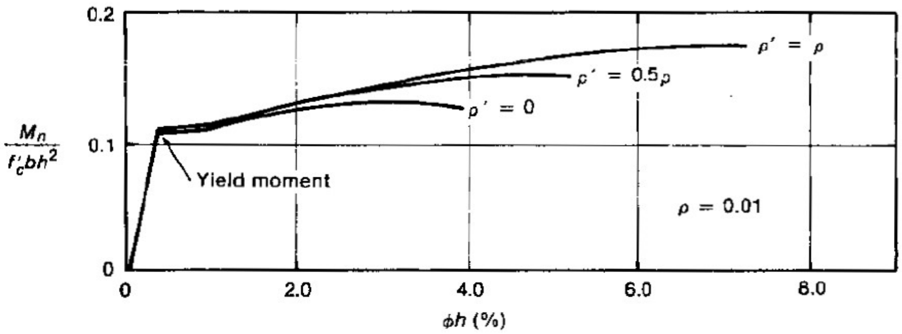


Figure 4.6-4: Ductility increase versus increasing in compression reinforcement.

- **Change the Mode of Failure from a Compression Failure to Secondary Compression Failure:**
  - When $\rho > \rho_b$, a beam fails in brittle manner through crushing of the compressive zone before the steel yields.
  - Adding of compression steel to such beam reduces the depth of the compression stress block "a".
  - As "a" decreases the strain in the tension reinforcement at failure increases, resulting in a more ductile behavior, see Figure 4.6-5 below.
  - The use of compression reinforcement for this reason has decreased markedly with use of strength design method.

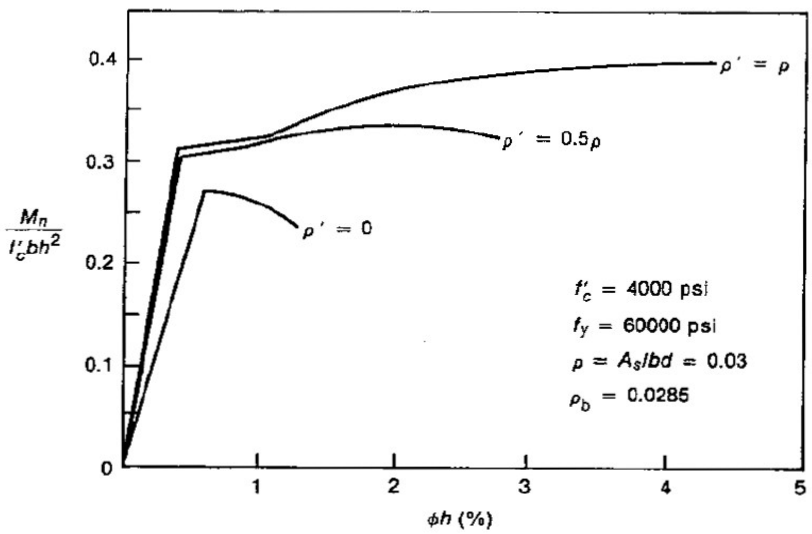


Figure 4.6-5: Changing in failure mode versus compression reinforcement.

- Analysis of a beam with tension and compression reinforcement starts with a checking to diagnose the cause for using the compression reinforcement based on the following argument. if

  $$\rho > \rho_{max} = 0.85\beta_1 \frac{f'_c}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004}$$

  then the compression reinforcement has been used to ***Change the Mode of Failure from Compression Failure to Secondary Compression Failure***. Then this reinforcement must be included in the beam analysis. Else, if

  $$\rho < \rho_{max} = 0.85\beta_1 \frac{f'_c}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004}$$

  then the compression reinforcement has been ***used either to reduce sustained-load deflection or to fabrication ease or to increase ductility*** and ***its effects can be neglected in the beam analysis***.

- Generally, compression reinforcement increases the value of maximum of steel ratio ρ_{max} and increases the value of nominal strength M_n. These effects will be discussed in paragraphs below.

4.6.2 Maximum Steel Ratio ($\bar{\rho}_{max}$) of a Rectangular Beam with Tension and Compression Reinforcement:

Based on basic tenets of ***Compatibility***, ***Stress-Strain Relation***, and ***Equilibrium***, one can prove that using of compression reinforcement increases the maximum permissible steel ratio from the value of:

$$\rho_{max} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004}$$

to a ratio of:

$$\bar{\rho}_{max} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} + \rho' \frac{f_s'}{f_y}$$

or

$$\bar{\rho}_{max} = \rho_{max} + \rho' \frac{f_s'}{f_y} \blacksquare$$

where f_s' is stress in the compression reinforcement at strains of ρ_{max}. It can be computed from strain distribution and as shown in relation below:

$$f_s' = E_s \left[\epsilon_u - \frac{d'}{d}(\epsilon_u + 0.004)\right] \leq f_y \blacksquare$$

4.6.3 Nominal Flexure Strength of a Rectangular Beam with Tension and Compression Reinforcement:

- The M_n relation of a doubly reinforced beam depends on the yielding of compression reinforcement.
- Then there are two relations for computing of M_n,
  - One for the doubly reinforced beam with compression steel at yield stress,
  - The other for the doubly reinforced beam with compression steel below yield stress.

4.6.3.1 M_n for a Beam with Compression Steel at Yield Stress

- Strains, stresses, and forces diagrams for a beam with tension and compression reinforcement at yield stress can be summarized in Figure 4.6-6 below.

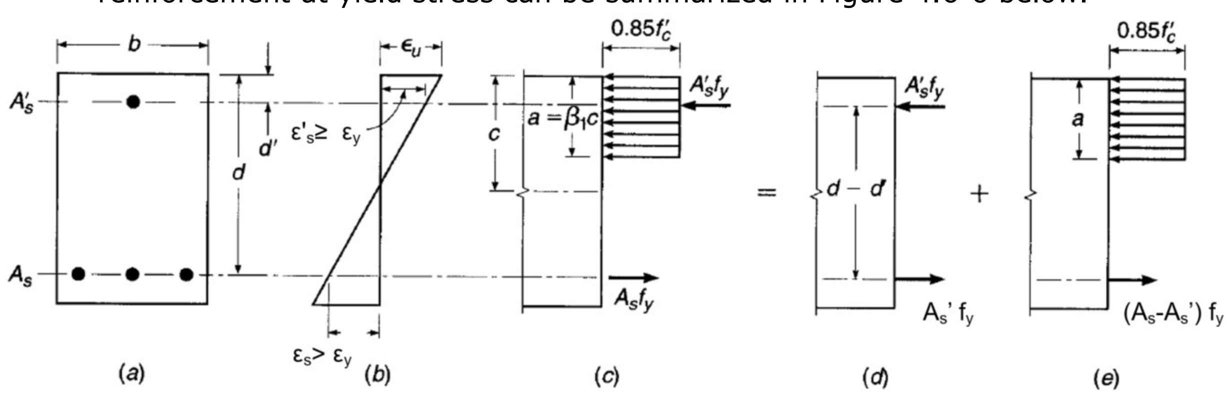


Figure 4.6-6: Strains and stresses for a doubly reinforced rectangular beam with yielded compression reinforcement.

- Then, based on superposition one can conclude that M_n for the section can be computed based on following relation:

$$\sum M_{about\ A_s'} = 0$$

$$M_n = M_{n1} + M_{n2} = A_s' f_y (d - d') + (A_s - A_s') f_y \left(d - \frac{a}{2}\right) \blacksquare$$

where

$$a = \frac{(A_s - A_s') f_y}{0.85 f_c' b} \blacksquare$$

4.6.3.2 M_n for a Beam with Compression Steel below Yield Stress

- Strains, stresses, and forces diagrams for a beam with compression reinforcement below yield stress can be summarized in Figure 4.6-7 below.

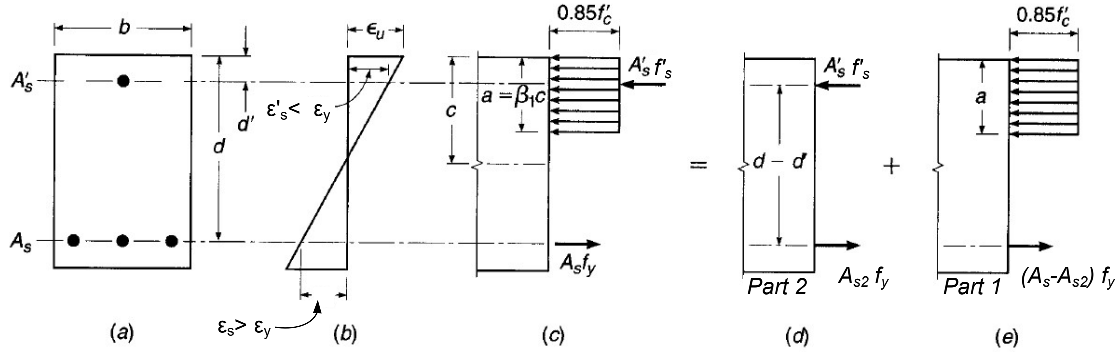


Figure 4.6-7: Strains and stresses for a doubly reinforced rectangular beam with compression reinforcement below the yield.

- Using the superposition, M_n for section can be computed based on following relation:

  $\sum M_{about\ A_s} = 0$

  $M_n = M_{n1} + M_{n2} = 0.85f'_c ab(d - \frac{a}{2}) + A'_s f'_s(d - d')$ ■

  where "a" and f_s' can be computed as follows:
  - From strain and stress diagram:

    $f'_s = \epsilon_u E_s \frac{(c-d')}{c}$ ■ (1)
  - From equilibrium:

    $\sum F_x = 0.0$

    $A_s f_y = 0.85\beta_1 f'_c bc + A'_s f'_s$ (2)

    Substitute of (1) into (2):

    $A_s f_y = 0.85\beta_1 f'_c bc + A'_s \epsilon_u E_s \frac{(c-d')}{c}$ ■ (3)
  - Solve this quadratic equation for "c" value:

    $c = \sqrt{Q + R^2} - R$ ■

    where:

    $Q = \frac{600d' A'_s}{0.85\beta_1 f'_c b}$ ■

    and

    $R = \frac{600A'_s - f_y A_s}{1.7\beta_1 f'_c b}$ ■
  - Then substitute "c" into equation (1) to obtain f'_s. Finally "a" value can be computed from:

    $a = \beta_1 c$

4.6.3.3 Criterion to Check if the Compression Steel is at Yield Stress or Not

- It is clear from above discussion; that the form of relation for computing of M_n is depended on checking of yielding of compression steel.
- Based on basic principles (Compatibility, Stress-Strain Relation, and Equilibrium), following criterion can be derived to check the yielding of compression reinforcement.

  If $\rho \geq \bar{\rho}_{cy}$, then $f'_s = f_y$ and the compression reinforcement is at yield stress.

  Else $f'_s < f_y$ and the compression reinforcement is below the yield stress.
- The minimum tensile reinforcement ratio $\bar{\rho}_{cy}$ that will ensure yielding of the compression reinforcement at failure can be computed as follows:

  $\bar{\rho}_{cy} = 0.85\beta_1 \frac{f'_c}{f_y}\frac{d'}{d}\frac{\epsilon_u}{\epsilon_u - \epsilon_y} + \rho'$ ■

4.6.4 Ties for Compression Reinforcement

- If compression bars are used in a flexural member, precautions must be taken to ensure that these bars will not buckle outward under load spelling off the outer concrete, see Figure 4.6-8 below.

Figure 4.6-8: Buckling of beam compression reinforcement.

- ACI Code (***25.7.2.1***) imposes the requirement that such bars be anchored in the same way that compression bars in columns are anchored by lateral ties. Such ties are designed based on the following procedures:
  - Select bar diameter for ties (**25.7.2.2**):
    All bars of tied columns shall be enclosed by lateral ties at least No 10 in size for longitudinal bars up to No. 32 and at least No. 13 in size for Nos. 36, 43, and 57 and bundled longitudinal bars.
  - The spacing of the ties shall not exceed (**25.7.2.1**):
    $S_{Maximum} = \min[16d_{bar}, 48d_{ties}, \text{Least dimension of column}]$
  - Ties Arrangement (**25.7.2.3**):
    According to ACI (**25.7.2.3**), *rectilinear ties shall be arranged to satisfy (a) and (b):*
    (a) Every corner and alternate longitudinal bar shall have lateral support provided by the corner of a tie with an included angle of not more than 135 degrees.
    (b) No unsupported bar shall be farther than 150 mm clear on each side along the tie from a laterally supported bar.

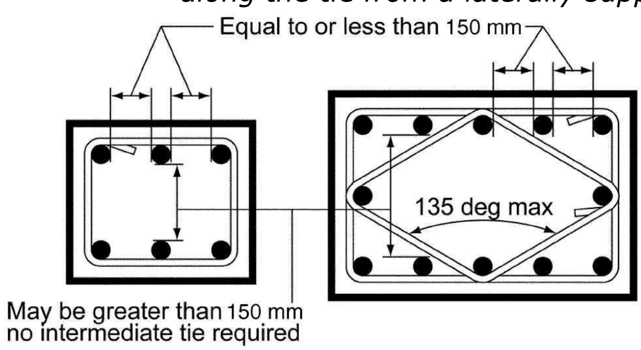


Figure 4.6-9: Ties arrangement according to requirements of ACI Code.

4.6.5 Examples

Example 4.6-1

Check the adequacy of beam shown in Figure 4.6-10 below and compute its design strength according to ACI Code. Assume that: $f_c' = 20\ \text{MPa}$ and $f_y = 300\ \text{MPa}$.

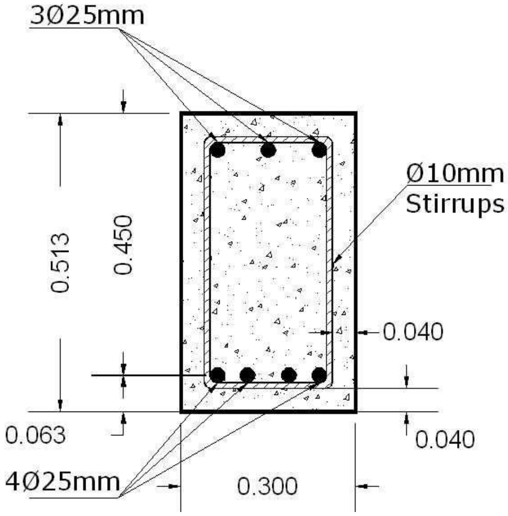


Figure 4.6-10: Cross section for beam of Example 4.6-1.

Solution

- Check the reason for using of compression reinforcement:

$$A_{s\,\text{Provided}} = 4 \times 490 = 1\,960\ \text{mm}^2 \Longrightarrow \rho_{\text{Provided}} = \frac{1\,960\ \text{mm}^2}{300 \times 450} = 14.5 \times 10^{-3}$$

$$\rho_{max} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} = 0.85 \times 0.85 \frac{20}{300} \frac{0.003}{0.003 + 0.004} = 20.6 \times 10^{-3} > \rho_{\text{Provided}}$$

Then, compression reinforcement has been added for a reason other than changing the failure mode from compression failure to secondary compression failure and its effects on section strength can be neglected. Therefore, the section can be analyzed as a singly reinforced section.

$$A_{s\,\text{minimum}} = \frac{0.25\sqrt{f_c'}}{f_y} b_w d \geq \frac{1.4}{f_y} b_w d$$

$$\because\ f_c' < 31\ MPa$$

$$\therefore A_{s\,\text{minimum}} = \frac{1.4}{f_y} b_w d = \frac{1.4}{300} \times 300 \times 450 = 630\ \text{mm}^2 < A_{s\,\text{Provided}}\ \text{Ok.} \blacksquare$$

- Compute section nominal strength M_n :

$$M_n = \rho f_y b d^2 \left(1 - 0.59\ \frac{\rho f_y}{f_c'}\right)$$

$$M_n = 14.5 \times 10^{-3} \times 300 \times 300 \times 450^2 \left(1 - 0.59\ \frac{14.5 \times 10^{-3} \times 300}{20}\right) = 230\ \text{kN.m}$$

- Compute strength reduction factor $\emptyset$:
  - Compute steel stain based on the following relations:

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{1\,960\ \text{mm}^2 \times 300\ \text{MPa}}{0.85 \times 20\ \text{MPa} \times 300\text{mm}} = 115\ \text{mm} \Longrightarrow c = \frac{a}{\beta_1} \frac{115\ \text{mm}}{0.85} = 135\ \text{mm}$$

$$\epsilon_t = \frac{d - c}{c} \epsilon_u = \frac{450 - 135}{135} \times 0.003 = 7.0 \times 10^{-3}$$

  - $\epsilon_t > 0.005$, then $\emptyset = 0.9$.

- Compute section design strength $\emptyset M_n$:

$$\emptyset M_n = \emptyset \times M_n = 0.9 \times 230\ \text{kN.m} = 207\ \text{kN.m}\blacksquare$$

Example 4.6-2

Check the adequacy of beam shown in Figure 4.6-11 below and compute its design strength according to ACI Code. Assume that: $f'_c = 20\ \text{MPa}$ and $f_y = 300\ \text{MPa}$.

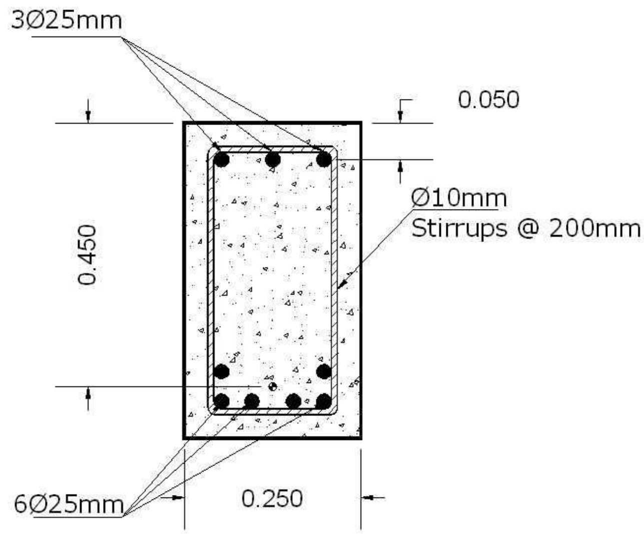


Figure 4.6-11: Cross section for beam of Example 4.6-2.

Solution

- Check the reason for using of compression reinforcement:

$A_{s\ \text{Provided}} = 6 \times 490 = 2\ 940\ \text{mm}^2 \Longrightarrow \rho_{\text{Provided}} = \dfrac{2\ 940\ \text{mm}^2}{250 \times 450} = 26.1 \times 10^{-3}$

$\rho_{max} = 0.85\beta_1 \dfrac{f'_c}{f_y} \dfrac{\epsilon_u}{\epsilon_u + 0.004} \Longrightarrow 0.85 \times 0.85 \dfrac{20}{300} \dfrac{0.003}{0.003 + 0.004} = 20.6 \times 10^{-3} < \rho_{\text{Provided}}$

Then, compression reinforcement has been added for changing the failure mode from compression failure to secondary compression failure and its effects on section strength must be included.

- Checking the Section Type (i.e., check the effect of compression reinforcement on maximum permissible steel ratio):

$\bar{\rho}_{max} = \rho_{max} + \rho' \dfrac{f'_s}{f_y}$ ■

where f'_s is stress in the compression reinforcement at strains of ρ_{max}. It can be computed from strain distribution and as shown in relation below:

$f'_s = E_s \left[\epsilon_u - \dfrac{d'}{d}(\epsilon_u + 0.004)\right] \leq f_y$ ■

$f'_s = 200\ 000\text{MPa}\left[0.003 - \dfrac{50}{450}(0.003 + 0.004)\right] = 444 > f_y$

$f'_s = f_y = 300\text{MPa}$

$\therefore\ \bar{\rho}_{max} = \rho_{max} + \rho'$

$A_s{}' = 3 \times 490 = 1\ 470\text{mm}^2$

$\rho' = \dfrac{1\ 470\text{mm}^2}{250 \times 450} = 13.1 \times 10^{-3}$

$\therefore\ \bar{\rho}_{max} = 20.6 \times 10^{-3} + 13.1 \times 10^{-3} = 33.7 \times 10^{-3} > \rho_{\text{Provided}}$ Ok.

- Compute Section Nominal Strength M_n:

First of all, check if the compression reinforcement is yielded on not.

$\bar{\rho}_{cy} = 0.85\beta_1 \dfrac{f'_c}{f_y}\dfrac{d'}{d}\dfrac{\epsilon_u}{\epsilon_u - \epsilon_y} + \rho' = 0.85 \times 0.85 \dfrac{20}{300}\dfrac{50}{450}\dfrac{0.003}{0.003 - \dfrac{300}{200\ 000}} + 13.1 \times 10^{-3}$

$\bar{\rho}_{cy} = 10.7 \times 10^{-3} + 13.1 \times 10^{-3} = 23.8 \times 10^{-3} < \rho_{\text{Provided}}$

$\therefore f'_s = f_y = 300\text{MPa}$

Then use the relation that derived for yielded compression reinforcement:

$M_n = M_{n1} + M_{n2} = A'_s f_y (d - d') + (A_s - A'_s) f_y \left(d - \dfrac{a}{2}\right)$

where,

$a = \dfrac{(A_s - A_s{}')f_y}{0.85 f'_c b} = \dfrac{(2\ 940 - 1\ 470) \times 300}{0.85 \times 20 \times 250} = 104\text{mm}$

$$M_n = M_{n1} + M_{n2} = 1\,470 \times 300 \times (450 - 50) + (2\,940 - 1\,470) \times 300 \times \left(450 - \frac{104}{2}\right)$$
$$M_n = M_{n1} + M_{n2} = 176.4 \times 10^6 N.mm + 175.5 \times 10^6 N.mm = 352\ kN.m$$

- Compute strength reduction factor $\emptyset$:
  - Compute steel stain based on the following relations:
  $$a = 104\text{mm} \Longrightarrow c = \frac{a}{\beta_1} = \frac{104\text{ mm}}{0.85} = 122\text{ mm}$$
  $$\epsilon_t = \frac{d - c}{c}\epsilon_u = \frac{450 - 122}{122} \times 0.003 = 8.06 \times 10^{-3}$$
  - $\epsilon_t > 0.005$, then $\emptyset = 0.9$.
- Compute section design strength $\emptyset M_n$:
  $\emptyset M_n = \emptyset \times M_n = 0.9 \times 352\text{ kN.m} = 317\text{ kN.m}$■
- Check Adequacy of Stirrups as Ties:
  $\because \emptyset_{for\ Longitudinal} = 25mm < No.32$
  $\therefore \emptyset_{for\ Ties} = 10mm\ Ok.$
  $S_{Maximum} = min[16d_{bar}, 48d_{ties}, Least\ dimension\ of\ column]$
  $S_{Maximum} = min[16 \times 25mm, 48 \times 10mm, 250mm] = min[400mm, 480mm, 250mm]$
  $S_{Maximum} = 250mm > S_{Provided} = 200mm\ \ Ok.$
  Checking if alternative rebar is supported or not
  $S_{Clear} = (250 - 40 \times 2 - 10 \times 2 - 3 \times 25) \times \frac{1}{2} = 37.5\ mm < 150mm\ Ok.$

Example 4.6-3

Recheck the adequacy of the beam of Example 4.6-2 above but with d'=65 mm.

Solution

- Check the reason for using of compression reinforcement:
  $$A_{s\ Provided} = 6 \times 490 = 2\,940\ mm^2 \Longrightarrow \rho_{Provided} = \frac{2\,940\ mm^2}{250 \times 450} = 26.1 \times 10^{-3}$$
  $$\rho_{max} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} = 0.85 \times 0.85\ \frac{20}{300}\ \frac{0.003}{0.003 + 0.004} = 20.6 \times 10^{-3} < \rho_{Provided}$$
  Then, compression reinforcement has been added for changing the failure mode from compression failure to secondary compression failure and its effects on section strength must be included.
- Checking the Section Type (i.e., check the effect of compression reinforcement on maximum permissible steel ratio):
  $$\bar{\rho}_{max} = \rho_{max} + \rho' \frac{f_s'}{f_y}\ ■$$
  where f_s' is stress in the compression reinforcement at strains of ρ_{max}. It can be computed from strain distribution and as shown in relation below:
  $$f_s' = E_s\left[\epsilon_u - \frac{d'}{d}(\epsilon_u + 0.004)\right] \le f_y\ ■$$
  $$f_s' = 200\,000MPa\left[0.003 - \frac{65}{450}(0.003 + 0.004)\right] = 398 > f_y \Longrightarrow f_s' = f_y = 300MPa$$
  $$\therefore\ \bar{\rho}_{max} = \rho_{max} + \rho'$$
  $$A_s' = 3 \times 490 = 1\,470mm^2 \Longrightarrow \rho' = \frac{1\,470mm^2}{250 \times 450} = 13.1 \times 10^{-3}$$
  $$\therefore\ \bar{\rho}_{max} = 20.6 \times 10^{-3} + 13.1 \times 10^{-3} = 33.7 \times 10^{-3} > \rho_{Provided}\ \ Ok.$$
- Compute of Section Nominal Strength M_n:
  First of all, check if the compression reinforcement is yielded on not.
  $$\bar{\rho}_{cy} = 0.85\beta_1 \frac{f_c'}{f_y}\frac{d'}{d}\frac{\epsilon_u}{\epsilon_u - \epsilon_y} + \rho' = 0.85 \times 0.85 \frac{20}{300}\frac{65}{450}\frac{0.003}{0.003 - \frac{300}{200\ 000}} + 13.1 \times 10^{-3}$$
  $$\bar{\rho}_{cy} = 13.9 \times 10^{-3} + 13.1 \times 10^{-3} = 27.0 \times 10^{-3} > \rho_{Provided}$$
  $$\therefore f_s' < f_y = 300MPa$$
  Compute of f_s' can be done based on following relations:
  - Compute "c" based on Quadratic Formula:
  $$c = \sqrt{Q + R^2} - R$$
  where:

$$Q = \frac{600d'A'_s}{0.85\beta_1 f'_c b} = \frac{600 \times 65\ \text{mm}\ \times 1\,470\text{mm}^2}{0.85 \times 0.85 \times 20\dfrac{\text{N}}{\text{mm}^2} \times 250\text{mm}} = 15\,870$$

and

$$R = \frac{600A'_s - f_y A_s}{1.7\beta_1 f'_c b} = \frac{600 \times 1\,470\text{mm}^2 - 300 \times 2\,940}{1.7 \times 0.85 \times 20 \times 250} = 0$$

$$c = \sqrt{15\,870\ + 0.0^2} - 0.0 = 126\text{mm}$$

- Compute $f_s^{'}$ can be computed based on following relation:

  $$f_s^{'} = \epsilon_u E_s \frac{(c - d')}{c} = 0.003 \times 200\,000 \times \frac{126 - 65}{126} = 290\ \text{MPa} < f_y\ \text{Ok.}$$

  Then use the relation that derived for yielded compression reinforcement:

  $$M_n = M_{n1} + M_{n2} = 0.85f'_c ab\left(d - \frac{a}{2}\right) + A'_s f'_s (d - d')\blacksquare$$

  where

  $$a\ = \beta_1 c = 0.85 \times 126\text{mm} = 107\text{mm}$$

  $$M_n = M_{n1} + M_{n2} = 0.85 \times 20 \times 107 \times 250\left(450 - \frac{107}{2}\right) + 1\,470 \times 290 \times (450 - 65)$$

  $$M_n = M_{n1} + M_{n2} = 180.3 \times 10^6\text{N.mm} + 164.1 \times 10^6\text{N.mm} = 344\ \text{kN.m}$$

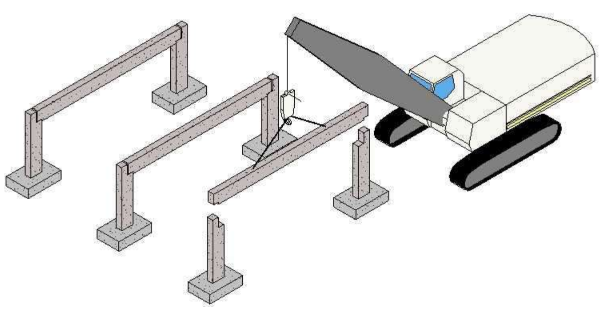
- Compute strength reduction factor $\emptyset$:
  - Compute steel stain based on the following relations:

    $$c = \ 126\text{mm} \Rightarrow \epsilon_t = \frac{d - c}{c}\epsilon_u = \frac{450 - 126}{126} \times 0.003 = 7.71 \times 10^{-3}$$

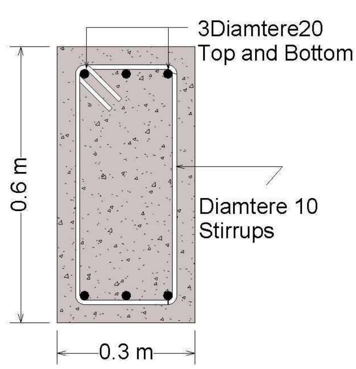
  - $\epsilon_t > 0.005$, then $\emptyset = 0.9$
- Compute section design strength $\emptyset M_n$:
  $\emptyset M_n = \ \emptyset \times M_n = \ 0.9 \times 344\ kN.m = 310\ kN.m\blacksquare$
- Check Adequacy of Stirrups as Ties:
  See previous example for stirrups checking when used as ties.

