Electrical and Electronic Technical College

Department of Computer Technical Engineering

Electrical Engineering Fundamental

First Class 2019- 2020

First Level

ASSISTANT LECTURER

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Symbols and Abbreviations:-

The system of units used in engineering and science is the (International system of units), usually abbreviated to SI units, and is based on the metric system.

Quantity	Quantity Symbol	Unit	Unit Symbol
Length	l	metre	m
Mass	m	kilogram	kg
Time	t	second	s
Velocity	v	metres per second	m/s or $m\,s^{-1}$
Acceleration	a	metres per second squared	m/s^2 or $m\,s^{-2}$
Force	F	newton	N
Electrical charge or quantity	Q	coulomb	C
Electric current	I	ampere	A
Resistance	R	ohm	Ω
Conductance	G	siemen	S
Electromotive force	E	volt	V
Potential difference	V	volt	V
Work	W	joule	J
Energy	E (or W)	joule	J
Power	P	watt	W

SI units may be made larger or smaller by using prefixes which denote multiplication or division by a particular amount.

Prefix	Name	Meaning
M	mega	multiply by 1 000 000 (i.e. $\times 10^6$)
k	kilo	multiply by 1000 (i.e. $\times 10^3$)
m	milli	divide by 1000 (i.e. $\times 10^{-3}$)
µ	micro	divide by 1 000 000 (i.e. $\times 10^{-6}$)
n	nano	divide by 1 000 000 000 (i.e. $\times 10^{-9}$)
p	pico	divide by 1 000 000 000 000 (i.e. $\times 10^{-12}$)

Charge: The unit of charge is the coulomb **(C)** where one coulomb is one ampere second. (1 coulomb = 6.24×10^{18} electrons).

Thus, charge, in coulombs $\boldsymbol{Q = It}$ where $\boldsymbol{I}$ is the current in amperes and $\boldsymbol{t}$ is the time in seconds.

Example: - if a current of (5 A) flows for (2 minutes), find the quantity of electricity transferred?

Quantity of electricity $Q = It$ coulombs

$$I = 5A, t = 2 \times 60 = 120s, hence\ Q = 5 \times 120 = 600C$$

Force: The unit of forces the Newton (N) where one newton is one-kilogram meter per second squared. Thus, force, in Newton's $\boldsymbol{F = ma}$

Where ***m*** is the ***mass*** in kilograms and ***a*** is the ***acceleration*** in meters per second squared. Gravitational force, or weight, is *mg*, where $\boldsymbol{g = 9.81\ m/s^2}$.

Example: A mass of 5000 g is accelerated at 2 m/s^2 by a force. Determine the force needed?

Force = mass x acceleration = 5kg x 2m/s^2 = 10 kg m/s^2 = 10 N.

Work: The unit of work or energy is the joule **(J)** where one joule is one Newton meter. Thus work done on a body, in joules, ***W=Fs*** where *F* is the force in Newton's and ***s*** is the distance in meters moved by the body in the direction of the force.

Power: The unit of power is the watt (*W*) where one watt is one joule per second. Power is defined as the rate of doing work or transferring energy. Thus, power, in watts, *P* = *W/t*, where *W* is the work done or energy transferred, in joules, and *t* is the time, in seconds. Thus, energy, in joules, *W=Pt*

$$\mathbf{P} = \frac{W}{t} = \frac{total\ work\ done\ (J)}{total\ time\ taken\ (S)} = \frac{J}{S}$$

$$\frac{J}{S} = 1\ watt = 1\ N.m/S$$

But $\mathbf{V} = \frac{W}{Q}$ $\ and\ I = \frac{Q}{t}$

$$\therefore \mathbf{P} = \frac{w}{t} = \frac{V.Q}{t} = V.I\ (watt).$$ **But V= I.R (Ohm's law).**

$P = V.\, I = I^2 .\ R = \frac{V^2}{R}\ watt.$

<u>Example:</u> A mass of 1000 kg is raised through a height of 10 m in 20 s. What is (a) the work done and (b) the power developed?

(a) Work done = force × distance

and force = mass × acceleration

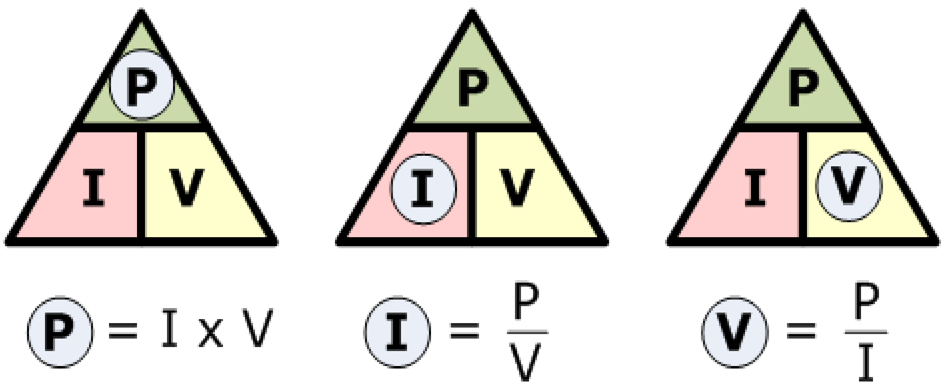
Hence, work done $= (1000\,\text{kg} \times 9.81\,\text{m/s}^2) \times (10\,\text{m})$

$= 98\,100\,\text{Nm}$

$= \mathbf{98.1\,kNm}$ or $\mathbf{98.1\,kJ}$

(b) Power $= \frac{\text{work done}}{\text{time taken}} = \frac{98100\,\text{J}}{20\,\text{s}}$

$= 4905\,\text{J/s} = \mathbf{4905\,W}$ or $\mathbf{4.905\,kW}$



<u>Example:</u> An electric heater consumes 1.8 M/J when connected to a 250 V supply for 30 minutes. Find the power rating of the heater and the current taken from the supply.

$$\text{Power} = \frac{\text{energy}}{\text{time}} = \frac{1.8 \times 10^6\,\text{J}}{30 \times 60\,\text{s}}$$

$$= 1000\,\text{J/s} = 1000\,\text{W}$$

i.e. **power rating of heater = 1 kW**

$$\text{Power } P = VI, \text{ thus } I = \frac{P}{V} = \frac{1000}{250} = 4\,\text{A}$$

Hence the current taken from the supply is 4 A.

1 What quantity of electricity is carried by 6.24×10^{21} electrons? [1000 C]

2 In what time would a current of 1 A transfer a charge of 30 C? [30 s]

3 A current of 3 A flows for 5 minutes. What charge is transferred? [900 C]

4 How long must a current of 0.1 A flow so as to transfer a charge of 30 C? [5 minutes]

5 What force is required to give a mass of 20 kg an acceleration of 30 m/s^2? [600 N]

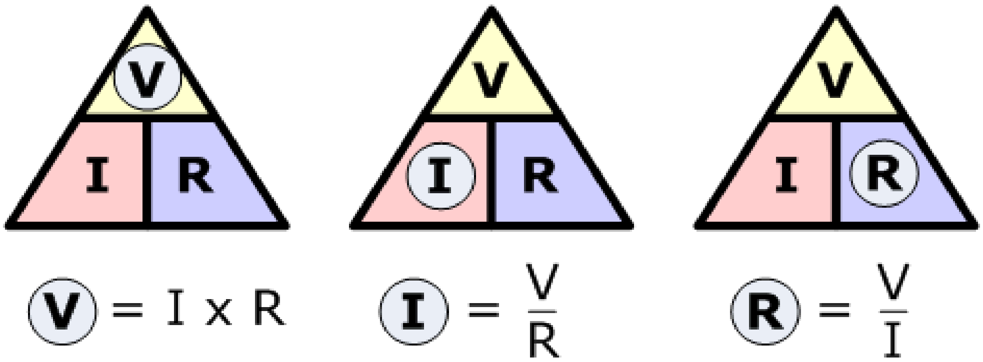
6 Find the accelerating force when a car having a mass of 1.7 Mg increases its speed with a constant acceleration of 3 m/s^2 [5.1 kN]

7 A force of 40 N accelerates a mass at 5 m/s^2. Determine the mass. [8 kg]

Ohm's Law:

The ratio of potential difference (V) between any two points on a conductor to the current (I) flowing between them, is constant. Provided the temperature of the conductor does not change.

In other words, $\frac{\text{V}}{\text{I}} = \text{constant}$ OR $\frac{V}{I} = R$



Resistance in Series:

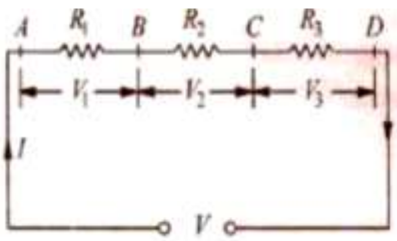


Fig. 1a

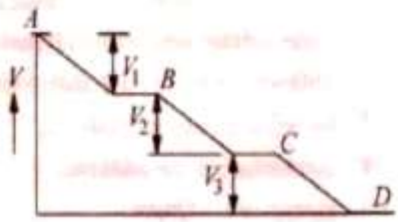


fig.1b

According to ohm's law;

V= V1+V2+V3

But $V = IR$ (Ohm's Law)

IR = IR1 + IR2 +IR3 =I (R1 + R2 + R3).

R eq =R1+R2+R3

As seen from above, the main characteristics of a series circuit are :

1. Same current flows through all parts of the circuit.

2. Different resistors have their individual voltage drops.

3. Voltage drops are additive.

4. Applied voltage equals the sum of different voltage drops.

5. Resistances are additive.

6. Powers are additive.

Voltage Divider Rule:

Since in a series circuit, same current flows through each of the given resistors, voltage drop varies directly with its resistance.
From fig 2:

The total resistance: $\boldsymbol{Rt} = R_1 + R_2 + R_3 = 12\ \Omega$

According to Voltage Divider Rule, various voltage drops are:

V1= V $\frac{R1}{R1+R2+R3}$. , V2= V $\frac{R2}{R1+R2+R3}$. , V3= V $\frac{R3}{R1+R2+R3}$.

