Lectured Four: Boolean Algebra &Logic Simplification

1.Rules of Boolean algebra.

2. The Boolean Expression for a Logic Circuit.

3. Implementation of a Logic Circuit Using a Boolean Expression. **4.**Implementation of a Logic Circuit via a Truth Table.

5. Converting a Boolean Expression to a Truth Table.

6. Simplification of Boolean Expressions Using Boolean Algebra.

7. Demorgan's Theorems.

8. Sum of Product (SOP)&Product of Sum (POS).

1. Rules of Boolean Algebra:

1. **Commutative Laws:**

   A+B=B+A

   A·B=B·A

2. **Associative Laws:**

   A+(B+C)=(A+B)+C

   A•(B•C)=(A•B) •C

3. **Distributive Laws:**

   A•(B+C)=(A•B)+(A•C)

   A+(B•C)=(A+B) •(A+C)

4. **Redundancy Laws:**

$$A+(A \bullet B) = A$$

$$A \bullet (A+B) = A$$

5. **Identity Laws:**

$$A+A = A$$

$$A \bullet A = A$$

6. **Inverse Laws:**

$$A+\bar{A}=1$$

$$A \bullet \bar{A}=0$$

7. **Zero and One Laws:**

A+1=1

A+0=A

A•1=A

A•0=0

8. **Negative Laws:**

9. **A+(¯ +B) = (A•B) A+(¯ •B) = (A+B)**

10. **De Morgans Laws:**

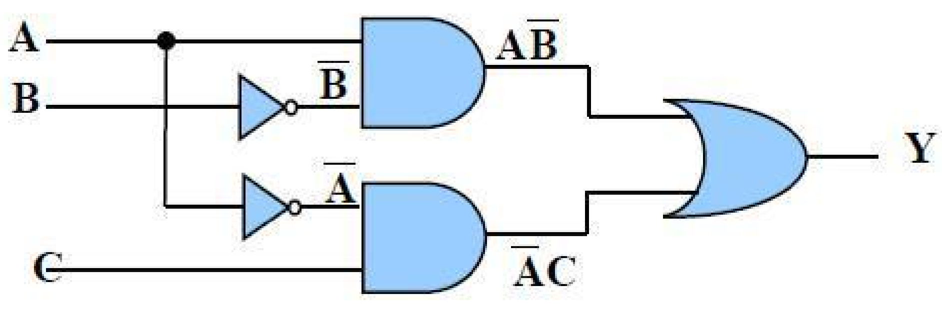
· 0

· 0

2. <u>The Boolean Expression for a Logic Circuit:</u>

To infer the Boolean Expression for any logical circuit we start from the left hand towards the right hand and typing the output of each circuit.

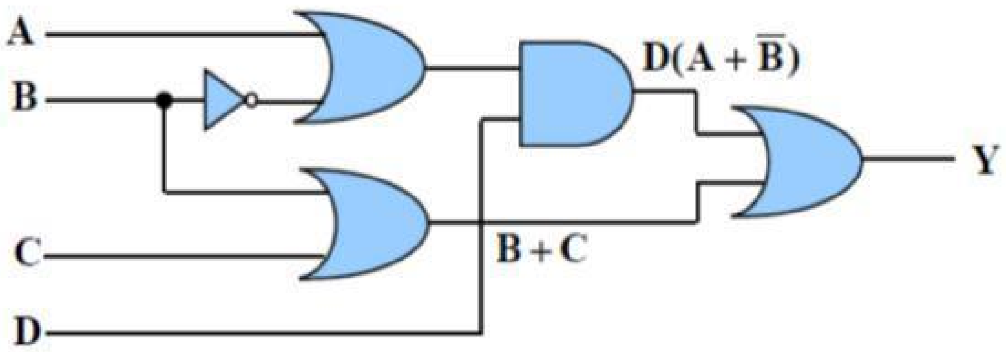
Ex1// Determine the Boolean expression for the following logic circuit:-