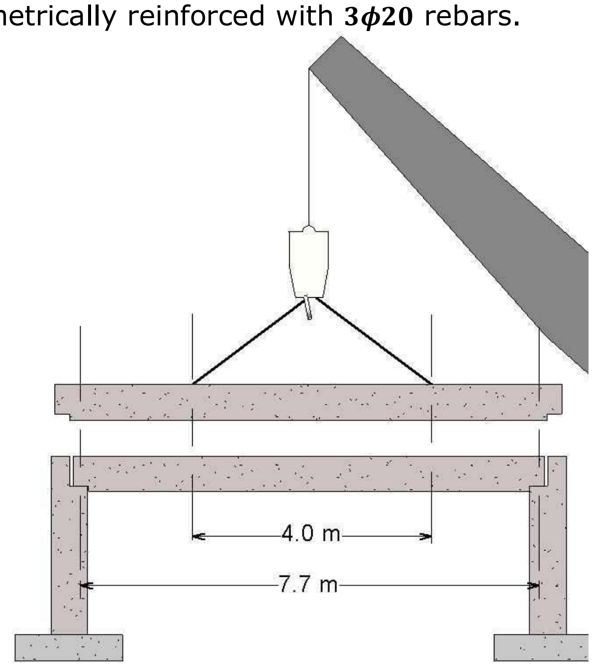
Example 4.6-4

To counteract stresses during lifting process, a simply supported precast concrete beam shown in Figure 4.6-12 below has been symmetrically reinforced with $\mathbf{3\phi20}$ rebars.



3D View.





Beam Cross Section **Elevation View.**

Figure 4.6-12: Precast beam of Example 4.6-4.

For this precast beam:

- With including effects of compressive reinforcement in your solution, compute section nominal flexural strength ϕM_n.

- What is the maximum uniformly distributed load "Wu" that could be applied on the beam during its work?
- Are the proposed reinforcement adequate during lifting process?

In your solution, assume that, $f_c' = 28\,MPa$ and $f_y = 420\,MPa$.

Solution

- **Section Flexural Strength:**

$$A_s = A_s' = 3 \times \frac{\pi \times 20^2}{4} = 942\,mm^2$$

$$d = 600 - 40 - 10 - \frac{20}{2} = 540\,mm, d' = 40 + 10 + \frac{20}{2} = 60\,mm$$

$$\rho = \rho' = \frac{942}{300 \times 540} = 5.81 \times 10^{-3}$$

$$\rho_{max} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} = 0.85^2 \times \frac{28}{420} \times \frac{3}{7} = 20.6 \times 10^{-3} > \rho$$

In spite of the compression, reinforcement has been used for a reason other than change failure mode; according to problem statement, the compression reinforcement should be included within solution.

$$\because \rho = \rho'$$

$$\therefore \bar{\rho}_{cy} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{d'}{d} \frac{\epsilon_u}{\epsilon_u - \epsilon_y} + \rho' > \rho$$

$$f_s' < f_y$$

$$c = \sqrt{Q + R^2} - R$$

$$Q = \frac{600 d' A_s'}{0.85\beta_1 f_c' b} = \frac{600 \times 60 \times 942}{0.85 \times 0.85 \times 28 \times 300} = 5588$$

$$R = \frac{600 A_s' - f_y A_s}{1.7\beta_1 f_c' b} = \frac{600 \times 942 - 420 \times 942}{1.7 \times 0.85 \times 28 \times 300} = 13.9$$

$$c = \sqrt{Q + R^2} - R = \sqrt{5588 + 13.9^2} - 13.9 = 62.1\text{ mm}$$

$$f_s' = \epsilon_u E_s \frac{(c - d')}{c} = 0.003 \times 200\,000 \times \frac{62.1 - 60}{62.1} = 20\text{ MPa} < f_y \text{ Ok.}$$

$$M_n = M_{n1} + M_{n2} = 0.85 f_c' a b \left(d - \frac{a}{2}\right) + A_s' f_s' (d - d')$$

$$a = \beta_1 c = 0.85 \times 62.1 = 52.8\,mm$$

$$M_n = 0.85 \times 28 \times 52.8 \times 300 \left(540 - \frac{52.8}{2}\right) + 942 \times 20 \times (540 - 60) = 203\text{ kN.m}$$

$$\epsilon_t = \frac{d - c}{c} \epsilon_u = \frac{540 - 62.1}{62.1} \times 0.003 = 23.1 \times 10^{-3} > 0.005 \Longrightarrow \phi = 0.9$$

$$\phi M_n = 0.9 \times 203 = 183\,kN.m \blacksquare$$

- **Maximum Permissible Factored Load W_u:**

$$M_u = \frac{W_u l^2}{8} = \phi M_n \Longrightarrow M_u = \frac{W_u \times 7.7^2}{8} = 183 \Longrightarrow W_u = 24.7 \frac{kN}{m} \blacksquare$$

- **Section Adequacy during Lifting Process:**

During lifting process, factored load is equal to factored dead load:

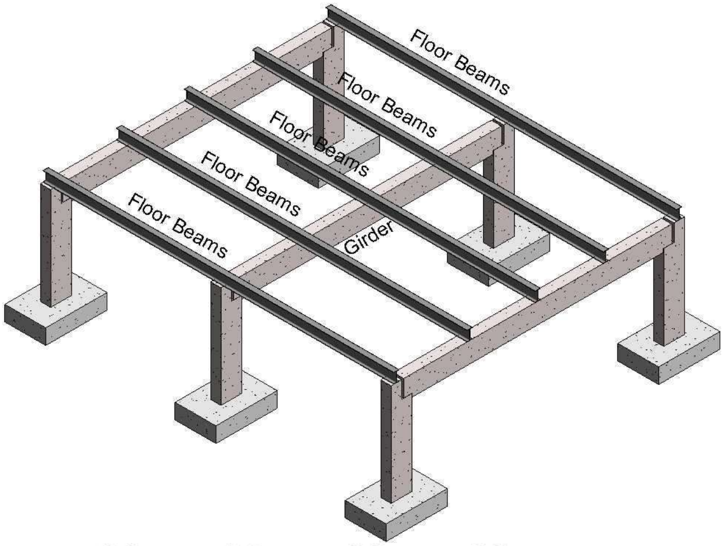
$$W_u = 1.4 W_d = 1.4 \times (24 \times 0.6 \times 0.3) = 6.05 \frac{kN}{m}$$

$$M_{u\,for\,cantilever\,part} = \frac{6.05 \times \left(0.5 \times (7.7 - 4)\right)^2}{2} = 10.4\,kN.m < \phi M_n \therefore Ok.$$

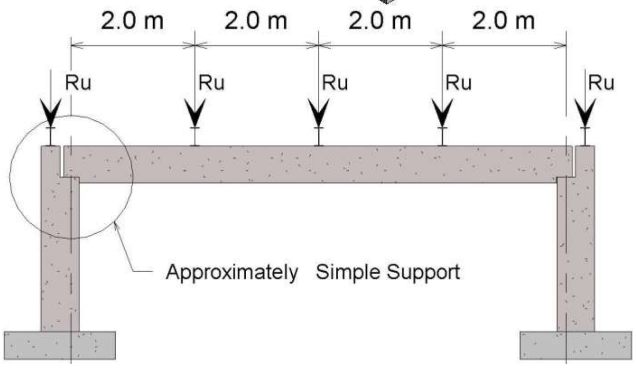
$$M_{u\,mid-span} = \frac{6.05 \times 4^2}{8} - 10.4 = 1.70 < \phi M_n \therefore Ok.$$

Example 4.6-5

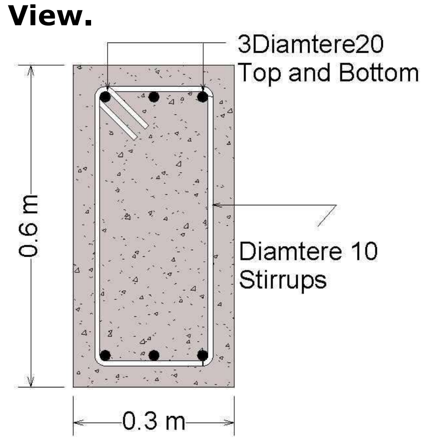
For a frame shown in Figure 4.6-13 below, with neglecting selfweight and with including the effects of compression rebars and based on flexural strength only; what is the maximum factored floor beam reaction "Ru" that could be supported by the girder? In your solution, assume that, $f_c' = 28\,MPa$ and $f_y = 420\,MPa$.



3D View.



Elevation View.



Beam Cross Section.

Figure 4.6-13: Frame system of Example 4.6-5.

Solution

- **Section Flexural Strength:**

$$A_s = A_s' = 3 \times \frac{\pi \times 20^2}{4} = 942\ mm^2$$

$$d = 600 - 40 - 10 - \frac{20}{2} = 540\ mm, d' = 40 + 10 + \frac{20}{2} = 60\ mm$$

$$\rho = \rho' = \frac{942}{300 \times 540} = 5.81 \times 10^{-3}$$

$$\rho_{max} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} = 0.85^2 \times \frac{28}{420} \times \frac{3}{7} = 20.6 \times 10^{-3} > \rho$$

In spite of the compression reinforcement has been used for a reason other than change failure mode, according to problem statement the compression reinforcement should be included within solution.

$$\because \rho = \rho'$$

$$\therefore \bar{\rho}_{cy} = 0.85\beta_1 \frac{f_c'}{f_y}\frac{d'}{d}\frac{\epsilon_u}{\epsilon_u - \epsilon_y} + \rho' > \rho \Rightarrow f_s' < f_y$$

$$\mathrm{c} = \sqrt{\mathrm{Q} + \mathrm{R}^2} - \mathrm{R}$$

$$\mathrm{Q} = \frac{600\mathrm{d}'\mathrm{A}_s'}{0.85\beta_1 f_c' b} = \frac{600 \times 60 \times 942}{0.85 \times 0.85 \times 28 \times 300} = 5588, \mathrm{R} = \frac{600\mathrm{A}_s' - f_y \mathrm{A}_s}{1.7\beta_1 f_c' b} = \frac{600 \times 942 - 420 \times 942}{1.7 \times 0.85 \times 28 \times 300} = 13.9$$

$$\mathrm{c} = \sqrt{\mathrm{Q} + \mathrm{R}^2} - \mathrm{R} = \sqrt{5588 + 13.9^2} - 13.9 = 62.1\ \mathrm{mm}$$

$$f_s' = \epsilon_u E_s \frac{(c - d')}{c} = 0.003 \times 200\,000 \times \frac{62.1 - 60}{62.1} = 20\ \mathrm{MPa} < f_y\ \ \mathrm{Ok.}$$

$$M_n = M_{n1} + M_{n2} = 0.85 f_c' a b \left(d - \frac{a}{2}\right) + A_s' f_s' (d - d')$$

$$a = \beta_1 c = 0.85 \times 62.1 = 52.8\ mm$$

$$M_n = 0.85 \times 28 \times 52.8 \times 300 \left(540 - \frac{52.8}{2}\right) + 942 \times 20 \times (540 - 60) = 203\ \mathrm{kN.m}$$

$$\epsilon_t = \frac{d - c}{c}\epsilon_u = \frac{540 - 62.1}{62.1} \times 0.003 = 23.1 \times 10^{-3} > 0.005 \Longrightarrow \phi = 0.9$$

$$\phi M_n = 0.9 \times 203 = 183\ kN.m \blacksquare$$

- **<u>Maximum Floor Beam Reaction:</u>**

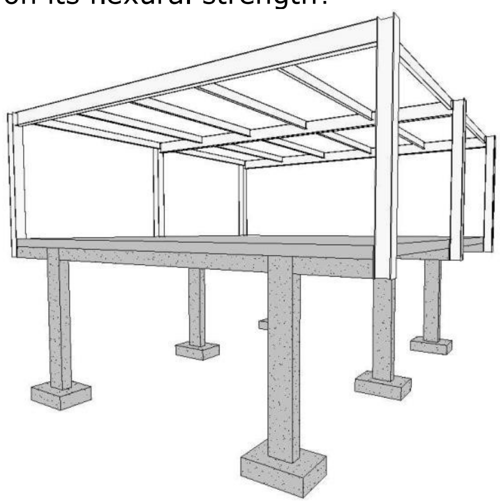
  Let $M_u = \phi M_n$

  With use of superposition principle, factored moment M_u is:

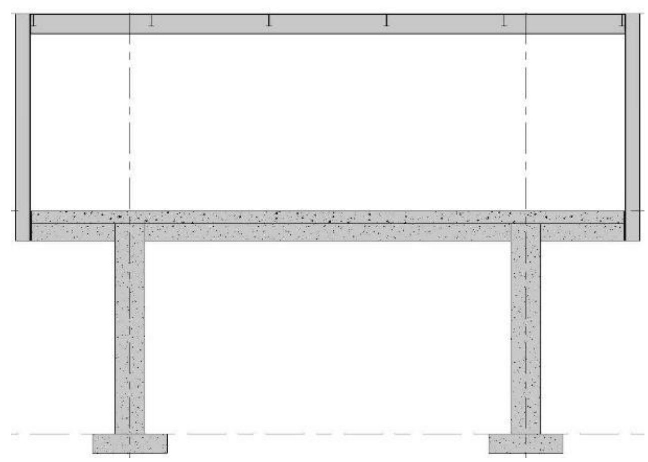
$$M_u = R_u a + \frac{R_u l}{4} = 2R_u + \frac{8}{4}R_u = 4R_u \Longrightarrow M_u = 4R_u = \phi M_n = 183 \Longrightarrow R_u = 45.8\ kN \blacksquare$$

Example 4.6-6

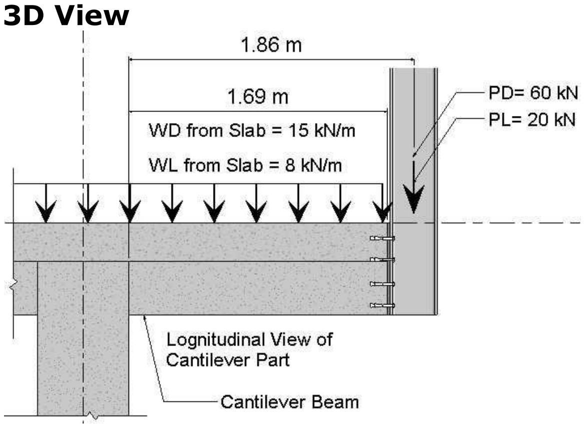
In an attempt to add a new floor for an existing reinforced concrete building, a steel frame shown in Figure 4.6-14 below has been proposed. The steel columns have been supported on cantilever concrete beams of the existing concrete floor. If the cantilever part of the beam is reinforced as shown; can it withstand the applied loads shown based on its flexural strength?

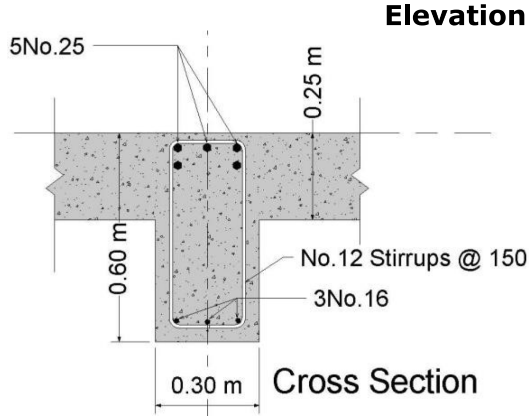


3D View



Elevation





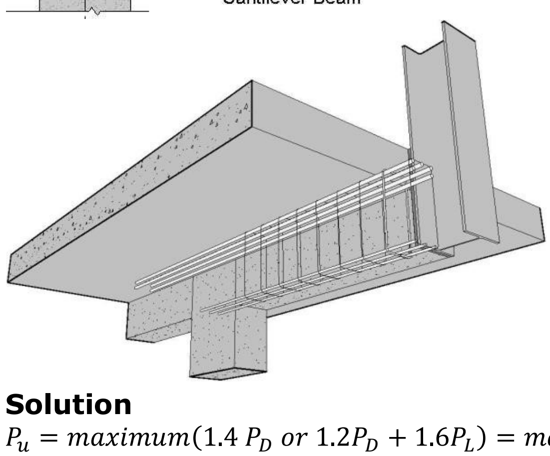


Figure 4.6-14: Building with additional new floor for Example 4.6-6.

Solution

$$P_u = maximum(1.4\ P_D\ or\ 1.2P_D + 1.6P_L) = maximum(1.4 \times 60\ or\ 1.2 \times 60 + 1.6 \times 20)$$

$$P_u = maximum(84\ or\ 104) = 104\ kN$$

$W_{self} = 0.3 \times (0.6 - 0.25) \times 24 = 2.52 \frac{kN}{m} \Longrightarrow W_D = 2.52 + 15 = 17.5 \frac{kN}{m}, W_L = 8 \frac{kN}{m}$

$W_u = maximum(1.4 \times 17.5\ or\ 1.2 \times 17.5 + 1.6 \times 8) = maximum(24.5\ or\ 33.8) = 33.8 \frac{kN}{m}$

$M_u = \frac{W_u l^2}{2} + P_u l = \frac{33.8 \times 1.69^2}{2} + 104 \times 1.86 = 242\ kN.m$

Check the reason for using of compression reinforcement:

$A_{Bar} = \frac{\pi \times 25^2}{4} \approx 490\ mm^2 \Longrightarrow \mathrm{A_{s\ Provided}} = 5 \times 490 = 2450\ \mathrm{mm^2}$

$d = 600 - 40 - 12 - 25 - \frac{25}{2} = 510\ mm \Longrightarrow \rho_{Provided} = \frac{2450}{300 \times 510} = 16.0 \times 10^{-3}$

$\rho_{max} = 0.85\beta_1 \frac{f'_c}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} = 0.85 \times 0.85 \ \frac{28}{420} \ \frac{0.003}{0.003 + 0.004} = 20.6 \times 10^{-3} > \rho_{Provided}$

Then, compression reinforcement has been added for a reason other than to chang the failure mode from compression failure to secondary compression failure and its effects on section strength can be neglected. Therefore, the section can be analyzed as a singly reinforced section.

As the flange is under tension, the span is a statically indeterminate one, and noting is mentioned about flange width, hence second term of second relation for $A_{s\ minimum}$ is adopted:

$\mathrm{A_{s\ minimum}} = \frac{0.5\sqrt{f'_c}}{f_y} b_w d = \left(\frac{0.5 \times \sqrt{28}}{420}\right) \times (300 \times 510) = 9640\ \mathrm{mm^2} < \mathrm{A_{s\ Provided}}\ \mathrm{Ok.}\ \blacksquare$

Compute section nominal strength M_n :

$M_n = \rho f_y b d^2 \left(1 - 0.59\ \frac{\rho f_y}{f'_c}\right) = 16.0 \times 10^{-3} \times 420 \times 300 \times 510^2 \times \left(1 - 0.59\ \frac{16.0 \times 10^{-3} \times 420}{28}\right)$
$= 450\ \mathrm{kN.m}$

Compute strength reduction factor $\emptyset$:

Compute steel stain based on the following relations:

$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{2450 \times 420}{0.85 \times 28\ \times 300} = 144\ \mathrm{mm} \Longrightarrow c = \frac{a}{\beta_1} = \frac{144}{0.85} = 169\ \mathrm{mm}$

$\epsilon_t = \frac{d - c}{c} \epsilon_u = \frac{510 - 169}{169} \times 0.003 = 6.05 \times 10^{-3} \Longrightarrow \emptyset = 0.9$

Compute section design strength $\emptyset M_n$:

$\emptyset M_n = \emptyset \times M_n = 0.9 \times 450 = 405\ \mathrm{kN.m} > M_u \therefore \mathrm{Ok.}$

Therefore, based on its flexural strength, cantilever part is adequate to support intended steel frame.

Example 4.6-7

Based on flexure strength of section A-A, computed the maximum value of P_u that could be supported by the beam presented in Figure 4.6-15 below. In Your solution, assume that:

- $f_c' = 21MPa$ and $f_y = 420MPa$.
- Selfweight could be neglected.
- $A_{Bar} = 500\ mm^2\ for\ \phi\ 25mm.$

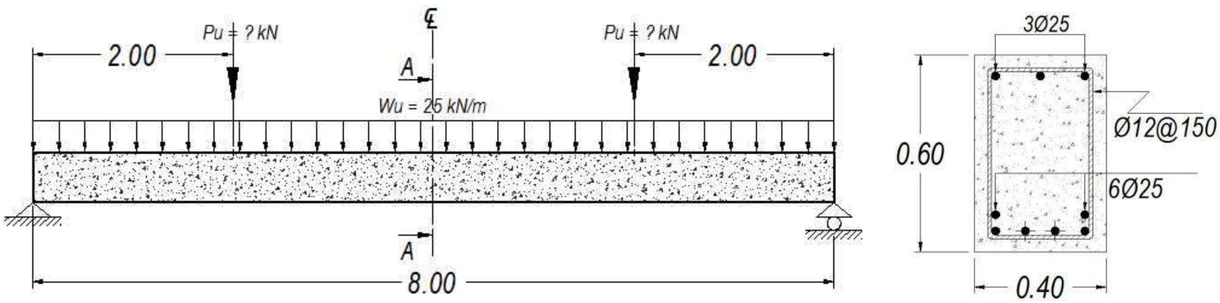


Figure 4.6-15: Simply supported beam for Example 4.6-7. **Section A-A**

Solution

- Check the reason for using of compression reinforcement:

$A_{s\ Provided} = 6 \times 500 = 3000\ \text{mm}^2, d = 600 - 40 - 12 - 25 - \frac{25}{2} = 510$

$\rho_{Provided} = \frac{3000}{400 \times 510} = 14.7 \times 10^{-3}$

$\rho_{max} = 0.85\beta_1 \frac{f'_c}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} = 0.85 \times 0.85\ \frac{21}{420}\ \frac{0.003}{0.003 + 0.004} = 15.5 \times 10^{-3} > \rho_{Provided}$

Then, compression reinforcement has been added for a reason other than changing the failure mode from compression failure to secondary compression failure and its effects on section strength can be neglected.
Then the section can be analyzed as a singly reinforced section.

$A_{s\ minimum} = \frac{0.25\sqrt{f'_c}}{f_y} b_w d \geq \frac{1.4}{f_y} b_w d$

$\because\ f'_c < 31\ MPa$

$\therefore A_{s\ minimum} = \frac{1.4}{f_y} b_w d = \frac{1.4}{420} \times 400 \times 510 = 680\ \text{mm}^2 < A_{s\ Provided}$ Ok. ■

- Compute section nominal strength M_n :

$M_n = \rho f_y b d^2 \left(1 - 0.59\ \frac{\rho f_y}{f'_c}\right)$

$M_n = 14.7 \times 10^{-3} \times 420 \times 400 \times 510^2 \left(1 - 0.59\ \frac{14.7 \times 10^{-3} \times 420}{21}\right) = 531\ \text{kN.m}$

- Compute strength reduction factor $\emptyset$:

Compute steel stain based on the following relations:

$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{3000\ \text{mm}^2 \times 420\ \text{MPa}}{0.85 \times 21\ \text{MPa}\ \times 400\text{mm}} = 176\ \text{mm}$

$c = \frac{a}{\beta_1} = \frac{176\ \text{mm}}{0.85} = 207\ \text{mm}$

$\epsilon_t = \frac{d - c}{c} \epsilon_u = \frac{510 - 207}{207} \times 0.003 = 4.39 \times 10^{-3}$

Then:

$\emptyset = 0.483 + 83.3\epsilon_t = 0.483 + 83.3 \times 4.39 \times 10^{-3} = 0.849$

- Compute section design strength $\emptyset M_n$:

$\emptyset M_n = \emptyset \times M_n = 0.849 \times 531\ \text{kN.m} = 451\ \text{kN.m}$ ■

- Compute Pu:

$M_u = P_u \times 2.0 + \frac{25 \times 8^2}{8} = 451\ \text{kN.m} \implies P_u = 125\ kN$ ■

Example 4.6-8

Compute the maximum factored load P_u that can be supported by a beam shown in Figure 4.6-16 below. In your solution:

- Neglect the selfweight.
- $f'_c = 21\ MPa$
- $f_y = 420\ MPa$.

