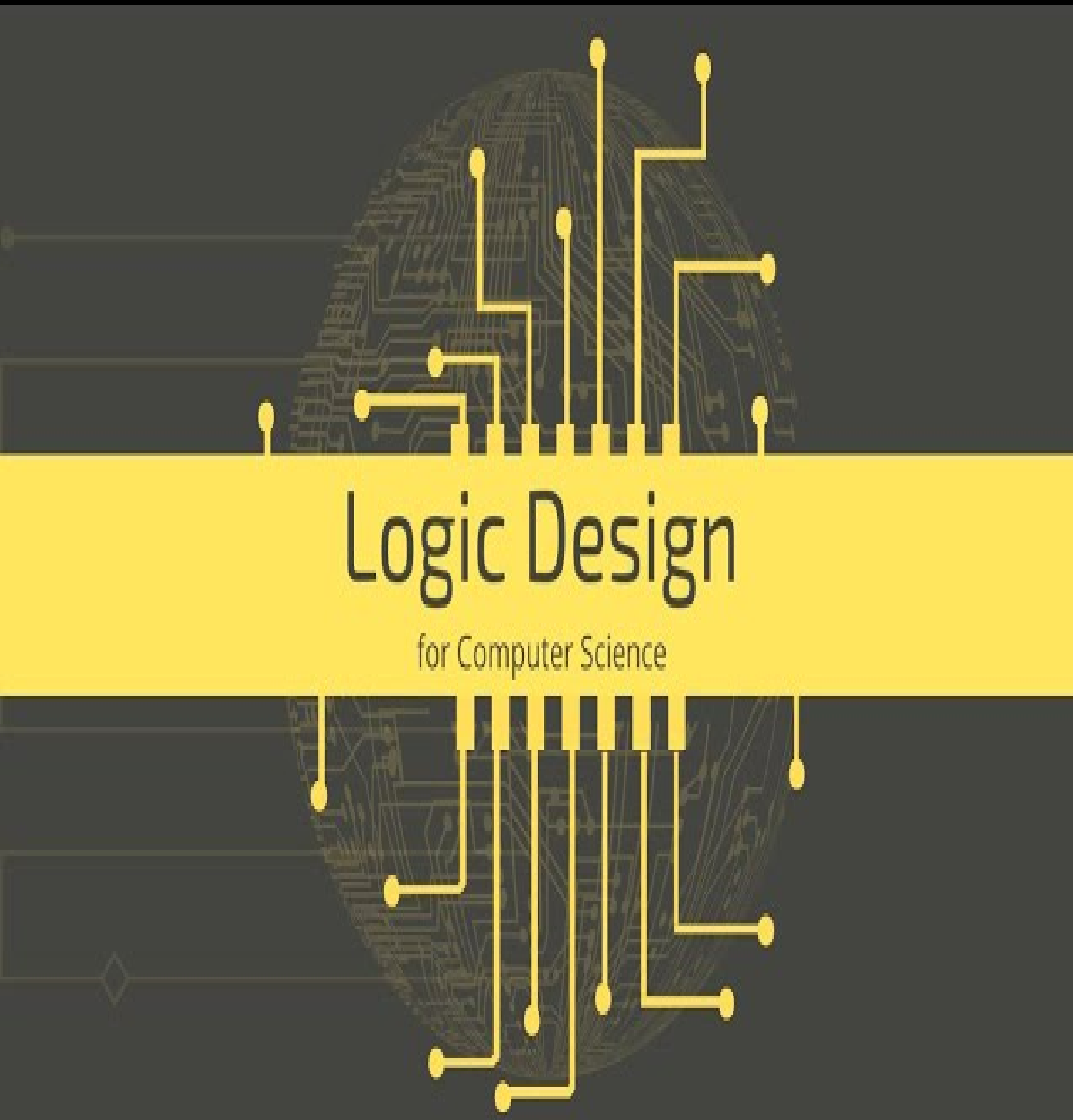

Logic Design

for Computer Science

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Lecture One / Number System Operation

1. Decimal Numbers System.
2. Binary Numbers System.
3. Octal Numbers System.
4. Hexadecimal Numbers System.

1. ***Decimal Number System:*** It is the oldest known system and it is consist of ten numbers (0,1,2,3,…..,9), (base 10).
   Decimal weight … …10^4 10^3 10^2 10^1 10^0. 10^{-1} 10^{-2} 10^{-3} ….