$V_1 = V.\frac{R_1}{Rt} = 24 \times \frac{2}{12} = 4\ V$

$V_2 = V.\frac{R_2}{Rt} = 24 \times \frac{4}{12} = 8\ V$

$V_3 = V.\frac{R_3}{Rt} = 24 \times \frac{6}{12} = 12\ V$

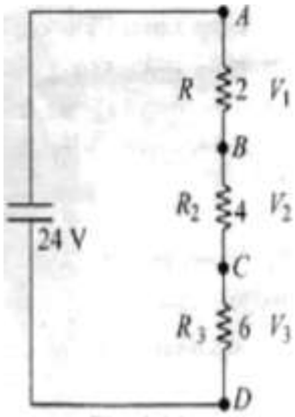


Fig. 2

<u>Example</u> : Using voltage divider rule (V.D.R.) ,find V_1 , V_2 , V_3 And V4 from fig. 5

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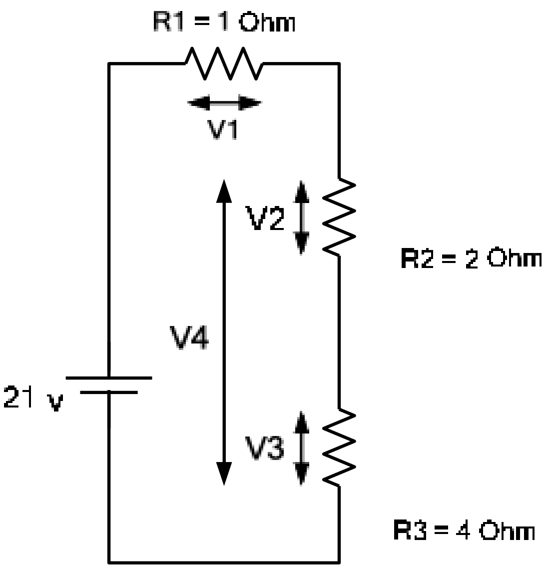


Fig. 5

$$V_1 = V \frac{R_1}{R_1 + R_2 + R_3} = 21 \frac{1}{1 + 2 + 4} = 3 \text{ v}$$

$$V_2 = V \frac{R_2}{R_1 + R_2 + R_3} = 21 \frac{2}{1 + 2 + 4} = 6 \text{ v}$$

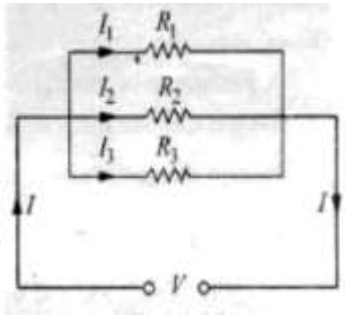
$$V_3 = V \frac{R_3}{R_1 + R_2 + R_3} = 21 \frac{4}{1 + 2 + 4} = 12 \text{ v}$$

$$V_4 = V \frac{R_2 + R_3}{R_1 + R_2 + R_3} = 21 \frac{2 + 4}{1 + 2 + 4} = 18 \text{ v}$$

Resistances in Parallel:

The main characteristics of a parallel circuit are:
I. same voltage acts across all parts of the circuit
2. Different resistors have their individual current.
3. Branch currents are additive.
4. Conductance are additive.
5. Powers are additive.

Fig.3



Three resistances, as joined in Fig. 3 are said to be connected in parallel. In this case (i) p.d. across all resistances is the same (ii) current in each resistor is different and is given by Ohm's Law and (iii) the total current i.. The sum of the three separate currents.

$$I = I_1 + I_2 + I_3, \qquad I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

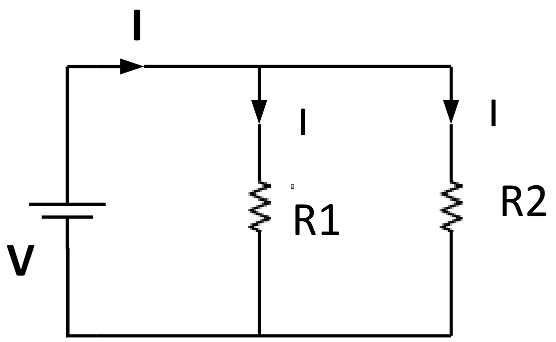
Now $I = \frac{V}{R}$ Where V is the applied voltage.

R = equivalent resistance of the parallel combination

$$\frac{V}{Req} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} \quad \text{Or} \quad \frac{1}{Req} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

R eq = $\frac{R1R2}{R1+R2}$

Current Divider Rule:



I1= $\frac{V}{R1}$, I2= $\frac{V}{R2}$

V= I. Req.

V = I $\frac{R1R2}{R1+R2}$

$\therefore$ I1 = I $\frac{R2}{R1+R2}$

I2 = I $\frac{R1}{R1+R2}$

Example : Using current divider rule (C.D.R.) , calculate I_1 , I_2 and I_3 from fig. 6 .

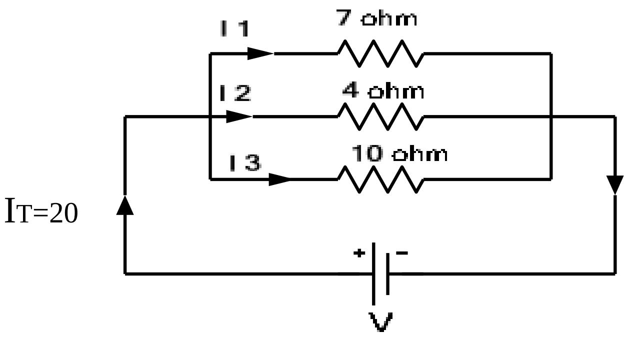


Fig. 6

To find I_1 , the other resistances are (4 // 10) .

$$\frac{4 \times 10}{4 + 10} = 2.857\ \Omega$$

$$I_1 = 20 \times \frac{2.857}{7 + 2.857} = 5.796A$$

To find I_2 , the other resistances are (7 // 10) .

$$\frac{7 \times 10}{7 + 10} = 4\ \Omega$$

$$I_2 = 20 \times \frac{4}{4 + 4} = 10\ A$$

To find I_3, the other resistances are (7 // 4) .

$$\frac{7 \times 4}{7 + 4} = 2.545\ \Omega$$

$$I_3 = 20 \times \frac{2.545}{2.545 + 10} = 4.057\ A$$

Example. *What is the value of the unknown resistor R in Fig.*4 *if the voltage drop across the 500* Ω *resistor is* 2.5 *volts? All resistances are in ohm.*

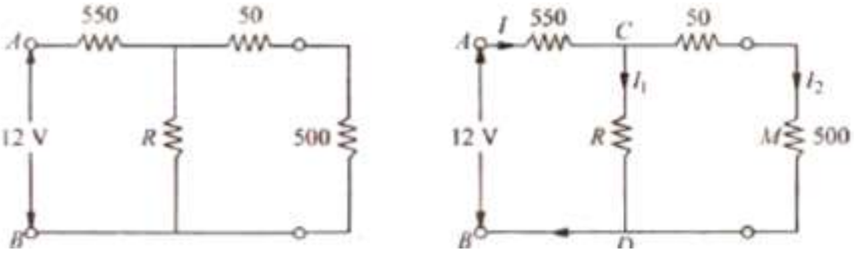


Fig. 4

Solution. By direct proportion, drop on 50 Ω resistance = 2.5 × 50/500 = 0.25 V

Drop across CMD or CD = 2.5 + 0.25 = 2.75 V

Drop across 550 Ω resistance = 12 − 2.75 = 9.25 V

I = 9.25/550 = 0.0168 A, I_2 = 2.5/500 = 0.005 A

I_1 = 0.0168 − 0.005 = 0.0118 A

∴ 0.0118 = 2.75/R; R = 233 Ω

Example. Calculate the effective resistance of the following combination of resistances and the voltage drop across each resistance when a P.D. of 60 V is applied between points A and B.

Resistance between A&C in fig. 5 is

$6\|3 = 2\ \Omega$

Resistance of branch ACD $= 18 + 2 = 20\ \Omega$

Now, there are two parallel paths between points *A* and *D* of resistances 20 Ω and 5Ω.

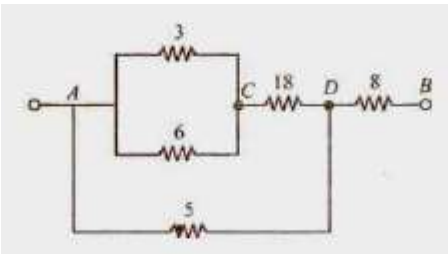


Fig 5

Hence, resistance between *A* and *D*= $20\|5 = 4\ \Omega$

:. Resistance between *A* and *B*= $4 + 8 = 12\ \Omega$

Total circuit current $= \frac{60}{12} = 5\ A$

Current in branch $ACD = 5 \times \frac{5}{25} = 1\ A$

. . P.D. across 3 Ω and 6 Ω resistors=1 x 2 =2 V

P.D. across 18 Ω resistors = 1 x 18=18 V
P.D. across 5 Ω resistors =4 x 5 = 20 V
P.D. across 8 Ω resistors = 5 x 8 =40 V

Example. Find R_{AB} in the circuit, given in Fig. 9.

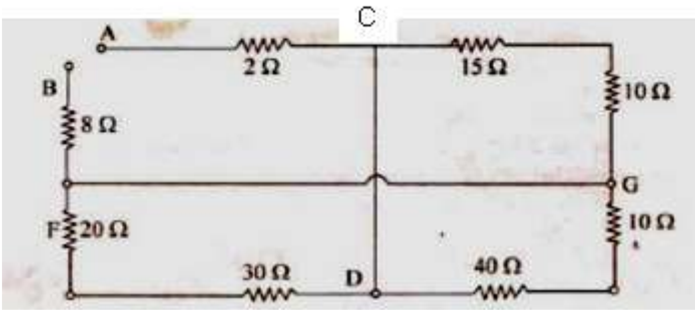


Fig.9

Ans. 22.5

Open and short circuit in series circuits

1. Open circuit :

In this case there are no current flows through the circuit as shown in fig. 1.

I = 0

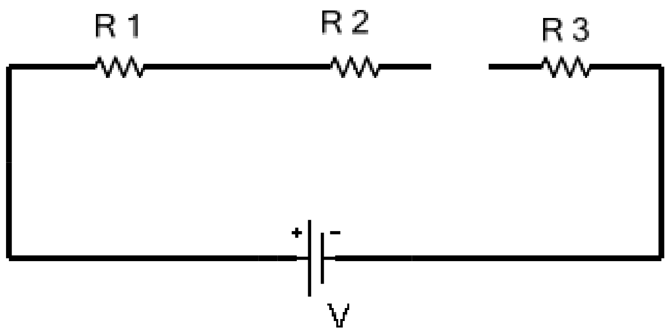


Fig. 1

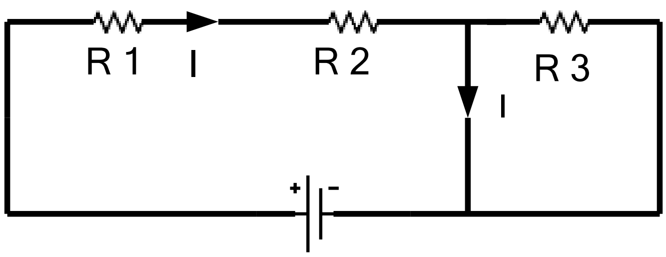
2. Short circuit :

If the resistance is short circuited, the current will flow through the short circuit. (No current flows through the shorted resistance)

As shown in fig. 2.

$$I = \frac{V}{R_1 + R_2}$$

Fig.2



Open and short circuit in parallel circuits

1. Open circuit:

In this case, there are no current flows in the open branch as shown in fig. 3.

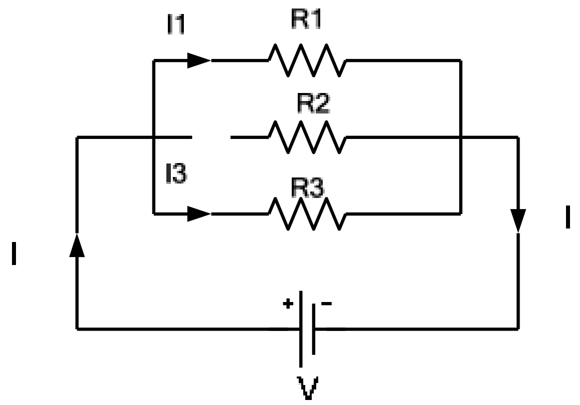


Fig. 3

$$I_2 = 0$$

$$I = I_1 + I_3$$

2. Short circuit:

In this case, there is no current flow through R_1, R_2 and R_3 because the total current. (I) pass through the short circuit as shown in fig. 4.

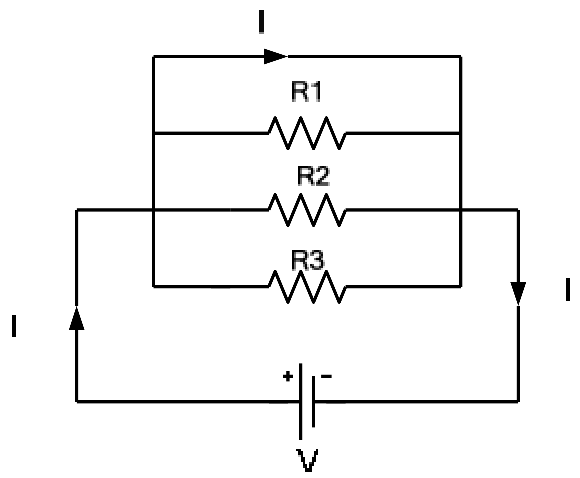


Fig. 4

$$I = \frac{V}{r_i}$$

Where r_i is the internal resistance of the battery.

Kirchhoff's laws

1. Kirchhoff's voltage law (KVL):

The algebraic sum of voltages in any closed loop is zero.

$\sum V = 0$

Now, from fig. 1, there are three equations according to Kirchhoff's voltage law.

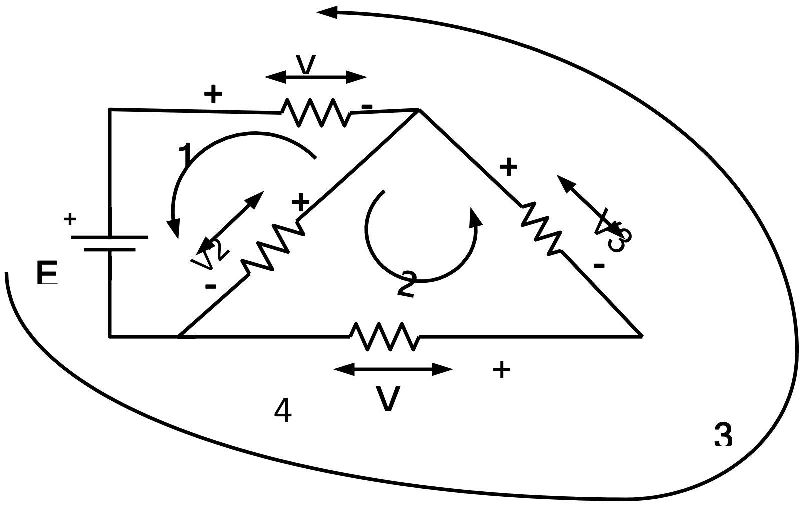


Fig. 1

Loop 1:

$E - V_1 - V_2 = 0$

$E = V_1 + V_2$ ------- (1)

Loop 2: $V_2 - V_3 - V_4 = 0$

$V_2 = V_3 + V_4$ -------- (2)

Loop 3:

$E - V_1 - V_3 - V_4 = 0$

$E = V_1 + V_3 + V_4$ --------- (3)

Example: For the circuit shown in fig. 2, using Kirchhoff's voltage law , find V_1 and V_2.

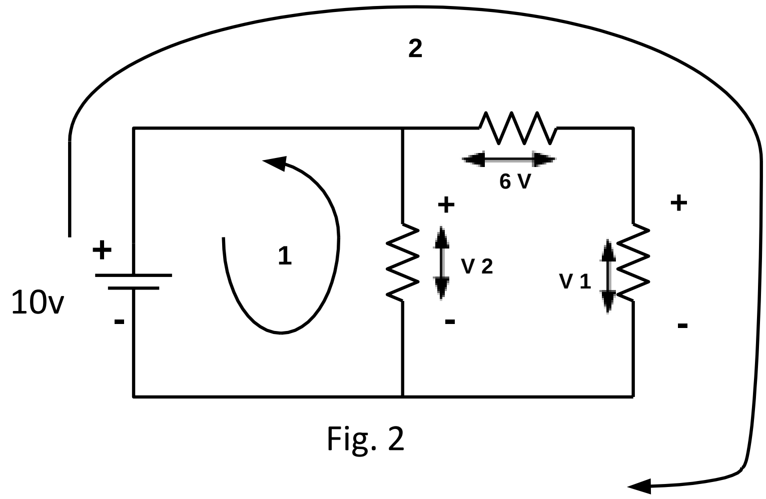


Fig. 2

Loop 1:

10 _ V2 = 0

V2 = 10V

Loop 2:

$10 - 6 - V_1 = 0$

$V_1 = 10 - 6 = 4V$

2- Kirchhoff's Current Law (KCL):

In any electrical network, the algebraic sum of currents meeting at a point (junction) is zero as shown in fig. 3.

$\sum I = 0$, OR $\Sigma I\ in = \Sigma I\ out$

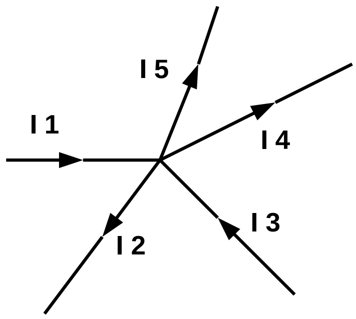


Fig. 3

$$I_1 + I_3 = I_2 + I_4 + I_5$$

$$I_1 + I_3 - I_2 - I_4 - I_5 = 0$$

Example : Using Kirchhoff's current law , find I_5 from fig. 4 .

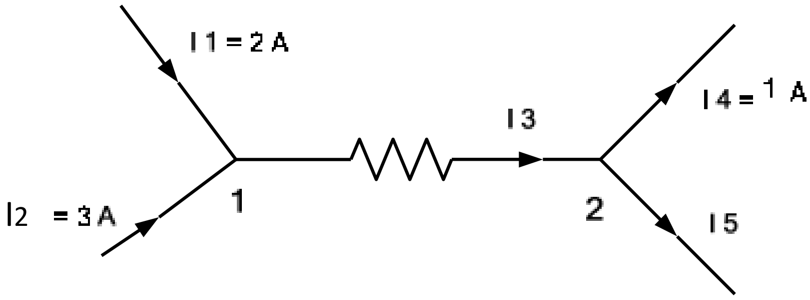


Fig. 4

At node 1:

$I_1 + I_2 = I_3$

$2 + 3 = 5$ A

$I_3 = I_4 + I_5$

$5 = 1 + I_5$

$I_5 = 5 - 1 = 4$ A

Example : Using Kirchhoff's law , find I_1 , I_2 and I_3 for the circuit shown in fig. 5 .

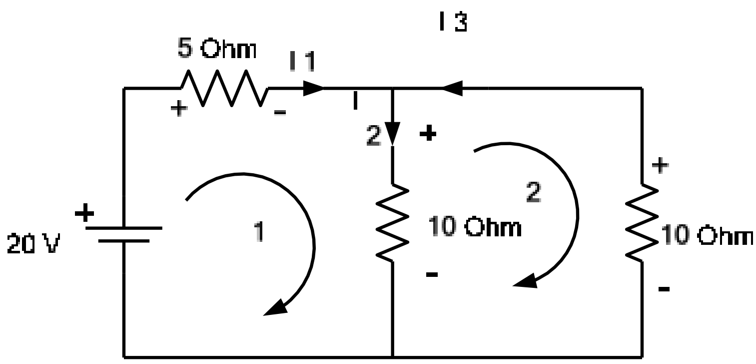


Fig.5

$I_1 = I_2 + I_3$ ---------- (1)

Loop 1:

$20 - 5 I_1 - 10 I_2 = 0$

$20 = 5 I_1 + 10 I_2$ /5

$I_1 + 2 I_2 = 4$ --------- (2)

Loop 2:

$- 10 I_2 + 10 I_3 = 0$

$I_2 = I_3$ -------------- (3)

From Equ. (2)

$I_1 = 4 - 2 I_2$ ------------- (4)

Sub. Equ. (3) And (4) in (1)

$4 - 2 I_2 = I_2 + I_2$

$4 I_2 = 4$

$I_2 = 1$ A

From Equ. (4)

$I_1 = 4 - (2 \times 1) = 2$ A

$I_3 = I_2$

$I_3 = 1$A

Delta/Star Transformation:

In solving networks (having considerable number of branches) by the application of Kirchhoff's Laws, one sometimes experiences great difficulty due to a large number of simultaneous equations that have to be solved. However, such complicated network can be simplified by successively replacing delta meshes by equivalent star system and vice versa.

The two arrangements shown in fig.10 will be electrically equivalent if the resistance as measured between any pair of terminals is the same in both the arrangements. Let us find this condition.

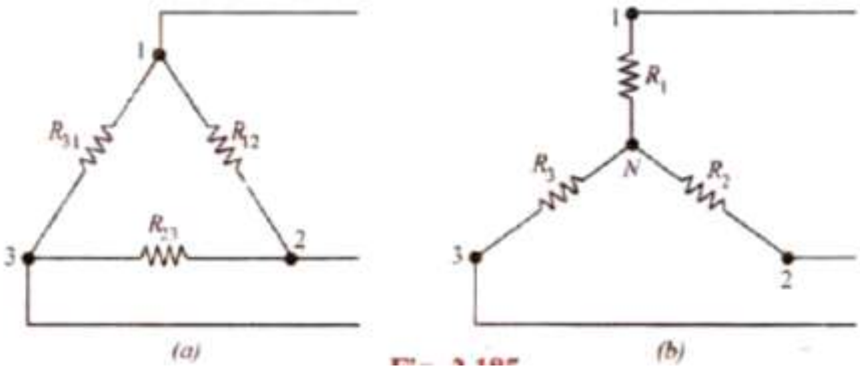


Fig.10

$$R_1 = \frac{R_{12}R_{31}}{R_{12}+R_{23}+R_{31}} \; ; R_2 = \frac{R_{12}R_{23}}{R_{12}+R_{23}+R_{31}} ; \; and$$

$$R_3 = \frac{R_{23}R_{31}}{R_{12}+R_{23}+R_{31}}$$

حاصل ضرب المقاومتين المحيطة على
مجموع المقاومات

Note that each resistor of the Y is equal to the product of the resistors in the two closest branches of the△ divided by the sum of the resistors in the△ .

Star/Delta Transformation:

This transformation can be easily done by using equations.

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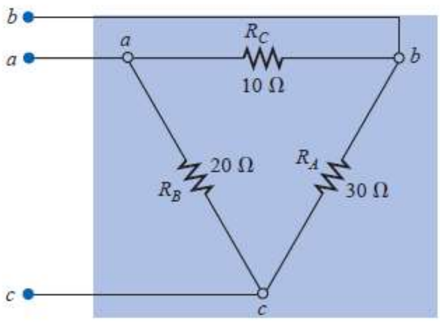
المقاومات على المقاومة البعيدة

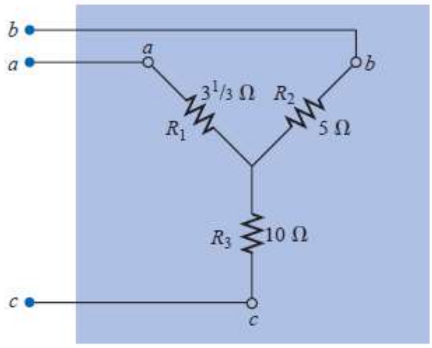
$$R_{12} = \frac{R1R2 + R1R3 + R2R3}{R_3} \; ; \quad R_{23} = \frac{R1R2 + R1R3 + R2R3}{R_1} \; ; \; and$$

$$R_{31} = \frac{R1R2 + R1R3 + R2R3}{R_2}$$

Note that the value of each resistor of the△ is equal to the sum of the possible product combinations of the resistances of the Y divided by the resistance of the Y farthest from the resistor to be determined.

EXAMPLE Convert the ▲ of Fig. To aY.





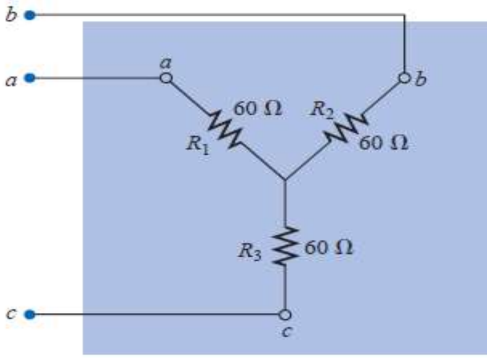
Solution:

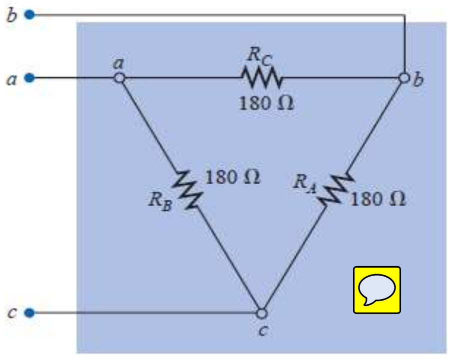
$$R_1 = \frac{R_B R_C}{R_A + R_B + R_C} = \frac{(20\ \Omega)(10\ \Omega)}{30\ \Omega + 20\ \Omega + 10\ \Omega} = \frac{200\ \Omega}{60} = \mathbf{3\tfrac{1}{3}\ \Omega}$$

$$R_2 = \frac{R_A R_C}{R_A + R_B + R_C} = \frac{(30\ \Omega)(10\ \Omega)}{60\ \Omega} = \frac{300\ \Omega}{60} = \mathbf{5\ \Omega}$$

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C} = \frac{(20\ \Omega)(30\ \Omega)}{60\ \Omega} = \frac{600\ \Omega}{60} = \mathbf{10\ \Omega}$$

EXAMPLE. Convert the Y of Fig. to a ▲.





Solution:

$$R_A = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_1}$$

$$= \frac{(60\ \Omega)(60\ \Omega) + (60\ \Omega)(60\ \Omega) + (60\ \Omega)(60\ \Omega)}{60\ \Omega}$$

$$= \frac{3600\ \Omega + 3600\ \Omega + 3600\ \Omega}{60} = \frac{10{,}800\ \Omega}{60}$$

$$R_A = \mathbf{180\ \Omega}$$

Example: *Find the input resistance of the circuit between the points A and B of Fig. 11.*

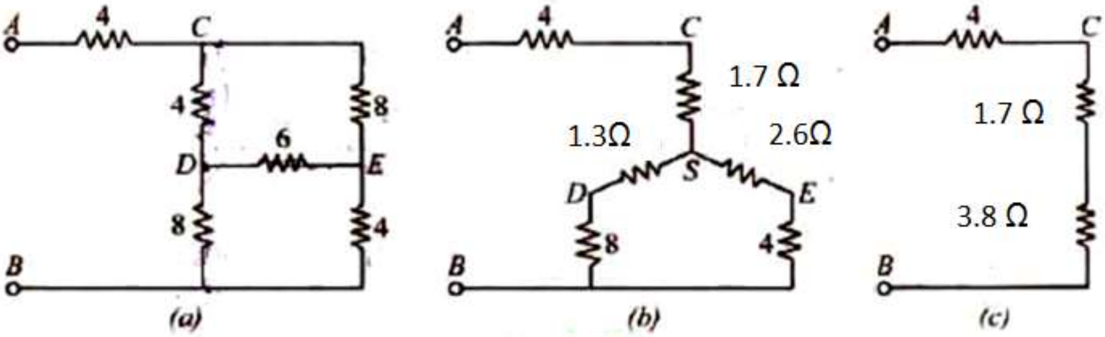


Fig.11

Solution. For finding R_{AB} we will convert the delta *CDE* of Fig. 11 *(a)* into its equivalent star as shown in Fig. 11 *(b).*

Rcs=$\frac{4x8}{18} = 1.7\ \Omega$, REs = $\frac{8x6}{18} = 2.6\ \Omega$, RDs = $\frac{4x6}{18} = 1.3\ \Omega$

2.6Ω + 4Ω = 6.6Ω , 1.3Ω +8Ω =9.3Ω, 6.6Ω//9.3Ω = $\frac{6.6x9.3}{6.6+9.3} = 3.8\ \Omega$

RT=4Ω+1.7Ω+3.8Ω = 9.5 Ω

Example. A bridge network ABCD has arms AB, BC, CD and DA of resistances 1, 1, 2 and 1 ohm respectively. If the detector AC has a resistance of 1 ohm, determine by delta / star transformation, the network resistance as viewed from the battery terminals.

Solution. As shown in Fig. 12 *(b),* delta *DAC* has been reduced to its equivalent star.

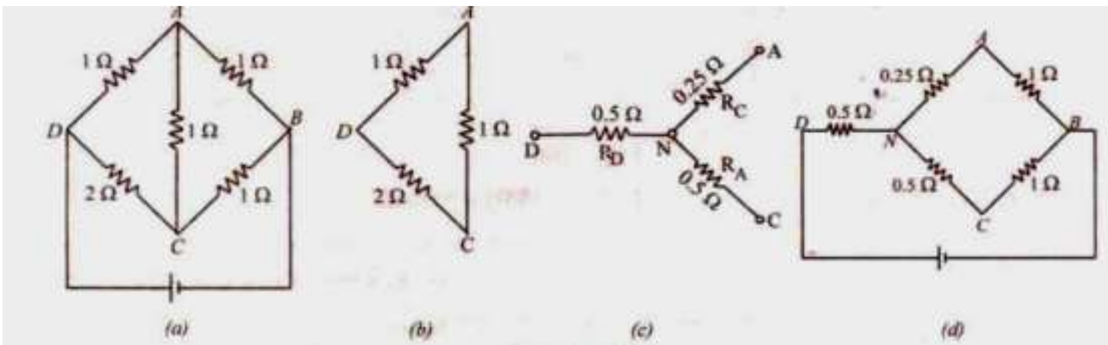


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Fig. 12

RD = $\frac{2x1}{2+1+1} = 0.5\Omega$, RC = $\frac{1}{4} = 0.25\Omega$, RA = $\frac{2}{4} = 0.5\Omega$

Hence, the original network of Fig. 12 *(a)* is reduced to the one shown in Fig. 12 *(d).* As seen, there are two parallel paths between points *N* and *B*, one of resistance 1.25Q and the other of resistance 1.5 Q. Their combined resistance is: 0.25Ω+1Ω=1.25Ω, P0.5Ω+1Ω=1.5Ω

∴ 1.25Ω // 1.5 Ω = $\frac{1.25\,x1.5}{1.25+1.5}$ = 0.68 Ω

Total resistance of the network between points *D* and *B* is:

∴**RT=** 0.5 Ω + 0.68 Ω = 1.18 Ω

Example. *Use delta-star conversion to find resistance between terminals 'AB' of the circuit shown in Fig.13 (a). All resistances are in ohms.*

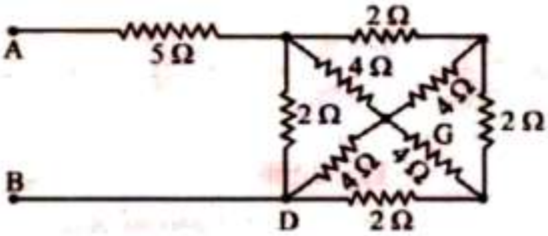
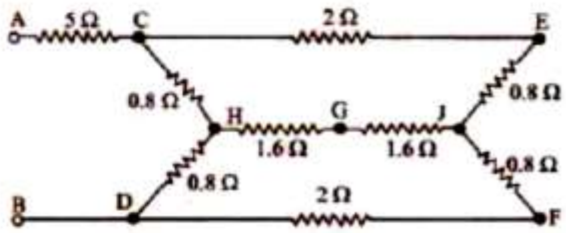
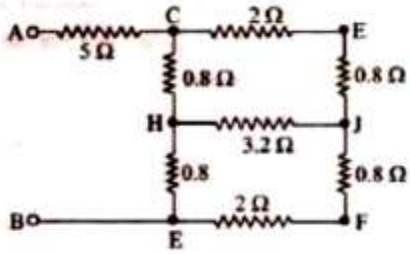


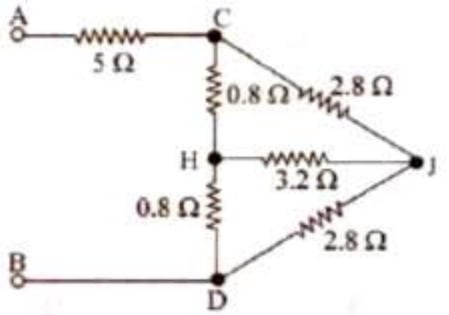
Fig.13 a



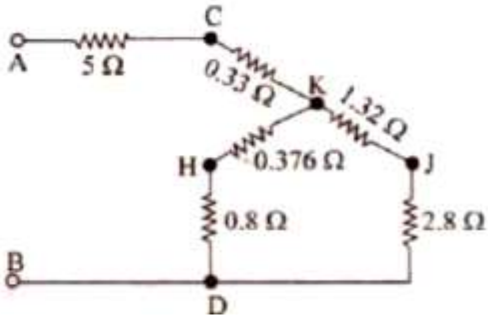
b



c



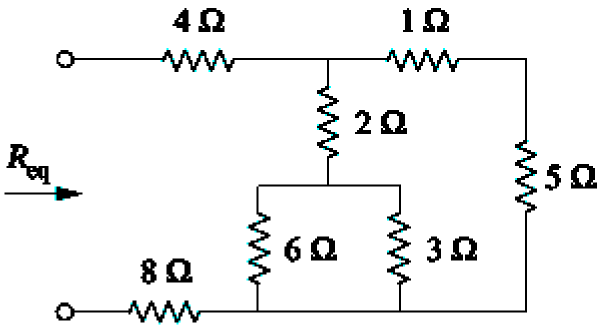
d

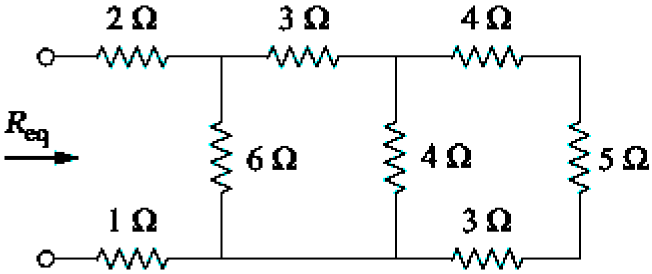


e

Exercises:

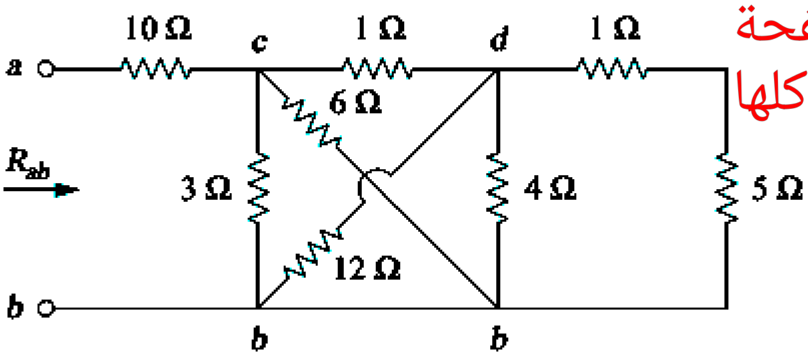
1- Find R_{eq} for the circuit shown: Ans. = 14.4Ω



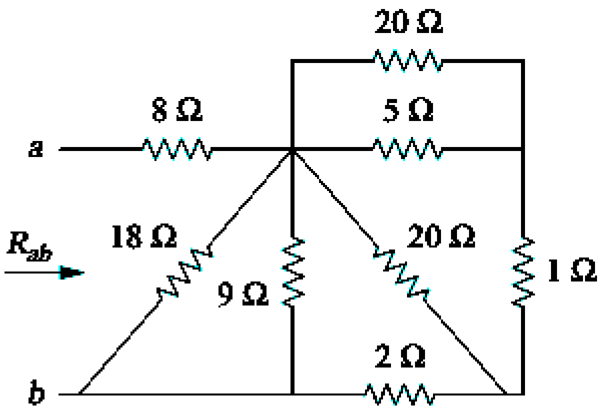


Ans. = 6Ω

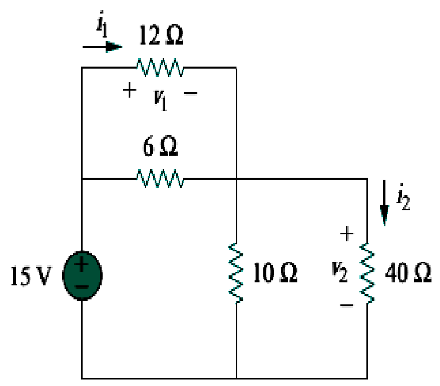
واجب الصفحة كلها



Ans.=11.2Ω

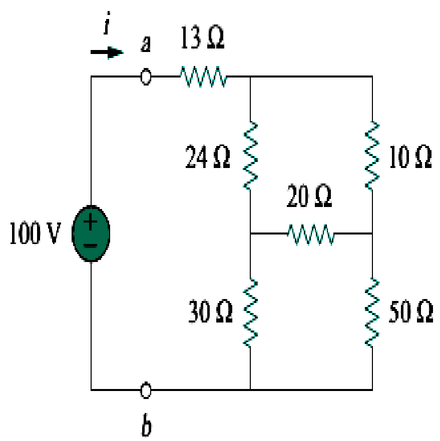


Ans. =11Ω



Find v_1 and v_2 in the circuit shown in Fig. 2.43. Also calculate i_1 and i_2 and the power dissipated in the 12-Ω and 40-Ω resistors.

Answer: $v_1 = 5$ V, $i_1 = 416.7$ mA, $p_1 = 2.083$ W, $v_2 = 10$ V, $i_2 = 250$ mA, $p_2 = 2.5$ W.



For the bridge network in Fig. 2.54, find R_{ab} and i.

Answer: 40 Ω, 2.5 A.

Thevenins Theorem:

The current flowing through load resistance R_L connected across any two terminals A and B of a network as shown in fig. 1 is given by:

$$I_L = \frac{V_{th}}{R_{th} + R_L}$$

Where:-

V_{th} is the open circuit voltage across the two terminals A and B where R_L is removed.

R_{th} is the internal resistance of the network as viewed back into the network from terminals A and B with voltage source replaced by its internal resistance , while current source replaced by open circuit .

R_L load resistor

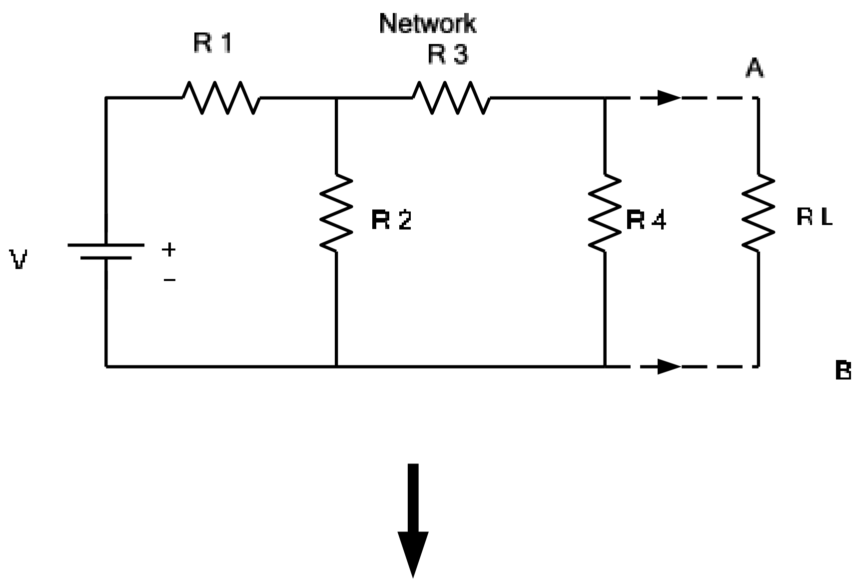
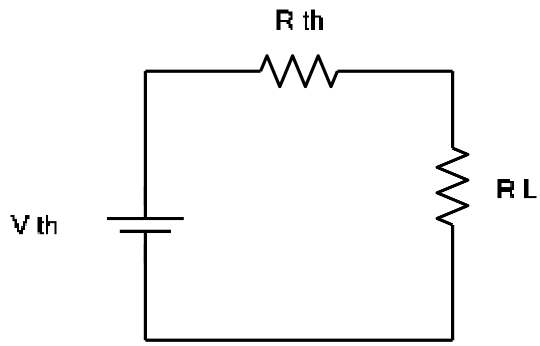


Fig. 1



Thevenins

equivalent
circuit

Now, R_{th} and V_{th} must be found. R_{th} could be

found as follows:

1. Replace voltage source by short circuit (if there is no internal resistance), while the current source replaced by open circuit.
2. Remove R_L from the circuit, then calculate R_{th} viewed from terminals A and B.

V_{th} could be found as follows:

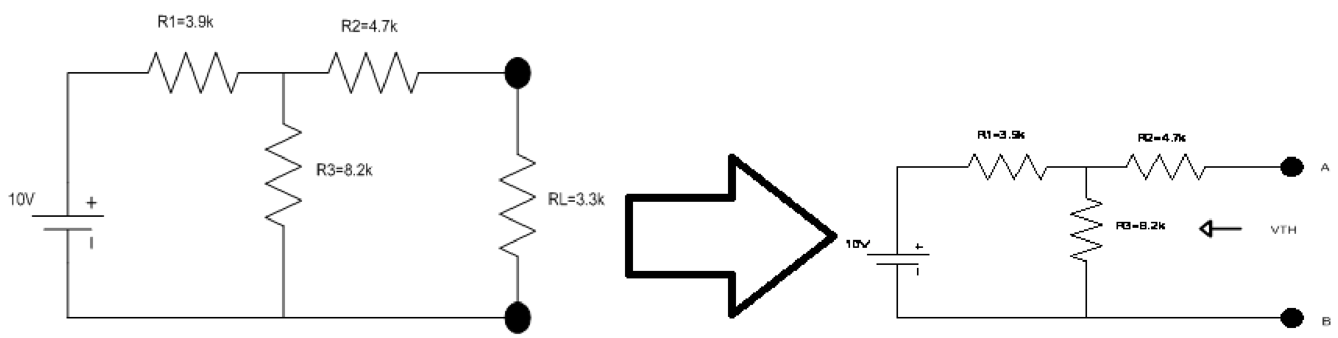
1. Remove R_L and make sure that the voltage or current source is connected.

2. Calculate V_{th} between points A and B.

Example1 : Using Thevenins theorem , find I_L in the circuit shown below .

Using Thevenins theorem:-

1. Remove R_L and Calculate V_{th} between points A and B.



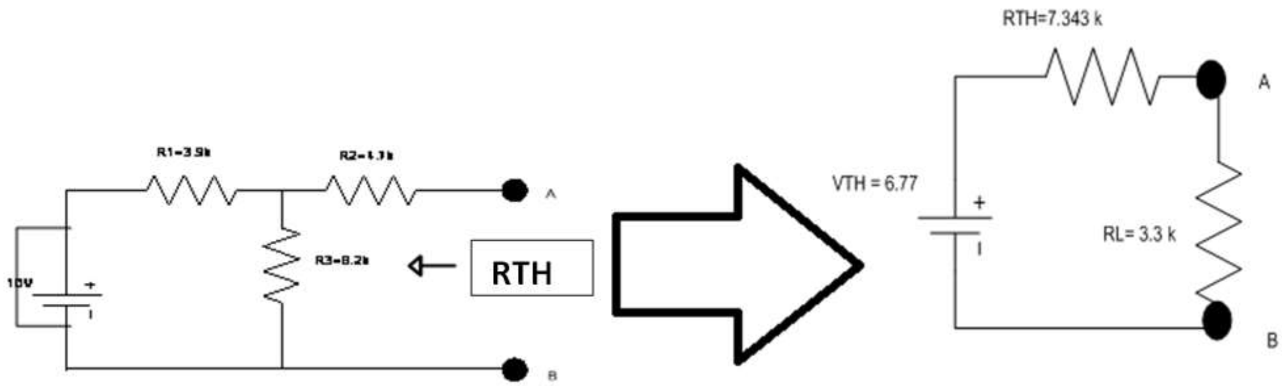
$$VTH = \frac{V}{R1+R3} R3 = \frac{10}{3.9+8.2} 8.2 = 6.77\ V$$

RTH:-

1. Replace voltage source by short circuit (if there is no internal resistance), while the current source replaced by open circuit.

RTH = R1 // R3 + R2

$$RTH = \frac{8.2*3.9}{8.2+3.9} + 4.7 = 7.343\ k\Omega$$



$$IL = \frac{VTH}{RTH+RL} = \frac{6.77}{7.343+3.3} = 0.636\ mA$$

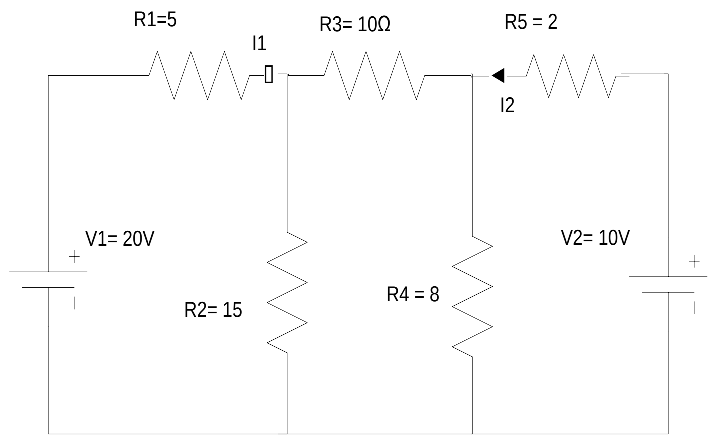
VL = IL * RL = 0.636mA * 3.3 kΩ = 1.908 V

Example 2: Using Thevenins theorem, find current flow R3 in the circuit shown below

Sol: find voltage between point A and B.

$$I_1 = \frac{V1}{R1+R2} = \frac{20}{5+15} = 1A$$

$V_{A,B}$ Left = I_1 * R2 = 1*15 = 15V

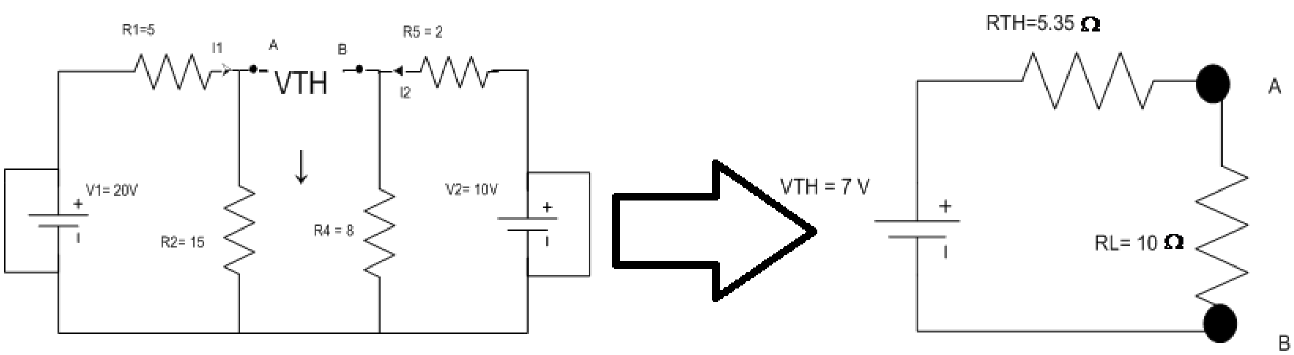


$$I_2 = \frac{V2}{R5+R4} = \frac{10}{2+8} = 1A$$

$V_{A.B}$ Right = I_2 * R4 = 1 * 8 = 8V

VTH = $V_{A,B}$ L - $V_{A.B}$ R = 15 – 8 = 7V

RTH = R1 // R2 + R5 // R4



$$RTH = \frac{5*15}{5+15} + \frac{2*8}{2+8} = 5.35\ \Omega$$

$$IL = \frac{VTH}{RTH+R3} = \frac{7}{5.35+10} = 0.46\ A$$

EXAMPLE3: - Find the Thevenins equivalent circuit for the network in the shaded area of the network of Fig. 9.27. Then find the current through *RL* for values of 2 Ω, 10Ω, and 100 **Ω.**

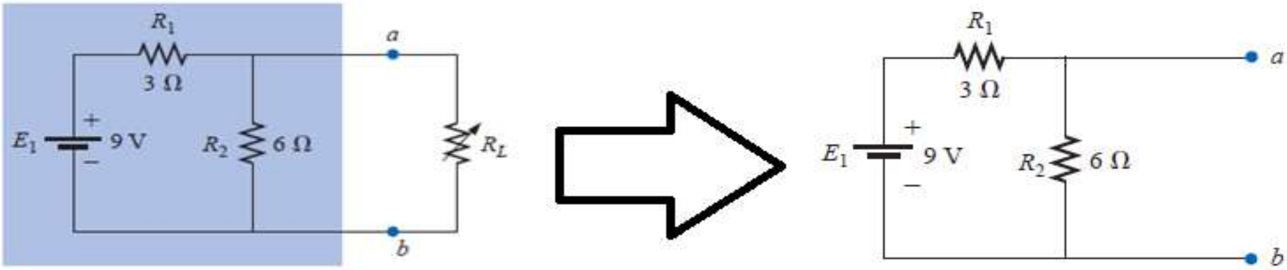
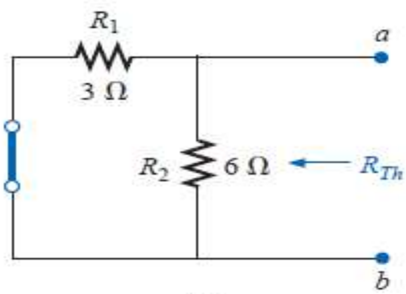
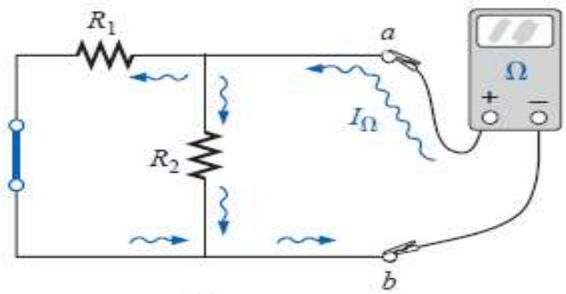


FIG. 9.27

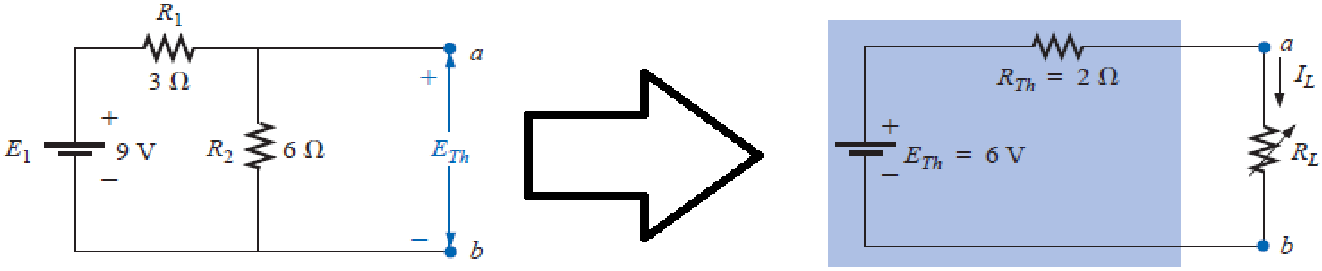


(a)



(b)

$$R_{Th} = R_1 \parallel R_2 = \frac{(3\ \Omega)(6\ \Omega)}{3\ \Omega + 6\ \Omega} = 2\ \Omega$$



$$E_{Th} = \frac{R_2 E_1}{R_2 + R_1} = \frac{(6\ \Omega)(9\ \text{V})}{6\ \Omega + 3\ \Omega} = \frac{54\ \text{V}}{9} = 6\ \text{V}$$

$$I_L = \frac{E_{Th}}{R_{Th} + R_L}$$

$R_L = 2\ \Omega$: $$I_L = \frac{6\text{ V}}{2\ \Omega + 2\ \Omega} = \mathbf{1.5\ A}$$

$R_L = 10\ \Omega$: $$I_L = \frac{6\text{ V}}{2\ \Omega + 10\ \Omega} = \mathbf{0.5\ A}$$

$R_L = 100\ \Omega$: $$I_L = \frac{6\text{ V}}{2\ \Omega + 100\ \Omega} = \mathbf{0.059\ A}$$

EXAMPLE 4 Find the Thevenins equivalent circuit for the network in the shaded area of the network of Fig. 9.33.

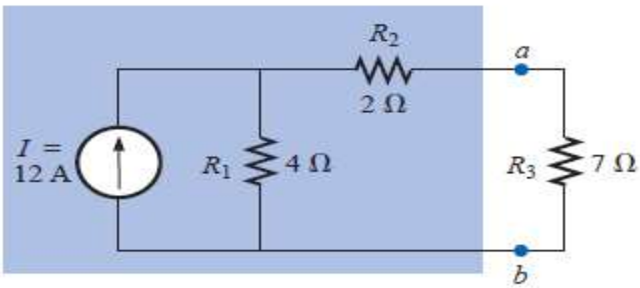
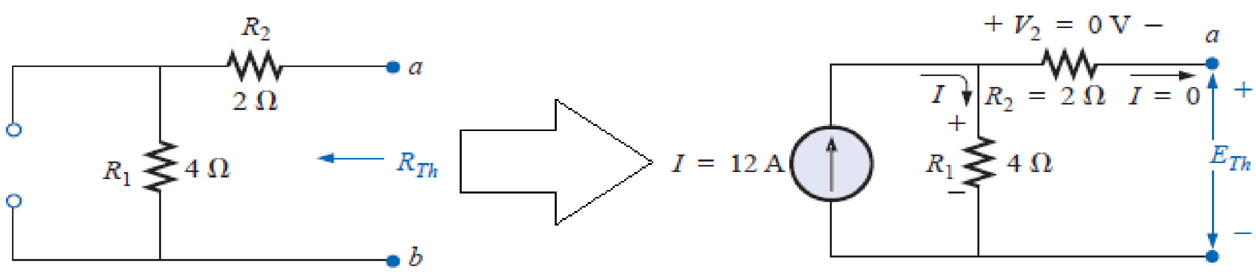


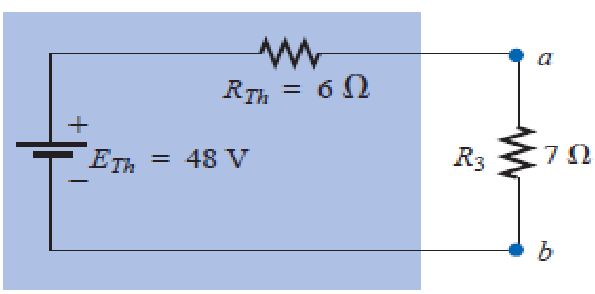
FIG. 9.33



$$R_{Th} = R_1 + R_2 = 4\ \Omega + 2\ \Omega = \mathbf{6\ \Omega}$$

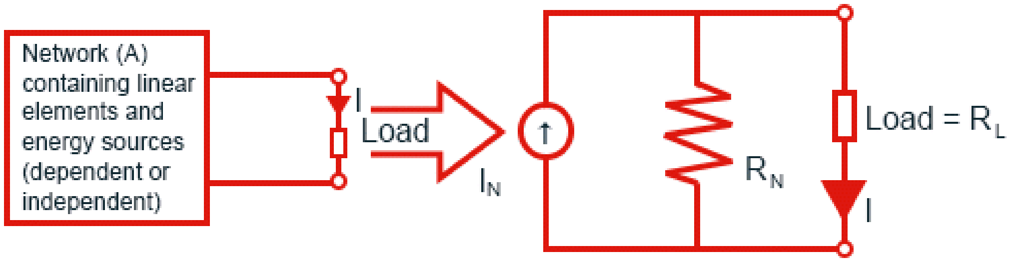
$$V_2 = I_2 R_2 = (0)R_2 = 0\text{ V}$$

$$E_{Th} = V_1 = I_1 R_1 = I R_1 = (12\text{ A})(4\ \Omega) = \mathbf{48\ V}$$



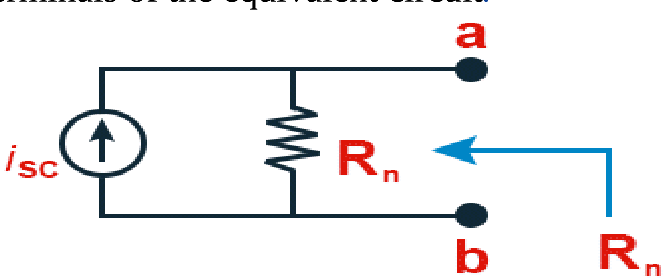
NORTON'S THEOREM

The theorem states the following:
Any two-terminal linear bilateral dc network can be replaced by an equivalent circuit consisting of a current source and a parallel resistor, as shown in Fig.
The discussion of Thevenins theorem with respect to the equivalent circuit can also be applied to the Norton equivalent circuit. The steps leading to the proper values of IN and RN are now listed.



Preliminary:
1. Remove that portion of the network across which the Norton equivalent circuit is found.
2. Mark the terminals of the remaining two-terminal network. RN:
3. Calculate RN by first setting all sources to zero (voltage sources are replaced with short circuits and current sources with open circuits) and then finding the resultant resistance between the two marked terminals. (If the internal resistance of the voltage and/or current sources is included in the original network, it must remain when the sources are set to zero.) Since RN _ RTh, the procedure and value obtained using the approach described for Thevenins theorem will determine the proper value of RN. IN:
4. Calculate IN by first returning all sources to their original position and then finding the short-circuit current between the marked terminals. It is the same current that would be measured by an ammeter placed between the marked terminals.

5. Draw the Norton equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the equivalent circuit.



The Norton and Thevenins equivalent circuits can also be found from each other by using the source transformation discussed earlier in this chapter and reproduced in Fig. 9.5

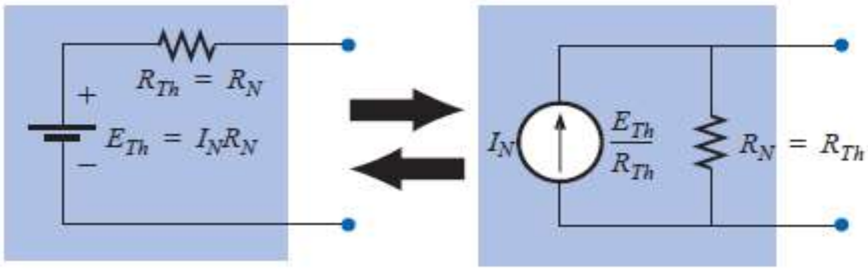


FIG. 9.59
Converting between Thévenin and Norton equivalent circuits.