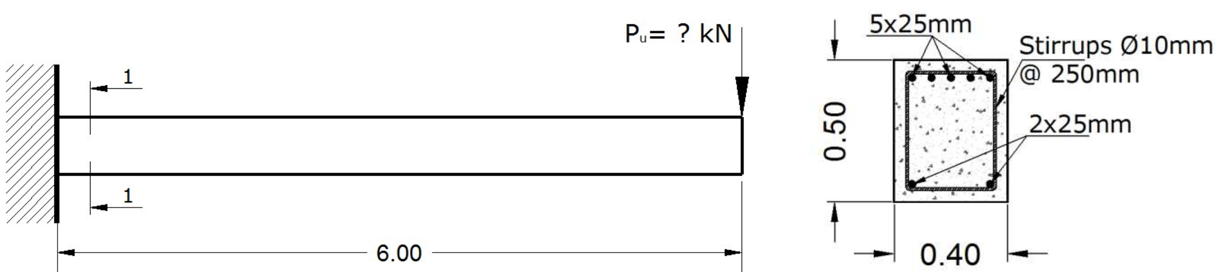


Figure 4.6-16: Cantilever beam for Example 4.6-8. **Section 1-1**

Solution

- Check the cause for using of compression reinforcement:

$$A_{Bar} = \frac{\pi \times 25^2}{4} = 490\ mm^2, d = 500 - 40 - 10 - 12.5 = 437.5\ mm$$

$$\rho_{Provided} = \frac{490 \times 5}{437.5 \times 400} = 14 \times 10^{-3}$$

$$\rho_{maximum} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{0.003}{0.003 + 0.004} = 0.85^2\ \frac{21}{420} \frac{0.003}{0.003 + 0.004} = 15.5 \times 10^{-3}$$

As $\rho_{Provided} < \rho_{max}$, then the compression reinforcement has been used to a cause other than the flexure strength. Then the section can be analyzed as singly reinforced section.

- Compute the section flexure nominal strength and design strength:

$$\sum F_x = 0$$

$$0.85 \times 21 \times a \times 400 = (490 \times 5) \times 420 \Longrightarrow a = 144\ mm$$

$$M_n = (490 \times 5) \times 420 \times \left(437.5 - \frac{144}{2}\right) = 376\ kN.m$$

- Strength Reduction Factor ϕ:

$$c = \frac{144\ mm}{0.85} = 169\ mm \Longrightarrow \epsilon_t = \frac{d - c}{c} \times \epsilon_u = \frac{437\ mm - 169\ mm}{169\ mm} \times 0.003 = 4.76 \times 10^{-3}$$

$$\emptyset = 0.483 + 83.3\epsilon_t = 0.483 + 83.3 \times 4.76 \times 10^{-3} = 0.879$$

- Compute ϕM_n:

$$\emptyset M_n = 0.879 \times 376\ kN.m = 331\ kN.m$$

- Compute the maximum permissible force P_u:

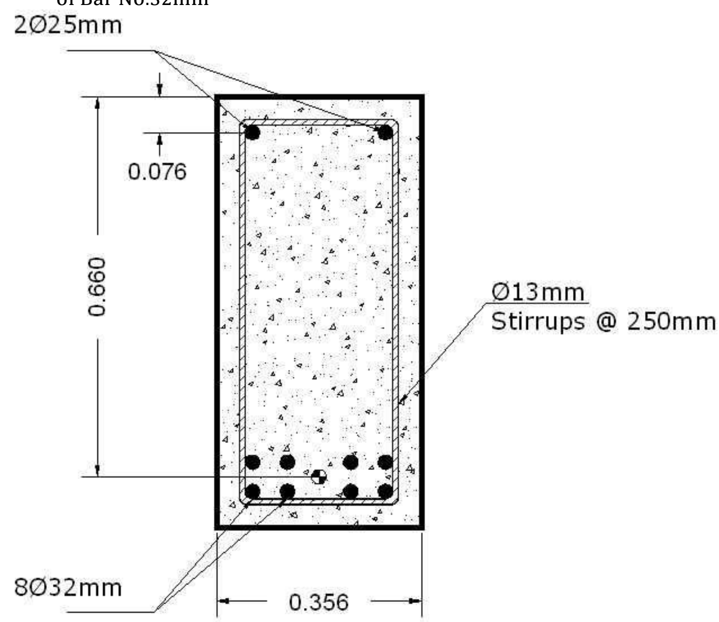
$$\emptyset M_n = 331\ kN.m = P_u \times l = P_u \times 6m \Longrightarrow P_u = 55.2\ kN.m \quad \blacksquare$$

4.6.6 Homework Problems

Problem 4.6-1

Check the adequacy of the beam shown below and compute its design strength according to ACI Code. Assume that:

1. $f_c' = 34.5$ MPa.
2. $f_y = 414$MPa.
3. $A_{of\ Bar\ No.25mm} = 510mm^2$.
4. $A_{of\ Bar\ No.32mm} = 819mm^2$.



Answers

- Check the reason for using of compression reinforcement:

$$A_{s\ Provided} = 6\ 552\ mm^2 \Longrightarrow \rho_{Provided} = 27.9 \times 10^{-3}$$

$$\beta_1 = 0.804 \Longrightarrow \rho_{max} = 24.4 \times 10^{-3} < \rho_{Provided}$$

Then, compression reinforcement has been added for changing the failure mode from compression failure to secondary compression failure and its effects on section strength must be included.

- Checking the Section Type (i.e., check the effect of compression reinforcement on maximum permissible steel ratio):

$\bar{\rho}_{max} = \rho_{max} + \rho' \frac{f_s'}{f_y}$ ■

$f_s' = f_y = 414MPa \Longrightarrow \bar{\rho}_{max} = \rho_{max} + \rho'$

$A_s' = 1\,020mm^2 \Rightarrow \rho' = 4.34 \times 10^{-3} \Rightarrow \therefore \bar{\rho}_{max} = 28.7 \times 10^{-3} > \rho_{Provided}$ Ok.

- Compute of Section Nominal Strength M_n:

First of all, check if the compression reinforcement is yielded on not.

$\bar{\rho}_{cy} = 25.5 \times 10^{-3} < \rho_{Provided} \Rightarrow \therefore f_s' = f_y = 414MPa$

Then use the relation that derived for yielded compression reinforcement:

$M_n = M_{n1} + M_{n2} = A_s' f_y (d - d') + (A_s - A_s') f_y \left(d - \frac{a}{2}\right)$

where

$a = 219mm$

$M_n = M_{n1} + M_{n2} = 247 \times 10^6 N.mm + 1\,261 \times 10^6 N.mm = 1\,508\ kN.m$

- Compute strength reduction factor $\emptyset$:

Compute steel stain based on the following relations:

$a = 219mm \Rightarrow c = 272\ mm \Rightarrow \epsilon_t = 4.28 \times 10^{-3}$

$\epsilon_t < 0.005$, then:

$\emptyset = 0.84$

- Compute section design strength $\emptyset M_n$:

$\emptyset M_n = 1\,267\ kN.m$ ■

- Check Adequacy of Stirrups as Ties:

$\because \emptyset_{for\ Longitudinal} = 25mm < No.32$

$\therefore \emptyset_{for\ Ties} = 13mm$ Ok.

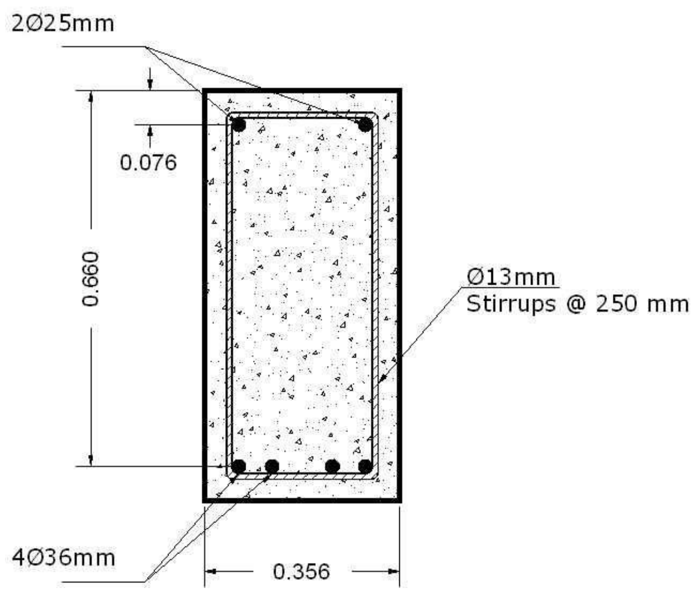
$S_{Maximum} = \min[16d_{bar}, 48d_{ties}, \text{Least dimension of column}]$

$S_{Maximum} = 356mm > S_{Provided} = 250mm$ Ok.

Problem 4.6-2

Check the adequacy of the beam shown below and compute its design strength according to ACI Code. Assume that:

1. $f_c' = 34.5$ MPa.
2. $f_y = 414$MPa.
3. $A_{of\ Bar\ No.25mm} = 510mm^2$.
4. $A_{of\ Bar\ No.36mm} = 1\,008mm^2$.



Answers

- Check the reason for using of compression reinforcement:

$A_{s\ Provided} = 4032mm^2$

$\rho_{Provided} = 17.2 \times 10^{-3}$

$\beta_1 = 0.804$

$\rho_{max} = 24.4 \times 10^{-3} > \rho_{Provided}$

Then, compression reinforcement has been added for a reason other than changing the failure mode from compression failure to secondary compression failure and its effects on section strength can be neglected.

Then the section can be analyzed as a singly reinforced section.

$\because f_c' > 31\ MPa\ \therefore A_{s\ minimum} = \frac{0.25\sqrt{f_c'}}{f_y} b_w d = 833\ mm^2 < A_{s\ Provided}$ Ok. ■

- Compute section nominal strength M_n:

$M_n = 970\ kN.m$

- Compute strength reduction factor $\emptyset$:
  Compute steel stain based on the following relations:
  $a = 160 \text{ mm} \Longrightarrow c = 199 \text{ mm} \Longrightarrow \epsilon_t = 6.95 \times 10^{-3}$
  $\epsilon_t > 0.005$, then:
  $\emptyset = 0.9$
- Compute section design strength $\emptyset M_n$:
  $\emptyset M_n = \emptyset \times M_n = 0.9 \times 970 \text{ kN.m} = 873 \text{ kN.m}$■

Problem 4.6-3

Re-compute design strength of beam above according to ACI Code with including the effect of compression reinforcement even it has been used for a purpose other than strength requirement.

Answers

- Compute of Section Nominal Strength M_n:
  First of all, check if the compression reinforcement is yielded on not.
  $\bar{\rho}_{cy} = 21.2 \times 10^{-3} + 4.31 \times 10^{-3} = 25.5 \times 10^{-3} > \rho_{Provided}$
  $\therefore f_s' < f_y$
  Compute of f_s' can be done based on following relations:
  a. Compute "c" based on Quadratic Formula:
     $c = \sqrt{Q + R^2} - R$
     where:
     $$Q = \frac{600d'A_s'}{0.85\beta_1 f_c' b} = 5\,541$$
     and
     $$R = \frac{600A_s' - f_y A_s}{1.7\beta_1 f_c' b} = -63.0$$
     $c = 160\text{mm}$
  b. Compute f_s' can be computed based on following relation:
     $$f_s' = \epsilon_u E_s \frac{(c - d')}{c} = 315 \text{ MPa} < f_y \text{ Ok.}$$
     Then use the relation that derived for not yielded compression reinforcement:
     $$M_n = M_{n1} + M_{n2} = 0.85 f_c' ab \left(d - \frac{a}{2}\right) + A_s' f_s' (d - d')$$■
     where
     $a = \beta_1 c = 129\text{mm}$
     $M_n = M_{n1} + M_{n2} = 802 \times 10^6 \text{N.mm} + 188 \times 10^6 \text{N.mm} = 990 \text{ kN.m}$
- Compute strength reduction factor $\emptyset$:
  a. Compute steel stain based on the following relations:
     $c = 160\text{mm} \Longrightarrow \epsilon_t = 9.38 \times 10^{-3}$
     It is useful to note, that using of compression reinforcement has increased strain of tensile reinforcement for $\epsilon_t = 6.95 \times 10^{-3}$ to a strain of $\epsilon_t = 9.38 \times 10^{-3}$. Then using of compression reinforcement has increased section ductility (as was discussed in reasons for using of compression reinforcement).
  b. $\epsilon_t > 0.005$, then $\emptyset = 0.9$
- Compute section design strength $\emptyset M_n$:
  $\emptyset M_n = \emptyset \times M_n = 891 \text{ kN.m}$■

4.7 DESIGN OF A DOUBLY REINFORCED RECTANGULAR SECTION

4.7.1 Essence of the Problem

- This article discusses the design of a doubly reinforced concrete beam to solve a problem related to the fourth one of the four reasons discussed in previous article, i.e. this article discusses the computing of compression reinforcement A_s' when the designer needs a reinforcement ratio greater than ρ_{max} to resist the applied factored moment M_u.
- Therefore, the knowns of the design problem are:
  - Applied factored moment that must be resisted "M_u".
  - Materials strength f_c' and f_y.
  - Pre-specified beam dimensions b and h determined based on architectural or other limitations. These dimensions have been selected relatively small such that the section cannot resist the required moment with tension reinforcement only.
- While, the main unknowns of the design problem are the tension and compression reinforcements and their details. Selection of adequate stirrups that can act as ties for compression reinforcement is a part of the design process.

4.7.2 Design Procedure

This procedure has been written assuming the designer has no previous indication that the proposed dimensions are inadequate and that the section should be designed as a doubly reinforced section.

1. Compute the required factored moment M_u based on the given spans and loads. As the dimensions have been pre-specified, then beam selfweight can be computed and added to applied loads.
2. Compute the required nominal moment based on following relation:
   $$M_n = \frac{M_u}{\emptyset}$$
   where $\emptyset$ will be assumed 0.9 to be checked later.
3. Check if the section can be designed as a singly reinforced section or not based on following reasoning:
   a. If the square roof of following relation has an imaginary value, then the section cannot be designed as singly reinforced section.
   $$\rho_{Required} = \frac{1 - \sqrt{1 - 2.36 \frac{M_n}{f_c' b d^2}}}{1.18 \times \frac{f_y}{f_c'}}$$
   b. If the required steel ratio
   $$\rho_{Required} = \frac{1 - \sqrt{1 - 2.36 \frac{M_n}{f_c' b d^2}}}{1.18 \times \frac{f_y}{f_c'}}$$
   is greater than the maximum steel ratio
   $$\rho_{max} = 0.85\beta_1 \ \frac{f_c'}{f_y} \ \frac{\epsilon_u}{\epsilon_u + 0.004}$$
   then the section cannot be designed as singly reinforced section.
4. Re-compute the required nominal moment for the section that must be designed as a doubly reinforced section based on:
   $$M_n = \frac{M_u}{\emptyset}$$
   As the section is at tensile strain range of ρ_{max}, i.e. at tensile strain "ϵ_t" of 0.004, then the strength reduction factor would be as indicated in Figure 4.7-1 below.
   $\emptyset = 0.816$

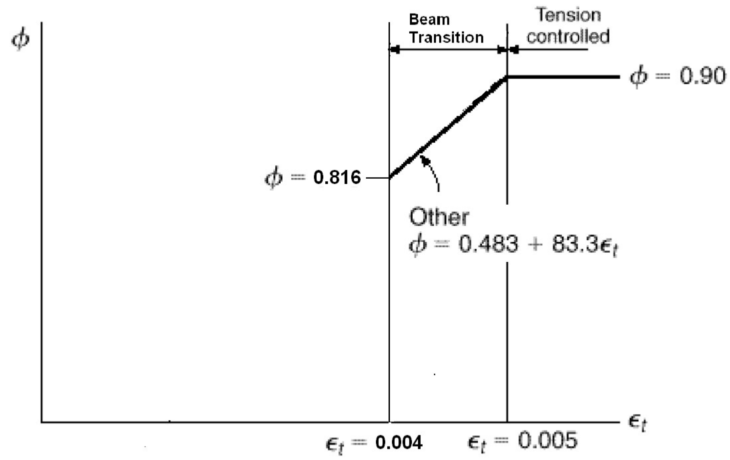


Figure 4.7-1: Strain versus strength reduction factor for beams according to ACI code, reproduced for convenience.

In design process of a doubly reinforced section, it is useful to imagine that the nominal flexure strength M_n is consisting of two parts shown below:

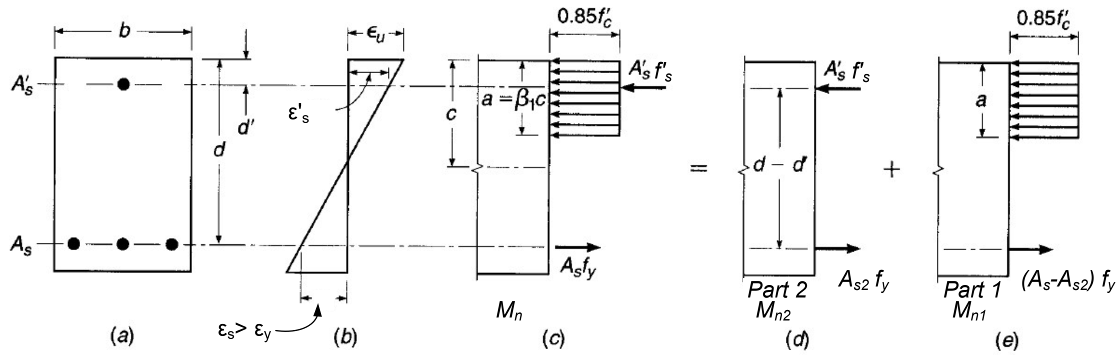


Figure 4.7-2: Strain, stress, and force distribution adopted in design of a doubly reinforced rectangular beam.

5. Compute of Tension Reinforcement A_s:
    a. Compute the nominal moment and tension reinforcement for part 1:
    $A_{s1} = A_{smax} = \rho_{max} bd$
    $$M_{n1} = \rho_{max} f_y bd^2 \left(1 - 0.59\,\frac{\rho_{max} f_y}{f_c'}\right)$$
    b. Compute the nominal moment and tension reinforcement for part 2:
    $M_{n2} = M_n - M_{n1}$
    $$A_{s2} = \frac{M_{n2}}{f_y(d - d')}$$
    c. Compute the ***Total Tension Reinforcement*** A_s:
    $A_s = A_{s1} + A_{s2}$ ■
6. Compute of Compression Reinforcement A_s':
    a. Check if compression reinforcement is yielded or not:
    Compute of "a" based on force diagram of Part 1:
    $$a = \frac{A_{s1} f_y}{0.85 f_c' b}$$
    then compute "c":
    $$c = \frac{a}{\beta_1}$$
    and compute of compressive stress in compression reinforcement:
    $$f_s' = \epsilon_u E_s \frac{c - d'}{c}$$
    b. If $f_s' \geq f_y$, then the compression reinforcement has yielded:
    $f_s' = f_y$
    $A_s' = A_{s2}$ ■
    c. Else, the compression reinforcement is not yielded:
    $A_s' = A_{s2} \frac{f_y}{f_s'}$ ■

7. Compute the Required Rebars Numbers.
8. Ties Design:
   a. Select bar diameter for ties:
      If single compression rebars with diameter of:
      $\emptyset_{Bar} \leq 32mm$
      then
      $\emptyset_{Tie} = 10mm$
      else, use:
      $\emptyset_{Tie} = 13mm$
   b. Compute the required spacing of the ties:
      $S_{Required\ for\ Ties} = \min[16d_{bar}, 48d_{ties}, \text{Least dimension of column}]$
      This spacing must be checked with the shear requirement also. Actual design practice is to select "S" based on shear requirement (As will be discussed in Chapter 5) and then check its adequacy for ties requirements.
   c. Use a suitable ties arrangement as discussed previously.
9. Draw the final section details.

4.7.3 Example

Example 4.7-1

A rectangular beam, that must carry a service live load of 36.0 kN/m and a dead load of 15.3 kN/m (including its selfweight) on a simple span of 5.49 m, is limited in cross section for architectural reasons to 250mm width and 500mm depth. Design this beam for flexure. In your design, assume the following:

- $f_y = 414$ Mpa, $f_c{}' = 27.5$ Mpa
- No. 29 for longitudinal tension reinforcement.
- No. 19 for compression reinforcement if required.
- No. 10 for stirrups (it's adequacy must be checked when used as a tie).
- Two layers of tension reinforcement.

Solution

- Compute the required factored moment M_u:

$$M_{Dead} = \frac{15.3\frac{kN}{m} \times 5.49^2 m^2}{8} = 57.6\ kN.m \quad M_{Live} = \frac{36.0\frac{kN}{m} \times 5.49^2 m^2}{8} = 136\ kN.m$$

$$M_u = \text{maximum of}\ [1.4M_{Dead} \text{or}\ 1.2M_{Dead} + 1.6M_{Live}]$$

$$M_u = maximum\ of\ [1.4 \times 57.6\ kN.m\ or\ 1.2 \times 57.6\ kN.m + 1.6 \times 136\ kN.m]$$

$$M_u = \text{maximum of}\ [80.6\ kN.m\ \text{ or }\ 287\ kN.m] = 287\ kN.m$$

- Compute the required nominal moment based on following relation:

$$M_n = \frac{M_u}{\emptyset} = \frac{287}{0.9} = 319\ kN.m$$

where ∅ will be assumed 0.9 to be checked later.

- Check if the section can be design as a singly reinforced section or not based on following reasoning:

$$d = 500 - 40 - 10 - 29 - \frac{25}{2} = 409mm$$

$$\rho_{Required} = \frac{1 - \sqrt{1 - 2.36\frac{M_n}{f_c' bd^2}}}{1.18 \times \frac{f_y}{f_c'}} = \frac{1 - \sqrt{1 - 2.36\frac{319 \times 10^6}{27.5 \times 250 \times 409^2}}}{1.18 \times \frac{414}{27.5}} = 23.2 \times 10^{-3}$$

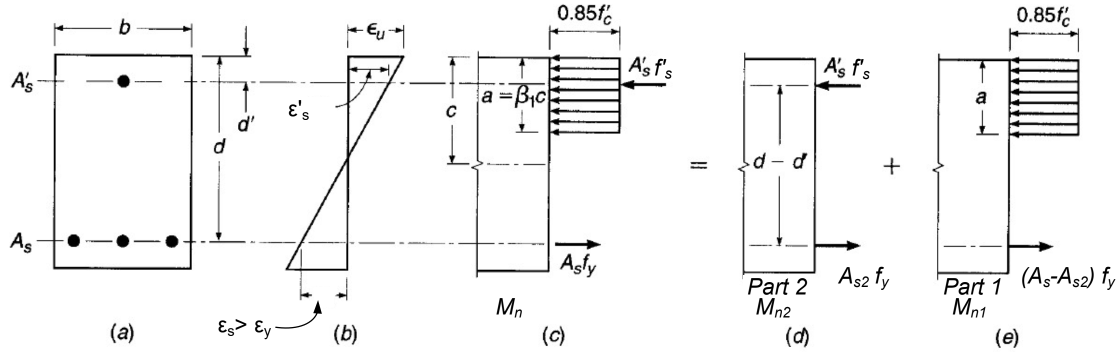
$$\rho_{max} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} = 0.85 \times 0.85\ \frac{27.5}{414}\ \frac{0.003}{0.003 + 0.004} = 20.6 \times 10^{-3} < \rho_{Required}$$

then the section must be design as a doubly reinforced section.

- Re-compute the required nominal for the section based on $\emptyset = 0.816$:

$$M_n = \frac{M_u}{\emptyset} = \frac{287}{0.816} = 352\ kN.m$$

It is useful to imagine that the nominal flexure strength M_n is consisting of two parts shown below:



- Compute of Tension Reinforcement As:
  - Compute the nominal moment and tension reinforcement for part 1:

    $A_{s1} = A_{smax} = \rho_{max} bd == 20.6 \times 10^{-3} \times 250 \times 409 = 2\,106\ mm^2$

    $M_{n1} = \rho_{max} f_y bd^2 \left(1 - 0.59\ \frac{\rho_{max} f_y}{f'_c}\right)$

    $M_{n1} = 20.6 \times 10^{-3} \times 414 \times 250 \times 409^2 \left(1 - 0.59\ \frac{20.6 \times 10^{-3} \times 414}{27.5}\right) = 291\ kN.m$
  - Compute the nominal moment and tension reinforcement for part 2:

    $M_{n2} = M_n - M_{n1} = 352\ kN.m - 291\ kN.m = 61\ kN.m$

    $d' = 40 + 10 + \frac{19}{2} = 59.2$

    $A_{s2} = \frac{M_{n2}}{f_y(d - d')} = \frac{61 \times 10^6}{414 \times (409 - 59.2)} = 421\ mm^2$
  - Compute the Total Tension Reinforcement A_s:

    $A_s = A_{s1} + A_{s2} = 2\,106\ mm^2 + 421\ mm^2 = 2\,527\ mm^2$
- Compute of Compression Reinforcement A_s':
  - Check if compression reinforcement is yielded or not:

    Compute of "a" based on force diagram of Part 1:

    $a = \frac{A_{s1} f_y}{0.85 f_c' b} = \frac{2\,106\ mm^2 \times 414\ MPa}{0.85 \times 27.5 MPa \times 250 mm} = 149\ mm$

    then compute "c":

    $c = \frac{a}{\beta_1} = \frac{149\ mm}{0.85} = 175\ mm$

    and compute of compressive stress in compression reinforcement:

    $f_s' = \epsilon_u E_s \frac{c - d'}{c} = 0.003 \times 200\,000 \times \frac{175\ mm - 59.5\ mm}{175\ mm} = 396\ MPa < f_y$
  - Then compression reinforcement is not yielded and compression reinforcement will be:

    $A_s' = A_{s2} \frac{f_y}{f'_s} = 421\ mm^2 \times \frac{414\ MPa}{396\ MPa} = 440\ mm^2$
- Compute the Required Rebars Numbers.

  $\text{Number of Tension Rebars} = \frac{(2\,527\ mm^2)}{\frac{\pi \times 29^2}{4}} = \frac{(2\,527\ mm^2)}{660\ mm^2} = 3.83$

  Then use $4\emptyset 29mm$ for tension reinforcement.

  Check if these rebars can be put in one layer:

  $b_{Required} = 40 \times 2 + 10 \times 2 + 29 \times 4 + 29 \times 3 = 303mm > b_{Provided}$

  Then, the rebars must be put in two layers as the designer has assumed.