Ex1// **128 =100 + 20 + 8**

$=1*10^2 + 2*10^1 + 8*10^0$

Ex2// **623 = 600 + 20 + 3**

$= 6*10^2 + 2*10^1 + 3*10^0$

$a_5\ a_4\ .\ a_3\ a_2\ a_1 = a_5*r^1 + a_4*r^0 + a_3*r^{-1} + a_2*r^{-2} + a_1*r^{-3}$

Where r =10.

2. ***Binary Number System:*** Its two digits a base-two system. The two binary digits (bits) are 1 and 0.

Binary weight 2^3 2^2 2^1 2^0

Weight value 8 4 2 1

Ex1// **$1011= 1*2^3 + 0*2^2 + 1*2^1 + 1*2^0$**

$= 1*8 + 0*4 + 1*2 + 1*1$

$= 8 + 0 + 2 + 1 = (11)_{10}$

Ex2// $11001 = 1*2^4 + 1*2^3 + 0*2^2 + 0*2^1 + 1*2^0$

$= 1*16 + 1*8 + 0*4 + 0*2 + 1*1$

$= 16+8+0+0+1 = (25)_{10}$

A. Binary-to-Decimal Conversion:

Ex1// Determine the decimal values of the following binary numbers:-

(a) 10110 **(b)** 110111

Sol// **(a)** $N = 1*2^4 + 0*2^3 + 1*2^2 + 1*2^1 + 0*2^0$

$= 1*16 + 0*8 + 1*4 + 1 + 2 + 0*1$

$= 16 + 0 + 4 + 2 + 0 = (22)_{10}$

(b) $N = 1*2^5 + 1*2^4 + 0*2^3 + 1*2^2 + 1*2^1 + 1*2^0$

$= 1*32 + 1*16 + 0*8 + 1*4 + 1 + 2 + 1*1$

$= 32+16+0+4+2+1 = (55)_{10}$

Ex2// Find the decimal value of the following binary number

$(1101101)_2$

1 1 0 1 1 0 1

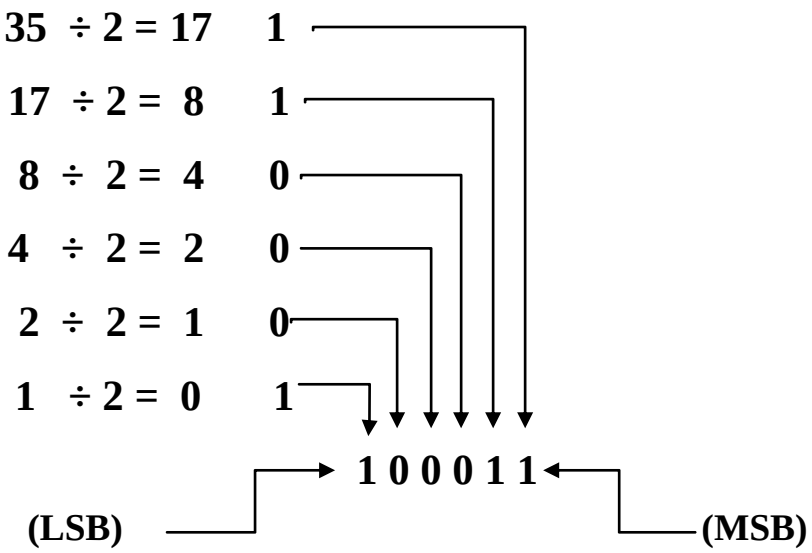
2^6 2^5 2^4 2^3 2^2 2^1 2^0 $= 2^6*1 + 2^5*1 + 2^4*0 + 2^3*1 + 2^2*1 + 2^1*0 + 2^0*1$

