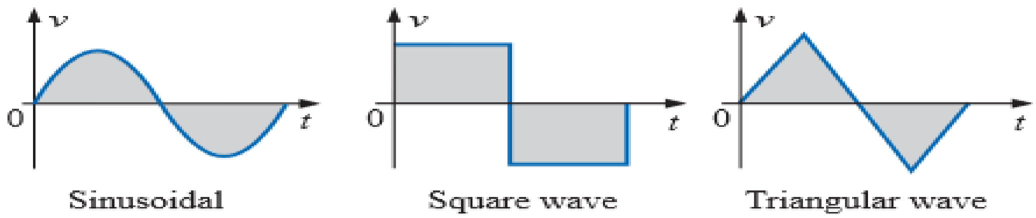
Chapter Two

Alternating Current Network

Types of Alternating Waveforms.



Alternating waveforms.

1- SINUSOIDAL:

Consider the sinusoidal voltage $v(t) = V_m \sin \omega t$ where

V_m = the *amplitude* of the sinusoid

ω = the *angular frequency* in radians/s

ωt = the *argument* of the sinusoid

It is evident that the sinusoid repeats itself every T seconds; thus, T is called the *period* of the sinusoid.

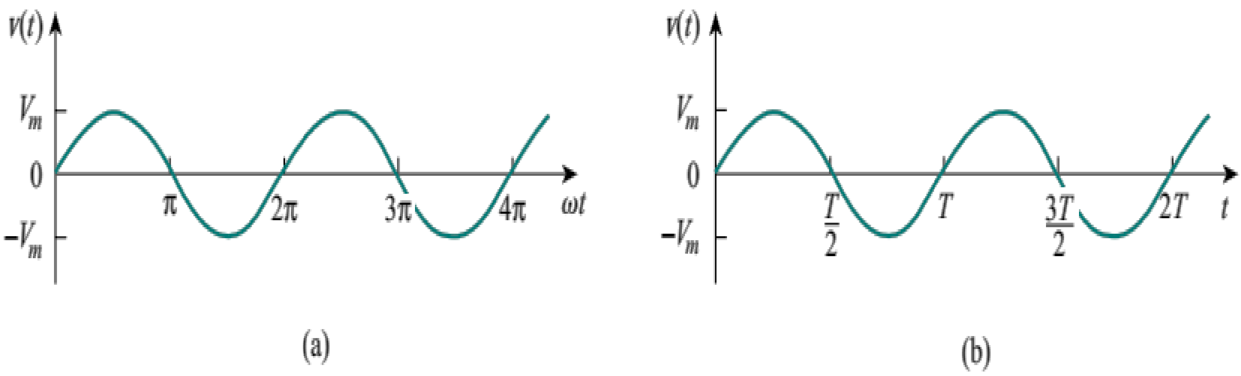
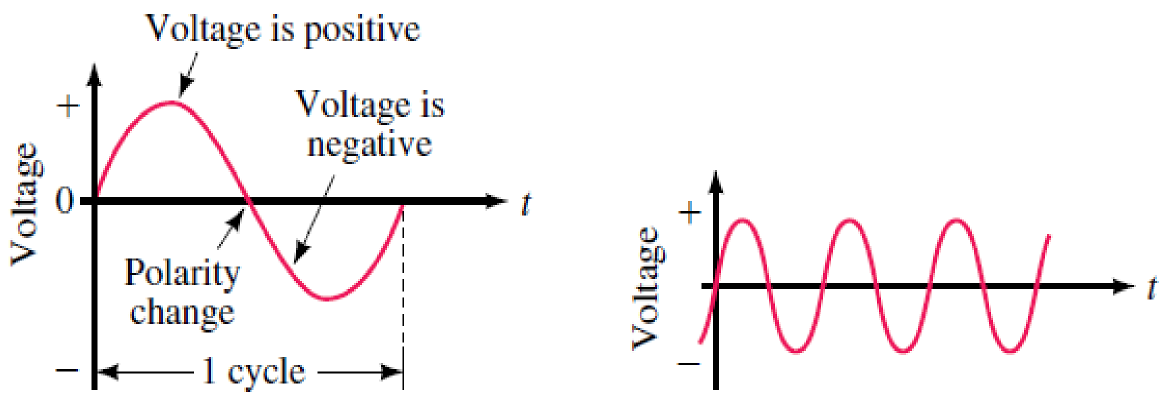


Figure 1: A sketch of $V_m \sin \omega t$: (a) as a function of ωt, (b) as a function of t.



(a) Variation of voltage versus time

(b) A continuous stream of cycles

$\alpha = \omega T$, $\omega = \frac{\alpha}{T}$

We observe that $\omega T = 2\pi$, $T = \frac{2\pi}{w}$, $\omega = \frac{2\pi}{T}$

$$v(t+T) = V_m \sin \omega (t+T) = V_m \sin \omega \left(t + \frac{2\pi}{\omega}\right)$$
$$= V_m \sin(\omega t + 2\pi) = V_m \sin \omega t = v(t)$$

Hence,

$$\boxed{v(t+T) = v(t)}$$

That is, v has the same value at t + T as it does at t and v (t) is said to be periodic. The reciprocal of this quantity is the number of cycles per second, known as the cyclic frequency f of the sinusoid. Thus,

$$f = \frac{1}{T} \quad and\ w = 2\pi f$$

While ω is in radians per second (rad/s), *f* is in hertz (Hz).

1 Hz = 1 cycle per second.

EXAMPLE: - Determine the angular velocity of a sine wave having a frequency of 60 Hz.
Solution:

$w = 2\pi f =$ (2π)(60 Hz) =≈ **377 rad/s**

EXAMPLE: - Determine the frequency and period of the sine wave of Fig.

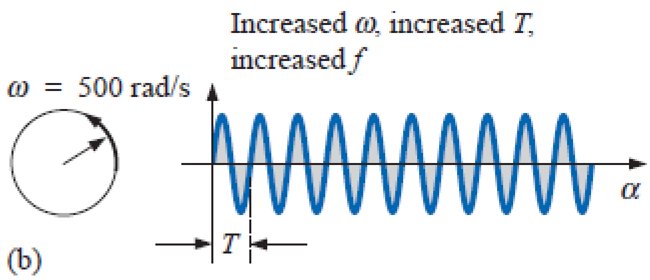


FIG.
Demonstrating the effect of ω on the frequency and period.

Solution: Since $\omega = \frac{2\pi}{T}$

$$T = \frac{2\pi}{\omega} = \frac{2\pi \text{ rad}}{500 \text{ rad/s}} = \frac{2\pi \text{ rad}}{500 \text{ rad/s}} = \mathbf{12.57\ ms}$$

and $$f = \frac{1}{T} = \frac{1}{12.57 \times 10^{-3}\text{ s}} = \mathbf{79.58\ Hz}$$

Let us examine the two sinusoids shown in Fig. 2:
$v_1(t) = V_m \sin \omega t$ and $v_2(t) = V_m \sin(\omega t + \varphi)$

Therefore, we say that v_2 leads v_1 by φ or that v_1 lags v_2 by φ. If $\varphi \neq 0$, we also say that v_1 and v_2 are out of phase as shown in Fig. 2. If $\varphi = 0$, then v_1 and v_2 are said to be in phase

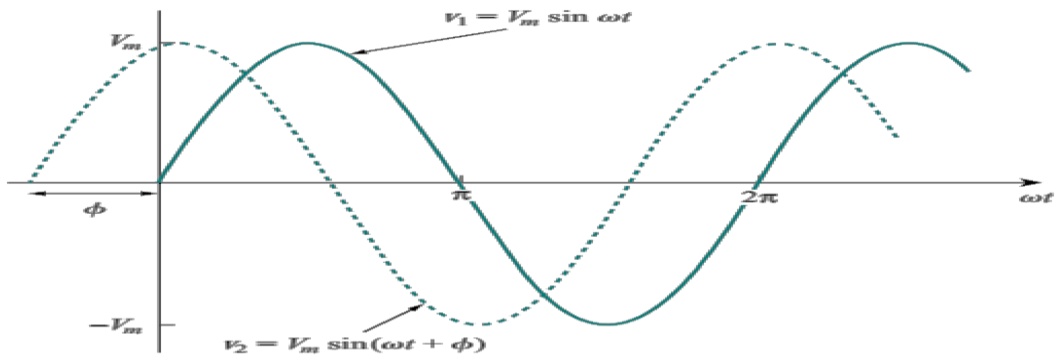


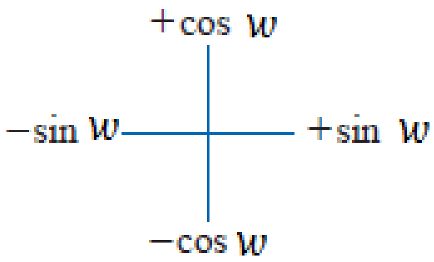
Figure 2

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

With these identities, it is easy to show that:

$$\sin(\omega t \pm 180^\circ) = -\sin \omega t$$
$$\cos(\omega t \pm 180^\circ) = -\cos \omega t$$
$$\sin(\omega t \pm 90^\circ) = \pm \cos \omega t$$

$$\cos(\omega t \pm 90^\circ) = \mp \sin \omega t$$



Graphic tool for finding the relationship between specific sine and cosine functions.

2- **Generating AC Voltages**:

One way to generate an AC voltage is to rotate a coil of wire at constant angular velocity in a fixed magnetic field, Figure 3. (Slip rings and brushes connect the coil to the load.) The magnitude of the resulting voltage is proportional to the rate at which flux lines are cut (Faraday's law), and its polarity is dependent on the direction the coil sides move through the field. Since the coil rotates continuously, the voltage produced will be a repetitive, periodic waveform as you saw in Figure 2. The time for one revolution of 600 rpm is one tenth of a second, i.e., 100 ms.