  $\text{Number of Compression Rebars} = \frac{(440\ mm^2)}{\frac{\pi \times 19^2}{4}} = \frac{(440\ mm^2)}{283\ mm^2} = 1.55$

  Then use $2\emptyset 19mm$ for compression reinforcement.
- Design of Required Ties:
  - Select bar diameter for ties:

    $\because \emptyset_{Bar} = 19mm < 32mm$ and single rebar.

    then $\emptyset_{Tie} = 10mm\ \ Ok.$

Compute the required spacing of the ties:

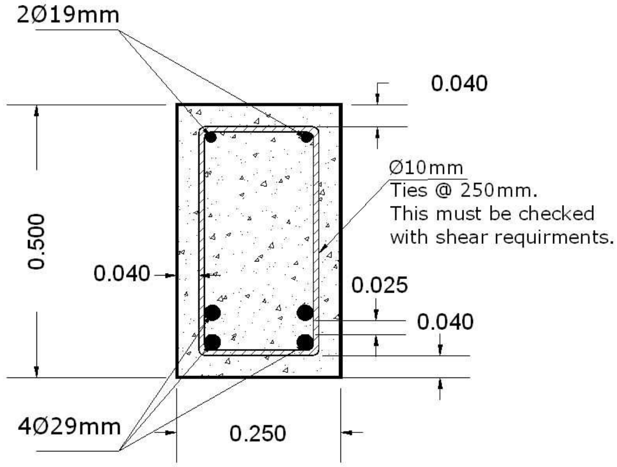
$S_{\text{Required for Ties}} = \min[16d_{bar}, 48d_{ties}\text{ , Least dimension of column}]$

$= \min[16 \times 19, 48 \times 10\ \ , 250]$

$S_{\text{Required for Ties}} = \min[394, 480, 250] = 250\text{ mm}$

Use $\emptyset 10\text{mm} @ 250\text{mm}$ for ties. This spacing must be checked with shear requirement as will be discussed in Chapter 4.

- Draw the final section details:



4.7.4 Homework Problems

Problem 4.7-1

Design a rectangular beam to carry a service live load moment of 561 kN.m and a service dead load of 317 kN.m (including moment due to beam selfweight). In your design assume the following:

1. A width of 350mm and a depth of 750mm (these dimensions have been determined based on architectural limitations).
2. Materials of $f_c' = 34.5\text{ MPa}$ and $f_y = 414\text{ MPa}$.
3. Two layers of longitudinal reinforcement.
4. Bar diameter of 25mm for longitudinal reinforcement.
5. Bar diameter of 10 mm for stirrups.

Answers

1. Compute the required factored moment M_u:

   $M_u = \text{ maximum of } [1.4M_{Dead} \text{ or } 1.2M_{Dead} + 1.6M_{Live}]$

   $M_u = maximum\ of\ [1.4 \times 317\ kN.m\ or\ 1.2 \times 317\ kN.m + 1.6 \times 561\ kN.m]$

   $M_u = \text{ maximum of } [444\text{ kN.m or }1\,278\text{kN.m}] = 1\,278\text{ kN.m}$

2. Compute the required nominal moment based on following relation:

   $$M_n = \frac{M_u}{\emptyset} = 1\,420\text{ kN.m}$$

   where $\emptyset$ will be assumed 0.9 to be checked later.

3. Check if the section can be design as a singly reinforced section or not based on following reasoning:

   $d = 662\text{ mm}$

   $$\rho_{Required} = \frac{1 - \sqrt{1 - 2.36\dfrac{M_n}{f_c' b d^2}}}{1.18 \times \dfrac{f_y}{f_c'}} = 27.9 \times 10^{-3}$$

   $\beta_1 = 0.8$

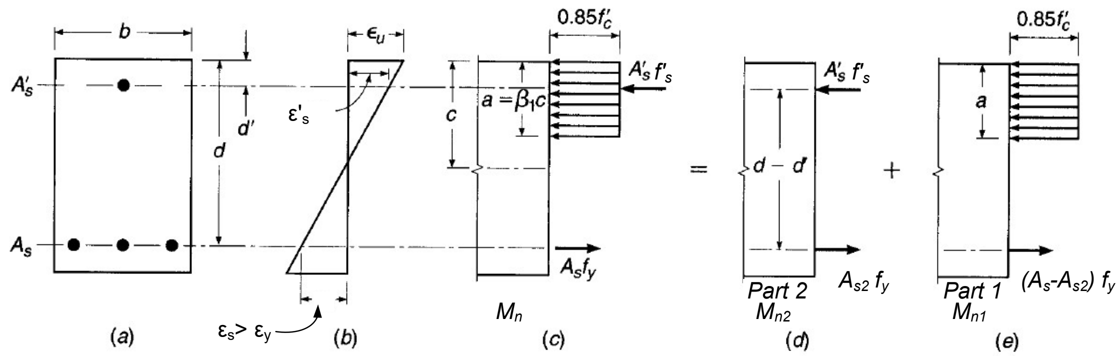
   $$\rho_{max} = 0.85\beta_1\ \frac{f_c'}{f_y}\ \frac{\epsilon_u}{\epsilon_u + 0.004} = 24.3 \times 10^{-3} < \rho_{Required}$$

   then the section must be design as doubly reinforced section

4. Re-compute the required nominal for the section based on $\emptyset = 0.816$:

   $$M_n = \frac{M_u}{\emptyset} = 1\,566\text{ kN.m}$$

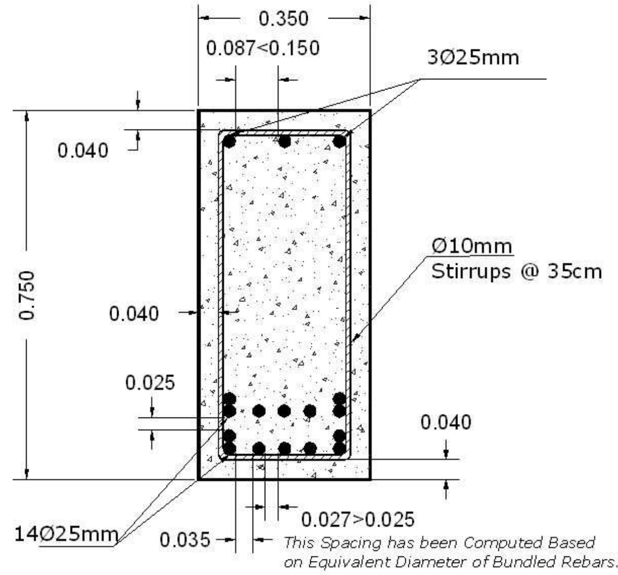
The nominal flexure strength M_n is considered to consist of the two parts shown below:



5. Compute of Tension Reinforcement As:
   a. Compute the nominal moment and tension reinforcement for part 1:
      $A_{s1} = A_{smax} = \rho_{max} bd = 5\,630\ mm^2$
      $M_{n1} = \rho_{max} f_y bd^2 \left(1 - 0.59\ \frac{\rho_{max} f_y}{f'_c}\right) = 1\,278\ kN.m$
   b. Compute the nominal moment and tension reinforcement for part 2:
      $M_{n2} = M_n - M_{n1} = 288\ kN.m$
      $d' = 62.5$
      $A_{s2} = \frac{M_{n2}}{f_y(d - d')} = 1\,160\ mm^2$
   c. Compute the Total Tension Reinforcement A_s:
      $A_s = A_{s1} + A_{s2} = 6\,790\ mm^2$
6. Compute of Compression Reinforcement A_s':
   a. Check if compression reinforcement is yielded or not:
      Compute of "a" based on force diagram of Part 1:
      $a = 227\ mm \Rightarrow c = 284\ mm \Rightarrow f_s' = \epsilon_u E_s \frac{c - d'}{c} = 468\ MPa > f_y$
   b. Then compression reinforcement is yielded and it's area will be:
      $f_s' = f_y = 414\ MPa \Rightarrow A_s' = A_{s2} = 1\,160\ mm^2$
7. Compute the Required Rebars Numbers.
   $\text{Number of Tension Rebars} = 13.8$
   Then use $14\emptyset 25mm$ for tension reinforcement.
   Check if these rebars can be put in two layers:
   $b_{Required} = 40 \times 2 + 10 \times 2 + 7 \times 25 + 6 \times 25 = 425mm > b_{Provided}$
   Then, the rebars must be put in more than two layers. This problem can be solved through using of Bundled Bars (See Section Details)
   $\text{Number of Compression Rebars} = 2.36$
   Then use $3\emptyset 25mm$ for compression reinforcement.
8. Design of Required Ties:
   a. Select bar diameter for ties:
      $\because \emptyset_{Bar} = 25mm \le 32mm$
      It is useful to note that ties design is depending on diameter of compression reinforcement and not on tension reinforcement. Therefore, the designer compare with diameter of compression reinforcement instead of comparison with equivalent diameter of Bundled Bars.
      Then
      $\emptyset_{Tie} = 10mm\ \text{Ok.}$
   b. Compute the required spacing of the ties:
      $S_{Required\ for\ Ties} = \min[16d_{bar}, 48d_{ties}, \text{Least dimension of column}]$
      $S_{Required\ for\ Ties} = \min[16 \times 25, 48 \times 10\ , 350] = 350\ mm$

Use $\emptyset 10\text{mm} @ 350\text{mm}$ for ties. This spacing must be checked with shear requirement as will be discussed in Chapter 5.

9. Draw the final section details:



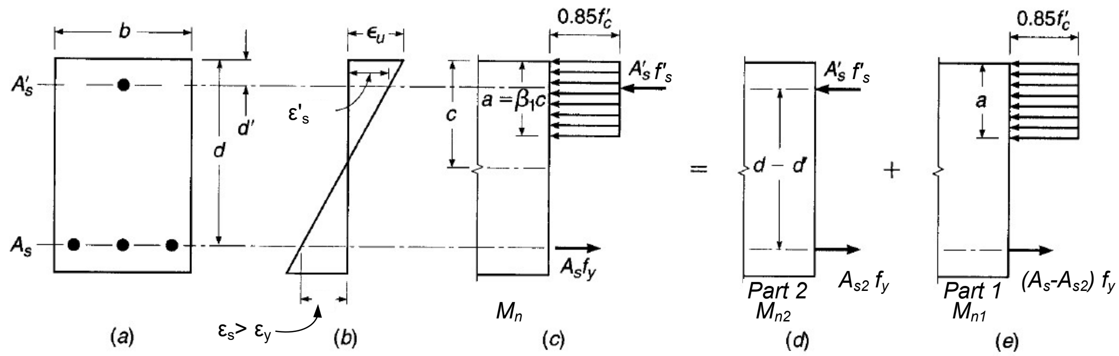
Problem 4.7-2

Resolve previous problem with using of:

1. Materials of $f_c' = 21\ \text{MPa}$ and $f_y = 420\ \text{MPa}$.
2. Two layers of longitudinal reinforcement.
3. Bar diameter of 32 mm for longitudinal reinforcement ($A_{bar} = 819\ \text{mm}^2$).
4. Bar diameter of 12 mm for stirrups.

Answers

1. Compute the required factored moment M_u:
   As for previous problem:
   $M_u = 1\,278\ kN.m$
2. Compute the required nominal moment based on following relation:
   $M_n = \frac{M_u}{\emptyset} = \frac{1\,278}{0.9} = 1\,420\ kN.m$
   where $\emptyset$ will be assumed 0.9 to be checked later.
3. Check if the section can be design as a singly reinforced section or not based on following reasoning:
   $d = 655\ mm$
   $$\rho_{Required} = \frac{1 - \sqrt{1 - 2.36\frac{M_n}{f_c' b d^2}}}{1.18 \times \frac{f_y}{f_c'}} = \frac{1 - 0.257i}{1.18 \times \frac{420}{21}}$$
   As the quantity under square root of above relation has a negative value, then the section cannot be designed as Singly Reinforced Section.
4. Re-compute the required nominal for the section based on $\emptyset = 0.816$:
   $M_n = \frac{M_u}{\emptyset} = 1\,566\ kN.m$
   The nominal flexure strength M_n is considered to consist of the two parts shown below:



5. Compute of Tension Reinforcement As:
   a. Compute the nominal moment and tension reinforcement for part 1:

$$\rho_{max} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} = 15.5 \times 10^{-3}$$

$$A_{s1} = A_{smax} = \rho_{max} bd = 3\,548\, mm^2$$

$$M_{n1} = \rho_{max} f_y bd^2 \left(1 - 0.59\, \frac{\rho_{max} f_y}{f_c'}\right) = 796\, kN.m$$

   b. Compute the nominal moment and tension reinforcement for part 2:

$$M_{n2} = M_n - M_{n1} = 770\, kN.m$$

$$d' = 64.5$$

$$A_{s2} = \frac{M_{n2}}{f_y(d - d')} = 3\,110\, mm^2$$

   c. Compute the Total Tension Reinforcement $\boldsymbol{A_s}$:

$$A_s = A_{s1} + A_{s2} = 3\,548\, mm^2 + 3\,110\, mm^2 = 6\,658\, \text{mm}^2$$

6. Compute of Compression Reinforcement A_s':
   a. Check if compression reinforcement is yielded or not:
   Compute of "a" based on force diagram of Part 1:

$$a = \frac{A_{s1} f_y}{0.85 f_c' b} = 239\, mm \Rightarrow c = \frac{a}{\beta_1} = 281\, mm$$

   And compute of compressive stress in compression reinforcement:

$$f_s' = \epsilon_u E_s \frac{c - d'}{c} = 462\, MPa > f_y$$

   b. Then compression reinforcement is yielded and it's area will be:

$$A_s' = A_{s2} = 3\,110\, mm^2$$

7. Compute the Required Rebars Numbers.
   Number of Tension Rebars

$$= \frac{6\,658\ \text{mm}^2}{819} = 8.13$$

   Then use $9\emptyset 32mm$ for tension reinforcement.
   Check if these rebars can be put in two layers:

$$b_{Required} = 40 \times 2 + 12 \times 2 + 5 \times 32 + 4 \times 32 = 392mm > b_{Provided} \text{ Not Ok.}$$

   To solve a problem that related to distribution Bundled Rebars will be used.
   Number of Compression Rebars

$$= \frac{3\,110\, mm^2}{819\, \text{mm}^2} = 3.80$$

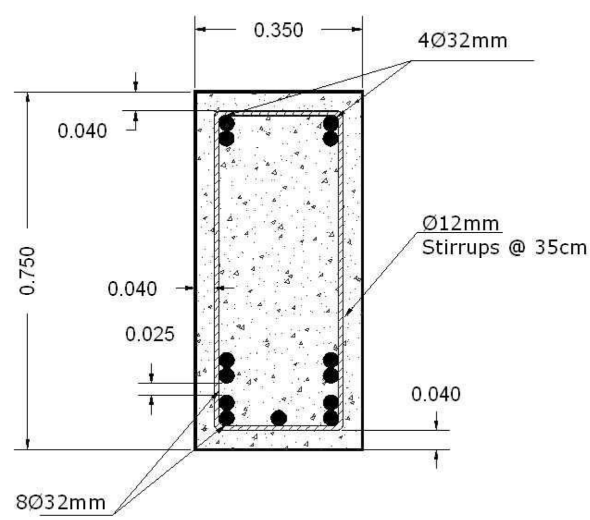


Figure 4.7-3: Detailed section for Problem 4.7-2.

Then use $4\emptyset 32mm$ for compression reinforcement.

8. Design of Required Ties:
    a. Select bar diameter for ties:
    As designer intend to use bundled compression rebars, then
    $\emptyset_{Tie} = 12\ mm\ \ Ok.$
    b. Compute the required spacing of the ties:
    $S_{Required\ for\ Ties} = \min[16d_{bar}, 48d_{ties}, Least\ dimension\ of\ column]$
    $\mathrm{S_{Required\ for\ Ties}} = \min[400, 576, 350]$
    $\mathrm{S_{Required\ for\ Ties}} = 350\ \mathrm{mm}$
    Use $\emptyset 12mm @ 350mm$ for ties. This spacing must be checked with shear requirement as will be discussed in Chapter 4.
9. Draw the final section details as indicated in Figure 4.7-3.

4.8 FLEXURE ANALYSIS OF A SECTION WITH T SHAPE

4.8.1 Construction Stages

- During construction, the concrete in columns is placed and allowed to harden before the concrete in the floor beam is placed. In next operation, concrete is placed in the slab and beams in a monolithic pour, article 26.5.7.2 of (ACI318M, 2014).
- As a result, the slab serves as the top flange of the beam as indicated by shading area in the Figure 4.8-1 below:

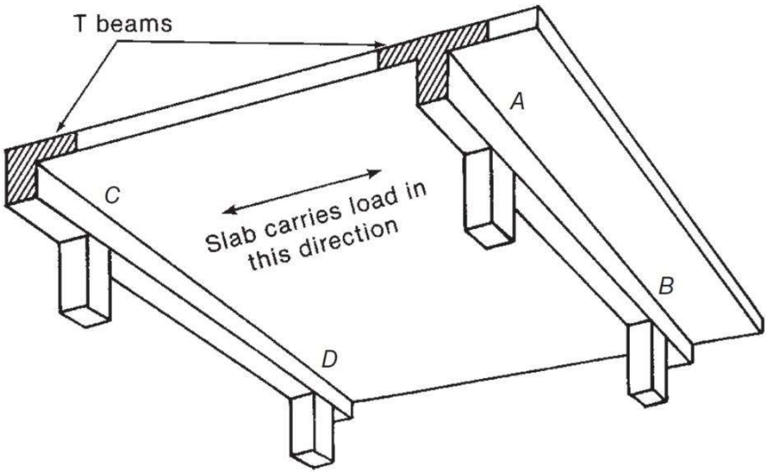


Figure 4.8-1: Slab beam interaction due to monolithic casting.

- Such a beam is referred to as a ***T Beam***. The interior beam, AB, of the Figure 4.8-1 above, has a flange on both sides. The spandrel beam, CD, with flange on one side only, is also referred to as a T Beam.

4.8.2 Behavior of Tee Beams

- An exaggerated deflected view of the interior beam "AB" is shown in Figure 4.8-2 below:

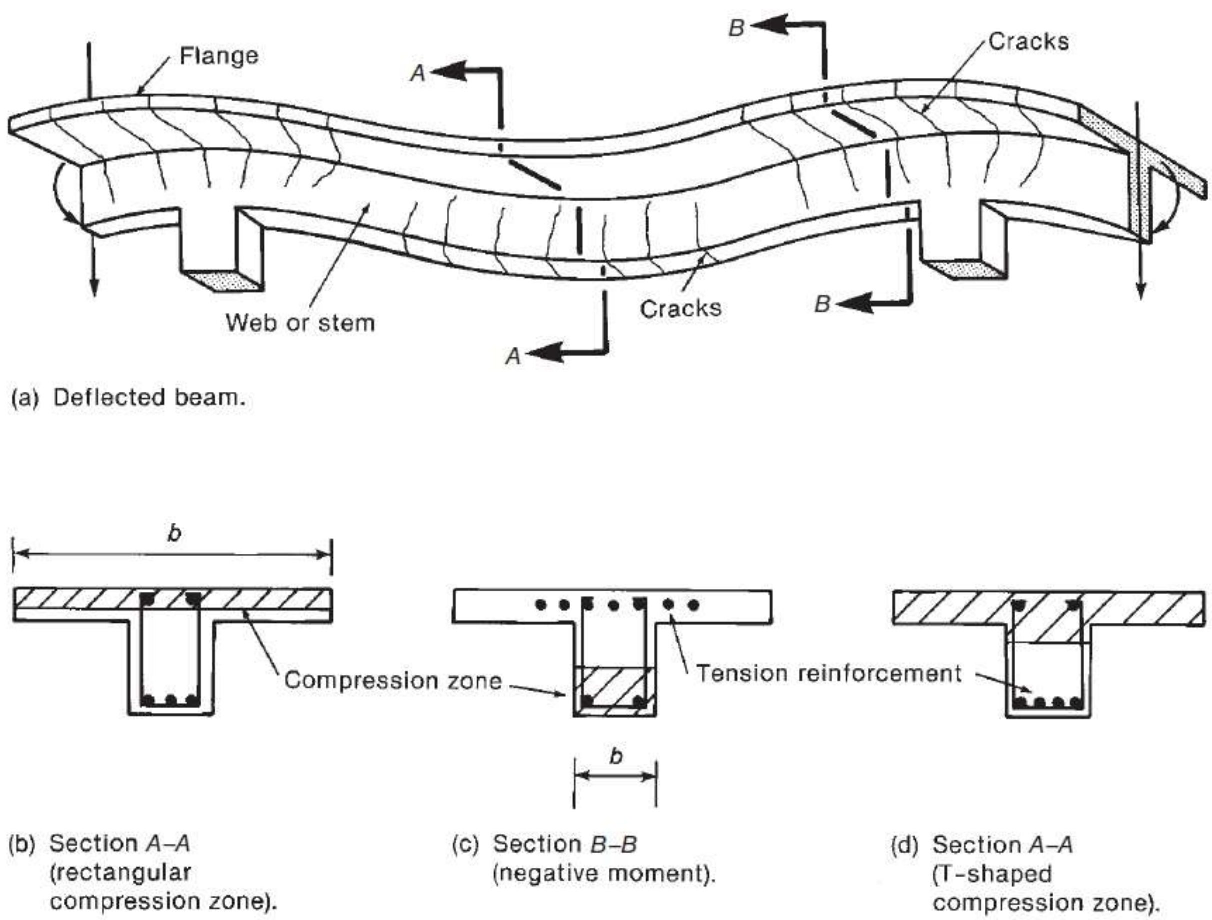


Figure 4.8-2: Exaggerated deflected view for a continuous beam with Tee shape.

- Form above deflected shape, following points can be concluded:
  - At mid-span, the compression zone is in flange as shown in Figure 4.8-2 "b" and "d" above.
  - Generally, it is rectangular as shown in Figure "b", although in a few cases, the neutral axis may shift down into the web, giving a T-shaped compression zone.
  - At the support, the compression zone is at the bottom of the beam and is rectangular, as shown in Figure "c"

4.8.3 Notations Adopted in Design of Tee Beams

Notations indicated in Figure 4.8-3 below are adopted in analysis and design of T Section.

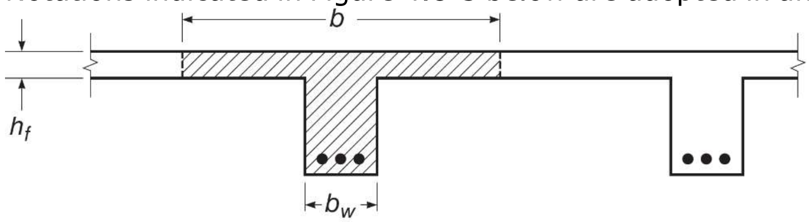


Figure 4.8-3: Notations adopted in analysis and design of Tee beams.

4.8.4 Procedure for Analysis of a Beam with T-Shape

Checking the adequacy of a T-Shape beam according to the requirements of ACI Code can be summarized as follows:

1. Definition of Section Dimensions:
   a. The first question that must be answered in the analysis of T section is "What is the part of the slab that will act as a compression flange for the T beam?"
   Due shearing deformation of the flange, which relieves the more remote elements of some compressive stress, ***shear-lag phenomenon***, actual compression stress in the beam flange varies as indicated in Figure 4.8-4 below.

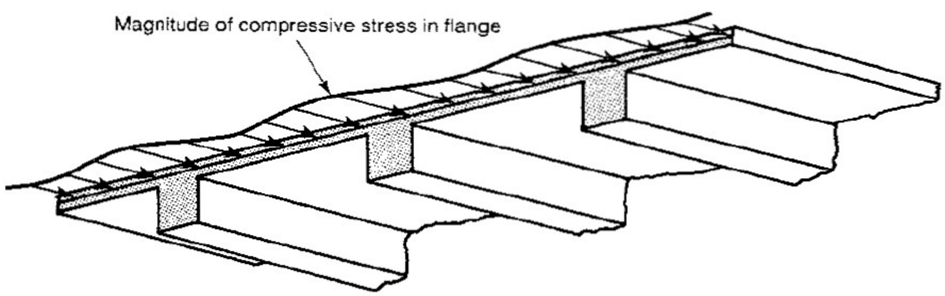


Figure 4.8-4: Actual distribution of compressive stresses in Tee flange.

According to ACI, the variable compressive stresses that acting on the overall width, b_o, in Figure 4.8-5 below can be replaced by an equivalent uniformly distributed compressive force that acting on an effective width, b.

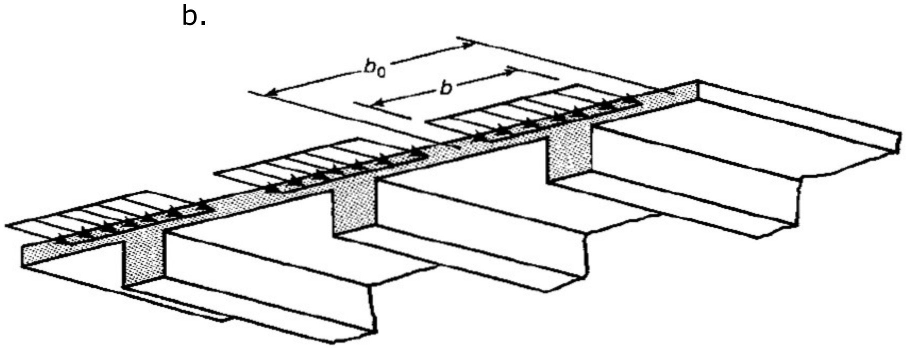


Figure 4.8-5: Equivalent uniform flange stresses adopted by the ACI code, 3D view.

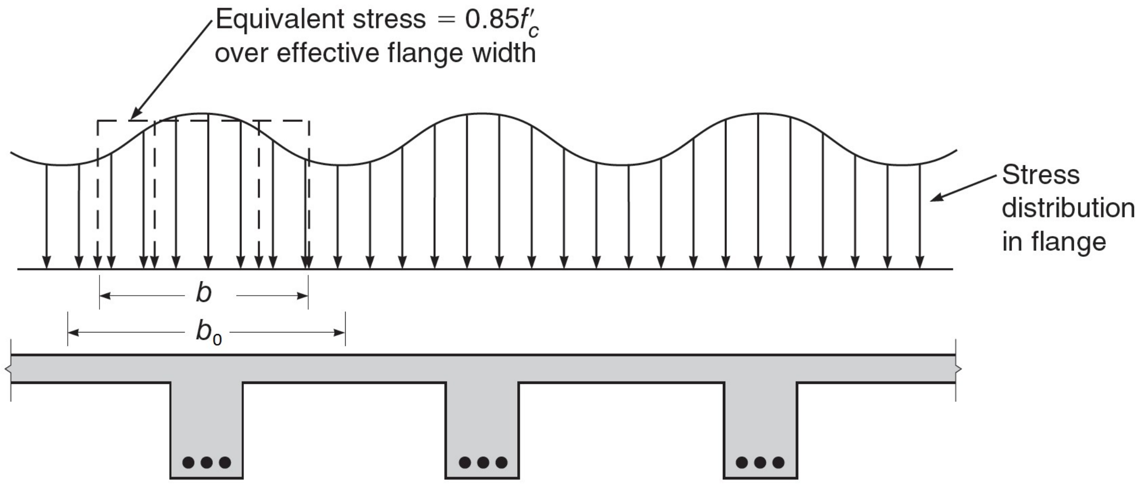


Figure 4.8-5: Equivalent uniform flange stresses adopted by the ACI code, a sectional view.

b. According to ACI Code (6.3.2.1), for nonprestressed T-beams supporting monolithic or composite slabs, the effective flange width, b, shall include the beam web width, b_w, plus an effective overhanging flange width in accordance with Table 4.8-1 below, where h is the slab thickness and s_w is the clear distance to the adjacent web:

Table 4.8-1: Dimensional limits for effective overhanging flange width for T-beams, Table 6.3.2.1 of the (ACI318M, 2014).