Sol//

$$Y = A\overline{B} + \overline{A}C$$

Ex2// Determine the Boolean expression for the following logic circuit:-



Sol//

$$Y = D(A + \overline{B}) + (B + C)$$

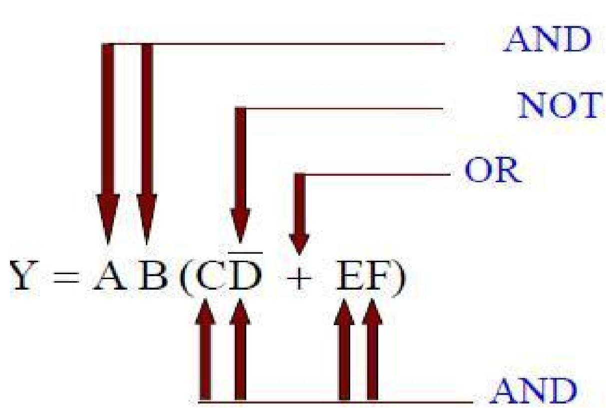
3. <u>Implementation of a Logic Circuit Using a Boolean Expression:</u>

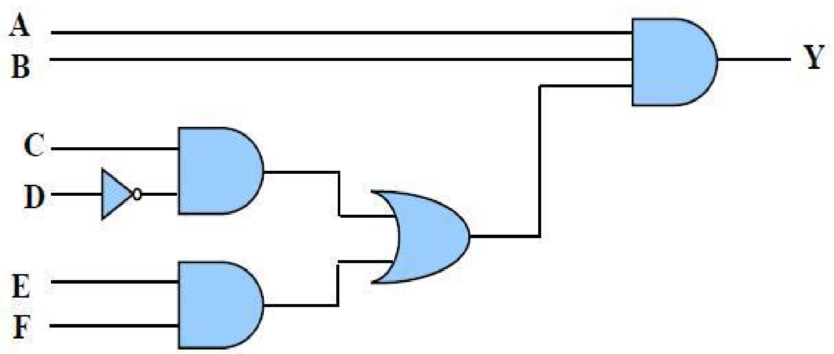
In this section, we explain how can use the Boolean expression to find the logic circuit diagram.

Ex// Determine the logic circuit diagram for the following Boolean expression

$$Y = AB(C\overline{D} + EF)$$

Sol//





4. Implementation of a Logic Circuit via a Truth Table:

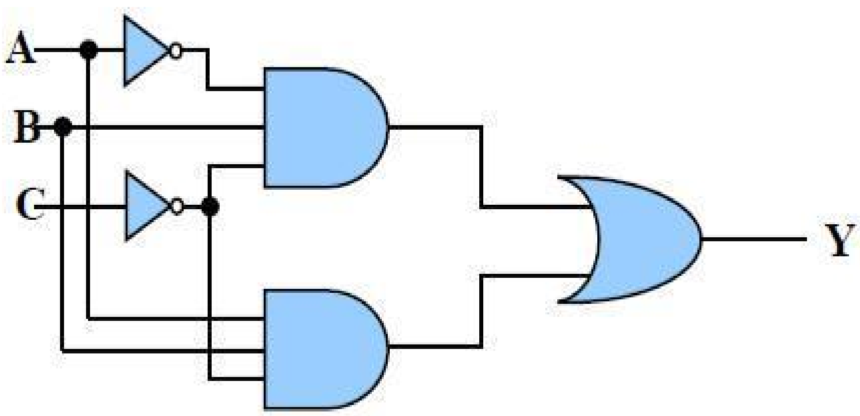
In this section, we explain using truth table as alternative to the Boolean expression to find the logic circuit diagram.