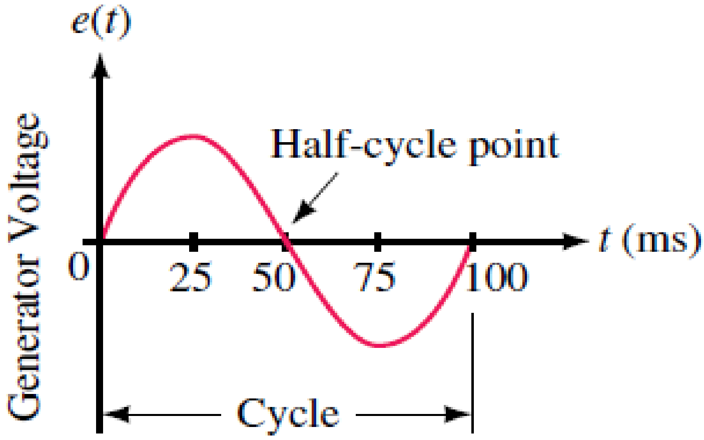
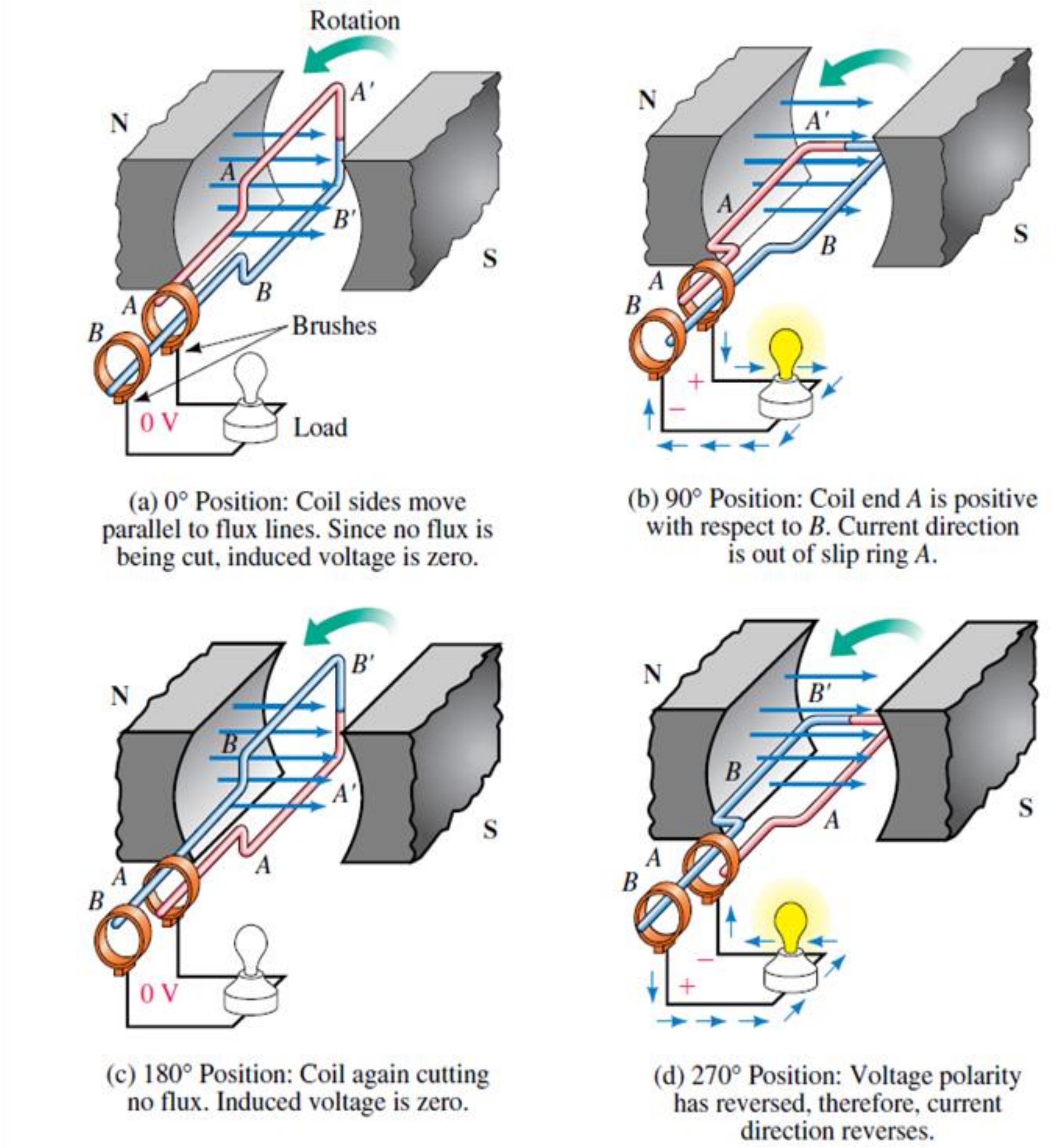


FIGURE 2 Cycle scaled in time. At 600 rpm, the cycle length is 100 ms.



(a) 0° Position: Coil sides move parallel to flux lines. Since no flux is being cut, induced voltage is zero.

(b) 90° Position: Coil end *A* is positive with respect to *B*. Current direction is out of slip ring *A*.

(c) 180° Position: Coil again cutting no flux. Induced voltage is zero.

(d) 270° Position: Voltage polarity has reversed, therefore, current direction reverses.

FIGURE 3. Generating an ac voltage. The 0° position of the coil is defined as in (a) Where the coil sides move parallel to the flux lines.

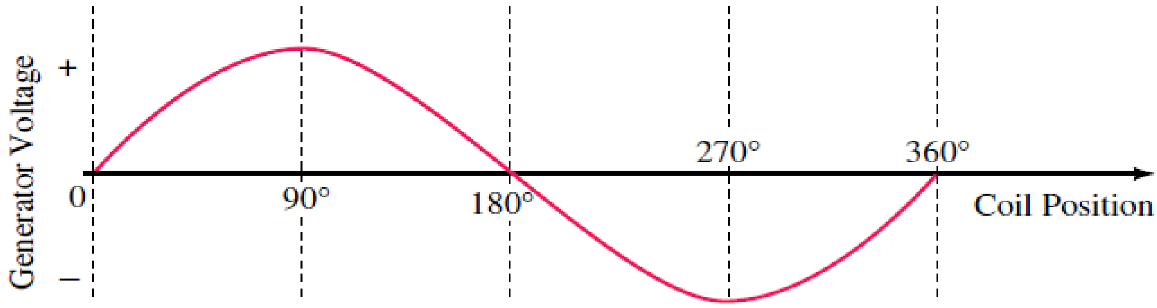


FIGURE 4. Coil voltage versus angular position.

As Figure 4 shows, the coil voltage changes from instant to instant. The value of voltage at any point on the waveform is referred to as its **instantaneous value.**

3- Definitions Related to Alternating Waveforms.

Definitions:-The sinusoidal waveform of Fig. 5 with its additional notation will now be used as a model in defining a few basic terms. These terms, however, can be applied to any alternating waveform. It is important to remember as you proceed through the various definitions that the vertical scaling is in volts or amperes and the horizontal scaling is *always* in units of time.

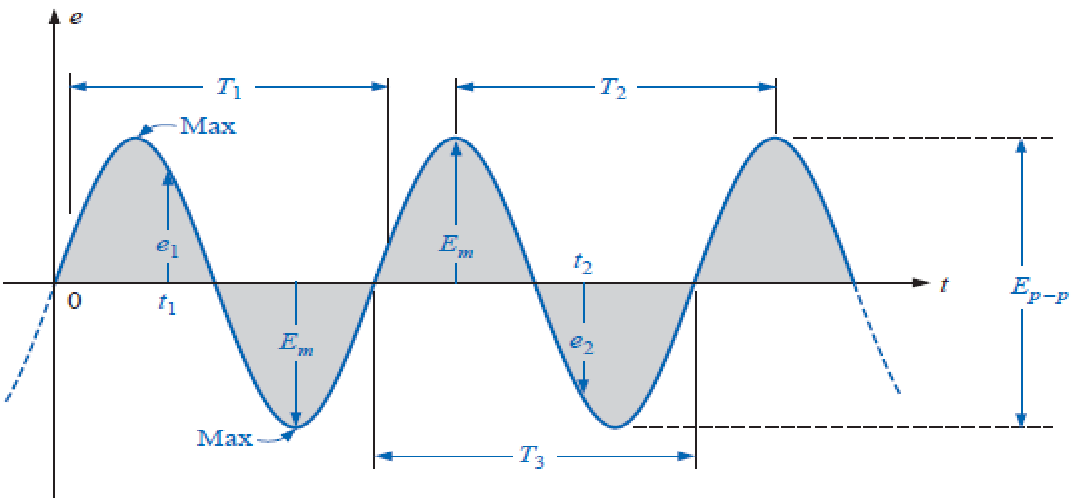


FIG. 5
Important parameters for a sinusoidal voltage.

Waveform: The path traced by a quantity, such as the voltage in Fig. 5, plotted as a function of some variable such as time (as above), position, degrees, radians, temperature, and so on.
Instantaneous value: The magnitude of a waveform at any instant of time; denoted by lowercase letters ($e1$, $e2$).

Peak amplitude: The maximum value of a waveform as measured from its *average*, or *mean*, value, denoted by uppercase letters (such as *Em* for sources of voltage and *Vm* for the voltage drop across a load). For the waveform of Fig.5, the average value is zero volts, and *Em* is as defined by the figure.

Peak value: The maximum instantaneous value of a function as measured from the zero-volt level. For the waveform of Fig. 5, the peak amplitude and peak value are the same, since the average value of the function is zero volts.

Peak-to-peak value: Denoted by *Ep-p* or *Vp-p*, the full voltage between positive and negative peaks of the waveform, that is, the sum of the magnitude of the positive and negative peaks.

Periodic waveform: A waveform that continually repeats itself after the same time interval. The waveform of Fig. 5 is a periodic waveform.

Period (*T*): The time interval between successive repetitions of a periodic waveform (the period *T*1 = *T*2 = *T*3 in Fig.5), as long as successive *similar points* of the periodic waveform are used in determining *T*.

Cycle: The portion of a waveform contained in *one period* of time. The cycles within *T*1, *T*2, and *T*3 of Fig.5 may appear different in Fig. 6, but they are all bounded by one period of time and therefore satisfy the definition of a cycle.

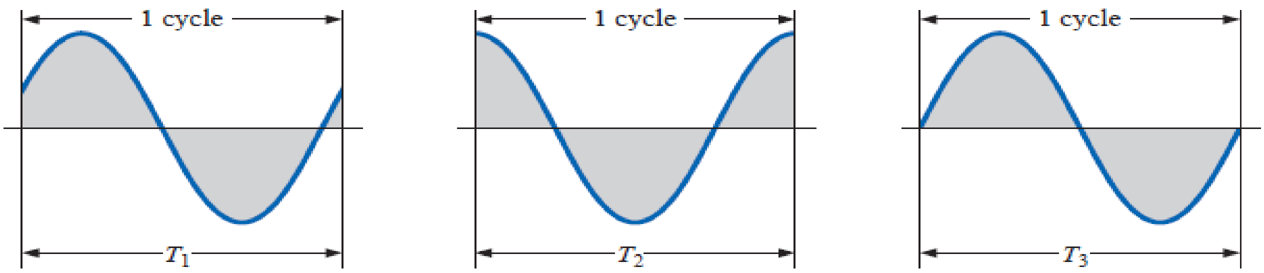


FIG. 6
Defining the cycle and period of a sinusoidal waveform.

Frequency (f): The number of cycles that occur in 1 s. The frequency of the waveform of Fig.7 (a) is 1 cycle per second, and for Fig. 7 (b), 21⁄2 cycles per second. If a waveform of similar shape had a period of 0.5 s [Fig. 7 (c)], the frequency would be 2 cycles per second.

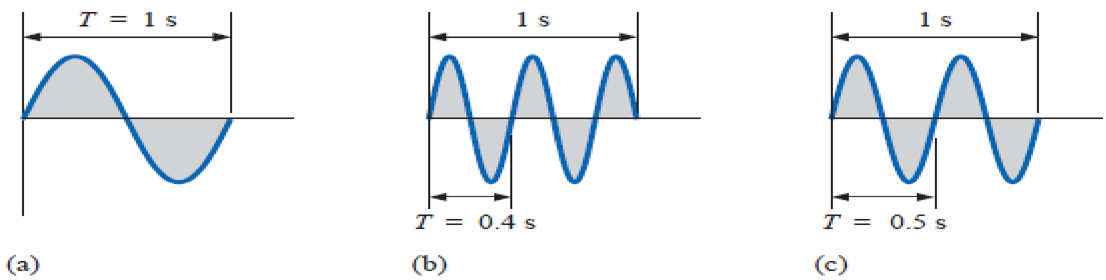


FIG. 7
Demonstrating the effect of a changing frequency on the period of a sinusoidal waveform.

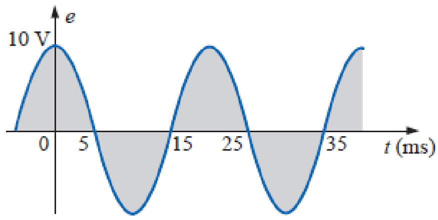
EXAMPLE: - Find the period of a periodic waveform with a frequency of?
a. 60 Hz.
b. 1000 Hz.

Solutions:

a. $T = \frac{1}{f} = \frac{1}{60\ HZ}$ =0.01667 s or **16.67 ms**
(a recurring value since 60 Hz is so prevalent)

b. $T = \frac{1}{f} = \frac{1}{1000\ HZ} = 10^{-3}$ s = **1 ms**

EXAMPLE Determine the frequency of the waveform of Fig.



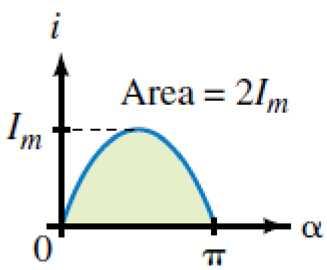
Example

Solution: From the figure, $T = (25\text{ ms} - 5\text{ ms}) = 20\text{ ms}$, and

$$f = \frac{1}{T} = \frac{1}{20 \times 10^{-3}\text{ s}} = \mathbf{50\ Hz}$$

4- AC Waveforms and Average Value.

Ac quantities are generally described by a group of characteristics, including instantaneous, peak, average, and effective values. To find the average value of a waveform, divide the area under the waveform by the length of its base. Areas above the axis are counted as positive, while areas below the axis are counted as negative Because a sine wave is symmetrical, its area below the horizontal axis is the same as its area above the axis; thus, over a full cycle its net area is zero, independent of frequency and phase angle. Thus, the average of sin wt, sin (wt ± θ), sin 2wt, cos wt, cos (wt ± θ), cos 2wt, and so on are each zero. The average of half a sine wave, however, is not zero. Consider Figure 8. The area under the half-cycle may be found using calculus as



$$\text{area} = \int_0^{\pi} I_m \sin\alpha \, d\alpha = -I_m \cos\alpha \Big[_0^{\pi} = 2I_m$$

$$\begin{aligned} \text{Area} &= I_m[-\cos\alpha]_0^{\pi} \\ &= -I_m(\cos\pi - \cos 0°) \\ &= -I_m[-1 - (+1)] = -I_m(-2) \end{aligned}$$

Figure 8

Two cases are important; full-wave average and half-wave average. The **full-wave** case is illustrated in Figure 9-a. The area from 0 to 2п is 2(2*Im*) and the base is 2п. Thus, the average is

$$I_{avg} = \frac{2(2I_m)}{2\pi} = \frac{2I_m}{\pi} = 0.637I_m$$

For the **half-wave** case (Figure 9 -b),

$$I_{avg} = \frac{2I_m}{2\pi} = \frac{I_m}{\pi} = 0.318I_m$$

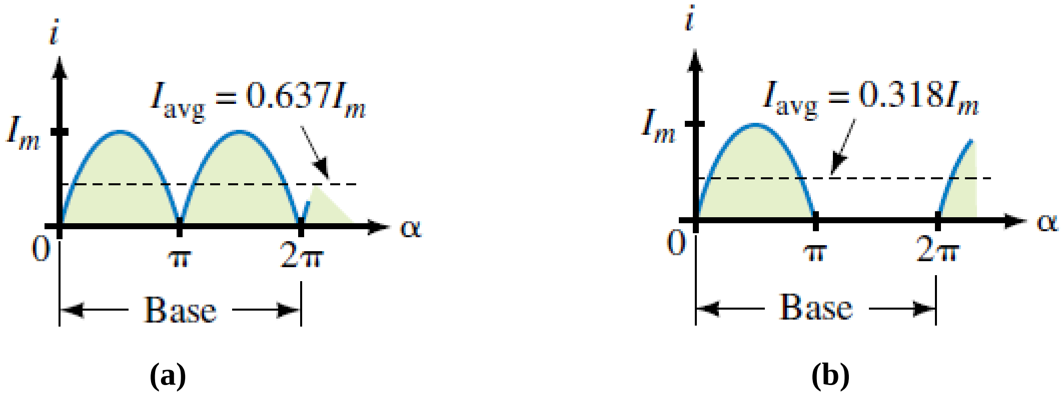


Figure (9)

The corresponding expressions for voltage are

$$V_{avg} = 0.637V_m \quad \text{(full-wave)}$$

$$V_{avg} = 0.318V_m \quad \text{(half-wave)}$$

Sometimes ac and dc are used in the same circuit. Figure 10 shows superimposed ac and dc. It does not alternate in polarity since it never changes polarity to become negative.

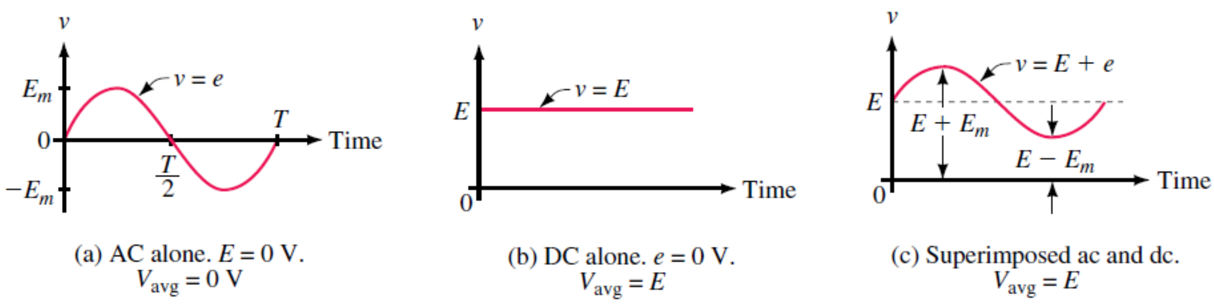


Figure 10

EXAMPLE: - Determine the average value of the sinusoidal waveform In Fig.

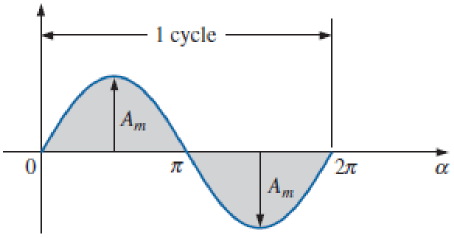


FIG.
Example

Solution: By inspection it is fairly obvious that *the average value of a pure sinusoidal waveform over one full cycle is zero.*

$$G = \frac{+2A_m - 2A_m}{2\pi} = \mathbf{0\ V}$$

EXAMPLE: - Determine the average value of the waveform in Fig.

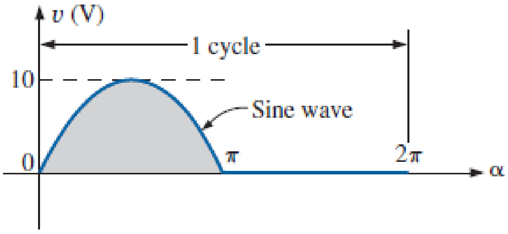


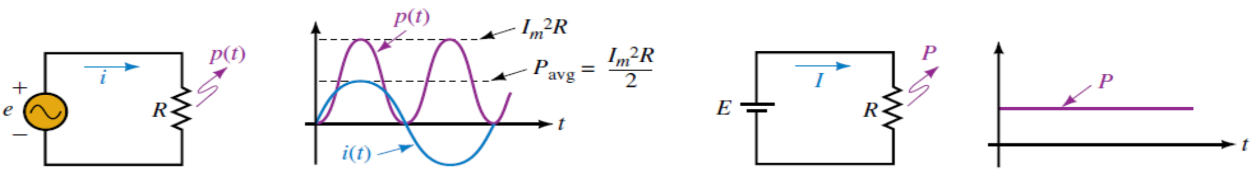
FIG.

Solution:

$$G = \frac{2A_m}{2\pi} = \frac{2(10\ \text{V})}{2\pi} \cong \mathbf{3.18\ V}$$

5-Effective Values.

An effective value is an equivalent dc value: it tells you how many volts or amps of dc that a time-varying waveform is equal to in terms of its ability to produce average power Effective values are also called **rms values root mean square** for reasons discussed shortly.



$p(t) = i^2R$. Therefore, $p(t)$ varies cyclically.

(a) AC circuit

$P = I^2R$. Therefore, P is constant.

(b) DC circuit

Figure 11

First, consider the dc case. Since current is constant, power is constant, and average power is.

$$P_{avg} = P = I^2R$$

Now consider the ac case. Power to the resistor at any value of time is $p(t) = i^2R$, where i is the instantaneous value of current.

Since $i = I_m \sin wt$,

$$\begin{aligned} p(t) &= i^2R \\ &= (I_m \sin \omega t)^2 R = I_m{}^2 R \sin^2 \omega t \\ &= I_m{}^2 R\left[\frac{1}{2}(1 - \cos 2\omega t)\right] \end{aligned}$$

$$p(t) = \frac{I_m{}^2 R}{2} - \frac{I_m{}^2 R}{2} \cos 2\omega t$$

Note that the average of cos 2w*t* is zero

$$P_{\text{avg}} = \text{average of } p(t) = \frac{I_m{}^2 R}{2}$$

Equate two eq.

$$I^2 = \frac{I_m{}^2}{2}$$

$$I_{\text{eff}} = \frac{I_m}{\sqrt{2}} = 0.707 I_m$$

$$E_{\text{eff}} = \frac{E_m}{\sqrt{2}} = 0.707 E_m$$

$$V_{\text{eff}} = \frac{V_m}{\sqrt{2}} = 0.707 V_m$$

The $\sqrt{2}$ relationship holds only for sinusoidal waveforms. For other waveforms, you need a more general formula. Using calculus, it can be shown that for any waveform

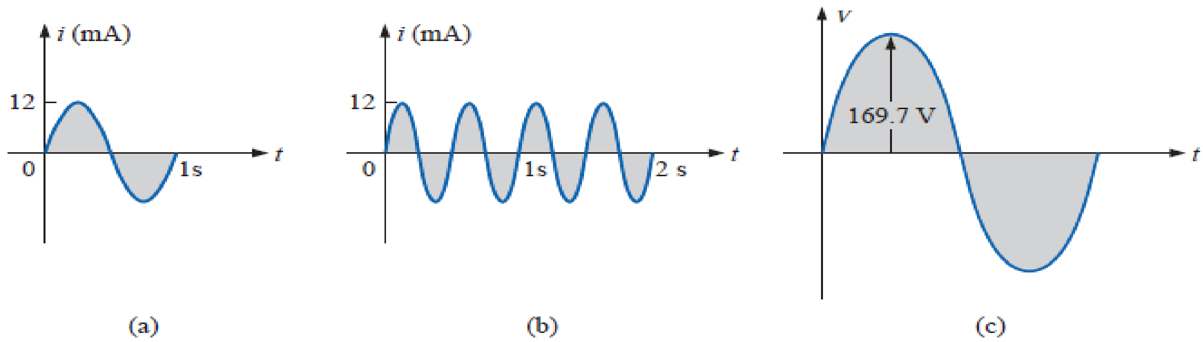
$$I_{eff} = \sqrt{\frac{1}{T}\int_0^T i^2 dt}$$

The integral of i^2 represents the area under the i^2 waveform. Thus,

$$I_{eff} = \sqrt{\frac{area\ under\ the\ i^2 curve}{base}}$$

To use this equation, we compute the root of the mean square to obtain the effective value. For this reason, effective values are called **root mean square** or **rms** values and **the terms *effective* and *rms* are synonymous.**

EXAMPLE: - Find the rms values of the sinusoidal waveform in each part of Fig.



Solution: For part (a), $I_{rms} = 0.707(12 \times 10^{-3}\ A) =$ **8.484 mA.** For part (b), again $I_{rms} =$ **8.484 mA.** Note that frequency did not change the effective value in (b) above compared to (a). For part (c), $V_{rms} = 0.707(169.73\ V) \cong$ **120 V,** the same as available from a home outlet.

EXAMPLE: - Find the rms value of the waveform in Fig. 13.60.

Solution: v^2 (Fig. 13.61).

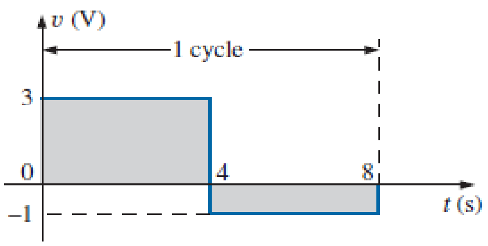


FIG. 13.60

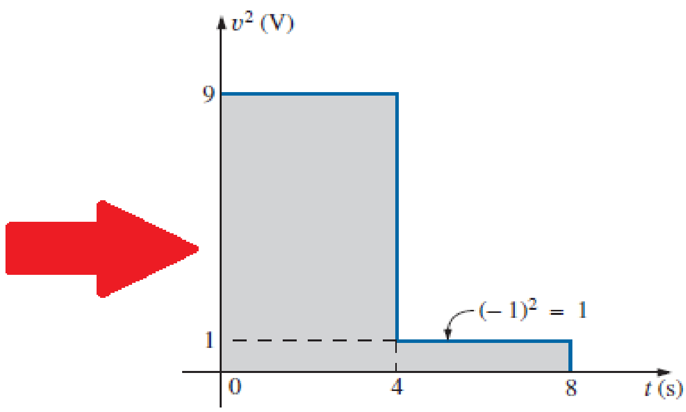


FIG. 13.61
The squared waveform of Fig. 13.60.

$$V_{rms} = \sqrt{\frac{(9)(4) + (1)(4)}{8}} = \sqrt{\frac{40}{8}} = \mathbf{2.24\ V}$$

EXAMPLE 13.23 Calculate the rms value of the voltage in Fig. 13.62.

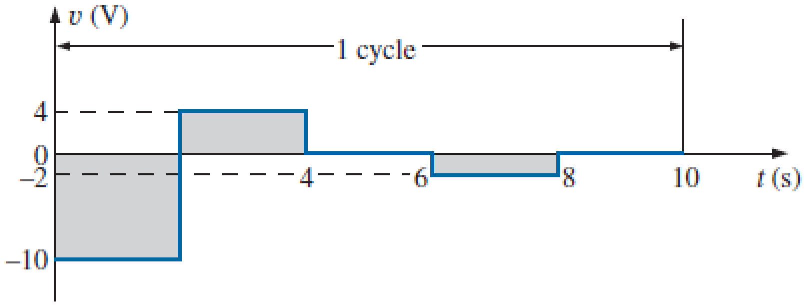


FIG. 13.62
Example 13.23.

Solution: v^2 (Fig. 13.63):

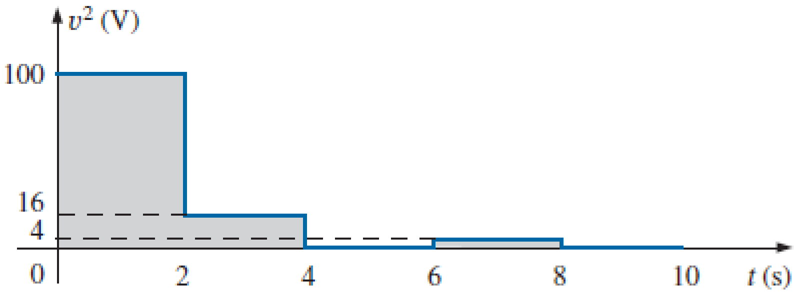


FIG. 13.63
The squared waveform of Fig. 13.62.

6- Representing AC Voltages and Currents by Complex Numbers.

The sinusoidal voltage $e(t)$ =200 sin(wt + 40°) of Figure 12(a) and (b) can be represented by its phase equivalent, $E = 200V\llcorner 40$, as in (c).

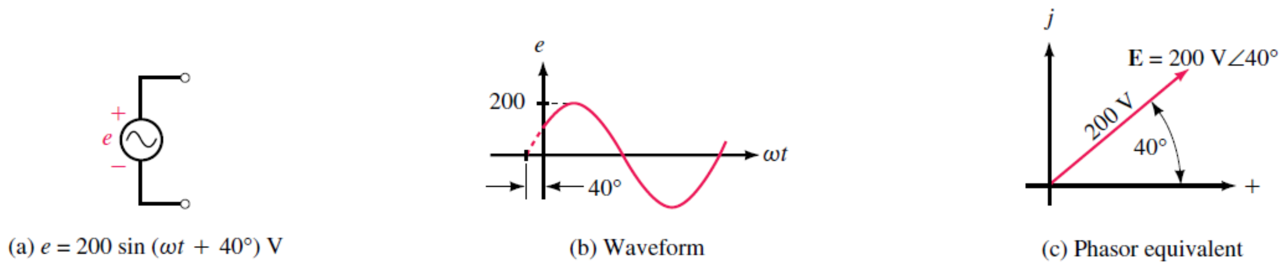


Figure 12

To convert between forms:

$$C = a + jb \qquad (rectangular\ form)$$

$$C = c\angle\theta \qquad (polar\ form)$$

$a = c\ cos\theta,\qquad b = c\ sin\theta,\qquad c = \sqrt{a^2 + b^2}$, and $\qquad \theta = tan^{-1}\frac{b}{a}$

$$j = \sqrt{-1},\qquad j^2 = -1,\qquad j^3 = -j,\qquad j^4 = 1, and \qquad j = \frac{-1}{j}$$

The conjugate of a complex number (denoted by an asterisk *) is a complex number with the same real part but the opposite imaginary part. Thus, the conjugate of $C = c\angle\theta = a + jb$ is $C^* = c\angle - \theta = a - jb$. For example, if $C = 3 + j4 = 5\angle 53.13^{\circ}$, then $C^* = 5\angle - 53.13^{\circ} = 3 - j4$.

EXAMPLE:- For Figure 13, $v_1 = \sqrt{2}(16)sinwt\ V, v_2 = \sqrt{2}(24)\sin(wt + 90^{\circ})\ V$, $v_3 = \sqrt{2}(15)\sin\ (wt - 90^{\circ})\ V$.
Determine source voltage *e*.

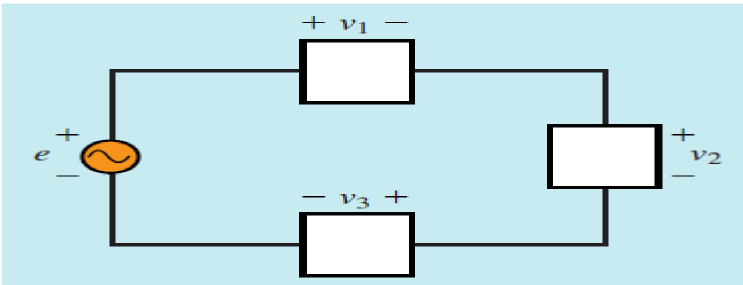


Figure 13

Solution:

Applying KVL:

$$e = V_1 + V_2 + V_3 = 16\angle 0^{\circ} + 24\angle 90^{\circ} + 15\angle{-90}^{\circ}$$
$$= 16 + j0 + 24j - 15j = 16 + 9j\ (rectangular\ form)$$
$$= \sqrt{16^2 + 9^2} = 18.36$$
$$\theta = \tanh^{-1}\frac{9}{16} = \angle 29.36^{\circ}$$
$$e = \sqrt{2}(18.36)\sin(wt + 29.36^{\circ})\ V$$
$$e = 18.36\angle 29.36^{\circ} \qquad (polar\ form)$$

7- Ohm's Law for AC Circuits.

-Resistors

When a resistor is subjected to a sinusoidal voltage as shown in Figure 15, the resulting current is also sinusoidal and in phase with the voltage.

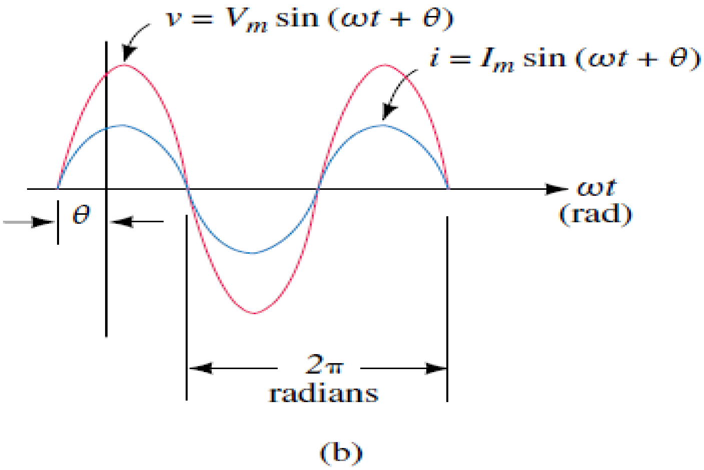


Figure 15

The sinusoidal voltage $v=V_m\sin(wt+\theta)$ may be written in phasor form as **V**$=V\angle\theta^{\circ}$.Whereas the sinusoidal expression gives the instantaneous value of voltage for awaveform having an amplitude of V_m (volts peak), the phasor form has a magnitude which is the effective (or rms) value.

The voltage and current phasors may be shown on a phasor diagram as In Figure 16.

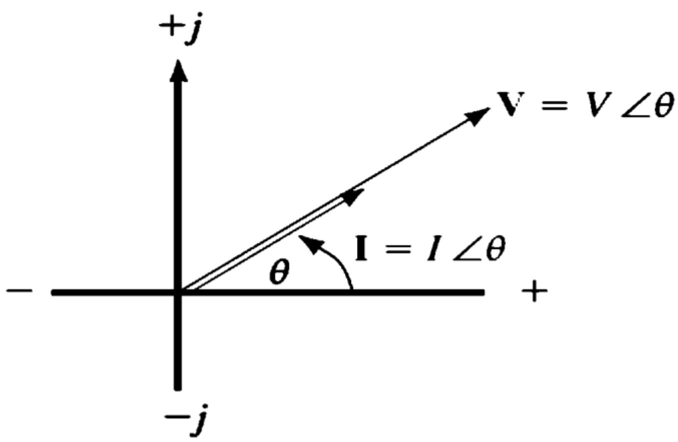


Figure 16

Example: - The current through a 5 Ω resistor is given. Find the sinusoidal expression for the voltage across the resistor for $i = 40 \sin(377t + 30°)$.

Solution: $V_m = I_m R = (40\ A)(5\ \Omega) = 200\ V$

(V and i are in phase), resulting in

$v = \mathbf{200 \sin(377t + 30°)}$

EXAMPLE: - The voltage across a resistor is indicated. Find the sinusoidal expression for the current if the resistor is 10 Ω. Sketch the curves for v and i.

a. $v = 100 \sin 377t$

b. $v = 25 \sin(377t + 60°)$

Solutions:

a.
$$I_m = \frac{V_m}{R} = \frac{100\ V}{10\ \Omega} = 10\ A$$

(v and i are in phase), resulting in

$$i = \mathbf{10 \sin 377t}$$

The curves are sketched in Fig.

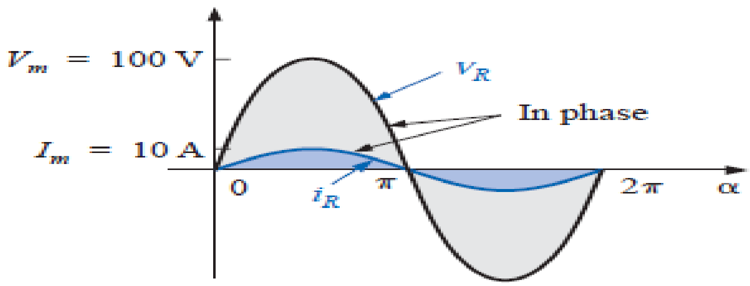


FIG.

b.
$$I_m = \frac{V_m}{R} = \frac{25\ V}{10\ \Omega} = 2.5\ A$$

(v and i are in phase), resulting in

$$i = \mathbf{2.5 \sin(377t + 60°)}$$

The curves are sketched in Fig.

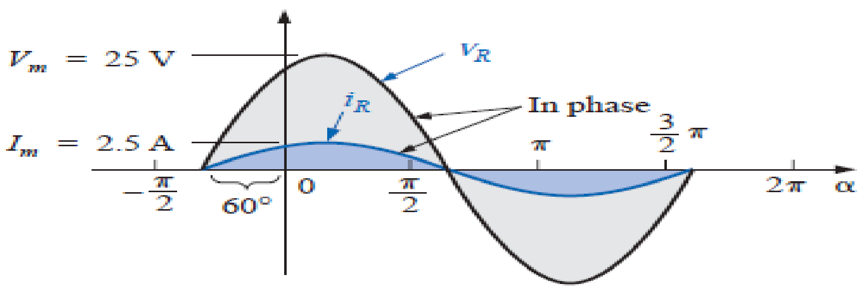
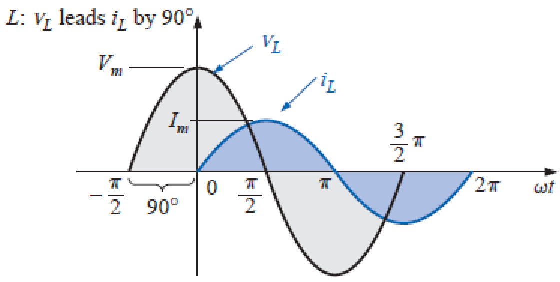


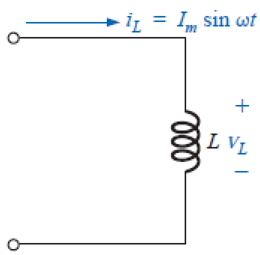
FIG.

-Inductors

When an inductor is subjected to a sinusoidal current, a sinusoidal voltage is induced across the inductor **such that the voltage across the inductor leads the current waveform by exactly 90°.** If we know the reactance of an inductor, then from Ohm's law the current in the inductor may be expressed in phases form as:



For a pure inductor, the voltage across the coil leads the current through the coil by 90°.



Investigating the sinusoidal response of an inductive element.

For an inductor, vL leads iL by 90°, or iL lags vL by 90°.

$iL = Im \sin(\omega t \pm \Theta)$

$vL = \omega LIm \sin(\omega t \pm \Theta \pm 90°)$

In vector form, the reactance of the inductor is given as:

$X_L = wL = 2\pi f L$ ohms, Ω

$$XL = \frac{Vm}{Im}, \text{ Vm} = \text{Im XL}$$

EXAMPLE: - The current through a 0.1-H coil is provided. Find the sinusoidal expression for the voltage across the coil. Sketch the *v* and *i* curves.

a. i =10 sin 377*t*
b. i =7 sin(377t -70°)

Solution:

a. XL =ωL = (377 rad/s)(0.1 H) = 37.7 Ω

   Vm = *Im XL* = (10 A) (37.7 Ω) = 377 V

   And we know that for a coil *v* leads *i* by 90°.

   v =377 sin (377*t* + 90°)

The curves are sketched in Fig.

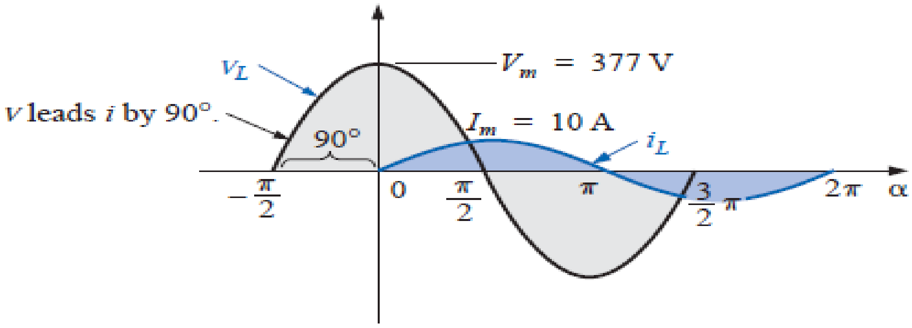


FIG.
Example (a).

b- XL remains at 37.7Ω

Vm =Im XL = (7A)(37.7Ω) = 263.9 V

And we know that for a coil V leads I by 90̊ . Therefore,

V= 263.9 sin(377t - 70̊ +90̊)

V= 263.9 sin(377t +20̊)

The curves are sketched in Fig.

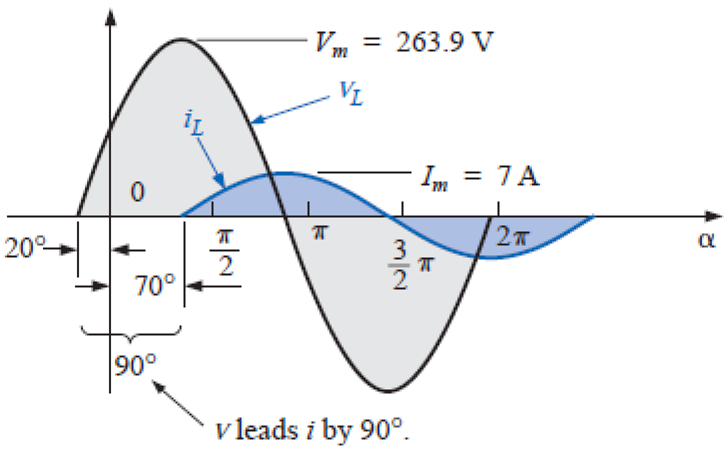


FIG.
Example (b).

H.W EXAMPLE:-The voltage across a 0.5-H coil is provided below. What is the sinusoidal expression for the current? $v = 100 \sin 20t$

- Capacitors:

When a capacitor is subjected to a sinusoidal voltage, a sinusoidal current Results. **The current through the capacitor leads the voltage by exactly 90°.** If We know the reactance of a capacitor, then from Ohm's law the current in the Capacitor expressed in phasor form is

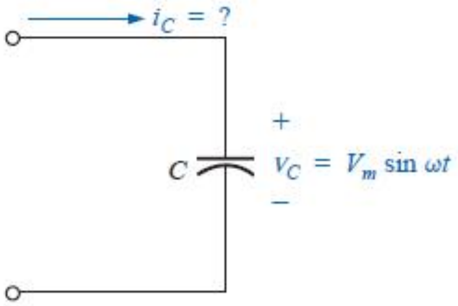


FIG.
Investigating the sinusoidal response of a capacitive element.

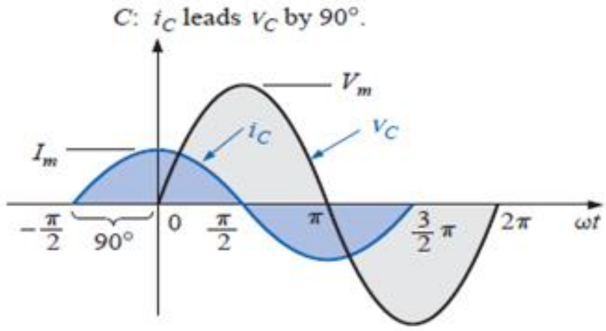


FIG.
The current of a purely capacitive element leads the voltage across the element by 90°.

For a capacitor, i_C leads v_C by 90^0, or vC lags iC by 90°.

$VC = Vm \sin(\omega t \pm \Theta)$

$iC = \omega C\, Vm \sin(\omega t \pm \Theta + 90°)$

$$\frac{Vm}{Im} = \frac{Vm}{\omega C\, Vm} = \frac{1}{\omega C}$$

$$X_C = \frac{1}{wC} = \frac{1}{2\pi f C} \text{ Ohms, } \Omega$$

$$\text{Xc} = \frac{Vm}{Im} \text{ Ohms, } \Omega$$

EXAMPLE The voltage across a 1-μF capacitor is provided below. What is the sinusoidal expression for the current? Sketch the v and i curves.

$$v = 30 \sin 400t$$

Solution:

$$\text{Eq. (14.6):} \quad X_C = \frac{1}{\omega C} = \frac{1}{(400 \text{ rad/s})(1 \times 10^{-6} \text{ F})} = \frac{10^6 \ \Omega}{400} = 2500 \ \Omega$$

$$\text{Eq. (14.7):} \quad I_m = \frac{V_m}{X_C} = \frac{30 \text{ V}}{2500 \ \Omega} = 0.0120 \text{ A} = 12 \text{ mA}$$

and we know that for a capacitor i leads v by 90°. Therefore,

$$\mathbf{i = 12 \times 10^{-3} \sin(400t + 90°)}$$

The curves are sketched in Fig.

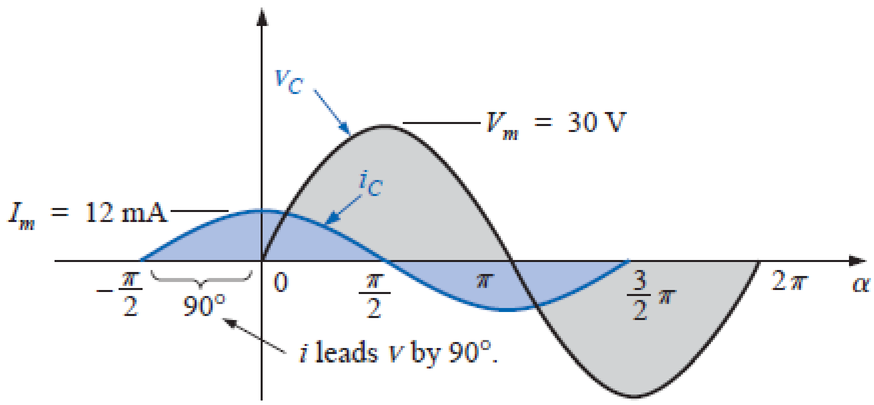


FIG.
Example

H.W

EXAMPLE: - The current through a 100-mF capacitor is given. Find the sinusoidal expression for the voltage across the capacitor. i =40 sin (500t +60°).

8. Power in AC Circuits*:*

At any given instant, the power to a load is equal to the product of voltage times current***:***

$$P = vi \qquad \text{Watts}$$

This is illustrated in Fig. 19, where we have multiplied voltage time's current point by point to get the power waveform.

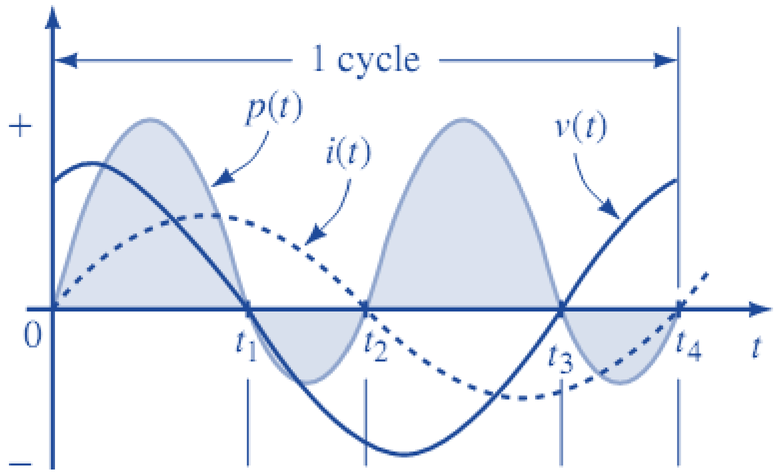


Figure 19

Thus, during positive parts of the power cycle, power flows from the source to the load, while during negative parts, it flows out of the load back into the circuit. Thus, if *P* has a positive value, it represents the power that is really dissipated by the load. For this reason, *P* is called **real power.** In modern terminology, real power is also called **active power.** Thus, *active power is the average value of the instantaneous power, and the terms real power, active power, and average power mean the same thing.*

First consider power to a purely resistive load (Fig. 20). Here, current is in phase with voltage. Assume $i = I_m \, sinwt, and \ v = V_m \, sinwt, then$

$$p = vi = (I_m \, sinwt)(V_m \, sinwt) = I_m \, V_m \, sin^2 wt = \frac{I_m \, V_m}{2}(1 - cos2wt)$$

Where $\frac{I_m \, V_m}{2}$ is called average power.

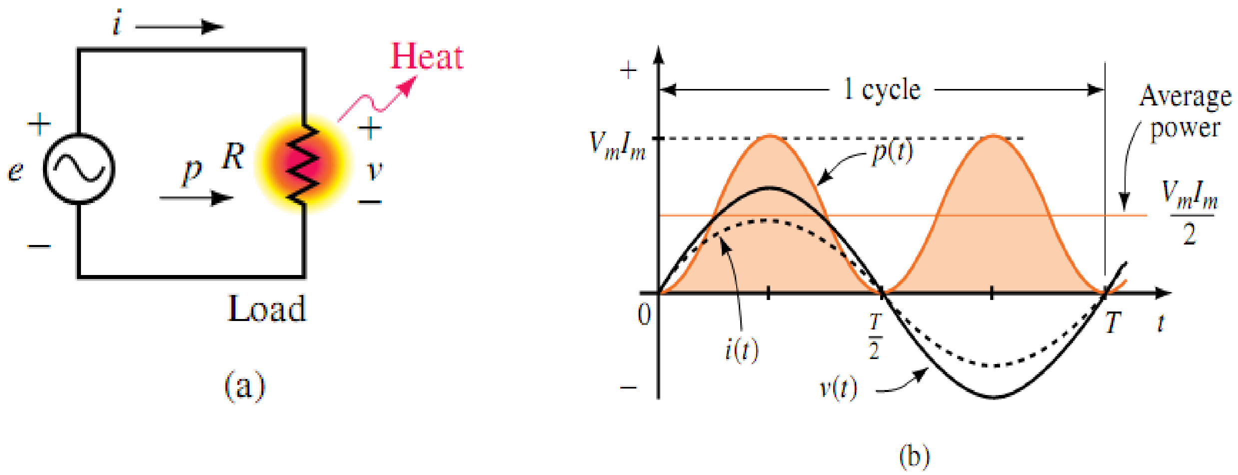
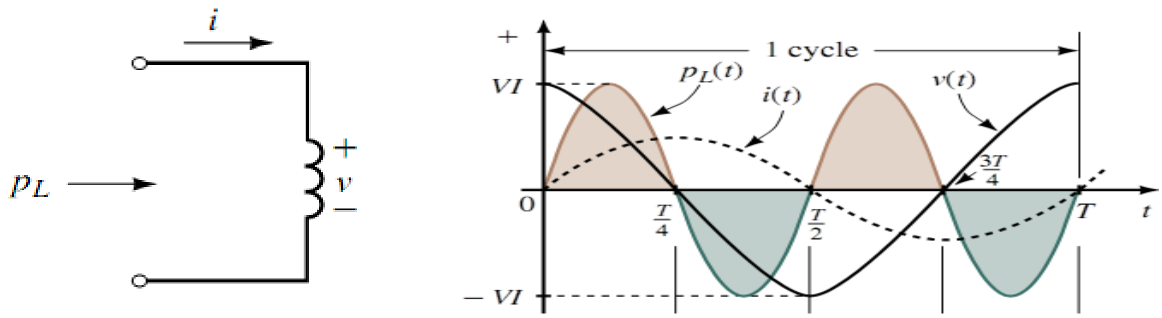


Figure 20

Note that *p* is always positive. This means that power flows only from the source to the load. We therefore conclude that *power to a pure resistance consists of active power only.*

For a purely inductive load as in Fig. 21 (a), current lags voltage by 90°. If we select current as reference $i = I_m\, sinwt, and\ v = V_m \sin{(wt + 90^{\circ})}$.A sketch of p versus time (obtained by multiplying *v* times *i*) then looks as shown in (b). Thus, *the average power to an inductance over a full cycle is zero, i.e., there are no power losses associated with a pure inductance.* Consequently, *P*=0 W and the only power flowing in the circuit is reactive power.



(a) ***Figure 21*** (b)

With$i = I_m\, sinwt,\ and\ v = V_m \sin{(wt + 90^{\circ})}, p = vi$ becomes

$$p_L = (I_m\, sinwt)(V_m \sin{(wt + 90^{\circ})})$$

After some trigonometric manipulation, this reduces to

$$p_L = IV \sin 2wt$$

The product VI is defined as **reactive power** and is given the symbol Q_L, its unit is VAR (volt-amps reactive).

$$Q_L = I^2 X_L = \frac{V^2}{X_L} \qquad (VAR)$$

For a purely capacitive load, current leads voltage by 90°. Taking current as reference $i = I_m\, sinwt, and\ v = V_m \sin(wt - 90^\circ)$. Multiplication of v times i yields the power curve of Fig. 22.

This means that *the average power to a capacitance over a full cycle is zero, i.e., there are no power losses associated with a pure capacitance.* Consequently, P=0 W and the only power flowing in the circuit is Reactive power.

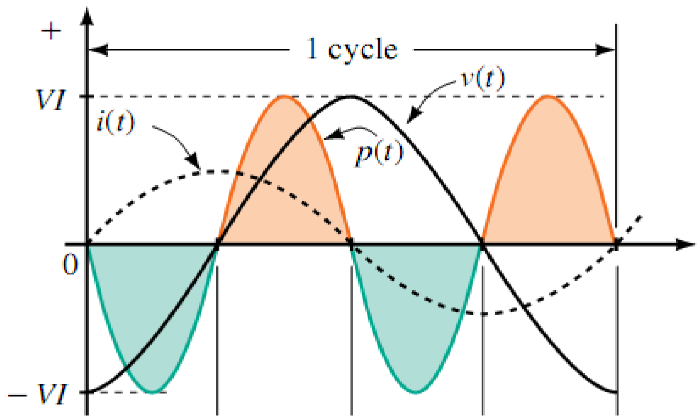


Figure 22

This reactive power is given by:

$$p_C = I_m V_m sinwt \sin(wt + 90^\circ)$$

Which reduces to:

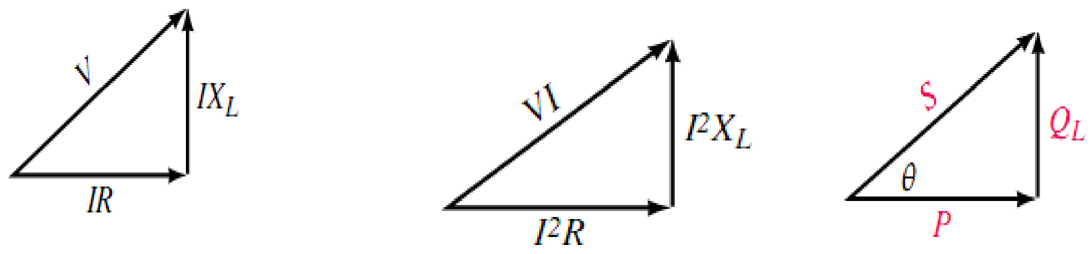
$$p_C = -VI\ sin2wt$$

Now define the product VI as Q_C which can be expressed as:

$$Q_C = I^2 X_C = \frac{V^2}{X_C} \qquad \text{(VAR)}$$

When a load has voltage V across it and current I through it, the power that appears to flow to it is VI. However, if the load contains both resistance and reactance, this product represents neither real power nor reactive power. Since it appears to represent power, it is called **apparent power.** Apparent power is given the symbol S and has units of volt-amperes(VA).Thus, $S = VI = I^2 Z = \frac{V^2}{Z} \qquad (VA)$

Note that these represent P, Q, and S respectively as indicated in figure 23. This is called the **power triangle.**



(a) Magnitudes only shown

(b) Multiplied by I

(c) Resultant power triangle

(d) Capacitive case

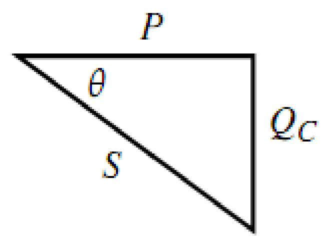


Figure 23

Q = Reactive power

P = Active power

S = apparent power.

From the geometry of this triangle, you can see that $=\sqrt{p^2+Q^2}$, and

$$P = VI\,cos\theta \;= S\,cos\theta \quad (W)$$

$$Q = VI\,sin\theta = S\,sin\theta \qquad (VAR)$$

The quantity $cos\theta$ is defined as **power factor** and is given the symbol F_P. Thus,

$$F_P = cos\theta, \qquad cos\theta = \frac{P}{S} \quad , and\ \theta = cos^{-1}\left(\frac{P}{S}\right)$$

Angle θ is the angle between voltage and current. Thus, an inductive circuit has a lagging power factor, while a capacitive circuit has a leading power factor. A load with a very poor power factor can draw excessive current.

Consider, for example, the simple R-L circuit in Fig. 1 , where

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}_T} = \frac{10\ \text{V}\angle 0^\circ}{3\ \Omega + j\,4\ \Omega} = \frac{10\ \text{V}\angle 0^\circ}{5\ \Omega\ \angle 53.13^\circ} = 2\ \text{A}\ \angle -53.13^\circ$$

The real power (the term *real* being derived from the positive real axis of the complex plane) is

$$P = I^2R = (2\ \text{A})^2(3\ \Omega) = 12\ \text{W}$$

and the reactive power is

$$Q_L = I^2X_L = (2\ \text{A})^2(4\ \Omega) = 16\ \text{VAR}\ (L)$$

with $\mathbf{S} = P + j\,Q_L = 12\ \text{W} + j\,16\ \text{VAR}\ (L) = 20\ \text{VA}\ \angle 53.13^\circ$

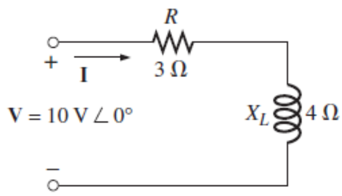


FIG. 1

OR.

$$\mathbf{S} = \mathbf{VI}^* = (10\ \text{V}\ \angle 0^\circ)(2\text{A}\ \angle +53.13^\circ) = 20\ \text{VA}\ \angle 53.13^\circ$$

as obtained above.

The angle θ associated with $\mathbf{S}$ and appearing in Figs. 2 is the power-factor angle of the network. Since

$$P = VI\cos\theta$$

or

$$P = S\cos\theta$$

then

$$\boxed{F_p = \cos\theta = \frac{P}{S}}$$

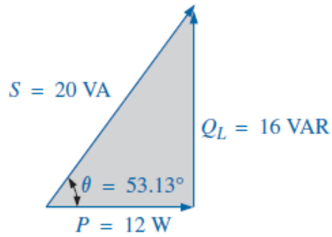


FIG. 2
The power triangle for the circuit

9. A.C Series Circuit:

Impedance is a term used to collectively determine how the resistance, capacitance, and inductance "impede" the current in a circuit. The symbol for impedance is the letter Z and the unit is the ohm (Ω). Because impedance may be made up of any combination of resistances and reactance's, it is written as a vector quantity **Z,** where

$$\boldsymbol{Z} = Z\angle\theta$$

Resistive impedance $\mathbf{Z_R}$ is a vector having a magnitude of R along the positive real axis. Inductive reactance $\mathbf{Z}_L$ is a vector having a magnitude of X_L along the positive imaginary axis, while the capacitive reactance $\mathbf{Z_C}$ is a vector having a magnitude of X_C along the negative imaginary axis as shown in figure 24.

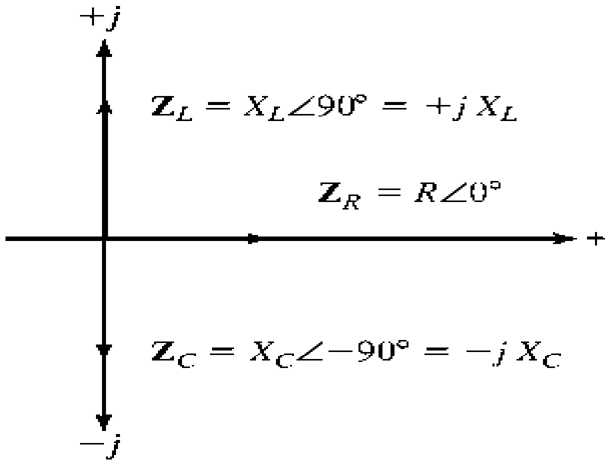


Figure 24

EX:-For a series ac circuit consisting of *n* impedances, the total impedance of the circuit is found as the vector sum:

$$Z_T = Z_1 + Z_2 + \cdots + Z_n$$

Thus we may determine the total impedance of figure 25:

$Z_T = 3 + j0 + 0 + j4 = (3 + j4)\Omega = 5\Omega\angle 53.13^{\circ}$

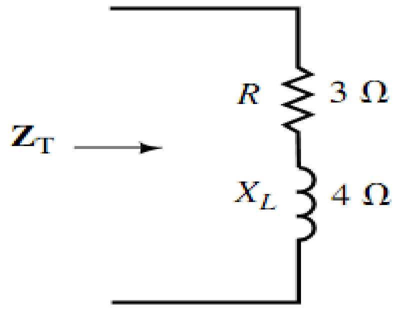


Figure 25

The above quantities are shown on an impedance Diagram as in Figure26.

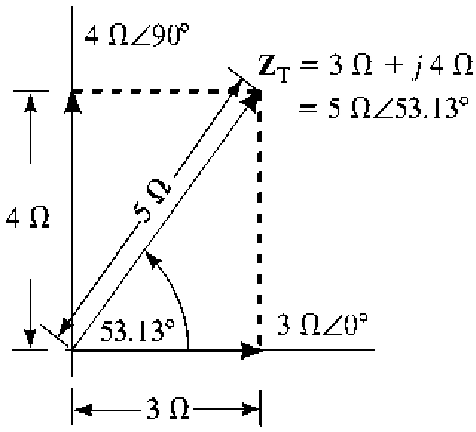


Figure 26

The rectangular form of an impedance is written as:

$$Z = R \pm jX$$

If we are given the polar form of the impedance, then we may determine the equivalent rectangular expression from

$$R = Z\cos\theta, \quad and \;\; X = Z\sin\theta$$

In the rectangular representation for impedance, the resistance term, *R*, is the total of all resistance looking into the network. The reactance term, *X*, is the difference between the total capacitive and inductive reactance's. The sign for the imaginary term will be positive if the inductive reactance is greater than the capacitive reactance. In such a case, the impedance vector will appear in the first quadrant of the impedance diagram and is referred to as being an **inductive** impedance. If the capacitive reactance is larger, then the sign for the imaginary term will be negative. In such a case, the impedance vector will appear in the fourth quadrant of the impedance diagram and the impedance is said to be **capacitive.**

The polar form of any impedance will be written in the form $\boldsymbol{Z} = Z\angle\theta$. The value Z is the magnitude (in ohms) of the impedance vector **Z** and is determined as follows: $Z = \sqrt{R^2 + X^2} \quad (\Omega)$

The corresponding angle of the impedance vector is determined as:

$$\theta = \pm tan^{-1}\left(\frac{X}{R}\right)$$

Example: Consider the network of Fig. 27.

a. Find $\mathbf{Z}_T$.

b. Sketch the impedance diagram for the network and indicate whether the total Impedance of the circuit is inductive, capacitive, or resistive.