Flange location	Effective overhanging flange width, beyond face of web	
Each side of web	Least of:	$8h$
		$s_w/2$
		$\ell_n/8$
One side of web	Least of:	$6h$
		$s_w/2$
		$\ell_n/12$

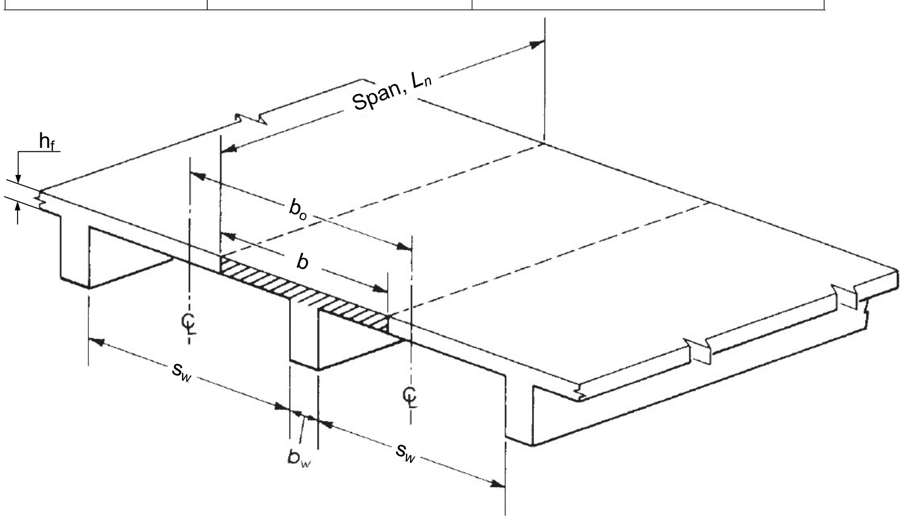
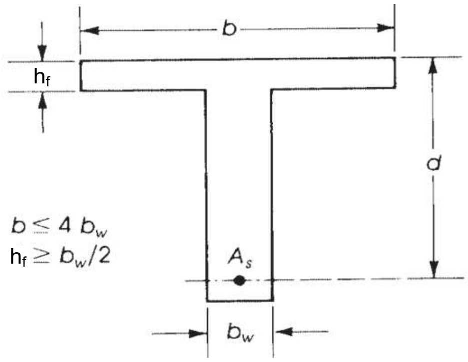


Figure 4.8-6: Notations of Table 4.8-1.

c. According to article 6.3.2.2 of the (ACI318M, 2014), isolated nonprestressed T-beams in which the flange is used to provide additional compression area shall have a flange thickness greater than or equal to $0.5b_w$ and an effective flange width less than or equal to $4b_w$.

Figure 4.8-7: Isolated T-shaped sections.



2. Checking the Section Type:
   Check if the failure is secondary compression failure or compression failure through following comparison:
   $\rho_w\ ?\ \rho_{w\,max}$
   where
   $$\rho_w = \frac{A_s}{b_w d}$$
   To derive a relation for computing of $\rho_{w\,max}$ it is useful to imagine that the T section is consists of following two parts indicated in Figure 4.8-8 below
   Then, based on $\sum F_x = 0$, one can show that:
   $$\rho_{w\,max} = \frac{A_{s\,max}}{b_w d} = 0.85\beta_1 \frac{f'_c}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} + \frac{A_{sf}}{b_w d}$$
   or $\rho_{w\,max} = \rho_{max} + \rho_f$■
   where $\rho_{w\,max} = \frac{A_{s\,max}}{b_w d}$
   $$\rho_f = \frac{A_{sf}}{b_w d}, A_{sf} = \frac{0.85 f'_c h_f (b - b_w)}{f_y}$$

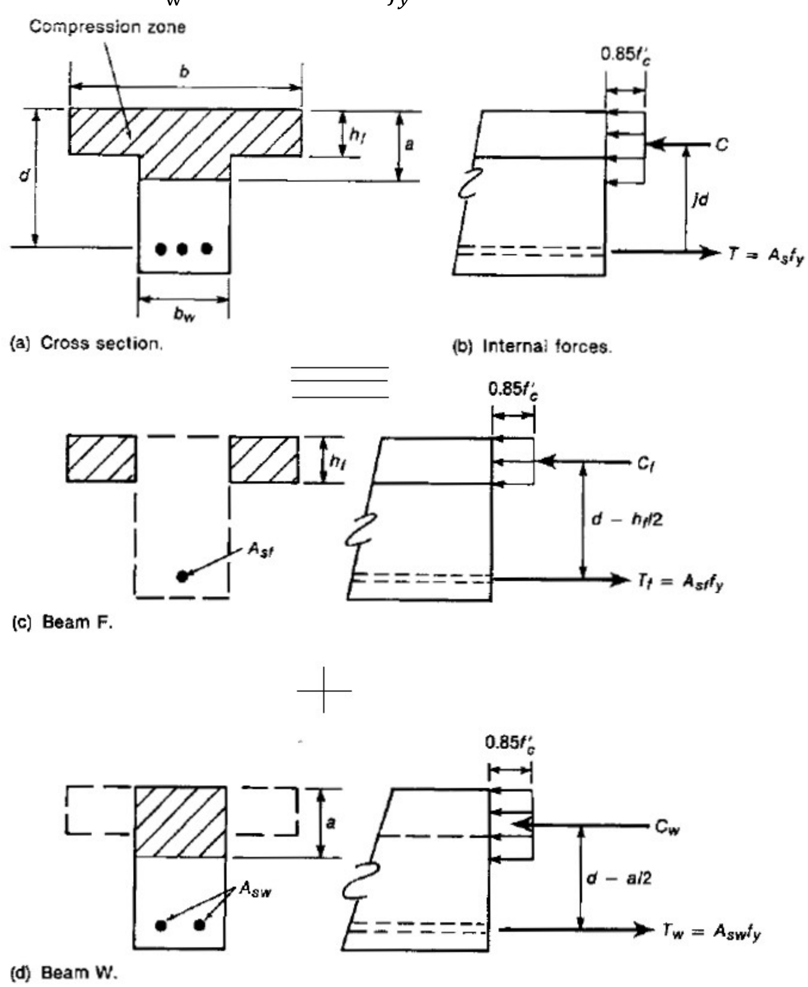


Figure 4.8-8: Imaginary two parts for Tee beam for analysis purpose.

3. Checking of $A_{s\,minimum}$ limitation
   As the flange is under compression stress, then the minimum steel area shall be compute based on ACI (9.6.1.2):
   $$A_{s\,minimum} = \frac{0.25\sqrt{f_c'}}{f_y} b_w d \geq \frac{1.4}{f_y} b_w d$$
4. Computing of Nominal Flexure Strength "M_n":
   As the relation for computing of M_n depends on location of compression block, if it is in the flange or extend to the web. Then the analyzer must first check to see if "a" is less than h_f or not (See Figure 4.8-9 below).

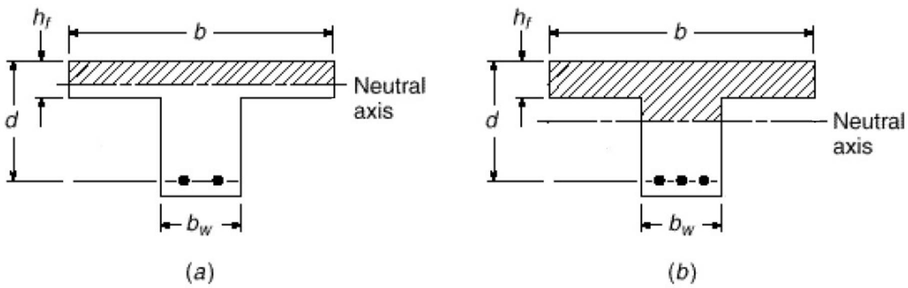


Figure 4.8-9: Possible location of compression block of Tee beams.

a. Assume that $a \leq h_f$ (based on experience, this can be considered as the general case):

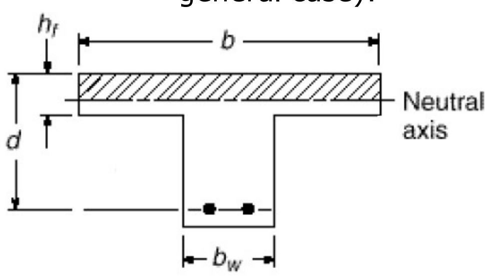


Figure 4.8-10: Compression block within section flange.

$$\because \sum F_x = 0$$
$$\therefore 0.85f_c'\, b\, a = A_s f_y$$
$$a = \frac{A_s f_y}{0.85 f_c' b}$$

b. If $a_{Computed} \leq h_f$, then above assumption is correct and nominal flexure strength M_n can be computed based on:
$$M_n = A_s f_y \left(d - \frac{a}{2}\right) \blacksquare$$

c. Else (i.e. $a_{Computed} > h_f$) then the nominal flexure strength M_n will be considered to be consist from two parts shown in Figure 4.8-11 below:
Compute A_{sf} based on:
$$\sum_{for\ Part\ Beam\ F} F_x = 0$$
$$A_{sf} f_y = 0.85 f_c' h_f (b - b_w)$$
$$A_{sf} = \frac{0.85 f_c' h_f (b - b_w)}{f_y} \blacksquare$$
Compute the correct value of "a" based on Part "Beam W":
$$\sum_{for\ Part\ Beam\ W} F_x = 0$$
$$(A_s - A_{sf}) f_y = 0.85 f_c' a (b - b_w)$$
$$a = \frac{(A_s - A_{sf}) f_y}{0.85 f_c' (b_w)}$$
Compute M_n based on following relation:
$$M_n = \left[0.85 f_c' h_f (b - b_w)\right] \left(d - \frac{h_f}{2}\right)_{M_n for\ Part\ Beam\ F} + \left[0.85 f_c' a b_w\right] \left(d - \frac{a}{2}\right)_{M_n for\ Part\ Beam\ W} \blacksquare$$

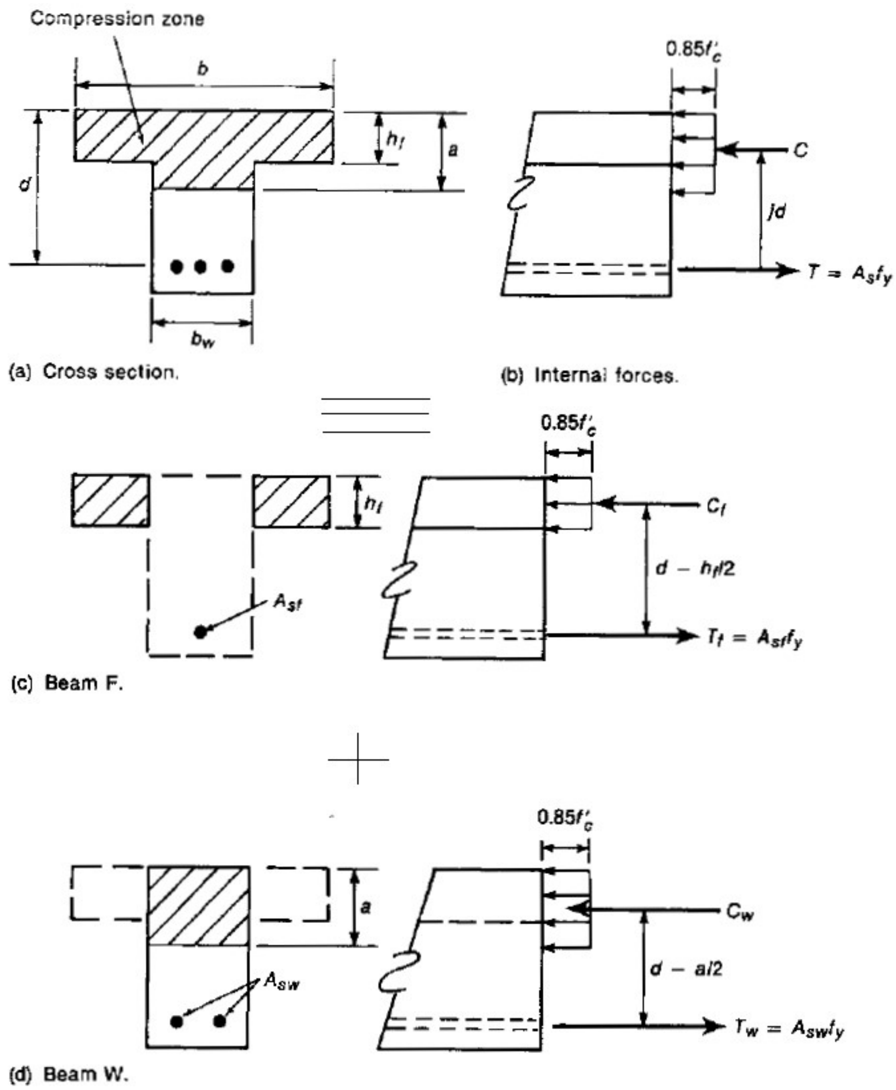


Figure 4.8-11: Analysis parts when compression block within flange.

5. Compute the Strength Reduction Factor ∅ Based on Following Relation:
    a. Compute steel stain based on the following relations:
       $c = \frac{a}{\beta_1}$
       $\epsilon_t = \frac{d - c}{c} \epsilon_u$
    b. If $\epsilon_t \geq 0.005$, then $\emptyset = 0.9$ Ok.
    c. If $\epsilon_t < 0.005$, then compute more accurate ∅:
       $\emptyset = 0.483 + 83.3\epsilon_t$
6. Finally Compute the Design Flexure Strength of Section $\emptyset M_n$:
   $\emptyset M_n = \emptyset \times M_n$

4.8.5 Examples

Example 4.8-1

Check the adequacy of the interior T-beam shown below for ACI Code requirements and determine its design strength.
Assume that:

- $f_y = 300$ Mpa
- $f_c' = 20$ Mpa.
- Beam Span 5.5m.
- $A_{Bar\ for\ \emptyset\ 19mm} = 284 mm^2$.

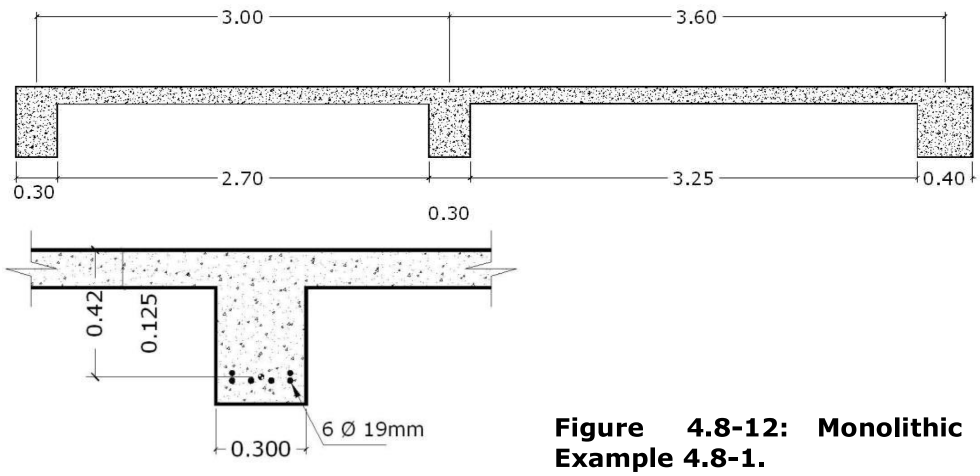
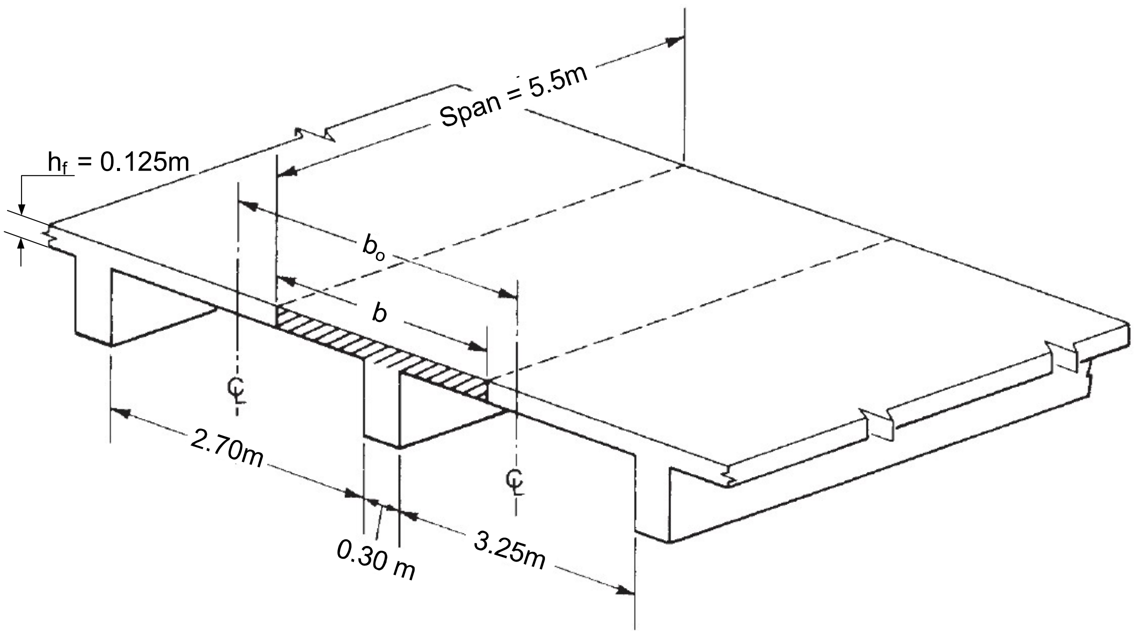


Figure 4.8-12: Monolithic Tee beam for Example 4.8-1.

Solution

1. Definition of Section Dimensions:



$$b = b_w + minimum\left[\frac{S_{w\,left}}{2}\ or\ 8h\ or\ \frac{l_n}{8}\right] + minimum\left[\frac{S_{w\,right}}{2}\ or\ 8h\ or\ \frac{l_n}{8}\right]$$

$$b = 0.3 + minimum\left[\frac{2.7}{2}\ or\ 8\times 0.125\ or\ \frac{5.5}{8}\right] + minimum\left[\frac{3.25}{2}\ or\ 8\times 0.125\ or\ \frac{5.5}{8}\right]$$

$$b = 0.3 + minimum\,[1.35\ or\ 1.0\ or\ 0.688] + minimum[1.63\ or\ 1.0\ or\ 0.688]$$

$$b = 0.3 + 0.688 + 0.688 = 1.68\ m$$

2. Checking the Section Type:
   Check if the failure is secondary compression failure or compression failure through following comparison:

$\rho_w\ ?\ \rho_{w\,max}$

$$\rho_{w\,max} = \frac{A_{s\,max}}{b_w d} = 0.85\beta_1\ \frac{f'_c}{f_y}\ \frac{\epsilon_u}{\epsilon_u + 0.004} + \frac{A_{sf}}{b_w d}$$

$$A_{sf} = \frac{0.85 f'_c h_f (b - b_w)}{f_y} = \frac{0.85\times 20\times 125\times(1680-300)}{300} = 9775\ mm^2$$

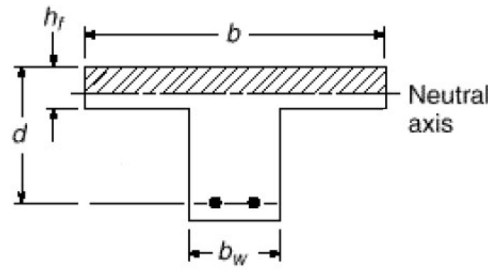
$$\rho_{w\,max} = 0.85^2 \times \frac{20}{300}\times\frac{0.003}{0.003+0.004} + \frac{9775}{300\times 420} = 20.6\times 10^{-3} + 77.6\times 10^{-3}$$
$$= 98.2\times 10^{-3}$$

$$\rho_w = \frac{A_s}{b_w d}$$

$$A_s = 6\times 284mm^2 = 1\,704mm^2$$

$$\rho_w = \frac{1704mm^2}{300\times 420} = 13.5\times 10^{-3} << \rho_{w\,max}\quad Ok.$$

3. Checking of $A_{s\,minimum}$ limitation:
   $\because f_c' < 31\,MPa$
   $\therefore A_{s\,minimum} = \frac{1.4}{f_y} b_w d = \frac{1.4}{300} \times 300 \times 420 = 588\,mm^2 \ll A_{s\,Provided}$ Ok.
4. Computing of Nominal Flexure Strength "M_n":
   Assume that $a \le h_f$ (based on experience, this can be considered as the general case):



   $a = \frac{A_s f_y}{0.85 f_c' b} = \frac{1704 \times 300}{0.85 \times 20 \times 1680} = 17.9\,mm < 125 mm\ Ok.$
   As $a_{Computed} \le h_f$, then above assumption is correct and nominal flexural strength, M_n, can be computed based on:
   $M_n = A_s f_y (d - \frac{a}{2}) = 1704 \times 300 \times \left(420 - \frac{17.9}{2}\right) = 210\,kN.m$
5. Compute the strength reduction factor, $\emptyset$, based on following relation:
   Compute steel stain based on the following relations:
   $c = \frac{a}{\beta_1} = \frac{17.9}{0.85} = 21.1 \Longrightarrow \epsilon_t = \frac{d-c}{c}\epsilon_u = \frac{420 - 21.1}{21.1} \times 0.003 = 56.7 \times 10^{-3}$
   As $\epsilon_t \gg 0.005$, then $\emptyset = 0.9$
6. Finally Compute the Design Flexure Strength of Section $\emptyset M_n$:
   $\emptyset M_n = \emptyset \times M_n = 0.9 \times 210 = 189\,kN.m$■

Example 4.8-2

Check the adequacy of isolated T-beam shown below for ACI Code requirements and determine its design strength.

Assume that:

- $f_y = 420$ Mpa
- $f_c' = 20$ Mpa.
- Reinforcement is $6\emptyset 25$mm with $A_{Bar} = 500mm^2$.