Ex1// Give the logic circuit diagram for the following truth table

A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

Sol//

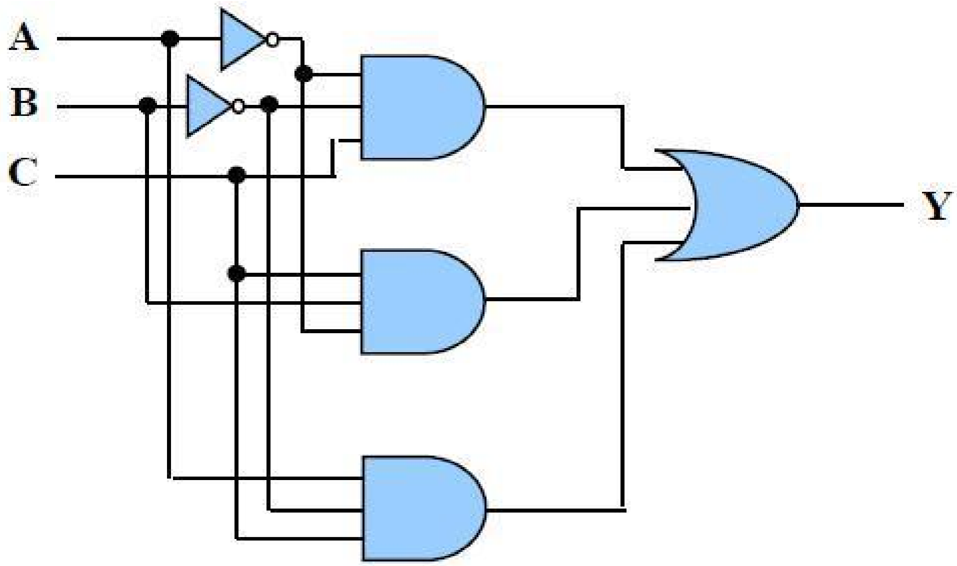
The Boolean Expression is:



Ex2//Give the logic circuit diagram for the following truth table

A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0

Sol// The Boolean expression is :



5. <u>**Converting a Boolean Expression to a Truth Table:**</u>

Ex1// Determine the truth table for the following Boolean expression

$$Y = \overline{AB}\,\overline{C} + \overline{A}B\overline{C} + AB\overline{C} + ABC$$

Sol//

$$\overline{A}\,\overline{B}\,\overline{C} = 000,\ \overline{A}B\overline{C} = 010,\ AB\overline{C} = 110,\ ABC = 111$$

The truth table •

A	B	C	Y
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

Ex2// Determine the truth table and logic diagram for the following Boolean expression

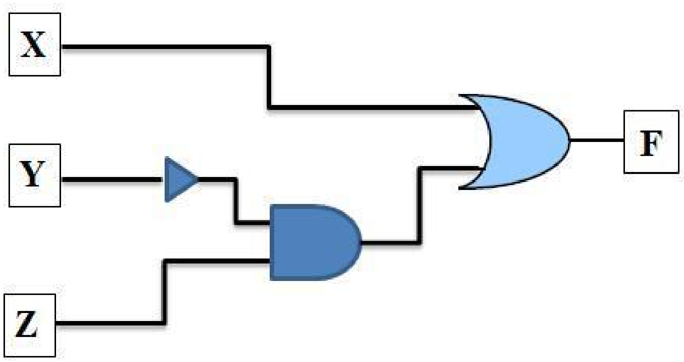
$$F = X + \overline{Y}Z$$

Sol//

• The truth table

X	Y	Z	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

- The logic circuit



6. Simplification of Boolean Expressions Using Boolean Algebra:

Ex1// Using the Boolean algebra laws, simplify the following expression:

Q = (A + B)(A + C)

Sol//

Q =	**(A + B)(A + C)**	
	AA + AC + AB + BC	**- Distributive law**
	A + AC + AB + BC	**- Identity AND law(A.A= A)**
	A(1 + C) + AB + BC	**- Distributive law**
	A.1 + AB + BC	**- Zero & One law (1 + C = 1)**
	A + AB + BC	**- Zero & One law (A.1 = A)**
	A(1 + B) + BC	**- Distributive law**
	A.1 + BC	**- Zero & One law (1 + B = 1)**
Q =	**A + BC**	**- Zero & One law (A.1 = A)**

Then the expression: (A + B)(A + C) can be simplified to A + BC

Ex2// Using the Boolean algebra laws, simplify the following expression:

Y = AB + A(A + C) + B(A + C). And draw the logic circuit diagram

Sol//

Y = AB + A(A + C) + B(A + C)

AB+ AA + AC + AB+ BC-Distributive law

AB + A + AC + AB + BC - Identity law (A.A= A)

