Al-Maarif University College
Computer Engineering Techniques

Lecture 2: Rectifier (AC to DC converter)

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2.1Introduction

Since the easily available voltage is a sinusoid, which alternates as a function of time, the first task is to convert it into a useful and reliable constant (dc) voltage for the successful operation of electronic circuits and direct current machines. The conversion process is called the rectification. Although there are other semiconductors devices suitable for rectification, diodes are frequently employed. A rectifier is a circuit that converts an ac signal into a dc signal or sometime is called ac to dc converter. Figure 2.1 illustrated the classification of rectifier

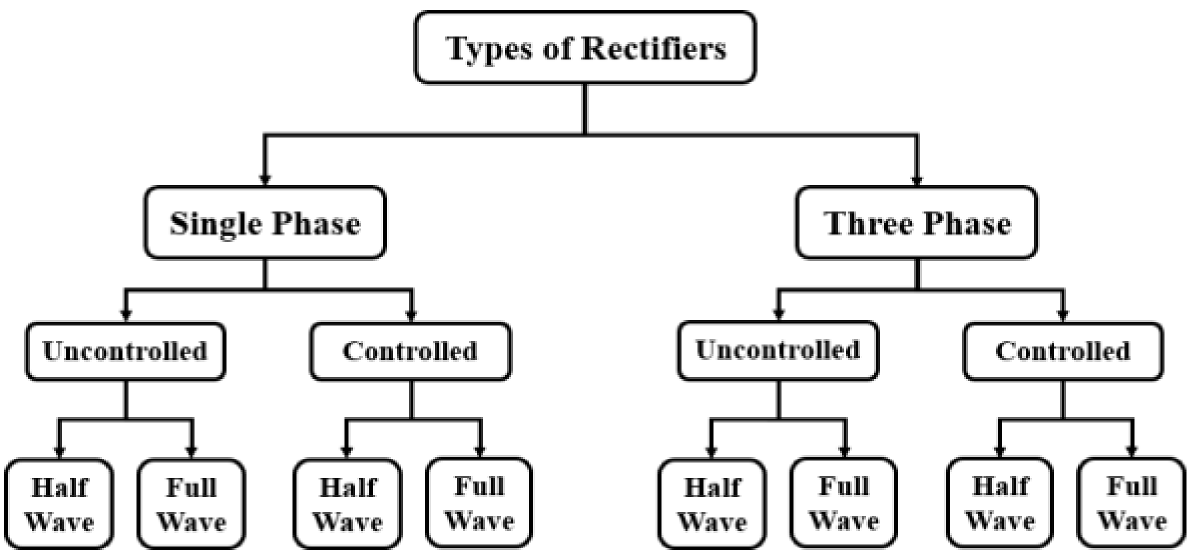


Figure 2.1 Classification of rectifier.

2.2 Single-Phase Half-Wave Uncontrolled Rectifier

2.2.1 Resistive Load

- A basic half-wave rectifier with a resistive load is shown in figure2.2. The source is ac, and the objective is to create a load voltage that has a nonzero dc component. The diode is a basic electronic switch that allows current in one direction only.
- For the positive half-cycle of the source in this circuit, the diode is on (forward-biased). Considering the diode to be ideal, the voltage across a forward-biased diode is zero and the current is positive.
- For the negative half-cycle of the source, the diode is reverse-biased, making the current zero. The voltage across the reverse-biased diode is the source voltage, which has a negative value.

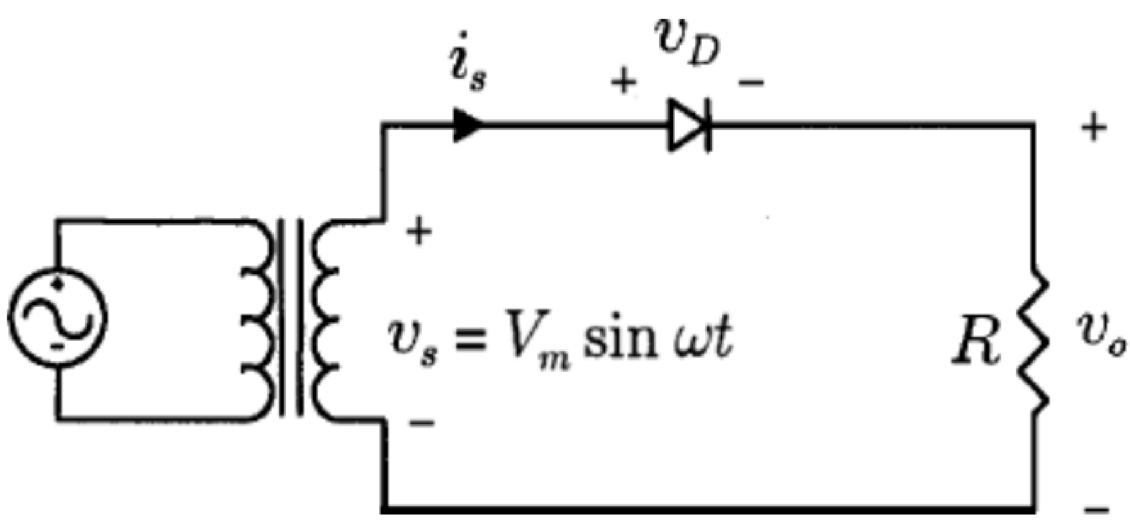


Figure 2.2 Circuit diagram.

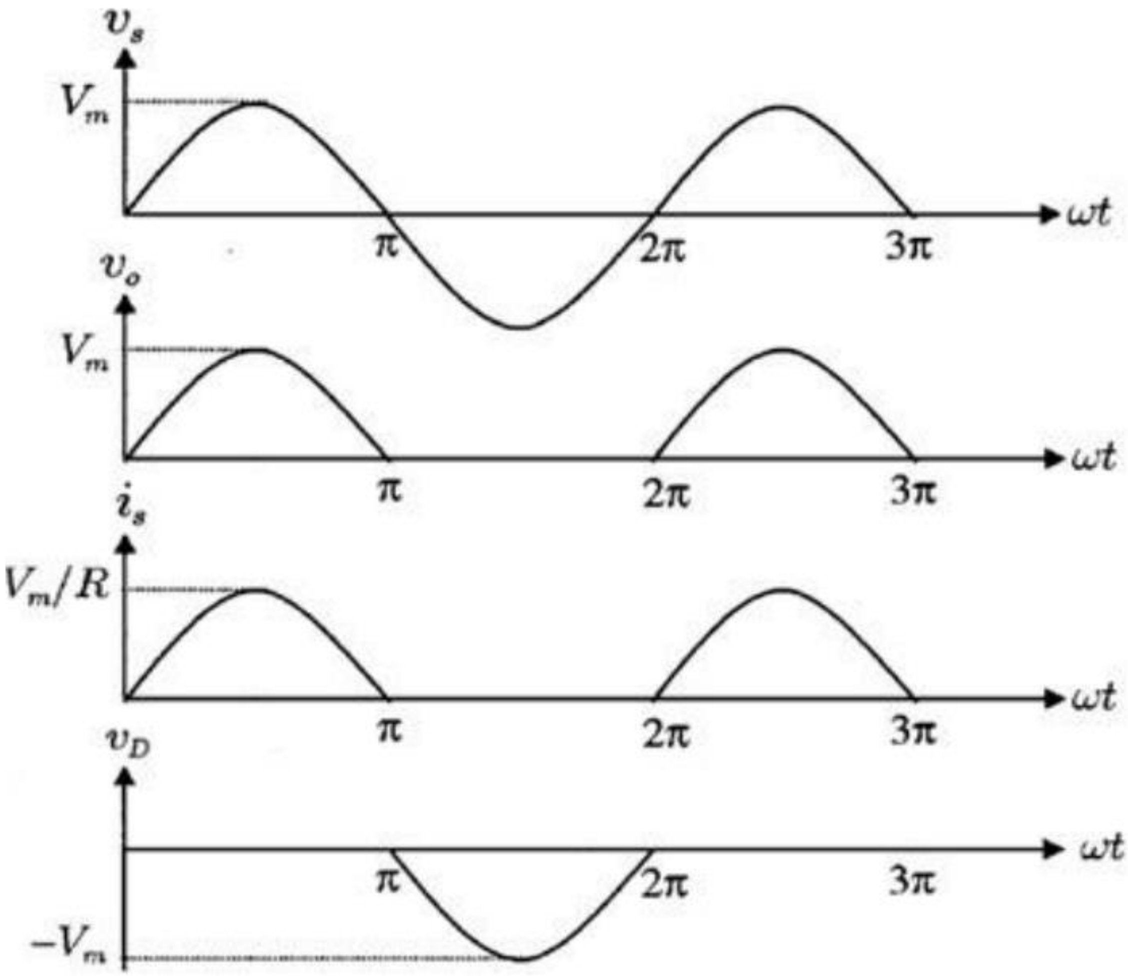


Figure 2.3 Waveforms.

$$V_{dc(Load)} = \frac{1}{2\pi}\left[\int_0^{\pi} V_m \sin wt\, dwt + \int_{\pi}^{2\pi} 0\, dwt\right]$$

$$V_{dc(Load)} = \frac{V_m}{2\pi}[-coswt]_0^{\pi}$$

$$V_{dc(Load)} = \frac{V_m}{\pi} \quad (1)$$

$$I_{dc(Load)} = \frac{V_m}{\pi\, R} \quad (2)$$

The rms values of V_o and I_o can be written as

$$V_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{\pi}[V_m \sin(\omega t)]^2 d(\omega t)}$$

$$V_{rms(Load)} = \frac{V_m}{2} \qquad (3)$$

$$I_{rms(Load)} = \frac{V_m}{2\,R} \qquad (4)$$

The Average output dc power is:

$$P_{dc(Load)} = V_{dc(Load)} I_{dc(Load)} = I_{dc}{}^2\,R = \frac{V_{dc}{}^2}{R} = \frac{V_m{}^2}{\pi^2\,R} \qquad (5)$$

The power delivered to resistive load:

$$P_{ac(Load)} = V_{rms(Load)} I_{rms(Load)} = I_{rms(Load)}{}^2 R = \frac{V_{rms(load)}{}^2}{R} = \frac{V_m{}^2}{4R} \qquad (6)$$

The input power factor is determined by:

$$PF = \frac{P_{ac}}{S} = \frac{P_{ac(Load)}}{V_{s,rms}\,I_{s,rms}} \qquad (7)$$

The efficiency of the rectifier is determined by

$$\eta = \frac{P_{dc}(load)}{P_{ac}\,(load)} \times 100\% \qquad (8)$$

Example: For the shown half-wave rectifier, the source is a sinusoid of 120 V_{rms} at a frequency of 60 Hz. The load resistor is 5 Ω. Determine (a) the average load current, (b) the dc and ac power absorbed by the load and (c) the power factor of the circuit (d)Efficiency.

Answer:

$V_{s,,rms=}$ 120V, F=60HZ, R=5 Ω

a) $I_{DC(Load)} = \frac{V_m}{\pi R} = \frac{120\sqrt{2}}{5\pi} = 10.8A$

b) $P_{dc(Load)} = \frac{{V_m}^2}{\pi^2 R} = \frac{169.7^2}{5\pi^2} = 583.57Watt$

$$V_{rms(Load)} = \frac{V_m}{2} = \frac{120\sqrt{2}}{2} = 84.9V$$

$$I_{rms(Load)} = \frac{V_{rms(Load)}}{R} = \frac{84.9}{5} = 17A$$

$$P_{ac(Load)} = V_{rms(Load)}\, I_{rms(Load)} = 84.9 \times 17 = 1443.3Watt$$

c) $PF = \frac{P_{ac(Load)}}{Vs{,}rms\ Is{,}rms} = \frac{1443.3}{120\times 17} = 0.707$

d) $\eta = \frac{P_{dc(Load)}}{P_{ac}(Load)} \times 100\% = \frac{583.57}{1443.3} \times 100\% = 40.4\%$

Single phase half wave with RL load

Industrial loads typically contain inductance as well as resistance. As the source voltage goes through zero, becoming positive in the circuit of figure 1, the diode becomes forward-biased. The Kirchhoff voltage law equation that describes the current in the circuit for the forward-biased ideal diode is

$$V_m \sin(wt) = Ri(t) + L\frac{di}{dt} \qquad (1)$$

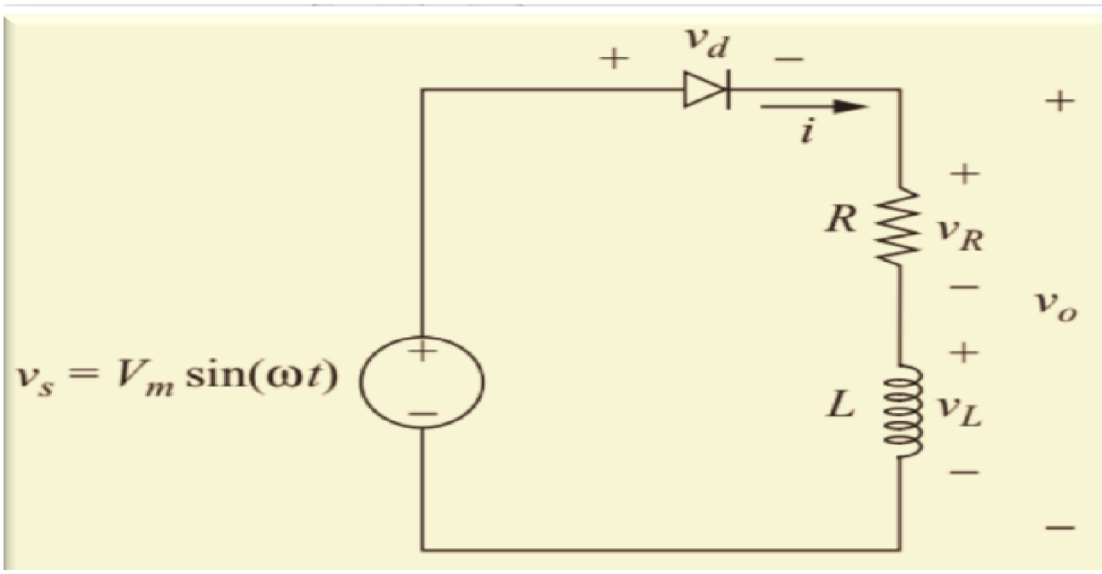


Figure1 Circuit single phase half wave with RL load.

The dc component of the output voltage is

$$V_{dc} = \frac{V_m}{2\pi}\int_0^{\beta} sin\omega t d\omega t = \frac{V_m}{2\pi}(1 - cos\beta) \qquad (2)$$

The dc component of the output current is

$$I_{dc} = \frac{V_m}{2\pi R}(1 - cos\beta) \qquad (3)$$

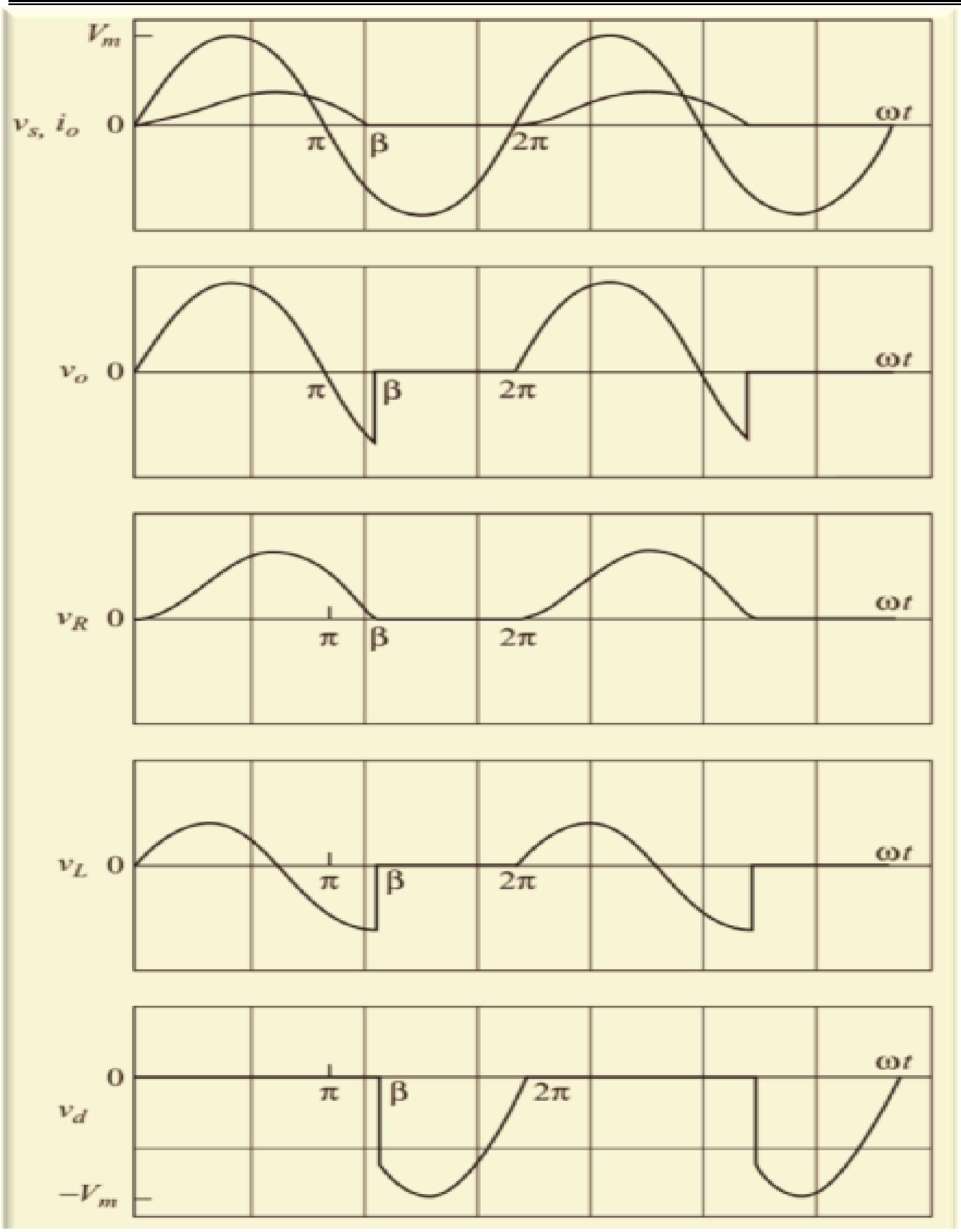


Figure2 Waveforms single phase half wave with RL load.

The solution of equation (1) can be obtained by expressing the current as the sum of the forced response and the natural response:

$$i(t) = i_f(t) + i_n(t) \tag{4}$$

- The forced response for this circuit is the current that exists after the natural response has decayed to zero. In this case, the forced response is the steady-state sinusoidal current that would exist in the circuit if the diode were not present.

- This steady-state current can be found from phasor analysis, resulting in

$$i_f(t) = \frac{V_m}{Z}\sin(\omega t - \theta) \tag{5}$$

Where

$$Z = \sqrt{R^2 + (\omega L)^2} \quad (6) \quad \text{and} \quad \theta = \tan^{-1}\left(\frac{\omega L}{R}\right) \quad (7)$$

- The natural response is the transient that occurs when the load is energized. It is the solution to the homogeneous differential equation for the circuit without the source or diode.

$$R\,i(t) + L\frac{di(t)}{dt} = 0 \tag{8}$$

For this first-order circuit, the natural response has the form

$$i_n(t) = Ae^{-t/\tau} \quad (9)$$

Where $\tau = \frac{L}{R}$ A=constant (10)

Adding the forced and natural responses gets the complete solution.

$$i(t) = i_f(t) + i_n(t) = \frac{V_m}{Z}\sin(\omega t - \theta) + Ae^{-t/\tau} \quad (11)$$

The constant A is evaluated by using the initial condition for current:

t=0, i(ωt)=0.

Using the initial condition and equation (11) to evaluate A yields

$$i(0) = \frac{V_m}{Z}\sin(0 - \theta) + Ae^0 = 0 \quad (12)$$

$$A = -\frac{V_m}{Z}\sin(-\theta) = \frac{V_m}{Z}\sin\theta \quad (13)$$

Substituting for A in equation (11) gives

$$i(t) = \frac{V_m}{Z}\sin(\omega t - \theta) + \frac{V_m}{Z}\sin(\theta)e^{-t/\tau} \tag{14}$$

$$= \frac{V_m}{Z}\left[\sin(\omega t - \theta) + \sin(\theta)e^{-t/\tau}\right] \tag{15}$$

The final current equation can be written as

$$i(\omega t) = \frac{V_m}{Z}\left[\sin(\omega t - \theta) + \sin(\theta)e^{-\omega t/\omega\tau}\right] \tag{16}$$

To find β, substitute ωt= β in equation (16)

$$i(\beta) = \frac{V_m}{Z}\left[\sin(\beta - \theta) + \sin(\theta)e^{-\beta/\omega\tau}\right] = 0 \tag{17}$$

Which reduces to

$$\sin(\beta - \theta) + \sin(\theta)e^{-\beta/\omega\tau} = 0 \tag{18}$$

To summarize, the current in the half-wave rectifier circuit with RL load is expressed as:

$$i(\omega t) = \begin{cases} \frac{V_m}{Z}\left[\sin(\omega t - \theta) + \sin(\theta) e^{-\omega t/\omega\tau}\right] & \text{for } 0 \leq \omega t \leq \beta \\ 0 & \text{for } \beta \leq \omega t \leq 2\pi \end{cases}$$

where $Z = \sqrt{R^2 + (\omega L)^2}$ $\theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$ and $\tau = \frac{L}{R}$

The dc component of the output current is

$$I_o = \frac{1}{2\pi}\int_0^{\beta} i(\omega t)\, d(\omega t) \tag{19}$$

Or it can be found as

$$V_{dc} = \frac{V_m}{2\pi}\int_0^{\beta} \sin\omega t\, d\omega t = \frac{V_m}{2\pi}(1 - \cos\beta) \tag{20}$$

$$I_{dc} = I_o = \frac{V_m}{2\pi R}(1 - \cos\beta) \tag{21}$$

The rms value of I_o can be written as

$$I_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i^2(\omega t)\, d(\omega t)} = \sqrt{\frac{1}{2\pi}\int_0^{\beta} i^2(\omega t)\, d(\omega t)} \tag{22}$$

Or it can be written as

$$V_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{\beta} [v_m \sin(\omega t)]^2 d\omega t} \tag{23}$$

$$= \sqrt{\frac{{V_m}^2}{4\pi}\left(\beta - \frac{1}{2}sin2\beta\right)} \tag{24}$$

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{V_{rms}}{\sqrt{R^2 + (\omega L)^2}} = \frac{1}{\sqrt{R^2 + (\omega L)^2}}\sqrt{\frac{{V_m}^2}{4\pi}\left(\beta - \frac{1}{2}sin2\beta\right)} \tag{25}$$

Example: For the RL half-wave rectifier, R=100Ω, L=0.1 H, ω=377 rad/s, and Vm=100 V. Determine (a) an expression for the current in this circuit, (b) the average current, (c) the rms current, (d) the power absorbed by the RL load, and (e) the power factor.

Solution:

$$Z = [R^2 + (\omega L)^2]^{0.5} = 106.9\,\Omega$$
$$\theta = \tan^{-1}(\omega L/R) = 20.7^\circ = 0.361 \text{ rad}$$
$$\omega t = \omega L/R = 0.377 \text{ rad}$$

(a) $$i(\omega t) = 0.936\sin(\omega t - 0.361) + 0.331e^{-\omega t/0.377} \quad \text{A} \quad \text{for } 0 \le \omega t \le \beta$$

$$\sin(\beta - 0.361) + \sin(0.361)e^{-\beta/0.377} = 0$$

β is found to be 3.50 rad, or 201^o

(b)

$$I_{dc} = I_o = \frac{V_m}{2\pi R}(1 - cos\beta) = \frac{100}{2\pi 100}(1 - cos201) = 0.308\ A$$

(c)

$$I_{rms} = \frac{1}{\sqrt{R^2 + (\omega L)^2}}\sqrt{\frac{{V_m}^2}{4\pi}(\beta - \frac{1}{2}sin2\beta)}$$

$$= \frac{1}{106.9}\sqrt{\frac{100^2}{4\pi}(3.5 - \frac{1}{2}sin7)} = 0.489\text{ A}$$

(d)