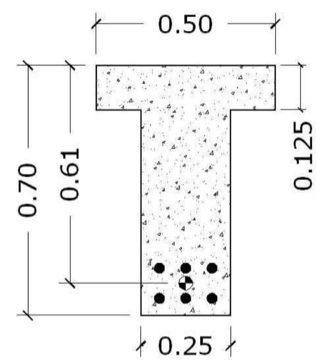


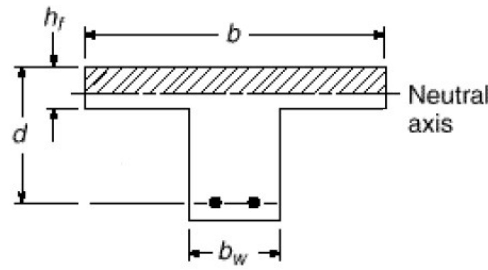
Figure 4.8-13: Isolated Tee beam for Example 4.8-2

Solution

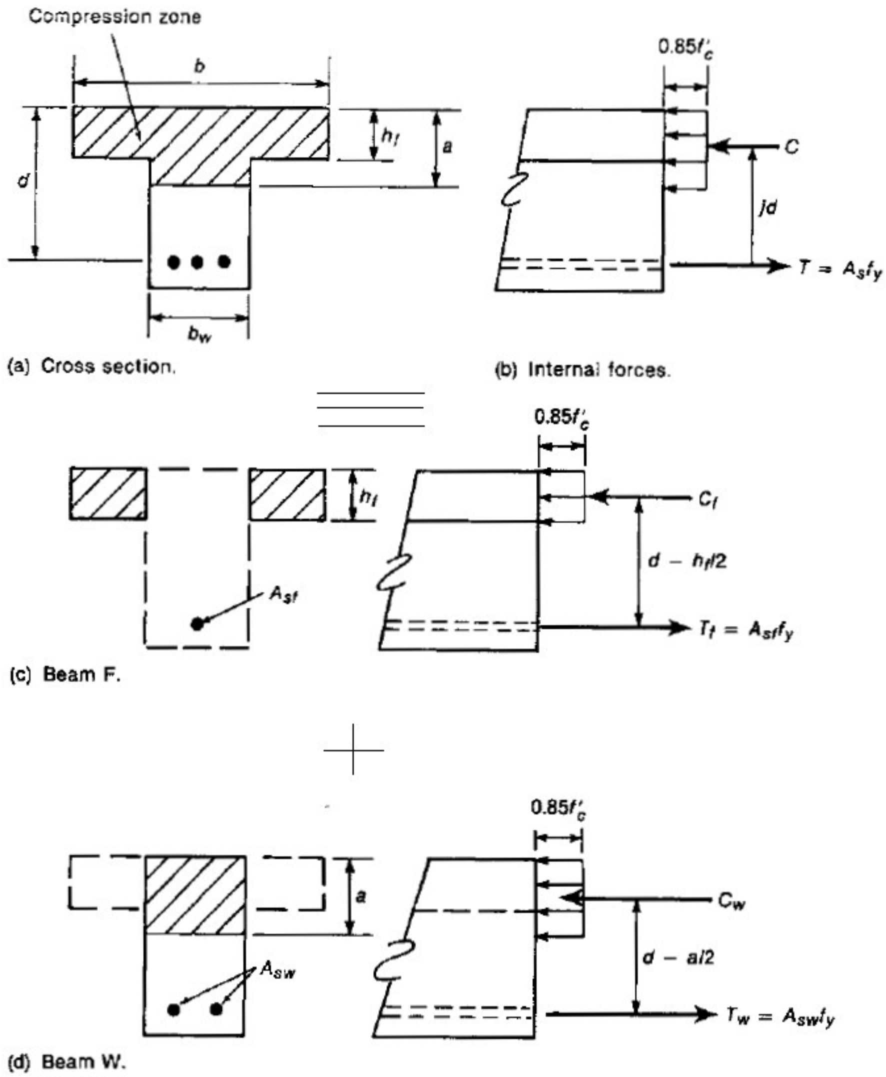
1. Check the Beam Dimensions:
   As the beam is an isolated T-beam, then its flange width and flange thickness must satisfy the following limitations:
   $h_f \not< \frac{b_w}{2} \Longrightarrow h_f = 125mm = \frac{b_w}{2} = \frac{250}{2}$ Ok.
   $b \not> 4b_w \Longrightarrow b = 500mm < 4 \times 250 mm\ Ok.$
2. Checking Section Type:
   $\rho_w\ ?\ \rho_{w\,max}$
   $\rho_{w\,max} = \frac{A_{s\,max}}{b_w d} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} + \frac{A_{sf}}{b_w d}$
   $A_{sf} = \frac{0.85 f_c' h_f (b - b_w)}{f_y} = \frac{0.85 \times 20 \times 125 \times (500 - 250)}{420} = 1\,265\,mm^2$
   $\rho_{w\,max} = 0.85^2 \frac{20}{420} \frac{0.003}{0.003 + 0.004} + \frac{1\,265\,mm^2}{250 \times 610} = 14.7 \times 10^{-3} + 8.30 \times 10^{-3} = 23 \times 10^{-3}$
   $A_s = 6 \times 500mm^2 = 3\,000mm^2$
   $\rho_w = \frac{A_s}{b_w d} = \frac{3\,000mm^2}{250 \times 610} = 19.7 \times 10^{-3} < \rho_{w\,max}$ Ok.
3. Checking of $A_{s\,minimum}$ limitation:
   $\because f_c' < 31\,MPa$
   $\therefore A_{s\,minimum} = \frac{1.4}{f_y} b_w d = \frac{1.4}{420} \times 250 \times 610 = 508\,mm^2 < A_{s\,Provided}$ Ok.
4. Computing of Nominal Flexure Strength "M_n":

Assume that $a \leq h_f$ (based on experience, this can be considered as the general case):

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{3\,000\text{mm}^2 \times 420\text{ MPa}}{0.85 \times 20\text{ MPa} \times 500\text{mm}} = 148\text{ mm} > 125 mm\ Not\ Ok.$$



As $a_{\text{Computed}} > h_f$ then the nominal flexure strength M_n will be considered to be consist from two parts shown below:



Compute the correct value of "a" based on Part "Beam W":

$$a = \frac{(A_s - A_{sf}) f_y}{0.85 f_c' (b_w)} = \frac{(3\,000 - 1\,265) \times 420}{0.85 \times 20(250)} = 171\text{ mm}$$

Compute M_n based on following relation:

$$M_n = \left[0.85 f_c' h_f (b - b_w)\right] \left(d - \frac{h_f}{2}\right)_{M_n for\ Part\ Beam\ F} + \left[0.85 f_c' a b_w\right] \left(d - \frac{a}{2}\right)_{M_n for\ Part\ Beam\ W}$$

$$M_n = [0.85 \times 20 \times 125 \times (500 - 250)] \left(610 - \frac{125}{2}\right)_{M_n for\ Part\ Beam\ F} + [0.85 \times 20 \times 171 \times 250] \left(610 - \frac{171}{2}\right)_{M_n for\ Part\ Beam\ W}$$

$M_n = 291\text{ kN.m} + 381\text{ kN.m} = 672\text{ kN.m}$■

5. Compute the Strength Reduction Factor ∅ Based on Following Relation:
   Compute steel stain based on the following relations:

$$c = \frac{a}{\beta_1} = \frac{171}{0.85} = 201 \Longrightarrow \epsilon_t = \frac{d-c}{c}\epsilon_u = \frac{610-201}{201} \times 0.003 = 0.006$$

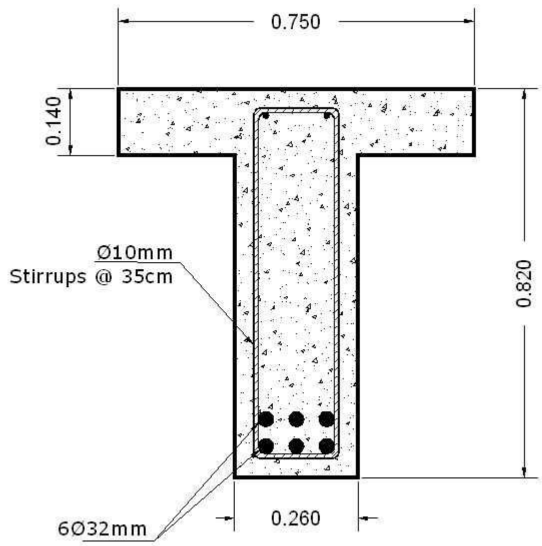
As $\epsilon_t \geq 0.005$, then $\emptyset = 0.9$ Ok.

6. Finally Compute the Design Flexure Strength of Section $\emptyset M_n$:
   $\emptyset M_n = \emptyset \times M_n = 0.9 \times 672 = 605 \text{ kN.m}$■

4.8.6 Problems for Solution

Problem 4.8-1

Check the adequacy of isolated T-beam shown below for ACI Code requirements and determine it's design strength.

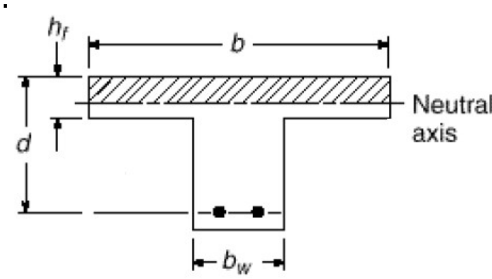


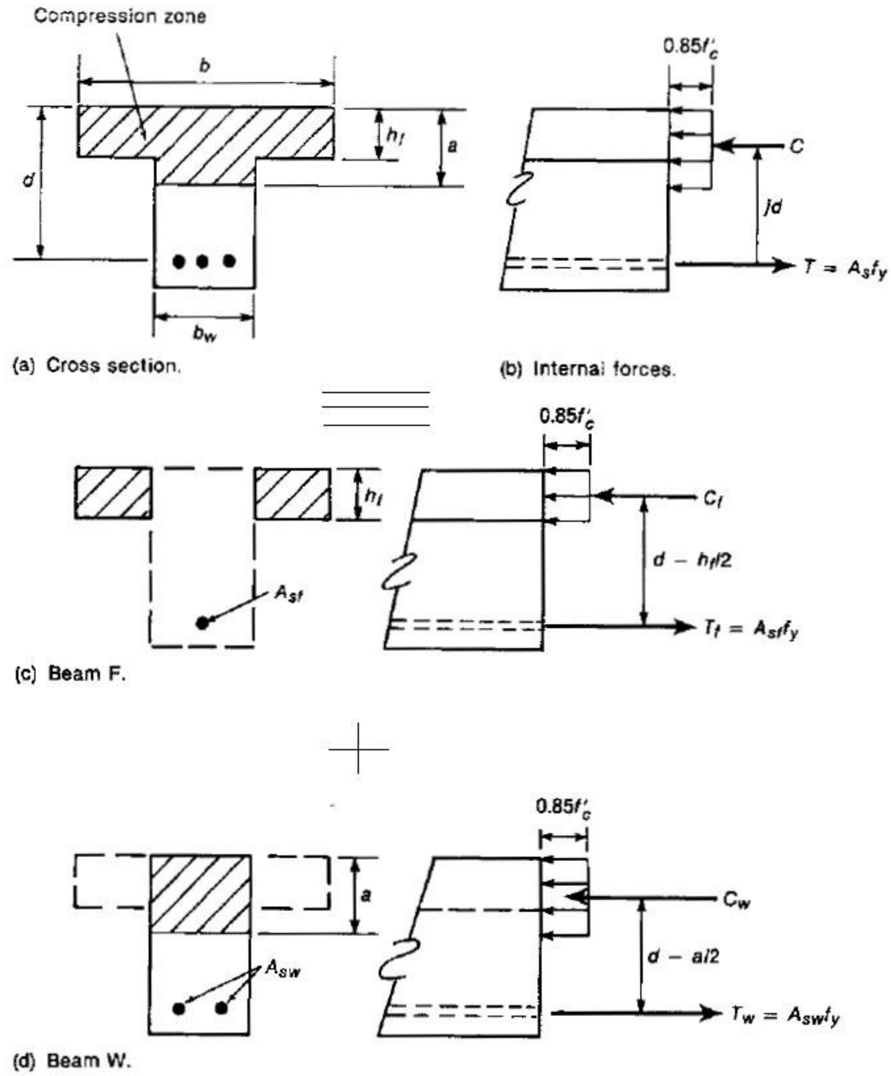
Assume that:

- $f_y = 400$ Mpa
- $f_c' = 20$ Mpa.
- Reinforcement is $6\emptyset 32$mm.

Answers

1. Check the Beam Dimensions:
   As the beam is an isolated T-beam, then its flange width and flange thickness must satisfy the following limitations:
   $$h_f \nless \frac{b_w}{2}$$
   $$h_f = 140\text{mm} > \frac{b_w}{2} = \frac{260}{2} \quad \text{Ok.}$$
   $$b \ngtr 4b_w \Longrightarrow b = 750\text{mm} < 4 \times 260 mm\ Ok.$$
2. Checking Section Type:
   $\rho_w \ ? \ \rho_{w\,max}$
   $$\rho_{w\,max} = \frac{A_{s\,max}}{b_w d} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} + \frac{A_{sf}}{b_w d}$$
   $$A_{sf} = \frac{0.85 f_c' h_f (b - b_w)}{f_y} = 2\,916 \text{ mm}^2$$
   $d = 725$ mm
   $$\rho_{w\,max} = 15.5 \times 10^{-3} + 15.5 \times 10^{-3} = 31.0 \times 10^{-3}$$
   $$\rho_w = \frac{A_s}{b_w d}$$
   $A_s = 4\,824\text{mm}^2$
   $\rho_w = 25.6 \times 10^{-3} < \rho_{w\,max}$ Ok.
3. Checking of $A_{s\,minimum}$ limitation:
   $\because f_c' < 31\ MPa$
   $$\therefore A_{s\,minimum} = \frac{1.4}{f_y} b_w d = 660 \text{ mm}^2 < A_{s\,Provided} \text{ Ok.}$$
4. Computing of Nominal Flexure Strength "M_n":
   Assume that $a \leq h_f$ (based on experience, this can be considered as the general case):
   $$a = \frac{A_s f_y}{0.85 f_c' b} = 151 \text{ mm} > 140 mm\ Not\ Ok.$$
   As $a_{Computed} > h_f$ then the nominal flexure strength M_n will be considered to be consist from two parts shown below:





Compute the correct value of "a" based on Part "Beam W":

$$a = \frac{(A_s - A_{sf})f_y}{0.85f'_c b_w} = 173\ \text{mm}$$

Compute M_n based on following relation:

$$M_n = \left[0.85f'_c h_f (b - b_w)\right]\left(d - \frac{h_f}{2}\right)_{M_n for\ Part\ Beam\ F} + \left[0.85f'_c a b_w\right]\left(d - \frac{a}{2}\right)_{M_n for\ Part\ Beam\ W}$$

$$M_n = 764\ \text{kN.m} + 488\ \text{kN.m} = 1\,252\ \text{kN.m} \blacksquare$$

5. Compute the Strength Reduction Factor $\emptyset$ Based on Following Relation:
   Compute steel stain based on the following relations:
   $$c = \frac{a}{\beta_1} = 204 \Longrightarrow \epsilon_t = \frac{d - c}{c}\epsilon_u = 7.66 \times 10^{-3}$$
   As $\epsilon_t \geq 0.005$, then $\emptyset = 0.9$ Ok.
6. Finally Compute the Design Flexure Strength of Section $\emptyset M_n$:
   $$\emptyset M_n = \emptyset \times M_n = 1\,127\ \text{kN.m} \blacksquare$$

Problem 4.8-2

Resolve previous problem but with $h_f = 180mm$.

Answers

1. Check the Beam Dimensions:
   As the beam is an isolated T-beam, then its flange width and flange thickness must satisfy the following limitations:
   $$h_f \not< \frac{b_w}{2} \Longrightarrow h_f = 180\text{mm} > \frac{b_w}{2} = \frac{260}{2}\ \text{Ok.}$$

$b \ngtr 4b_w \Longrightarrow b = 750mm < 4 \times 260mm\ Ok.$

2. Checking Section Type:

$\rho_w\ ?\ \rho_{w\,max}$

$$\rho_{w\,max} = \frac{A_{s\,max}}{b_w d} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} + \frac{A_{sf}}{b_w d}$$

$$A_{sf} = \frac{0.85 f_c' h_f (b - b_w)}{f_y} = 3\,749\ mm^2$$

$d = 725\ mm$

$\rho_{w\,max} = 15.5 \times 10^{-3} + 19.9 \times 10^{-3} = 35.4 \times 10^{-3}$

$$\rho_w = \frac{A_s}{b_w d}$$

$A_s = 4\,824 mm^2$

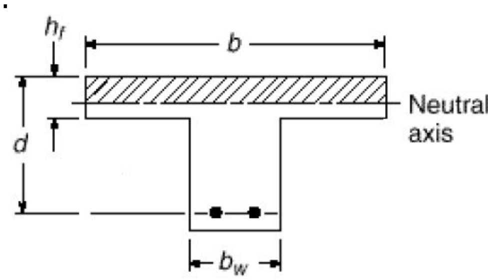
$\rho_w = 25.6 \times 10^{-3} < \rho_{w\,max}$ Ok.

3. Checking of $A_{s\,minimum}$ limitation:

$\because f_c' < 31\ MPa \Longrightarrow \therefore A_{s\,minimum} = \frac{1.4}{f_y} b_w d = 660\ mm^2 < A_{s\,Provided}$ Ok.

4. Computing of Nominal Flexure Strength "M_n":

Assume that $a \leq h_f$ (based on experience, this can be considered as the general case):



$$a = \frac{A_s f_y}{0.85 f_c' b} = 151\ mm < 180mm\ Ok.$$

As $a_{Computed} \leq h_f$, then above assumption is correct and nominal flexure strength M_n can be computed based on:

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = 1\,253\ kN.m$$

5. Compute the Strength Reduction Factor $\emptyset$ Based on Following Relation:

Compute steel stain based on the following relations:

$$c = \frac{a}{\beta_1} = 178mm \Longrightarrow \epsilon_t = \frac{d - c}{c}\epsilon_u = 9.22 \times 10^{-3}$$

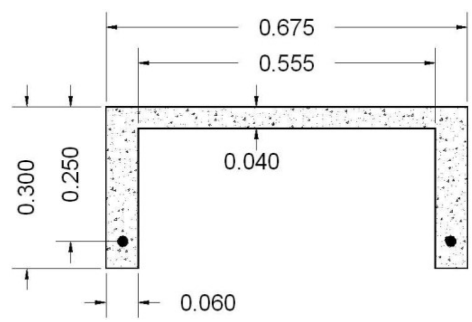
As $\epsilon_t \geq 0.005$, then $\emptyset = 0.9$ Ok.

6. Finally Compute the Design Flexure Strength of Section $\emptyset M_n$:

$\emptyset M_n = \emptyset \times M_n = 1\,128\ kN.m$ ■

Problem 4.8-3

Check the adequacy of the precast beam shown below according to ACI Code requirements and compute its flexure design strength. Assume that:

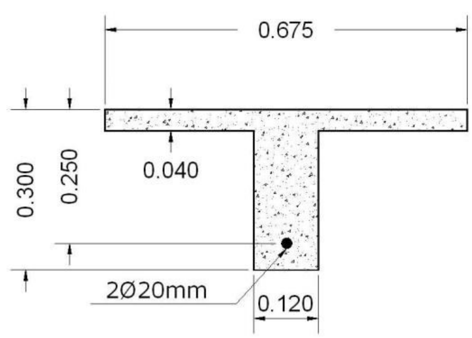


- $f_y = 400$ Mpa
- $f_c{}' = 20$ Mpa.
- Each leg has been reinforced with one of 20 mm rebar.

Answers

Note: It is easily to show that the horizontal movements of an area in a reinforced concrete beam has no effects on strain or stress distribution if the section remains to have a vertical axis of symmetry.

Then the section will be transformed for the shape below and analyzed as a T shape with web width of 120 mm:



1. Checking Section Type:

$\rho_w \; ? \; \rho_{w\,max}$

$\rho_{w\,max} = \frac{A_{s\,max}}{b_w d} = 0.85\beta_1 \frac{f'_c}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} + \frac{A_{sf}}{b_w d}$

$A_{sf} = \frac{0.85 f'_c h_f (b - b_w)}{f_y} = 945 \text{ mm}^2$

$d = 250 \text{ mm}$

$\rho_{w\,max} = 15.5 \times 10^{-3} + 31.5 \times 10^{-3} = 47.0 \times 10^{-3}$

$\rho_w = \frac{A_s}{b_w d}$

$A_s = 628 \text{ mm}^2$

$\rho_w = 20.9 \times 10^{-3} < \rho_{w\,max}$ Ok.

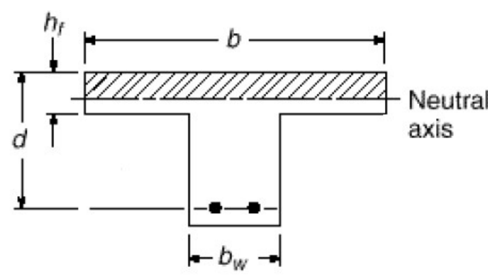
Checking of $A_{s\,minimum}$ limitation:

$\because f'_c < 31\,MPa \Longrightarrow \therefore A_{s\,minimum} = \frac{1.4}{f_y} b_w d = \frac{1.4}{400} \times (60 \times 2) \times 250 = 105 \text{ mm}^2$

$< A_{s\,Provided}$ Ok.

2. Computing of Nominal Flexure Strength "M_n":

Assume that $a \leq h_f$ (based on experience, this can be considered as the general case):



$a = \frac{A_s f_y}{0.85 f'_c b} = 21.9 \text{ mm} < 40 mm\ Ok.$

As $a_{Computed} \leq h_f$, then above assumption is correct and nominal flexure strength M_n can be computed based on:

$M_n = A_s f_y (d - \frac{a}{2}) = 60.0 \text{ kN.m}$

3. Compute the Strength Reduction Factor $\emptyset$ Based on Following Relations:

Compute steel stain based on the following relations:

$c = \frac{a}{\beta_1} = 25.8 \text{ mm} \Rightarrow \epsilon_t = \frac{d - c}{c} \epsilon_u = 26.1 \times 10^{-3}$

As $\epsilon_t \geq 0.005$, then $\emptyset = 0.9$ Ok.

4. Finally Compute the Design Flexure Strength of Section $\emptyset M_n$:

$\emptyset M_n = \emptyset \times M_n = 54.0 \text{ kN.m}$■

4.9 DESIGN OF A BEAM WITH T-SHAPE

4.9.1 Essence of Problem

- Generally, all design problems for T-section can be classified as a design of a section with pre-specified dimensions (h_f, b, b_w, and h). Usually these dimensions have been determined as follows:
    - h_f, b are both determined from slab design that logically be executed before the design of supporting beams.
    - b_w, and h are determined based on one of following criteria:
        - Based on architectural requirements.
        - Based on strength or deflection requirements in supports region (i.e., region of negative moment), in a continuous T beam.
        - Based on shear requirements (as will be discussed in Chapter 4).
- Therefore, the main unknown of design problem is to determine the required reinforcement and its details.

4.9.2 Design Procedures

Based on above known and unknown, the design procedure can be summarized as follows:

- Computed of M_u:
  Based on given loads and spans the applied factored moment M_u can be computed. As slab weight has been already included in the applied dead load, therefore only selfweight of beam stem should be added.

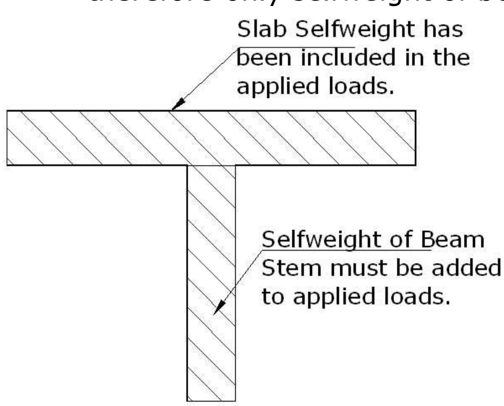


Figure 4.9-1: Selfweight of Tee beams.

- Based on slab and beam data, determine the effective flange width "b" and as was discussed in previous article. For isolated T beam, beam dimensions must be checked based on ACI requirements.
- Compute M_n based on following relation:
  $$M_n = \frac{M_u}{\emptyset}$$
  where $\emptyset$ will be assumed 0.9 to be checked later. This assumption is generally satisfied in the design of T section.
- Check if this section can be design with a compression block in section flange or extend to section web based on following comparison:
  If
  $$M_n \leq 0.85 f'_c h_f b \left(d - \frac{h_f}{2}\right)$$
  then $a \leq h_f$. Else $a > h_f$

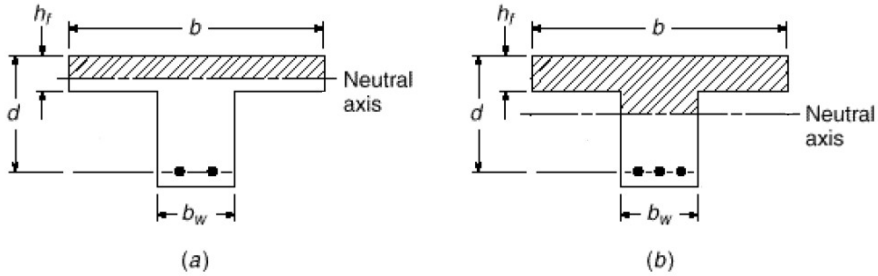


Figure 4.9-2: Possible location of compression block of Tee beams, reproduction of Figure 4.8-9 for quick reference.

- Design of a section with $a \le h_f$:
  This section can be designed as a rectangular section with dimensions of b and d.

$$\rho_{Required} = \frac{1 - \sqrt{1 - 2.36 \frac{M_n}{f'_c b d^2}}}{1.18 \times \frac{f_y}{f'_c}}$$

$A_{S\ Required} = \rho_{Required} bd$ ■

- Design of a section with $a > h_f$:
  - Compute the nominal moment that can be supported by flange overhangs:

    $$M_{n1} = 0.85 f'_c h_f (b - b_w)(d - \frac{h_f}{2})$$

    Steel reinforcement for this part will be:

    $$A_{sf} = \frac{0.85 f'_c h_f (b - b_w)}{f_y}$$ ■

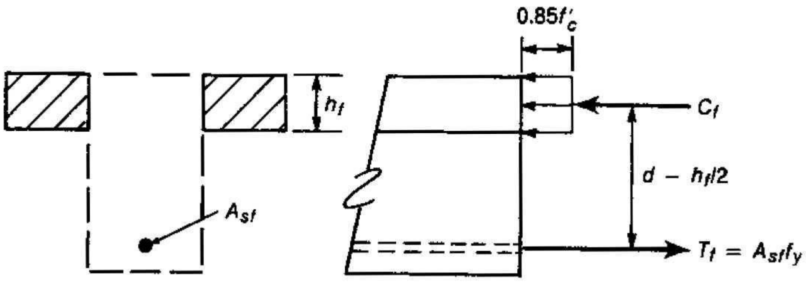


Figure 4.9-3: Forces acting on overhang parts and corresponding streel area.

  - Compute the remaining nominal strength that must be supported by section web:

    $M_{n2} = M_n - M_{n1}$

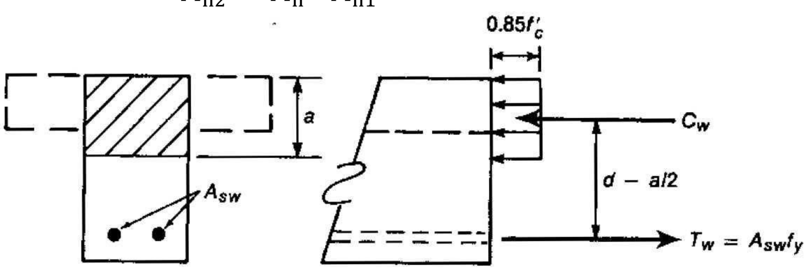


Figure 4.9-4: Forces acting on web part and corresponding steel area.

For this moment "M_{n2}", the section can be designed as a rectangular section with dimensions of b_w and d:

$$\rho_{Required} = \frac{1 - \sqrt{1 - 2.36 \frac{M_{n2}}{f'_c b_w d^2}}}{1.18 \times \frac{f_y}{f'_c}}$$

$A_{s2} = \rho_{Required} b_w d$

then:

$A_{s\ Required} = A_{sf} + A_{s2}$ ■

- Check $A_{s\ Required}$ with minimum steel area permitted by the ACI Code:

$$A_{s\ minimum} = \frac{0.25\sqrt{f'_c}}{f_y} b_w d \ge \frac{1.4}{f_y} b_w d$$

If $A_{s\ Required} > A_{s\ minimum}$ Ok. Else, use:

$A_{s\ Required} = A_{s\ minimum}$

- Check the $A_{s\ Required}$ with the maximum steel area permitted by ACI Code:

$$\rho_w = \frac{A_{s\ Required}}{b_w d} \ ?\ \rho_{w\ max} = 0.85\beta_1 \frac{f'_c}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} + \frac{A_{sf}}{b_w d}$$

If

$\rho_w \le \rho_{w\ max}$ Ok.

Else, the designer must increase one or more of beam dimensions, i.e., in practice, compression reinforcement is not used in T sections.

- Check the assumption of $\emptyset = 0.9$:
  - Compute "a":
    If $a \le h_f$
    $$a = \frac{A_s f_y}{0.85 f_c' b}$$
    If $a > h_f$
    $$\mathrm{a} = \frac{(\mathrm{A_s} - \mathrm{A_{sf}})\mathrm{f_y}}{0.85\mathrm{f_c'}(\mathrm{b_w})}$$
  - Compute steel stain based on the following relations:
    $$\mathrm{c} = \frac{\mathrm{a}}{\beta_1}$$
    $$\epsilon_t = \frac{\mathrm{d - c}}{\mathrm{c}} \epsilon_u$$
  - If $\epsilon_t \ge 0.005$, then $\emptyset = 0.9$ Ok.
  - If $\epsilon_t < 0.005$, then re-compute a more accurate $\emptyset$:
    $\emptyset = 0.483 + 83.3\epsilon_t$
    and return to step of computing M_n.
- Finally, compute the required number of rebars and reinforcement layers and draw section details.