C. Use Ohm's law to determine **I,** $\mathbf{V}_R$, and $\mathbf{V}_C$.

Solution:

a. $$Z_T = 25 + j200 - j225$$
$$= (25 - j25)\Omega$$
$$= 35.35\Omega\angle{-45^\circ}$$

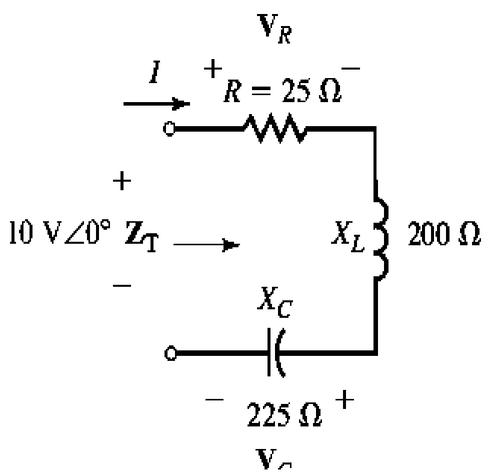


Fig. 27.

b. The corresponding impedance diagram is shown in Fig. 28.
Because the total impedance has a negative.

Reactance term ($j25$), $\mathbf{Z}_T$ is capacitive.

$$C.\quad I = \frac{10V\angle 0^\circ}{35.35\Omega\angle -45^\circ} = 0.2828A\angle 45^\circ$$

$$V_R = (0.2828A\angle 45^\circ)(25\Omega\angle 0^\circ)$$

$$= 7.07V\angle 45^\circ$$

$$V_C = (0.2828A\angle 45^\circ)(225\Omega\angle -90^\circ)$$

$$= 63.63V\angle -45^\circ$$

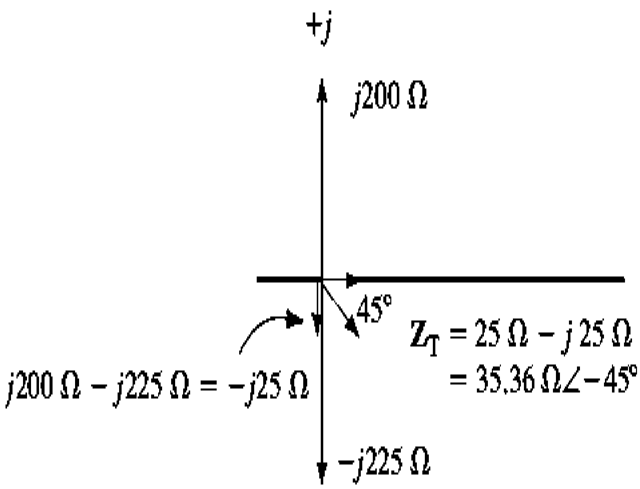


Figure28

In the simple series circuit shown in Fig. 31, we know that only the resistor will dissipate power.

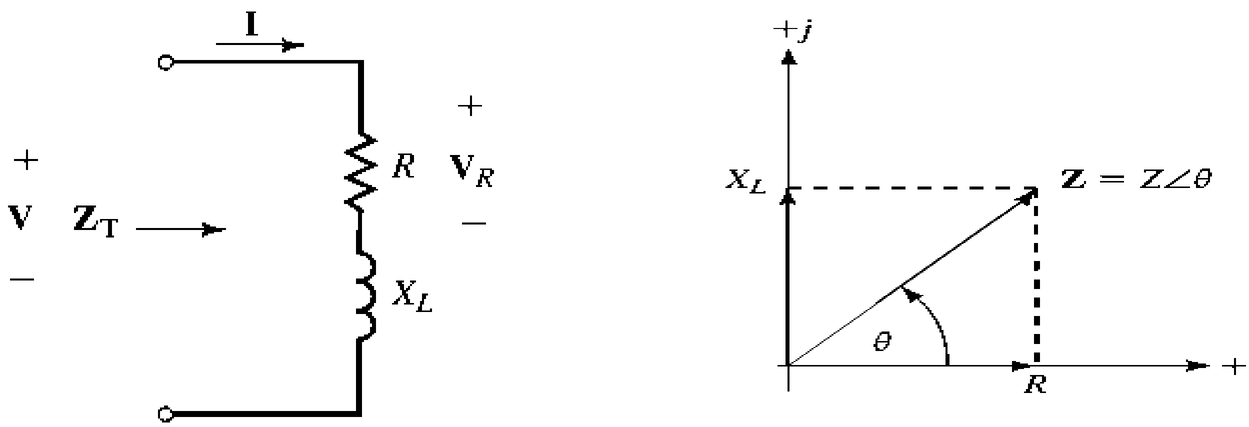


Figure 31

$$P = V_R I = \frac{V_R^2}{R} = I^2 R, we\ have\ I = \frac{V}{Z}, \qquad thus$$

$$P = \frac{V^2}{Z^2} R = \frac{V^2}{Z} = \frac{R}{Z}$$

From figure 32 we have $cos\theta = \frac{R}{Z}$ $\quad thus\ \ P = VI\ cos\theta = I^2 Z\ cos\theta = \frac{V^2}{Z}\ cos\theta$

Example:

Refer to the circuit of Figure 32 .

a. Find the impedance **Z**.

b. Calculate the power factor of the circuit.

c. Determine **I.**

d. Sketch the phasor diagram for **E** and **I.**

e. Find the average power delivered to the circuit by the voltage source.

f. Calculate the average power dissipated by both the resistor and the capacitor

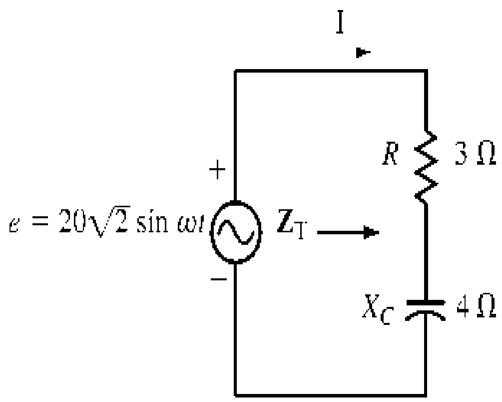


Figure 32

Solutions: $a.\ \ Z_T = R + X_C = (3 - j4)\Omega = 5\Omega\angle{-53.13^{\circ}}$

b. $F_P = cos\theta = \frac{R}{Z} = \frac{3}{5} = 0.6\ \ leading$

c. The phasor form of the applied voltage is:

$$E = \frac{\sqrt{2}\times 20}{\sqrt{2}}\angle 0 = 20V\angle 0$$

$$I = \frac{20V\angle 0}{5\Omega\angle{-53.13^{\circ}}} = 4A\angle 53.13^{\circ}$$

d.The phasor diagram is shown in figure 33

$e.\ \ P = VI\ cos\theta = 20\times 4\times\cos(53.13)$

$= 48\ \ W$

f.

$$P_R = 4^2\times 3\times\cos(0) = 48\ \ W$$

$$P_C = 4^2\times 4\times\cos(90) = 0\ \ W$$

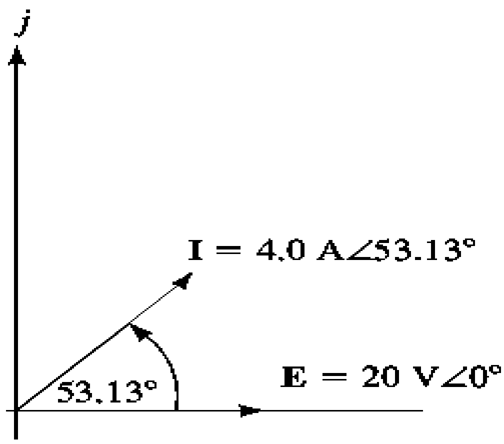


Figure 34

Example: Consider the circuit of Figure 35. a. Find $\mathbf{Z}_T$.

b. Determine the voltages $\mathbf{V}_R$ and $\mathbf{V}_L$ using the voltage divider rule.

c. Verify Kirchhoff's voltage law around the closed loop.

Solution: a. $Z_T = 5k\Omega + j12k\Omega = 13k\Omega\angle 67.38^\circ$

b.

$$V_R = \frac{5k\Omega\angle 0^\circ}{13k\Omega\angle 67.38^\circ} \times 26V\angle 0^\circ = 10V\angle -67.38^\circ$$

$$V_L = \frac{12k\Omega\angle 90^\circ}{13k\Omega\angle 67.38^\circ} \times 26V\angle 0^\circ = 24V\angle 22.62^\circ$$

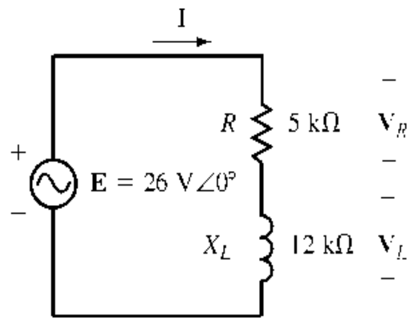


Figure 35

C.

V (t) = $\sqrt{Vr^2 + Vl^2} = 26 = \sqrt{10^2 + 24^2} = \sqrt{100 + 576} = 26$

$$26V\angle 0^\circ - 10V\angle -67.38^\circ - 24V\angle 22.62^\circ = 0$$

$$(26 + j0) - (3.846 - j9.231) - (22.145 + j9.231) = 0$$

$$(26 - 3.846 - 22.145) + j(0 - 9.231 + 9.231) = 0$$

$$0 + j0 = 0$$