$= 64+32+0+8+4+0+1 = 96+13 = 109 \rightarrow (109)_{10}$

B. Decimal-to-Binary Conversion

Ex1// Convert the decimal number $(35)_{10}$ to binary number

Sol// $(35)_{10} = (10011)_2$

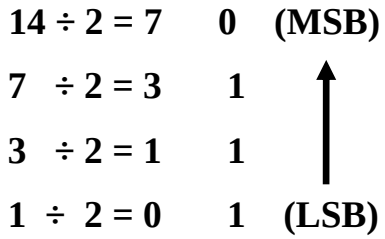


Least Significant Bit ***(LSB)***

Most Significant Bit ***(MSB)***

Ex2// Convert the decimal number $(14)_{10}$ to binary number

Sol// $(14)_{10} = (1110)_2$



C. Conversion of Fractional Binary-to-Decimal

Ex1// $(0.1011)_2 = 1*2^{-1} + 0*2^{-2} + 1*2^{-3} + 1*2^{-4}$

$= 0.5 + 0 + 0.125 + 0.0625 = (0.6875)_{10}$

Ex2// $(0.10001)_2 = 1*2^{-1} + 0*2^{-2} + 0*2^{-3} + 0*2^{-4} + 1*2^{-5}$

$= 0.5 + 0 + 0 + 0 + 0.03125 = (0.53125)_{10}$

D. Conversion of Fractional Decimal-to-Binary

Ex1// Convert $(0.57251)_{10}$ to binary number

Sol//

0.57251 * 2 = 1.14502 1 (LSB)
0.14502 * 2 = 0.29004 0
0.29004 * 2 = 0.58008 0
0.58008 * 2 = 1.16016 1
0.16016 * 2 = 0.32032 0
0.32032 * 2 = 0.64064 0
0.64064 * 2 = 1.28128 0 (MSB)
. .
. .

Thus, $(0.57251)_{10} = (0.1001000)_2$

Ex2// Convert $(0.65625)_{10}$ to binary number

0.65625 * 2 = 1.31250 1 (LSB)
0.31250 * 2 = 0.65200 0
0.65200 * 2 = 1.25000 1
0.25000 * 2 = 0.5000 0
0.5 * 2 = 1 1 (MSB)

Thus, $(0.65625)_{10} = (0.10101)_2$

E. Converting a Mixed Binary Number to Decimal Number

Ex1// $(11010.10110)_2 = 1*2^4 + 1*2^3 + 0*2^2 + 1*2^1 + 0*2^0 \,.\, 1*2^{-1} + 0*2^{-2}$

$$+ 1*2^{-3} + 1*2^{-4} + 0*2^{-5}$$

$$= 16 + 8 + 0 + 2 + 0 + \frac{1}{2} + 0 + \frac{1}{8} + \frac{1}{16} = (26.6875)_{10}$$

Ex2// $(10110.1101)_2 = 1*2^4 + 0*2^3 + 1*2^2 + 1*2^1 + 0*2^0 . 1*2^{-1} + 1*2^{-2}$

$$+ 0*2^{-3} + 1*2^{-4}$$

$$= 16 + 0 + 4 + 2 + 0 + \frac{1}{2} + \frac{1}{4} + 0 + \frac{1}{16} = (22.8125)_{10}$$

F. *Converting a Mixed Decimal Number to Binary Number*

Ex1// Convert $(274.1875)_{10}$ to binary number

Sol//

274 ÷ 2 = 137	0	(MSB)
137 ÷ 2 = 68	1	
68 ÷ 2 = 34	0	
34 ÷ 2 = 17	0	
17 ÷ 2 = 8	1	
8 ÷ 2 = 4	0	
4 ÷ 2 = 2	0	
2 ÷ 2 = 1	0	
	1	(LSB)

0.1875 * 2 = 0.3750	0	(LSB)
0.3750 * 2 = 0.750	0	
0.750 * 2 = 1.500	1	
0.500 * 2 = 1.000	1	(MSB)

Thus, $(274.1875)_{10} = (100010010.0011)_2$

3. ***Octal Numbers System:*** The octal number system is composed of eight digits, which are 0, 1, 2, 3, 4, 5, 6, and 7.

To count above 7, begin another column and start over:

10, 11, 12, 13, 14, 15, 16, and 17.

20, 21, 22, 23, 24, 25, 26, and 27.

30, 31, … … … …… and 37.

A. Octal-to-Decimal Conversion:

Ex1// $(37)_8 = 3*8^1 + 7*8^0$

$= 3*8 + 7*1$

$= 24 + 7$

$= (31)_{10}$

Ex2// $(63)_8 = 6*8^1 + 3*8^0$

$= 6*8 + 3*1$

$= 48 + 3$

$= (51)_{10}$

Ex3// $(0.23)_8 = 2*8^{-1} + 3*8^{-2}$

$= 2 * \frac{1}{8} + 3 * \frac{1}{64}$

$= 2*0.125 + 3*0.015625$

$= 0.25 + 0.046875$

$= (0.296875)_{10}$

B. Decimal-to-Octal Conversion

Ex// Determine the octal values of the following decimal numbers:

(a) $(127)_{10}$ **(b)** $(254)_{10}$ **(c)** $(0.1875)_{10}$

(a) 127 ÷ 8 = 15 7 **(MSB)**

15 ÷ 8 = 1 7

1 ÷ 8 = 0 1 **(LSB)**

Thus, $(127)_{10} = (177)_8$

(b) 254 ÷ 8 = 31 6 **(MSB)**

31 ÷ 8 = 3 7

3 ÷ 8 = 0 3 **(LSB)**

Thus, $(254)_{10} = (376)_8$

(c) 0.1875 * 8 = 1.5 1 (LSB)

0.5 * 8 = 4.0 4 (MSB)

Thus, $(0.1875)_{10} = (0.14)_8$

C. ***Octal-to-Binary Conversion:*** each digit in the octal numbering system can be represented by 3-bits in the binary numbering system as explained in the table (1.1).

Table (1.1) : Representation of octal number as a binary number

Octal Digit	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*
Binary Digit	*000*	*001*	*010*	*011*	*100*	*101*	*110*	*111*

Ex1// **$(357)_8$ = 3 5 7**

011 101 111 = $(011\ 101\ 111)_2$

Ex2// **$(460)_8$ = 4 6 0**

100 110 000 = $(100\ 110\ 000)_2$

D. ***Binary-to-Octal Conversion:*** Conversion binary number to octal number is start with right most group of three bits and moving from right to left.

Ex1

$(110101)_2$

110 101

6 5

$(6\ 5)_8$

$(65)_8$

Ex2

$(101111001)_2$

101 111 001

5 7 1

$(5\ 7\ 1)_8$

$(571)_8$

4. <u>Hexadecimal Numbers:</u> the hexadecimal number system has a base of sixteen; it is composed of 16 digits and alphabetic characters as explained in the table (1.2).

Table (1.2): Representation of hexadecimal number as a decimal & binary number.

Decimal	Binary	Hexadecimal
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7
8	1000	8
9	1001	9
10	1010	A
11	1011	B
12	1100	C
13	1101	D
14	1110	E
15	1111	F

A. <u>Hexadecimal-to-Decimal Conversion:</u> there are two methods to convert the hexadecimal number to decimal number:

- ***First Method:***

1- Convert to binary number.

2- Convert from binary number to decimal number.

Ex//

A 8 5

1010 1000 0101 =

$2^{11}*1+2^{10}*0+2^{9}*1+2^{8}*0+2^{7}*1+2^{6}*0+2^{5}*0+2^{4}*0+2^{3}*0+2^{2}*1+2^{1}*0+2^{0}*1=$

$2^{11}+2^{9}+2^{7}+2^{2}+2^{0}=2048+512+128+4+1=2693=(2693)_{10}$

- ***Second Method***

Ex1// $(23)_{16} = 2*16^{1} + 3*16^{0}$

$= 2*16 + 3*1 = 32 + 3 = (35)_{10}$

Ex2// $(3B)_{16} = 3*16^{1} + B*16^{0}$

$= 3*16 + 11*1 = 48 + 11 = (59)_{10}$

<u>B. Decimal-to-Hexadecimal Conversion:</u>

Ex// Determine the hexadecimal values for following decimal numbers

(a) $(152)_{10}$ **(b)** $(249)_{10}$ **(c)** $(567.1875)_{10}$

Sol//

(a) **152 ÷ 16 = 9 8 (MSB)**

9 ÷ 16 = 0 9 (LSB)

Thus, $(152)_{10} = (98)_{16}$

(b) **249 ÷ 16 = 15 9**

15 ÷ 16 = 0 F

Thus, $(249)_{10} = (F9)_{16}$

(c)

567 ÷ 16 = 35	**7**		**0.1875 * 16 = 3.000**	**3**
35 ÷ 16 = 2	**3**			
2 ÷ 16 = 0	**2**			

Thus, $(567.1875)_{10} = (237.3)_{16}$

C. Hexadecimal-to-Binary Conversion:

Ex// **$(10A4)_{16}$ ⟶ $(1000010100100)_2$**

1	0	A	4
0001	0000	1010	0100

D. Binary-to-Hexadecimal Conversion:

Ex// Determine the Hexadecimal values of the following binary numbers:

(a) $(1101010010)_2$ **(b)** $(011110100 . 10111111)_2$

Sol//

(a) **1101 0100 1000 = $(D48)_{16}$**

(b) **0111 1010 0000 .1011 1111= $(7A0.BF)_{16}$**