4.9.3 Examples

Example 4.9-1

Design the T-beam for the floor system shown in Figure 4.9-5 below. The floor slab supported by 6.71 m simple span beams. Service loads are: $\mathrm{W_{Live}} = 14.6\frac{\mathrm{kN}}{\mathrm{m}}$ and $\mathrm{W_{Dead}} = 29.2\frac{\mathrm{kN}}{\mathrm{m}}$.

Assume that the designer intends to use:

- $\mathrm{f_y} = 414\ \mathrm{Mpa}\ f_c' = 21\ Mpa$.
- $\emptyset 25mm$ for longitudinal reinforcement ($A_{Bar} = 510mm^2$) and $\emptyset 10mm$ for stirrups.
- One layer of reinforcement.

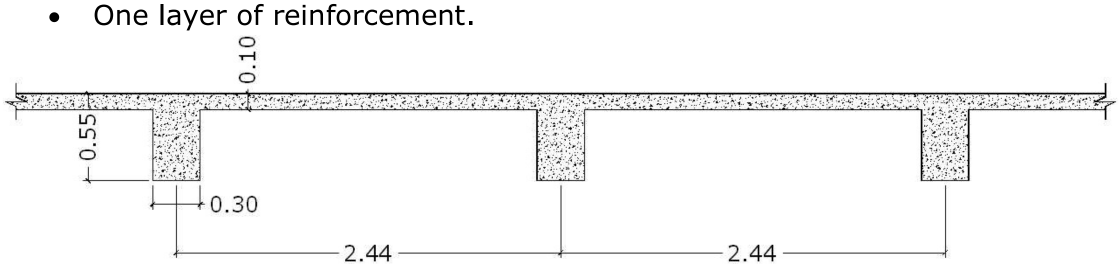
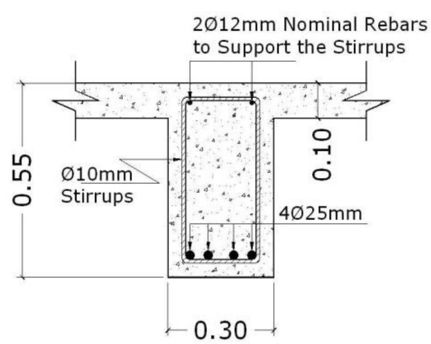


Figure 4.9-5: Floor system for Example 4.9-1.

Solution

- Computed of M_u
$$\mathrm{W_{Self}} = (0.55 - 0.1)\mathrm{m} \times 0.3\mathrm{m} \times 24\frac{\mathrm{kN}}{\mathrm{m^3}} = 3.24\frac{\mathrm{kN}}{\mathrm{m}} \Longrightarrow \mathrm{W_{Dead}} = 29.2\frac{\mathrm{kN}}{\mathrm{m}} + 3.24\frac{\mathrm{kN}}{\mathrm{m}} = 32.4\frac{\mathrm{kN}}{\mathrm{m}}$$
$$\mathrm{M_{Dead}} = \frac{32.4\frac{\mathrm{kN}}{\mathrm{m}} \times 6.71^2\mathrm{m}^2}{8} = 182\ \mathrm{kN.m}, \mathrm{M_{Live}} = \frac{14.6\frac{\mathrm{kN}}{\mathrm{m}} \times 6.71^2\mathrm{m}^2}{8} = 82.2\ \mathrm{kN.m}$$
$$\mathrm{M_u} = \text{maximum of } [1.4\mathrm{M_{Dead}} \text{ or } 1.2\mathrm{M_{Dead}} + 1.6\mathrm{M_{Live}}]$$
$$\mathrm{M_u} = \text{maximum of } [1.4 \times 182\ \mathrm{kN.m} \text{ or } 1.2 \times 182\ \mathrm{kN.m} + 1.6 \times 82.2\ \mathrm{kN.m}]$$
$$\mathrm{M_u} = \text{maximum of } [255\ \mathrm{kN.m} \text{ or } 350\mathrm{kN.m}] = 350\ \mathrm{kN.m}$$
- Compute of Required Nominal Flexure Strength $\mathrm{M_n}$:
$$\mathrm{M_n} = \frac{\mathrm{M_u}}{\emptyset} = \frac{350\ \mathrm{kN.m}}{0.9} = 389\ \mathrm{kN.m}$$
where $\emptyset$ will be assumed 0.9 to be checked later.
- Compute the effective flange width "b"
$$b = \mathrm{b_w} + minimum\ \left[\frac{s_w}{2}\ or\ 8h\ or \frac{l_n}{8}\right] \times 2$$
$$b = 300 + minimum\ \left[\frac{(2440 - 300)}{2}\ or\ 8 \times 100\ or\ \frac{6710}{8}\right] \times 2$$
$$b = 300 + minimum\ [1070\ or\ 800\ or\ 839] \times 2 = 300 + 800 \times 2 = 1900$$

- Check if this section can be design with a compression block in section flange or extend to section web based on following comparison:
  $M_n ?\ 0.85f_c'\ h_f b\left(d-\frac{h_f}{2}\right)$
  $d = 550mm - 40mm - 10mm - \frac{25}{2}mm = 487mm$
  $M_n = 389\ kN.m\ ?\ 0.85f_c'\ h_f b\left(d-\frac{h_f}{2}\right) = 0.85 \times 21 \times 100 \times 1900\left(487-\frac{100}{2}\right) = 1482\ kN.m$
  $M_n = 389\ kN.m < 0.85f_c'\ h_f b\left(d-\frac{h_f}{2}\right) = 1482\ kN.m$
  Then $a < h_f$
- Design of a section with $a \le h_f$:
  This section can be designed as a rectangular section with dimensions of b and d.
  $$\rho_{Required} = \frac{1-\sqrt{1-2.36\frac{M_n}{f_c'bd^2}}}{1.18\times\frac{f_y}{f_c'}} = \frac{1-\sqrt{1-2.36\frac{389\times10^6}{21\times1900\times487^2}}}{1.18\times\frac{414}{21}} = 2.13\times10^{-3}$$
  $A_{S\ Required} = \rho_{Required}bd = 2.13\times10^{-3}\times1900\times487 = 1971\ mm^2$
  $\text{No of Rebars} = \frac{1971}{510} = 3.86$
  Try $4\emptyset25mm$
  $A_{s\ provided} = 4\times510mm^2 = 2040\ mm^2$
  $b_{Required} = 40\times2+10\times2+4\times25+3\times25 = 275\ mm < 300mm\ Ok.$
- Check $A_{s\ Provided}$ with minimum steel area permitted by the ACI Code:
  $$A_{s\ minimum} = \frac{0.25\sqrt{f_c'}}{f_y}b_w d \ge \frac{1.4}{f_y}b_w d \Rightarrow A_{s\ minimum} = \frac{1.4}{f_y}b_w d = \frac{1.4}{414}\times300\times487 = 494\ mm^2$$
  As $A_{s\ Provided} > A_{s\ minimum}$ Ok.
- Check the $A_{s\ Provided}$ with the maximum steel area permitted by ACI Code:
  $$\rho_w = \frac{A_{s\ Provided}}{b_w d}\ ?\ \rho_{w\ max} = 0.85\beta_1\frac{f_c'}{f_y}\frac{\epsilon_u}{\epsilon_u+0.004}+\frac{A_{sf}}{b_w d}$$
  $$A_{sf} = \frac{0.85f_c'h_f(b-b_w)}{f_y} = \frac{0.85\times21\times100\times(1900-300)}{414} = 6899\ mm^2$$
  $$\rho_w = \frac{2\ 040\ mm^2}{300\times487}\ ?\ \rho_{w\ max} = 0.85\times0.85\times\frac{21}{414}\frac{0.003}{0.003+0.004}+\frac{6899}{300\times487}$$
  $\rho_w = 13.9\times10^{-3} \ll \rho_{w\ max} = 15.7\times10^{-3}+47.2\times10^{-3}62.9\times10^{-3}$ Ok.
- Check the assumption of $\emptyset = 0.9$:
  - Compute "a":
    $\sum F_x = 0 \Rightarrow 0.85\times21\times a\times1900$
    $= 2040\ \times414 \Rightarrow a = 24.9\ mm$
  - Compute steel stain based on the following relations:
    $c = \frac{a}{\beta_1} = \frac{24.9}{0.85} = 29.3mm \Rightarrow \epsilon_t$
    $= \frac{487-29.3}{29.3}\times0.003$
    $= 46.9\times10^{-3}$
  - As $\epsilon_t > 0.005$, then $\emptyset = 0.9$ Ok.
- Draw the Section Details:



Example 4.9-2

Design a T-beam having a cross section shown in Figure 4.9-6 below to support a total factored moment M_u of 461 kN.m. Assume that the effective flange width has been computed and as shown in Figure below.

Assume that the designer intends to use:

- $f_y = 414\ Mpa\ f_c' = 21\ Mpa.$
- $\emptyset35mm$ for longitudinal reinforcement ($A_{Bar} = 1000mm^2$) and $\emptyset10mm$ for stirrups.
- One layer of reinforcement.

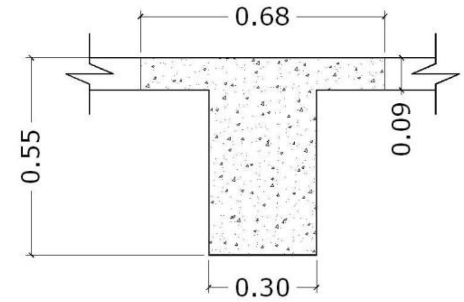


Figure 4.9-6: T section for Example 4.9-2.

Solution

- Compute of Required Nominal Flexure Strength M_n:

$$M_n = \frac{M_u}{\emptyset} = \frac{461\ kN.m}{0.9} = 512\ kN.m$$

where $\emptyset$ will be assumed 0.9 to be checked later.

- Check if this section can be design with a compression block in section flange or extend to section web based on following comparison:

$$M_n?\ 0.85f_c'\ h_f b\left(d - \frac{h_f}{2}\right)$$

$$d = 550mm - 40mm - 10mm - \frac{35}{2}mm = 482mm$$

$$M_n = 512\ kN.m > 0.85f_c'\ h_f b\left(d - \frac{h_f}{2}\right) = 0.85 \times 21 \times 90 \times 680\left(482 - \frac{90}{2}\right) = 477\ kN.m$$

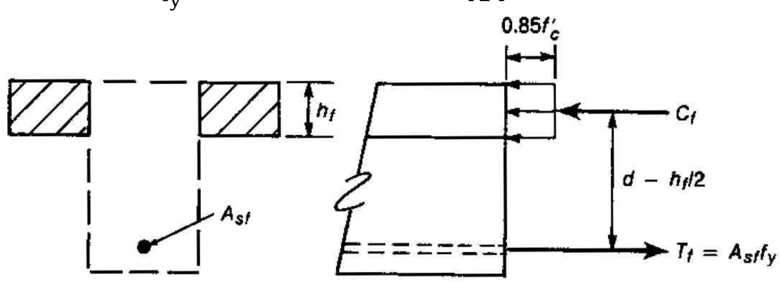
- Design of a section with $a > h_f$:
  - Compute the nominal moment that can be supported by flange overhangs:

$$M_{n1} = 0.85f_c' h_f(b - b_w)\left(d - \frac{h_f}{2}\right) = 0.85 \times 21 \times 90 \times (680 - 300)(482 - \frac{90}{2})$$

$$M_{n1} = 267\ kN.m$$

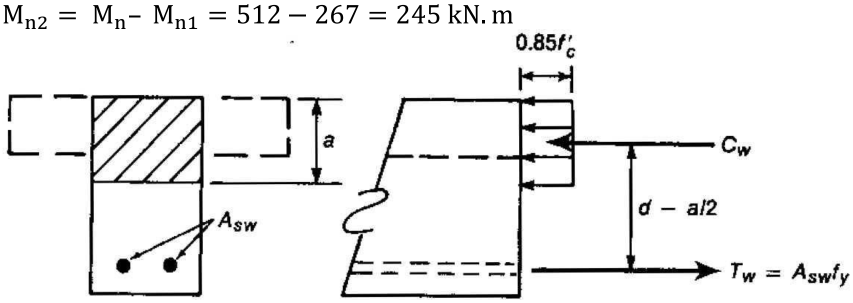
Steel reinforcement for this part will be:

$$A_{sf} = \frac{0.85f_c' h_f(b - b_w)}{f_y} = \frac{0.85 \times 21 \times 90 \times (680 - 300)}{414} = 1\ 474\ mm^2$$



  - Compute the remaining nominal strength that must be supported by section web:

$$M_{n2} = M_n - M_{n1} = 512 - 267 = 245\ kN.m$$



For this moment "M_{n2}", the section can be designed as a rectangular section with dimensions of b_w and d:

$$\rho_{Required} = \frac{1 - \sqrt{1 - 2.36\frac{M_{n2}}{f_c' b_w d^2}}}{1.18 \times \frac{f_y}{f_c'}} = \frac{1 - \sqrt{1 - 2.36\frac{245 \times 10^6}{21 \times 300 \times 482^2}}}{1.18 \times \frac{414}{21}} = 9.55 \times 10^{-3}$$

$$A_{s2} = \rho_{Required} b_w d = 9.55 \times 10^{-3} \times 300 \times 482 = 1\ 381\ mm^2$$

Then:

$A_{s\,Required} = A_{sf} + A_{s2} = 1\,474\ mm^2 + 1\,381\ mm^2 = 2\,855\ mm^2$

$\text{No. of Rebars} = \dfrac{2855mm^2}{1000mm^2} = 2.86$

Try $3\emptyset35mm$

$A_{S\,Provided} = 3\,000\ mm^2$

$b_{Required} = 40 \times 2 + 10 \times 2 + 3 \times 35 + 2 \times 35 = 275mm < 300mm\ Ok.$

- Check $A_{S\,Provided}$ with minimum steel area permitted by the ACI Code:

$A_{s\,minimum} = \dfrac{0.25\sqrt{f_c'}}{f_y} b_w d \geq \dfrac{1.4}{f_y} b_w d$

$A_{s\,minimum} = \dfrac{1.4}{f_y} b_w d = \dfrac{1.4}{414} \times 300 \times 482 = 489\ mm^2 < A_{S\,Provided}\ \text{Ok.}$

- Check the $A_{S\,Provided}$ with the maximum steel area permitted by ACI Code:

$\rho_w = \dfrac{A_{S\,Provided}}{b_w d}\ ?\ \rho_{w\,max} = 0.85\beta_1 \dfrac{f_c'}{f_y} \dfrac{\epsilon_u}{\epsilon_u + 0.004} + \dfrac{A_{sf}}{b_w d}$

$\rho_w = \dfrac{3000}{300 \times 482} = 20.7 \times 10^{-3}\ ?\ \rho_{w\,max} = 0.85^2 \dfrac{21}{414} \dfrac{0.003}{0.003 + 0.004} + \dfrac{1\,474\ mm^2}{300 \times 482}$

$\rho_w = 20.7 \times 10^{-3} < \rho_{w\,max} = 15.7 \times 10^{-3} + 10.2 \times 10^{-3} = 25.9 \times 10^{-3}\ \text{Ok.}$

- Check the assumption of $\emptyset = 0.9$:
  - Compute "a":

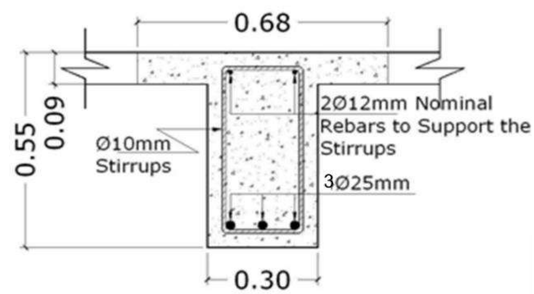
    $a = \dfrac{(A_s - A_{sf})f_y}{0.85 f_c'(b_w)} = \dfrac{(3\,000 - 1\,474) \times 414}{0.85 \times 21 \times 300} = 118\ mm$

  - Compute steel stain based on the following relations:

    $c = \dfrac{a}{\beta_1} = \dfrac{118\ mm}{0.85} = 139\ mm$

    $\epsilon_t = \dfrac{d - c}{c}\epsilon_u = \dfrac{482 - 139}{139} \times 0.003$
    $= 7.40 \times 10^{-3}$

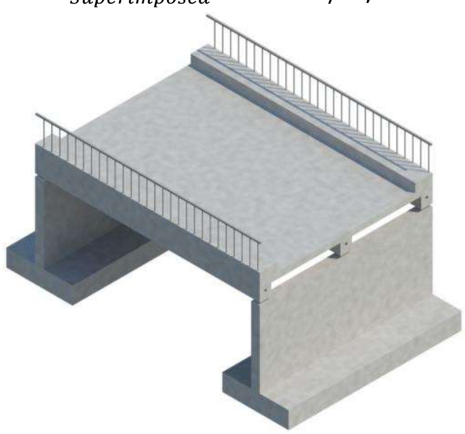
  - As $\epsilon_t > 0.005$, then $\emptyset = 0.9$ Ok.
- Draw the Section Details:



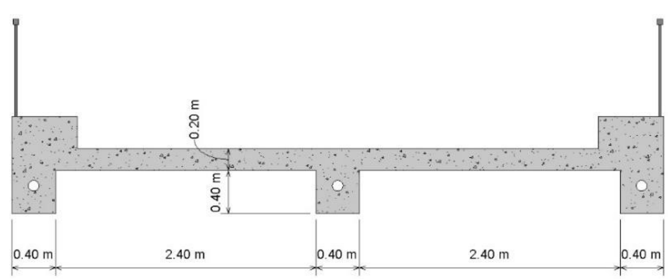
Example 4.9-3

For a pedestrian bridge indicated in ***Figure*** 4.9-7 below, a structural designer includes sleeve with diameter of 100mm for communication and electrical cables. Design for flexural the central supporting beam. In your design, assume that:

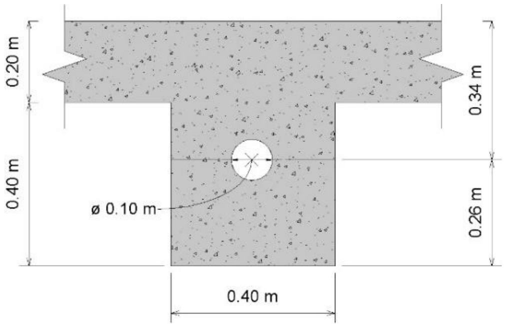
- Materials strength are $f_c' = 28\ MPa$ and $f_y = 420\ MPa$.
- Two layers of reinforcement with a bar diameter of 20mm for longitudinal reinforcement,
- Rebar with a diameter of 10mm for stirrups,
- $W_{Superimposed} = 20\ kN/m$, not including beam own weight, $W_{Live} = 12\ kN/m$



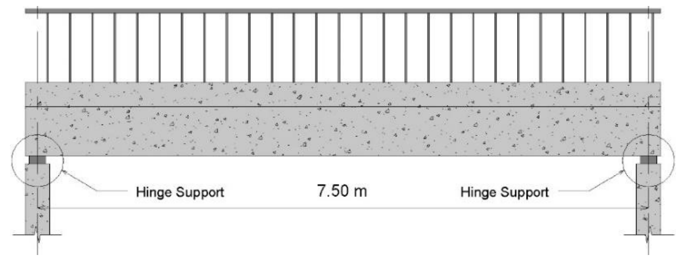
3D view.



Cross sectional view.



Callout view.



Elevation view.

Figure 4.9-7: A pedestrian bridge for Example 4.9-3.

Solution

- <u>Compute factored load $\boldsymbol{W_u}$:</u>
  Assuming that selfweight of the beam flange has been included in the superimposed dead load of $W_{Superimposed} = 20\,kN/m$, therefore what should be included as a selfweight would include stem selfweight only.
  $$W_{Selfweight} = 0.4 \times 0.4 \times 24 = 3.84\frac{kN}{m}$$
  Reducing in selfweight due to pipe conduit has been conservatively neglected. The total dead load would be:
  $$W_{Dead} = W_{Selfweight} + W_{Superimposed} = 3.84 + 20 = 23.8\frac{kN}{m}$$
  The factored uniformly distributed load that acting on the beam would be:
  $$W_u = \max(1.4W_D, 1.2W_D + 1.6W_L) = \max(1.4 \times 23.8, 1.2 \times 23.8 + 1.6 \times 12) = 47.8\frac{kN}{m}$$
  With the indicated simple supports, the maximum factored moment, M_u, at beam mid-span would be:
  $$M_u = \frac{W_u l^2}{8} = \frac{47.8 \times 7.50^2}{8} = 336\,kN.m$$
- <u>Compute $\mathrm{M_n}$:</u>
  $$\mathrm{M_n} = \frac{\mathrm{M_u}}{\emptyset}$$
  where $\emptyset$ will be assumed 0.9 to be checked later. This assumption is generally satisfied in the design of T section.
  $$M_n = \frac{336}{0.9} = 373\,kN.m$$
- <u>Beam effective depth, d:</u>
  With adopting of two layers of $No.20$ for longitudinal reinforcement and $No.10$ for stirrup reinforcement, the effective depth, d, would be:
  $$d = 600 - 40 - 10 - 20 - \frac{25}{2} \approx 517\,mm$$
- <u>Effective flange width:</u>
  With referring to Figure 4.9-8:
  $$b = \mathrm{b_w} + minimum\left[\frac{s_{w\,left}}{2}\ or\ 8h\ or\ \frac{l_n}{8}\right] + minimum\left[\frac{s_{w\,right}}{2}\ or\ 8h\ or\ \frac{l_n}{8}\right]$$
  $$b = 0.4 + \min\left(\left(\frac{2.4}{2}\right), (8 \times 0.2), \left(\frac{7.5}{8}\right)\right) + \min\left(\left(\frac{2.4}{2}\right), (8 \times 0.2), \left(\frac{7.5}{8}\right)\right) = 2.275\ \mathrm{m}$$
- <u>Check type of section:</u>
  Check to see if the stress block would be in the flange or extends to the web:
  $$\mathrm{M_n} = 373\,kN.m \ll 0.85\mathrm{f_c'}\,\mathrm{h_f b}\left(\mathrm{d} - \frac{\mathrm{h_f}}{2}\right) = \frac{0.85 \times 28 \times 200 \times 2.275 \times 10^3 \times \left(517 - \frac{200}{2}\right)}{10^6}$$
  $$= 4516\,kN.m$$
  Then $\mathrm{a} < \mathrm{h_f}$ and the section behaves as a rectangular section with dimensions of b by d.

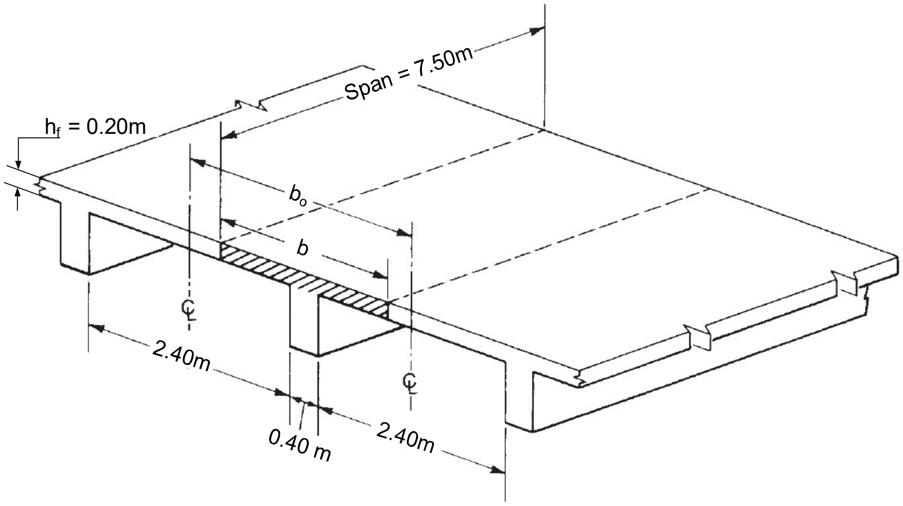


Figure 4.9-8: Flange computation parameters for Example 4.9-3.

- <u>Effect of conduit hole:</u>
  According to aforementioned argument, stress block is located at flange; therefore, the conduit hole has no effect on beam strength, as it is located at the tension zone that completely neglected in traditional concrete theory.
- <u>Required reinforcement ratio, $\rho_{Required}$:</u>

$$\rho_{Required} = \frac{1-\sqrt{1-2.36\frac{M_n}{f_c' b d^2}}}{1.18\times\frac{f_y}{f_c'}} = \frac{\left(1-\sqrt{1-2.36\times\left(\frac{373\times 10^6}{28\times(2.275\times 10^3)\times 517^2}\right)}\right)}{\left(1.18\times\frac{420}{28}\right)}$$
$$= 1.48\times 10^{-3}$$

$A_{s\,Required} = 1.48\times 10^{-3}\times(2.275\times 10^3)\times 517 = 1741\,mm^2$

- <u>Check with $A_{s\,minmum}$:</u>
  With conservative neglecting of the conduit hole, minimum required reinforcement, $A_{s\,minimum}$ would be:
  $\because f_c' < 31\,MPa$

$$A_{s\,minimum} = \frac{1.4}{f_y} b_w d = \frac{1.4}{420}\times 400\times 517 = 689\,mm^2 < A_{s\,Required} \therefore Ok.$$

- <u>Check with $\rho_{maximum}$:</u>

$$\rho_{w\,max} = 0.85\beta_1\,\frac{f_c'}{f_y}\,\frac{\epsilon_u}{\epsilon_u+0.004}+\frac{A_{sf}}{b_w d}$$

$$A_{sf} = \frac{0.85 f_c' h_f (b-b_w)}{f_y} = \frac{\left(0.85\times 28\times 200\times\left((2.275\times 10^3)-400\right)\right)}{420} = 21250\,mm^2$$

$$\rho_{w\,maximum} = 0.85\times 0.85\times\left(\frac{28}{420}\right)\times\left(\frac{0.003}{0.003+0.004}\right)+\left(\frac{21250}{400\times 517}\right) = 123\times 10^3$$

$$\rho_{w\,Required} = \frac{1741}{400\times 517} = 8.42\times 10^{-3}$$

$\rho_{w\,maximum} \gg \rho_{w\,Required} \therefore Ok.$

As expect, the Tee section has very high ductility level and would fail as a tension control section. This implicitly indicates that ***the assumption of*** $\phi = 0.9$ ***is valid and there is no need to be checked explicitly***.

- <u>Details of the section:</u>

$$No.of\ Reabrs = \frac{1741}{\frac{\pi\times 20^2}{4}} = 5.54$$

Therefore use $6No.20$ in two layers as indicated in below.