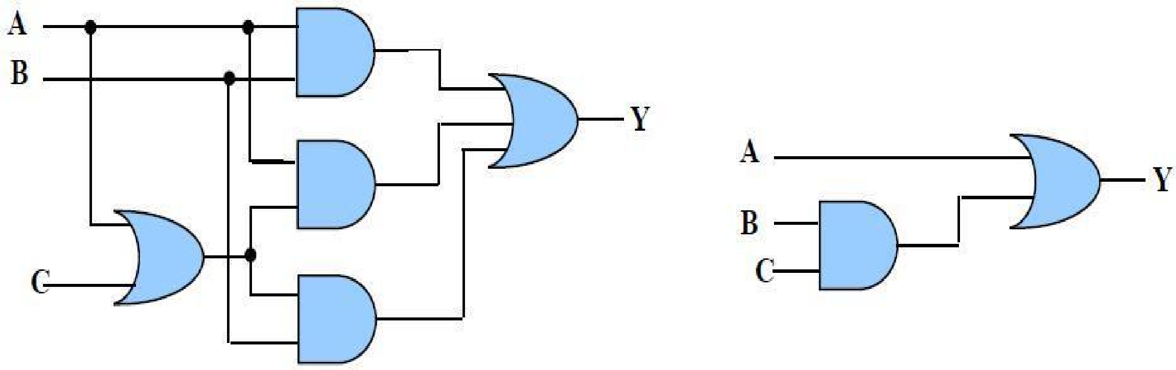
AB+ A + AC + BC- Identity law (AB+AB)

A(B +1+ C) + BC- Distributive law

A . 1 + BC - Zero & One law (B + 1+ C =1)

Y = A + BC - Zero & One law (A . 1= A)

The following figure explain the logic circuit diagram before and after simplification. We must ensure when give any value to the input variables we will obtain the same value in the both two cases.



Ex3// Using the Boolean algebra laws, simplify the following expression:

Y = A. (A . B + C)

Sol//

Y = A . (A . B + C)

A . A . B + A . C **- Distributive law**

A . B + A . C **- Identity law (A.A= A)**

Y = A . (B + C) **- Distributive law**

7. **<u>Demorgan's Theorems</u>:** Is an important part from the Boolean algebra laws to transform the AND state to OR state and vice versa.

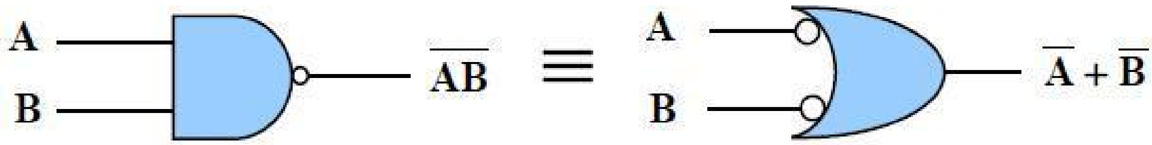
- **The First Theorem:** ()

The Block Diagram:

Truth Table:

Inputs		Output	
A	B		
0	0	1	1
0	1	0	0
1	0	0	0
1	1	0	0

- **The Second Theorems:** $\overline{()}$
- **The Block Diagram:**
- **The Truth Table:**



Inputs		Output	
A	B		
0	0	1	1
0	1	1	1
1	0	1	1
1	1	0	0

Ex1// Simplify the following Boolean expression by using the Demorgan's theorems

$$Y = \overline{(A + \overline{B} + \overline{C}) \bullet (\overline{A} + B + \overline{C})}$$

$$Y = \overline{(A + \overline{B} + \overline{C}) \bullet (\overline{A} + B + \overline{C})}$$

$$= \overline{(A + \overline{B} + \overline{C})} + \overline{(\overline{A} + B + \overline{C})}$$

$$= \overline{A}\,\overline{\overline{B}}\,\overline{\overline{C}} + \overline{\overline{A}}\,\overline{B}\,\overline{\overline{C}} = \overline{A}BC + A\overline{B}C$$

Ex2// Simplify the following Boolean expression by using the Demorgan's theorems

$$Y = \overline{(\overline{A} + B) + CD}$$
$$= \overline{(\overline{A} + B)}.\overline{CD}$$
$$= (\overline{\overline{A}}.\overline{B})(\overline{C} + \overline{D})$$
$$= A\overline{B}(\overline{C} + \overline{D})$$

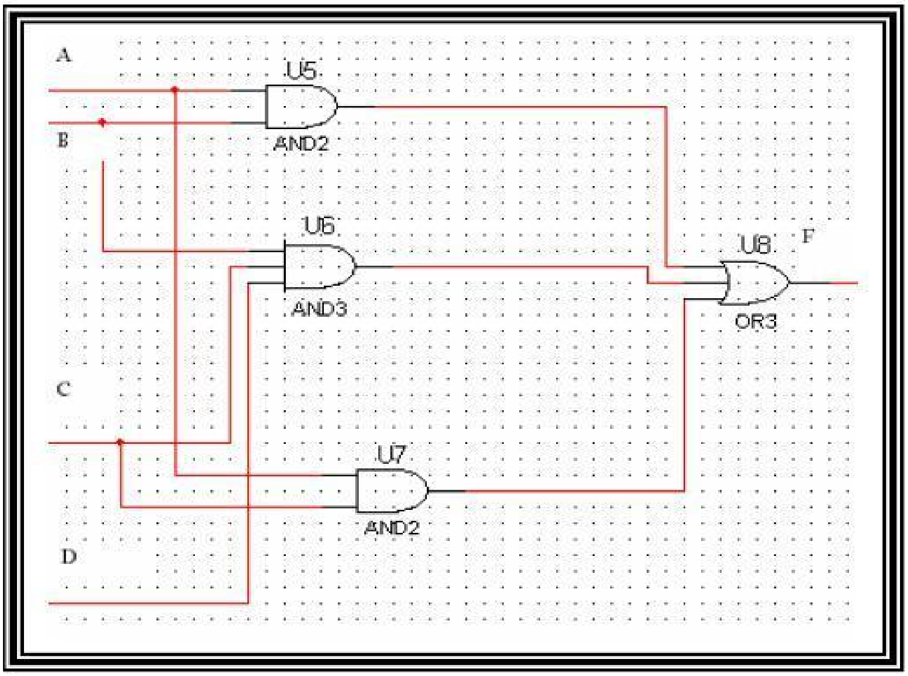
8. **<u>Sum of Product (SOP)& Product of Sum (POS):</u>** are logic expression can be obtained from truth table. The term which the output is **"1"** results in **SOP** form. . The term which the output is **"0"** results in **POS** form.

Inputs			Min Terms		Max Terms	
A	B	C	Expression	Designation	Expression	Designation
0	0	0		M_0	A+B+C	M_0
0	0	1	C	M_1	A+B+	M_1
0	1	0		M_2	A+ +C	M_2
0	1	1	C	M_3	A+ +	M_3
1	0	0	A.	M_4	+B+C	M_4
1	0	1	A.	M_5	+B+	M_5
1	1	0	A.B.	M_6		M_6
1	1	1	A. B.C	M_7		M_7

<u>Sum-Of-Products (SOP):</u>

Ex1// Draw the logic circuit diagram for the following expression

X=AB+BCD+AC



Ex2// Design the logic circuit to compare between two bits and have one output if the inputs are identical

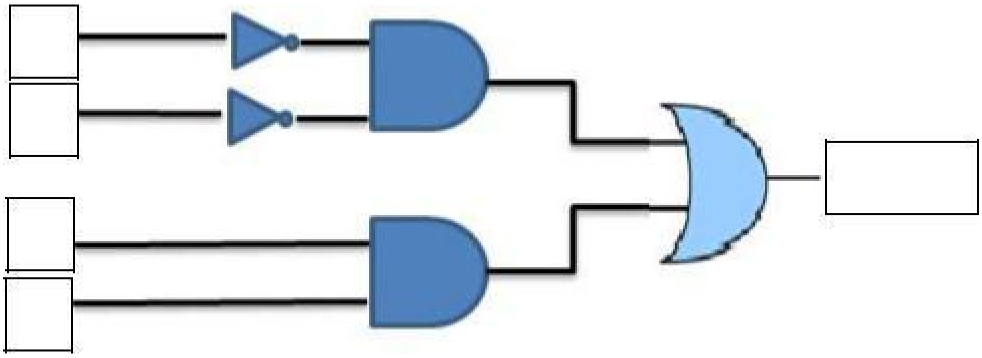
Sol// We must first find the truth table, write the equation and draw the logic circuit diagram:

A	B	F
0	0	1
0	1	0
1	0	0
1	1	1

SOP

POS

( ) ( )



A

B

F

SOP

A

B

- ***:(Product-Of-Sum (POS***

Ex1// Draw the logic circuit diagram for the following (expression (A+B)(B+C+D)(A+C