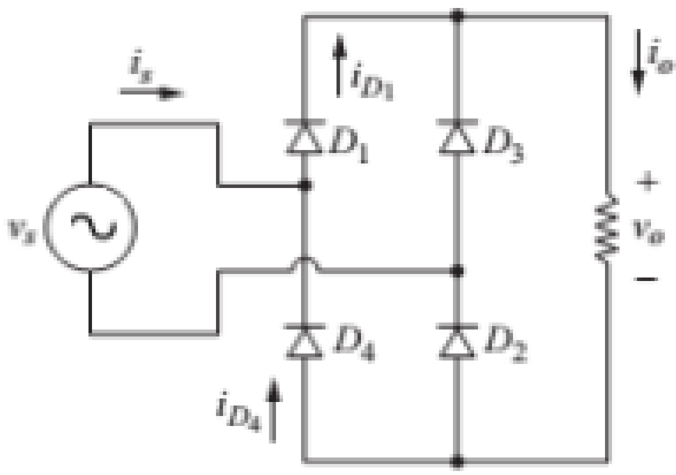
$$P_{ac} = {I_{rms}}^2 R = 0.489^2 \times 100 = 23.9121W$$

(e)

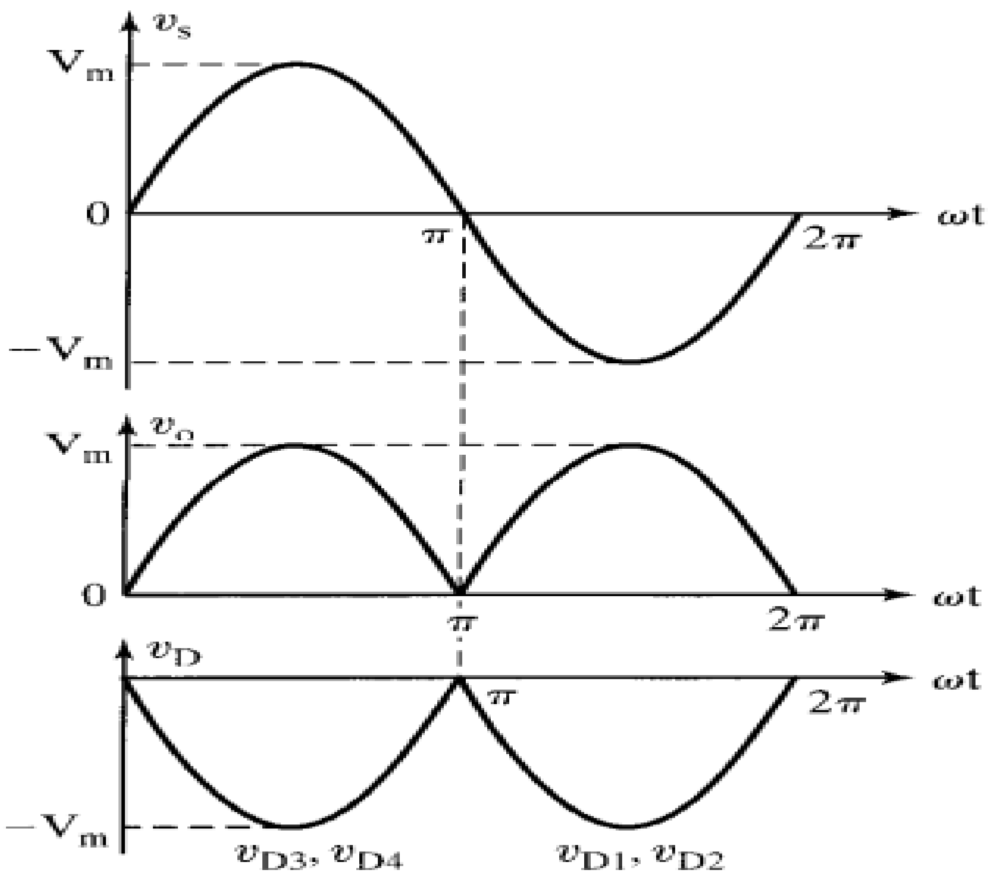
$$Pf = \frac{P_{ac}}{S} = \frac{P_{ac}}{Vs,rms\ Is,rms} = \frac{23.9121}{70.71 \times 0.489} = 0.7$$

Single-Phase Full-Wave Rectifiers

We could use four diodes, as shown in Figure1 (a). During the positive half-cycle of the input voltage, the power is supplied to the load through diodes D1 and D2. During the negative cycle, diodes D3 and D4 conduct. The waveform for the output voltage is shown in Figure1 (b). The peak-inverse voltage of a diode is only V_m. This circuit is known as a bridge rectifier, and it is commonly used in industrial applications.



(a)Circuit diagram



(b) Waveforms

Figure (1) Full-wave bridge rectifier

The dc component of the output voltage is the average value, and load current is the resistor voltage divided by resistance

$$V_{dc} = \frac{2V_m}{\pi} \quad (1)$$

$$I_{dc} = \frac{2V_m}{\pi R} \quad (2)$$

The rms value of the output voltage determined by:

$$V_{o,rms} = \frac{V_m}{\sqrt{2}} \quad (3)$$

The rms value of the output current determined by:

$$I_{o,rms} = \frac{V_{o,rms}}{R} \quad (4)$$

Power absorbed by the load resistor can be determined by:

$$P_{ac} = {I_{o,rms}}^2 R \quad (5)$$

Output DC power can be determined by:

$$P_{dc} = {I_{dc}}^2 R \quad (6)$$

The efficiency can be determined by:

$$\eta = \frac{P_{dc}}{P_{ac}} \times 100\% \quad (7)$$

The power factor can be determined by:

$$PF = \frac{P_{ac}}{V_{s,rms} I_{s,rms}} \quad (8)$$

The ripple factor can be determined by:

$$RF = \frac{V_{ac}}{V_{dc}} = \frac{\sqrt{{V_{o,rms}}^2 - {V_{dc}}^2}}{V_{dc}} \quad (9)$$

Example:

For the circuit in Figure (1), the supply voltage has a peak value of 250 V, and frequency of 50 Hz. If the load is R=100 Ω,

a) determine the average load current, b) output DC power, c) efficiency of rectification, d) ripple factor of output voltage. e) calculate the power factor

Answer:

a) $I_{dc} = \frac{2V_m}{\pi R} = \frac{2\times250}{100\pi} = 1.591A$

b)$P_{dc} = {I_{dc}}^2 R = 1.519^2 \times 100 = 253.302W$

c) $\eta = \frac{P_{dc}}{P_{ac}} \times 100$

$$P_{ac} = {I_{o,rms}}^2 R$$

$$I_{o,rms} = \frac{V_m}{R\sqrt{2}} = \frac{250}{100\sqrt{2}} = 1.76A$$

$$P_{ac} = 1.76^2 \times 100 = 309.76W$$

$$\eta = \frac{253.302}{309.76} \times 100\% = 81\%$$

d) Ripple factor

$$RF = \frac{\sqrt{{V_{o,rms}}^2 - {V_{dc}}^2}}{V_{dc}} = 0.483$$

e) Power factor

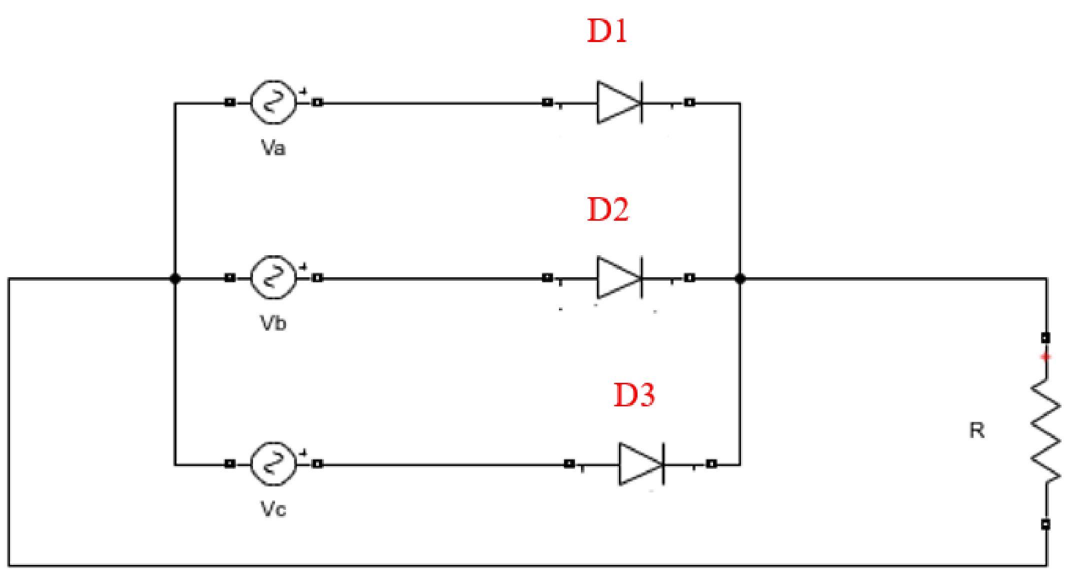
$$PF = \frac{309.76}{176.777\times1.767} = 0.996$$

Three phase uncontrolled half wave rectifier with R load

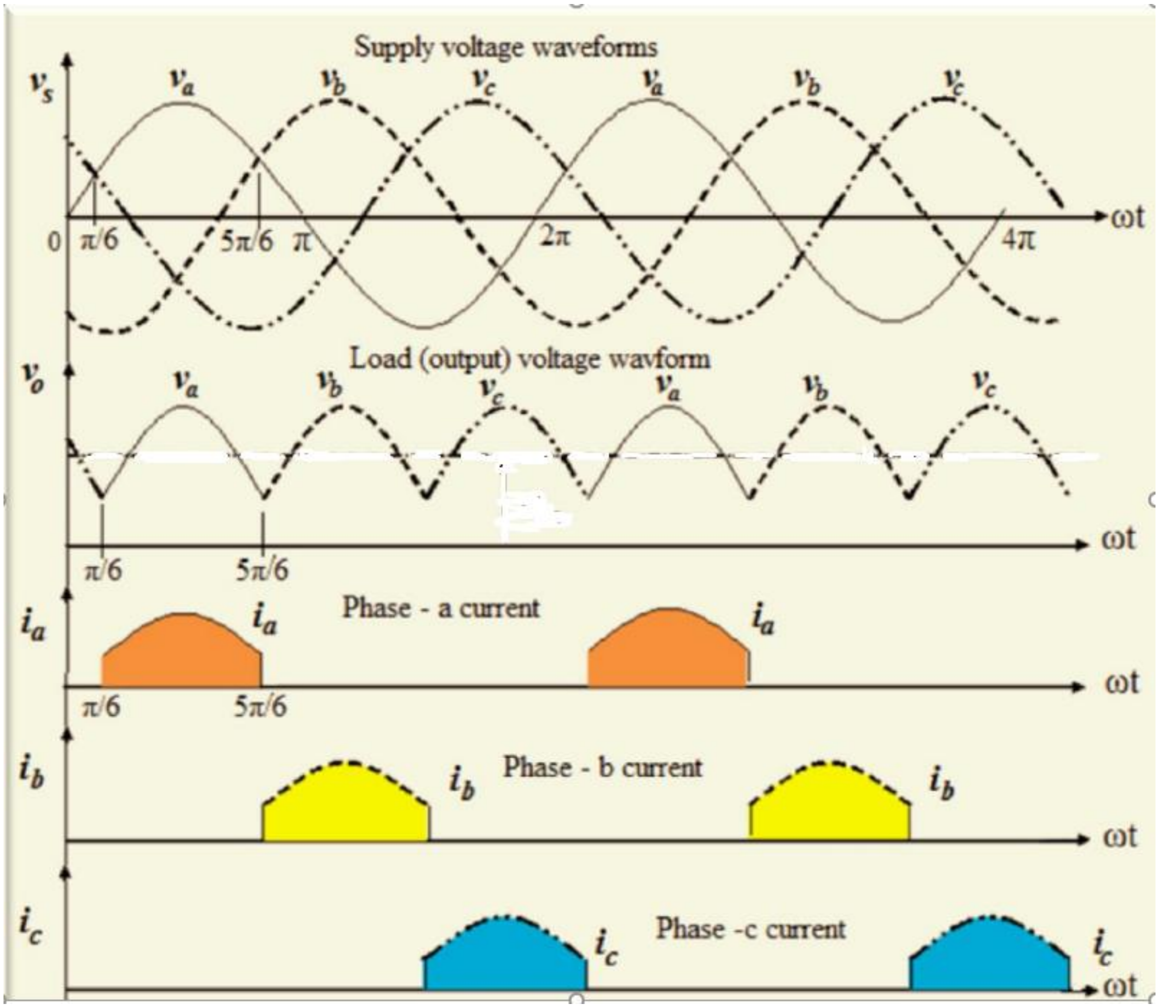
Figure 1(a) and (b) illustrated the circuit and waveforms of three phase uncontrolled half wave rectifier

The principle of operation of this convertor can be explained as follows:

- The diode in a particular phase conducts during the period when the voltage on that phase is higher than that on the other two phases. For example: from $\pi/6$ to $5\pi/6$, D1 has a more positive voltage at its anode, in this period D2 and D3 are off. The neutral wire provides a return path to the load current.
- The conduction sequence is: D1, D2, D3
- It is clear that, unlike the single-phase rectifier circuit, the conduction angle of each diode is $2\pi/3$, instead of π.



(a) Circuit of three phase uncontrolled half wave rectifier



(b) Waveforms of three phase uncontrolled half wave rectifier

Figure (1) three phase uncontrolled half wave rectifier

$V_a = Vm\sin\omega t$, $V_b = Vm\sin(\omega t\text{-}4\pi/6)$, $V_c = Vm\sin(\omega t\text{-}8\pi/6)$

$$V_{dc} = \frac{1}{\frac{2\pi}{3}} \int_{\pi/6}^{5\pi/6} V_m \, sin\omega t \, d\omega t = \frac{3V_m}{2\pi} \left[-cos\omega t\right]_{\pi/6}^{5\pi/6}$$

$$= \frac{3V_m}{2\pi}\left[-(\cos\frac{5\pi}{6} - cos\frac{\pi}{6})\right] = \frac{3V_m}{2\pi}\left[-(-\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2})\right]$$

$$= \frac{3\sqrt{3}V_m}{2\pi} \quad =0.827V_m$$

V_a start from 0
V_b start from 120
V_c start from 240
……………………

$\frac{4\pi}{6} = 120$

$\frac{8\pi}{6} = 240$

$\frac{5\pi}{6} = 150$

$\frac{\pi}{6} = 30$

Example: Three phase uncontrolled half wave rectifier. If the supply phase voltage is 220 V and the resistance of the load is 100 Ω, determine the DC load voltage and current.

ANS:

$V_{s,RMS}$=220 V, R=100 Ω

$$V_{dc} = 0.827V_m$$
$$V_m = 220\sqrt{2} = 311.127V$$
$$V_{dc} = 0.827 \times 311.127 = 257.302V$$
$$I_{dc} = \frac{V_{dc}}{R} = \frac{257.302}{100} = 2.573A$$