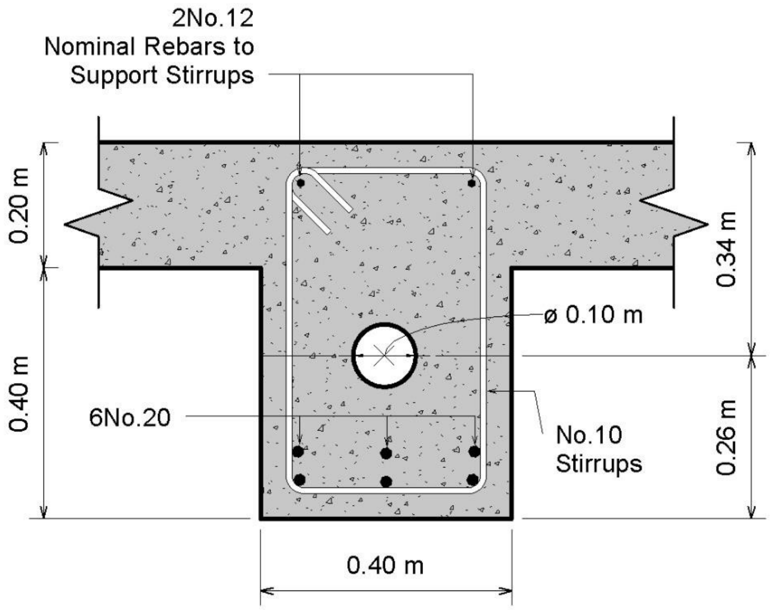


Figure 4.9-9: Final detailed section for Example 4.9-3.

Example 4.9-4

The vertical reaction at end B of an indeterminate propped cantilever beam has been computed as shown in ***Figure 4.9-10*** below:

- Design flexure reinforcement for section at support end (A).
- Check adequacy if same amount of flexure reinforcement calculated at end (A) is used for section of maximum positive bending moment.

Assume that:

- Beam selfweight is neglected.
- Use a rebar of $\phi 25mm$ with A_{bar} of 510 mm^2 and stirrups of $\phi 10mm$.
- $f_c' = 28\ MPa$ and $f_y = 420\ MPa$.

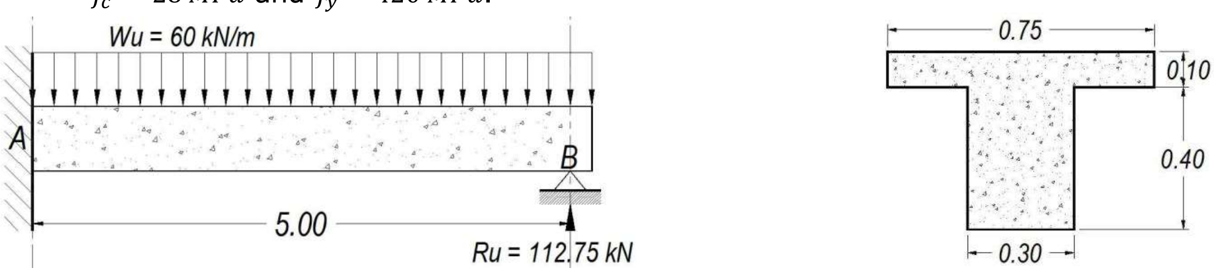


Figure 4.9-10: Propped cantilever beam Example 4.9-4. **Beam Section**

Solution

- **Design flexure reinforcement for section at support end (A):**
  In negative region, section behaves as rectangular section as flange is already cracked.
  - Compute M_n:

$$M_u = -60 \times 5.00 \times \frac{5.00}{2} + 112.75 \times 5 = -186\ kN.m$$

Assume ϕ to be 0.9 to be checked later:

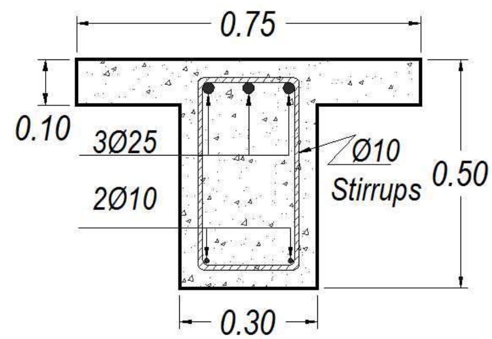
$$M_n = \frac{186}{0.9} = 207$$

  - Compute ρ:

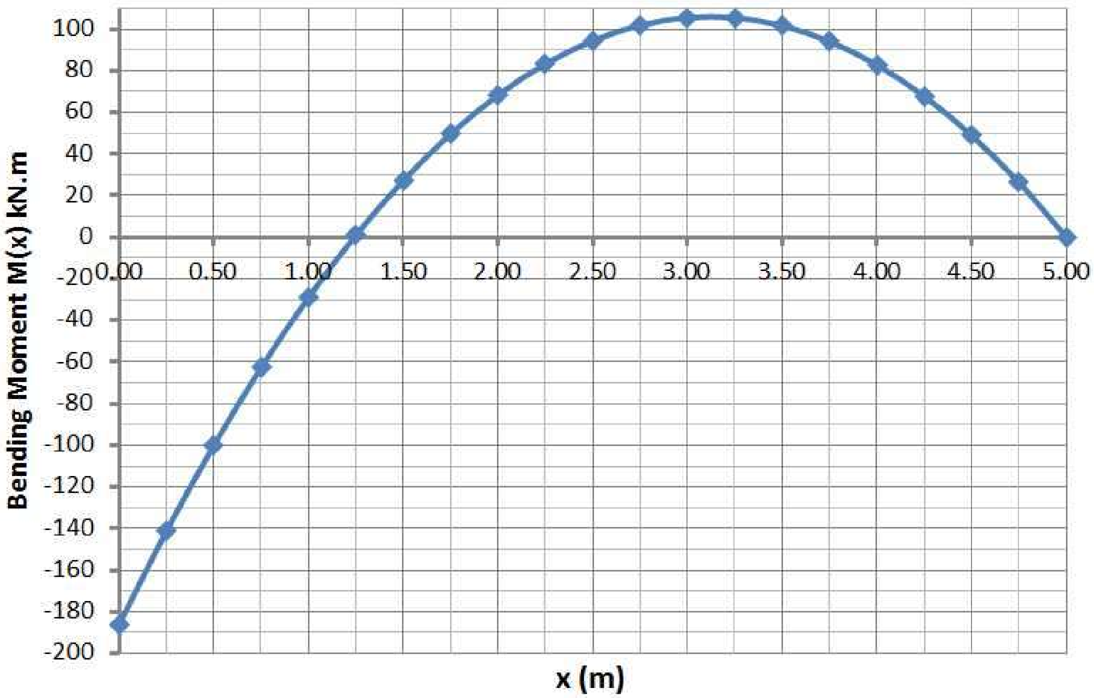
$$d_{one\ layer} = 500 - 40 - 10 - \frac{25}{2} = 438$$

$$\rho_{Required} = \frac{1 - \sqrt{1 - 2.36 \frac{M_n}{f_c' b d^2}}}{1.18 \times \frac{f_y}{f_c'}} = \frac{1 - \sqrt{1 - 2.36 \frac{207 \times 10^6}{28 \times 300 \times 438^2}}}{1.18 \times \frac{420}{28}} = 9.33 \times 10^{-3}$$

- Check $\rho_{Maximum}$:
  $$\rho_{max} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} = 0.85^2 \times \frac{28}{420} \times \frac{0.003}{0.007} = 20.6 \times 10^{-3} > \rho\ Ok.$$
- Compute As:
  $$A_s = 9.33 \times 10^{-3} \times 438 \times 300 = 1226\ mm^2$$
  $$No. of\ rebars = \frac{1226}{510} = 2.4$$
  Try $3\phi25$.
- Check b:
  $$b_{Required} = 40 \times 2 + 10 \times 2 + 25 \times 3 + 25 \times 2 = 225 < 300\ Ok.$$
- Check A_{smin}:
  Since the span is a statically indeterminate span and $f_c' < 31.4\ MPa$, then A_{smin} will be:
  $$A_{smin} = \frac{1.4}{fy} b_w d = \frac{1.4}{420} \times 300 \times 438 = 438\ mm^2 < A_{s\ provided}\ Ok.$$
- Check ϕ Assumption:
  $$a = \frac{3 \times 510 \times 420}{0.85 \times 28 \times 300} = 90\ mm \Rightarrow c = \frac{a}{\beta_1} = \frac{90}{0.85} = 106\ mm$$
  $$\epsilon_t = \frac{438 - 106}{106} \times 0.003 = 9.40 \times 10^{-3}$$
  $$> 5 \times 10^{-3} Ok.$$
- Draw final section. Drawing shown is a preliminary one as instead of adding two rebars with nominal diameter for lower side, specific amount of positive reinforcement should be extended into support region according to ACI code requirements. This aspect will be discussed in Chapter 5 of the course.



- **Check adequacy if same amount of flexure reinforcement calculated at end (A) is used for section of maximum positive bending moment:**
  - Intuitively one can conclude that reinforcement computed for the negative region would be adequate when used on the bottom side for the positive moment. This is due to the facts that:
    - As will be discussed in ***Chapter 11***, the positive moment is lower than the negative moment for regular spans that subjected to uniformly distributed loads.
    - The flange is effective in the positive region while it is neglected in the negative region.
  - Compute the maximum positive moments:
    The maximum positive moment could either be computed by:
    $$R_{u\ left} = 60 \times 5.0 - 112.75 = 187.25\ kN$$
    $$M(x) = -186.25 + 187.25x - \frac{60x^2}{2}$$
    $$\frac{dM}{dx} = 187.25 - 60x$$
    $$x_{maximum} = 3.121$$
    $$M(3.121) = -186.25 + 187.25 \times 3.121 - \frac{60 \times 3.121^2}{2}$$
    $$M_{u+ve\ Max} = 106\ kN.m$$
    Or by:
    $$\Sigma F_y = 0_{for\ right\ side}$$
    $$60x = 112.75$$
    $$x = 1.88\ m\ from\ right\ support$$
    $$M_{u\ +v\ \ maximum} = 112.75 \times 1.88 - 60 \times \frac{1.88^2}{2} = 106\ kN.m$$



- Check $\rho_{wmax.}$:

  $\rho_{wmax} = \rho_{max} + \frac{A_{sf}}{b_w d}$

  $A_{sf} = \frac{0.85 \times 28 \times (750 - 300) \times 100}{420} = 2550\, mm^2$

  $\rho_{wmax.} = 20.6 \times 10^{-3} + \frac{2550}{300 \times 438} = 40.0 \times 10^{-3} \Longrightarrow \rho_w = \frac{3 \times 510}{300 \times 438} = 11.6 \times 10^{-3}$
  $< \rho_{w\,max.}\ Ok.$

- Check A_{smin}:

  $A_{smin} = \frac{1.4}{fy} b_w d = \frac{1.4}{420} \times 300 \times 438 = 438\, mm^2 < A_{s\,provided}\ Ok.$

- Compute M_n:

  Assume $a \leq h_f$:

  $a = \frac{3 \times 510 \times 420}{0.85 \times 28 \times 750} = 36mm < 100mm\ Ok.$

  $M_n = (3 \times 510 \times 420) \times \left(438 - \frac{36}{2}\right) = 270\ kN.m$

- Compute ϕ:

  $a = 36\ mm \Longrightarrow c = \frac{a}{\beta_1} = \frac{36}{0.85} = 42.3\ mm$

  $\epsilon_t = \frac{438 - 42.3}{42.3} \times 0.003 = 28.1 \times 10^{-3} > 5 \times 10^{-3} Ok.$

- ϕM_n:

  $\phi M_n = 0.9 \times 270 = 243\ kN.m > M_u\ Ok.$

Example 4.9-5

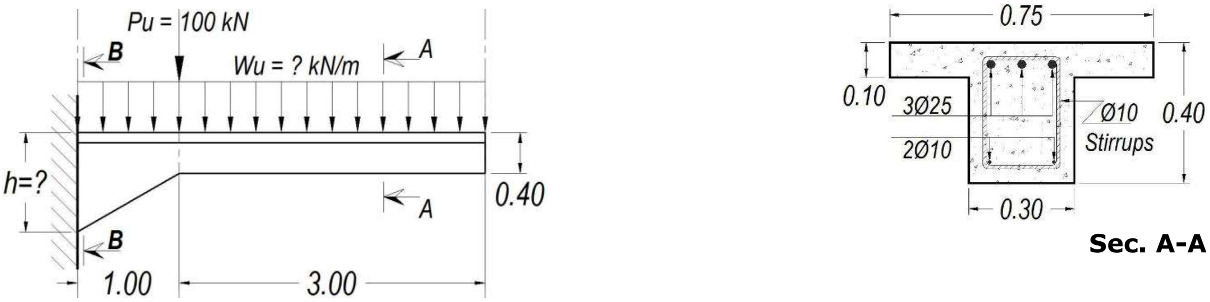
A structural designer has proposed dimensions and reinforcement for the cantilever T-beam shown in Figure 4.9-11 below.

Based on flexure strength for the beam at Section A-A and at Section B-B find:

- Maximum factored uniform load (W_u) that could be supported by the beam.
- Minimum beam depth (h) for Section B-B.

In your solution, assume that:

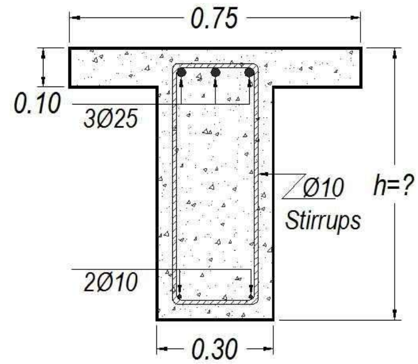
- Beam selfweight could be neglected.
- As = 510mm^2 for $\phi 25mm$ rebars.
- $f_c' = 28\ MPa$ and $f_y = 420\ MPa$.



Sec. A-A

Figure 4.9-11: Cantilever T-beam for Example 4.9-5.

Sec. B-B



Solution

As flange is on the tension side, then the beam can be analyzed as a rectangular section except in computing $A_{s\,minimum}$ where flange should be considered.

Find Wu:

$$d = 400 - 40 - 10 - \frac{25}{2} = 338mm, A_s = 3 \times 510 = 1530\ mm^2$$

$$\rho_{provided} = \frac{1530}{300 \times 338} = 15.0 \times 10^{-3}$$

$$\rho_{maximum} = 0.85^2 \times \frac{28}{420} \times \frac{0.003}{0.003 + 0.004} = 20.6 \times 10^{-3} > \rho_{provided}\ Ok$$

For this statically determinate span with a flange in tension, minimum flexure reinforcement should be computed based on:

$$A_{s\,min} = minimum\ \left(\frac{0.25\sqrt{f_c'}}{f_y} bd, \frac{0.50\sqrt{f_c'}}{f_y} b_w d\right)$$

As

$b = 750\ mm > 2b_w = 2 \times 300 = 600\ mm$

then, the second term governs.

$$A_{s\,min} = \frac{0.50\sqrt{28}}{f_y} b_w d = \frac{0.50\sqrt{28}}{420} \times 300 \times 338 = 639\ mm^2 < A_s \therefore Ok.$$

$$M_n = 15.0 \times 10^{-3} \times 420 \times 300 \times 338^2 \times \left(1 - 0.59 \times \frac{15.0 \times 10^{-3} \times 420}{28}\right) = 187\ kN.m$$

Check ϕ:

$$a = \frac{1530 \times 420}{0.85 \times 28 \times 300} = 90\ mm \Rightarrow c = \frac{90}{0.85} = 106\ mm \Rightarrow \epsilon_t = \frac{d - c}{c} \epsilon_u = \frac{338 - 106}{106} \times 0.003 = 0.0065 > 0.005$$

then:

$\phi = 0.9$

$M_u = \phi M_n = 0.9 \times 187 = 168\ kN.m$

$$M_u = \frac{W_u l^2}{2} \Rightarrow W_u = \frac{2M_u}{l^2} = \frac{2 \times 168}{3^2} = 37.3 \frac{kN}{m}\ ■$$

Find beam depth "d":

$$M_u = \frac{37.3 \times 4^2}{2} + 100 \times 1.0 = 398\ kN.m$$

$$\Sigma F_x = 0 \Rightarrow a = \frac{1530 \times 420}{0.85 \times 28 \times 300} = 90mm$$

Compute M_n (assume that $\phi = 0.9$, to be checked later):

$$M_n = \frac{398}{0.9} = 442\ kN.m \Longrightarrow 442 \times 10^6 = 1530 \times 420 \times \left(d - \frac{90}{2}\right) \Rightarrow d = 733\ mm$$

Check ϕ:

$$a = 90\ mm \Rightarrow c = \frac{90}{0.85} = 106\ mm \Longrightarrow \epsilon_t = \frac{d-c}{c}\epsilon_u = \frac{733-106}{106} \times 0.003 = 0.0177 > 0.005$$

then:

$\phi = 0.9$

Beam depth:

$$h = 733 + \frac{25}{2} + 10 + 40 = 796\ mm$$

Use

$h = 800\ mm$ ■

4.9.4 Problems for Solution

A T-beam having a span of 6.0 m, a web thickness of 300mm, and an overall depth of 645 mm. The beams spacing is 1.2m center to center and the slab thickness is 100 mm. Design this beam for flexure to carries a total factored moment of 1300 kN.m.

Assume that the designer intends to use:

- $f_y = 400$ Mpa $f_c' = 28$ Mpa
- $\emptyset 32$mm for longitudinal reinforcement ($A_{Bar} = 819 mm^2$) and $\emptyset 10$mm for stirrups.
- Two layers of reinforcement.

Answers

- Compute of Required Nominal Flexure Strength M_n:

  $$M_n = \frac{M_u}{\emptyset} = 1\,444\ kN.m$$

  where $\emptyset$ will be assumed 0.9 to be checked later.
- Compute the effective flange width "b":

  $$b = b_w + minimum\ \left[\frac{s_w}{2}\ or\ 8h\ or \frac{l_n}{8}\right] \times 2$$

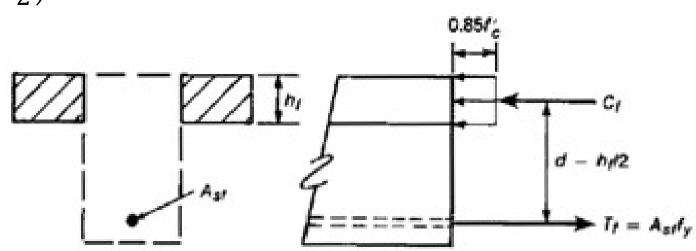
  $$b = 300 + minimum\ \left(\frac{1200-300}{2}\ or\ 8 \times 100\ or \frac{6000}{8}\right) \times 2$$

  $$= 300 + minimum\ (450\ or\ 800\ or 750) \times 2 = 300 + 450 \times 2 = 1200\ mm$$
- Check if this section can be design with a compression block in section flange or extend to section web based on following comparison:

  $$M_n?\ 0.85f_c'\ h_f b\ \left(d - \frac{h_f}{2}\right)$$

  $$d = 550mm$$

  $$M_n = 1\,444\ kN.m >\ 0.85f_c'\ h_f b\ \left(d - \frac{h_f}{2}\right) = 1\,428\ kN.m$$
- Design of a section with $a > h_f$:



  - Compute the nominal moment that can be supported by flange overhangs:

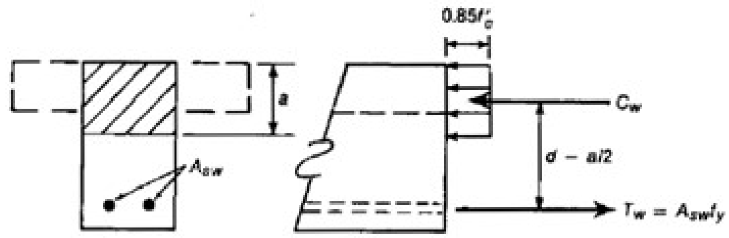
    $$M_{n1} = 0.85f_c' h_f (b - b_w)(d - \frac{h_f}{2}) = 1\,071\ kN.m$$

    Steel reinforcement for this part will be:

    $$A_{sf} = \frac{0.85f_c' h_f (b - b_w)}{f_y} = 5\,355\ mm^2$$
  - Compute the remaining nominal strength that must be supported by section web:

    $$M_{n2} = M_n - M_{n1} = 373\ kN.m$$

For this moment "M_{n2}", the section can be designed as a rectangular section with dimensions of b_w and d:



$$\rho_{Required} = \frac{1-\sqrt{1-2.36\frac{M_{n2}}{f_c' b_w d^2}}}{1.18\times\frac{f_y}{f_c'}} = 11.4\times10^{-3} \Rightarrow A_{s2} = \rho_{Required} b_w d$$
$$= 1\,881\ \mathrm{mm^2}$$

Then:

$A_{s\,Required} = A_{sf} + A_{s2} = 7\,236\ \mathrm{mm^2} \Rightarrow \mathrm{No.\,of\,Rebars} = 8.84$

Try 9Ø32mm $\Rightarrow A_{S\,Provided} = 7\,371\ \mathrm{mm^2}$

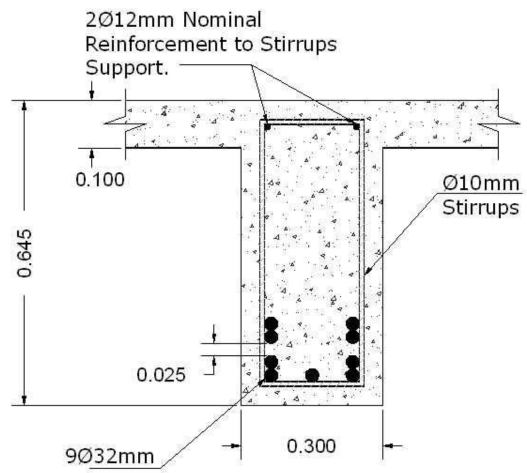
- Check $A_{S\,Provided}$ with minimum steel area permitted by the ACI Code:

$$A_{s\,minimum} = \frac{0.25\sqrt{f_c'}}{f_y} b_w d \geq \frac{1.4}{f_y} b_w d \Rightarrow A_{s\,minimum} = 578\ \mathrm{mm^2} < A_{S\,Provided}\ \mathrm{Ok.}$$

- Check the $A_{S\,Provided}$ with the maximum steel area permitted by ACI Code:

$$\rho_w = \frac{A_{S\,Provided}}{b_w d}\ ?\ \rho_{w\,max}$$
$$= 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} + \frac{A_{sf}}{b_w d}$$
$$\rho_w = 44.7\times10^{-3}\ ?\ \rho_{w\,max}$$
$$= 21.7\times10^{-3} + 32.5\times10^{-3}$$
$$\rho_w = 20.7\times10^{-3} < \rho_{w\,max} = 54.2\times10^{-3}\ \mathrm{Ok.}$$

- Check the assumption of $\emptyset = 0.9$:
  - Compute "a":
    $$a = \frac{(A_s - A_{sf})f_y}{0.85f_c'(b_w)} = 113\ \mathrm{mm}$$
  - Compute steel stain based on the following relations:
    $$c = \frac{a}{\beta_1} = 133\ \mathrm{mm} \Rightarrow \epsilon_t = \frac{d-c}{c}\epsilon_u = 9.41\times10^{-3}$$
  - As $\epsilon_t > 0.005$, then $\emptyset = 0.9$ Ok.
- Draw the Section Details:



Problem 4.9-1

A reinforced concrete T-beam is to be designed for tension reinforcement. The beam width is 250mm and total depth of 490mm. The flange thickness is 100mm and its effective width has been computed to be 900mm. The applied total factored moment is 300kN.m

Assume that the designer intends to use:

- $f_y = 414\ \mathrm{Mpa}, f_c' = 21\ \mathrm{Mpa}$
- Ø28mm for longitudinal reinforcement and Ø10mm for stirrups.
- Two layers of reinforcement.

Answers

- Compute of Required Nominal Flexure Strength M_n:

$$M_n = \frac{M_u}{\emptyset} = 333\ \mathrm{kN.m}$$

where Ø will be assumed 0.9 to be checked later.

- Check if this section can be design with a compression block in section flange or extend to section web based on following comparison:

$$M_n ?\ 0.85f_c' h_f b\left(d - \frac{h_f}{2}\right)$$

$d = 400 \text{ mm}$

$M_n = 333 \text{ kN.m} > 0.85f_c' h_f b \left(d - \frac{h_f}{2}\right) = 562 \text{ kN.m}$

- Design of a section with $a \leq h_f$:
  This section can be designed as a rectangular section with dimensions of b and d.

$$\rho_{Required} = \frac{1 - \sqrt{1 - 2.36\frac{M_n}{f_c' bd^2}}}{1.18 \times \frac{f_y}{f_c'}} = \frac{1 - \sqrt{1 - 2.36 \times \frac{333 \times 10^6}{21 \times 900 \times 400^2}}}{1.18 \times \frac{414}{21}} = 6.01 \times 10^{-3}$$

$A_{S\,Required} = \rho_{Required} bd = 6.01 \times 10^{-3} \times 900 \times 400 = 2164 \text{ mm}^2$

$A_{Bar} = 615 \text{ mm}^2$

$\text{No of Rebars } = \frac{2164}{615} = 3.52$

Try $4\emptyset 28\text{mm}$

$A_{s\,provided} = 2\,460 \text{ mm}^2$

$b_{Required} = 296 \text{ mm} > 250\text{mm}$

Then the reinforcement must be put in two layers as the designer is assumed.

- Check $A_{s\,Provided}$ with minimum steel area permitted by the ACI Code:

$$A_{s\,minimum} = \frac{0.25\sqrt{f_c'}}{f_y} b_w d \geq \frac{1.4}{f_y} b_w d \Rightarrow A_{s\,minimum} = 338 \text{ mm}^2$$

As $A_{s\,Provided} > A_{s\,minimum}$ Ok.

- Check the $A_{s\,Provided}$ with the maximum steel area permitted by ACI Code:

$$\rho_w = \frac{A_{s\,Provided}}{b_w d} \;?\; \rho_{w\,max} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} + \frac{A_{sf}}{b_w d}$$

$$A_{sf} = \frac{0.85 f_c' h_f (b - b_w)}{f_y} = 2\,802 \text{ mm}^2$$

$\rho_w = 24.6 \times 10^{-3} \;?\; \rho_{w\,max} = 15.7 \times 10^{-3} + 28.0 \times 10^{-3}$

$\rho_w = 24.6 \times 10^{-3} \ll \rho_{w\,max}$
$= 43.7 \times 10^{-3}$ Ok.

- Check the assumption of $\emptyset = 0.9$:
  - Compute "a":
    $\sum F_x = 0 \Rightarrow a = 63.4 \text{ mm}$
  - Compute steel stain based on the following relations:
    $c = \frac{a}{\beta_1} = 74.6\text{mm} \Rightarrow \epsilon_t = 13.1 \times 10^{-3}$
  - As $\epsilon_t > 0.005$, then $\emptyset = 0.9$ Ok.
- Draw the Section Details: