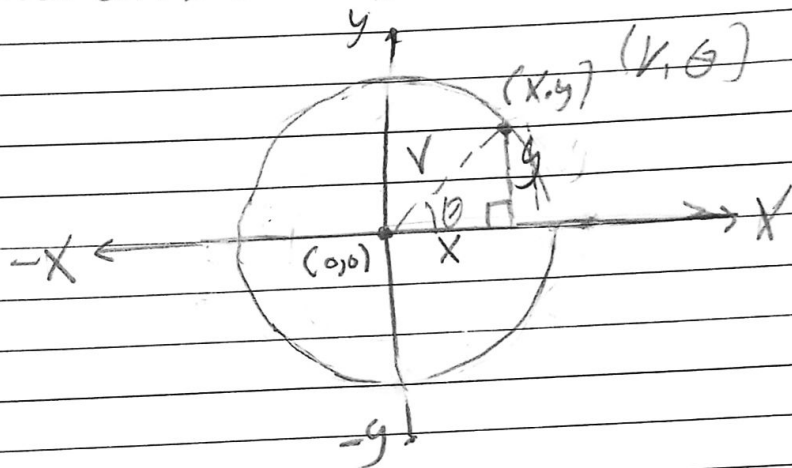


Polar Coordinates

Polar coordinates - Relation Between Polar coordinates and Cartesian coordinates, Finally Polar Equations



r = the length of vector between a point (x,y) and origin point

θ = the angle what create it the vector (r) with positive x axis
you need to identify

$$r = \sqrt{x^2 + y^2} \quad \text{--- 1}$$

$$x = r \cos \theta \quad \text{--- 2}$$

$$y = r \sin \theta \quad \text{--- 3}$$

To clarify (2 & 3)

Remember that from triangle laws above

$$\sin \theta = \frac{\text{opposite}}{\text{chord}} = \frac{y}{r}$$

$$y = r \sin \theta$$

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{x}{r} \Rightarrow \boxed{x = r \cos \theta}$$

notice that

Polar Coordinates is (r, θ)

طول القوس بين القطب والقطب = r
الزاوية التي بين القطب والمحور = θ

also we have

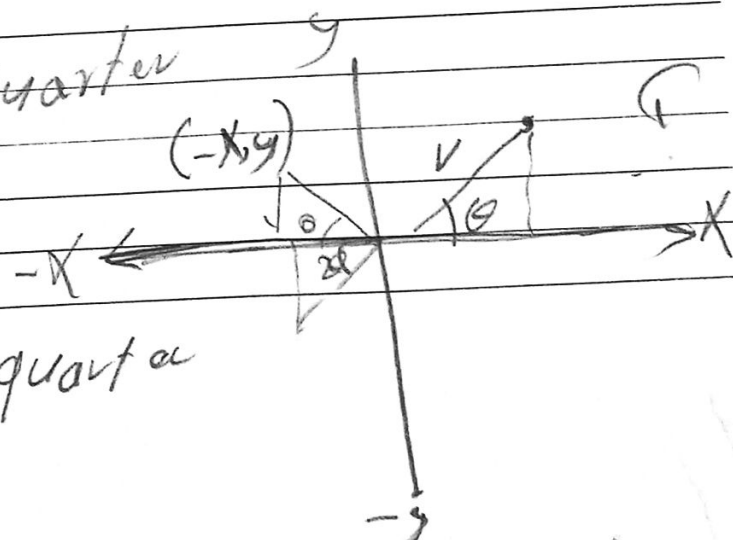
$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{y}{x}$$

(4, 3) to find Polar Coordinate (r, θ) while
(2, 3) ,, ,, to Cartesian Coordinate

Remark, Remember that to find θ uses

$\tan \theta = \frac{y}{x}$ this when (x, y) are positive
but if one of them negative should use

1) $(x, y) = (+x, +y)$ first Quarter



2) $(-x, y)$ in second quadrant

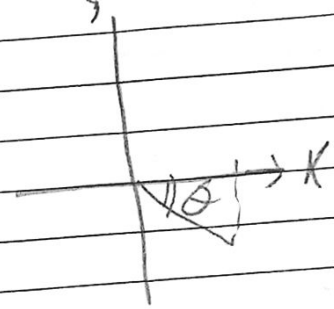
$$\theta = 180 - \theta$$

3) $(-x, -y) \Rightarrow$ third quarter then

$$\theta = 180 + \theta$$

4) $(x, -y) \Rightarrow$ Four quarter then

$$\theta = 360 - \theta$$



Example (11)
If the cartesian coordinates for a point in (x, y) -axis is $(8, 6)$ determine a Polar coordinate for this point. Notice that $\tan 37^\circ = \frac{3}{4}$

Sol) $r = \sqrt{x^2 + y^2} \Rightarrow r = \sqrt{8^2 + 6^2} \Rightarrow r = \sqrt{64 + 36}$

$\therefore r = 10$

$$\tan \theta = \frac{y}{x} \Rightarrow \tan \theta = \frac{6}{8}$$

$$\tan \theta = \frac{3}{4}$$

$\therefore \theta = 37^\circ$

$\therefore (r, \theta) = (10, 37^\circ)$

Example 2

If a Polar Coordinates for a Point is $(5, 53^\circ)$, find a cartesian coordinates.
Note that

$$\sin 53^\circ = 0.8$$

$$\cos 53^\circ = 0.6$$

Sol)

$$x = r \cos \theta$$

$$= 5 \cos 53^\circ \Rightarrow x = 5 \cdot (0.6) = 3$$

$$y = r \sin \theta$$

$$= 5 \cdot \sin 53^\circ \Rightarrow y = 5 \cdot (0.8) = 4$$

$$\text{So } (x, y) = (3, 4)$$

$$\Rightarrow (5, 53^\circ) \rightarrow (3, 4)$$