Example 1 *Using Norton's theorem, find the constant-current equivalent of the circuit shown in Fig. 2.204 (a).*

Solution. When terminals *A* and *B* are short-circuited as shown in Fig. 2.204 (*b*), total resistance of the circuit, as seen by the battery, consists of a 10 Ω resistance in series with a parallel combination of 10 Ω and 15 Ω resistances.

$$\therefore \quad \text{total resistance} = 10 + \frac{15 \times 10}{15 + 10} = 16\ \Omega$$

$$\therefore \quad \text{battery current} \quad I = 100/16 = 6.25\ \text{A}$$

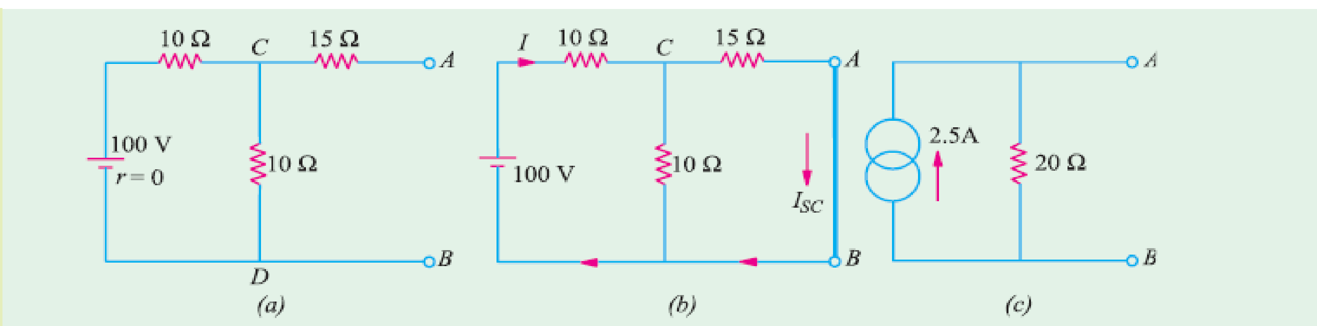


Fig. 2.204

This current is divided into two parts at point *C* of Fig. 2.204 (*b*).

Current through *A B* is $I_{SC} = 6.25 \times 10/25 = 2.5$ A

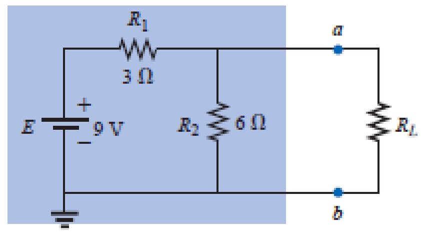
Since the battery has no internal resistance, the input resistance of the network when viewed from *A* and *B* consists of a 15 Ω resistance in series with the parallel combination of 10 Ω and 10 Ω Hence, $R_1 = 15 + (10/2) = 20\ \Omega$

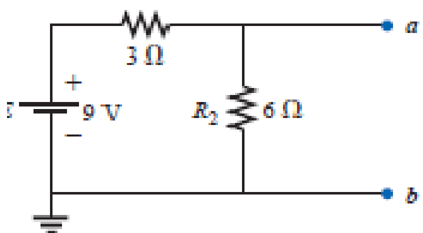
Hence, the equivalent constant-current source is as shown in Fig. 2.204 (*c*).

EXAMPLE:-2 find the Norton equivalent circuit for the network in the shaded area of Fig.

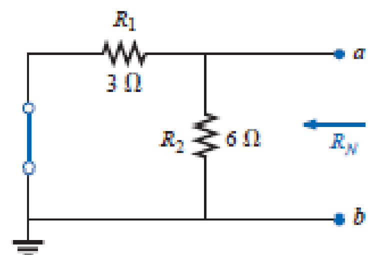
Solution:

Steps 1 and 2 are shown in Fig.





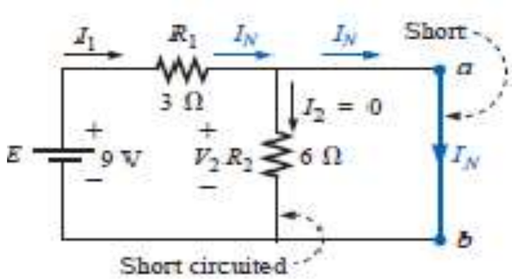
Step 3 is shown in Fig.



R_N R_1 // R_2 = $\frac{3*6}{3+6} = 2\ \Omega$

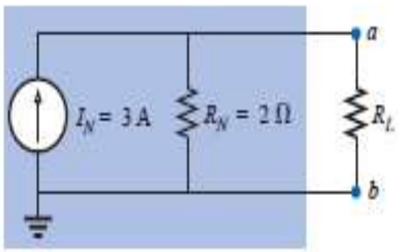
Step 4 is shown in Fig, clearly indicating that the short-circuit Connection between terminals *a* and *b* is in parallel with *R*2 and eliminates its effect. *IN* is therefore the same as through *R*1, and the full battery voltage appears across *R*1 since

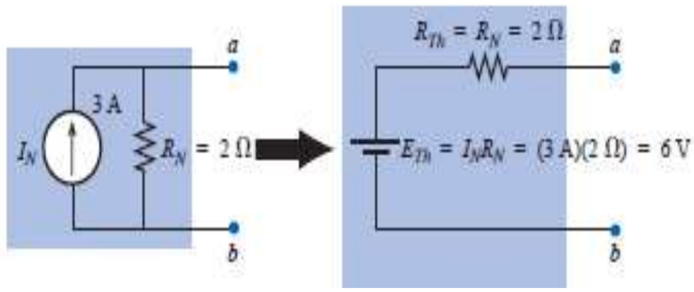
$V_2 = I_2 R_2 = 0 * 6 = 0$ V



Therefore, $I_N = \frac{E}{R1} = \frac{9v}{3\Omega} = 3A$

Step 5: See Fig. 9.64. This circuit is the same as the first one considered in the development of Thevenins theorem. A simple conversion indicates that the Thevenins circuits are, in fact, the same (Fig.).





EXAMPLE:-3 find the Norton equivalent circuit for the network external to the 9-Ω Resistor in Fig. 9.66.

Solution:

Steps 1 and 2: See Fig. 9.67.

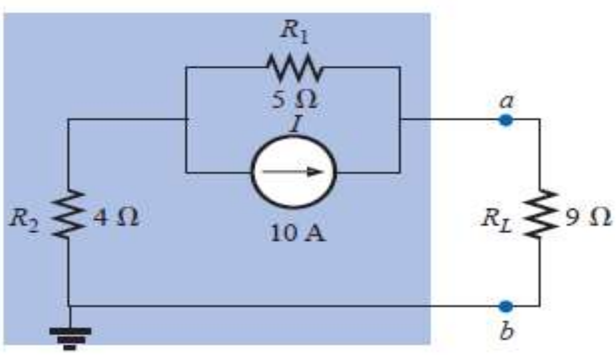


FIG. 9.66

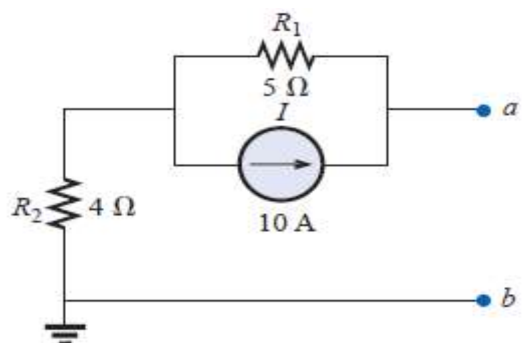


FIG. 9.67

Step 3: See Fig. 9.68

$R_N = R_1 + R_2 = 5\Omega + 4\Omega = \mathbf{9}\ \Omega$

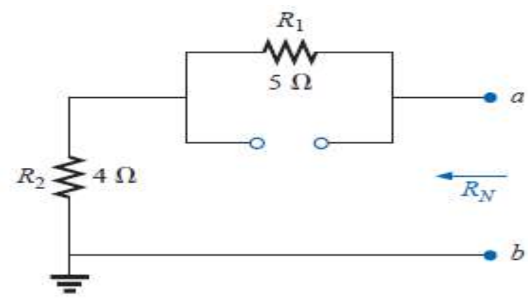


FIG. 9.68

Step 4: As shown in Fig. 9.69, the Norton current is the same as the current through the 4-Ω resistor. Applying the current divider rule,

$$I_N = \frac{R_1 I}{R_1 + R_2} = \frac{(5\ \Omega)(10\ \text{A})}{5\ \Omega + 4\ \Omega} = \frac{50\ \text{A}}{9} = \mathbf{5.556\ A}$$

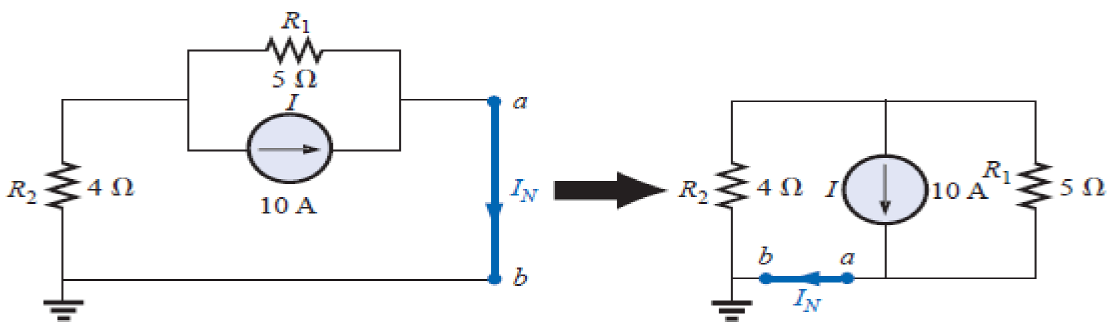


FIG. 9.69

Step 5: See Fig. 9.70.

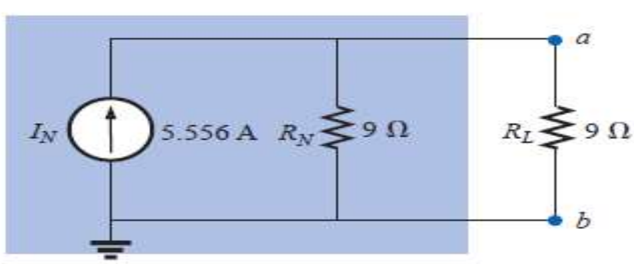


FIG. 9.70

EXAMPLE:-4 (Two sources) Find the Norton equivalent circuit for the portion of the network to the left of *a*-*b* in Fig. 9.71.

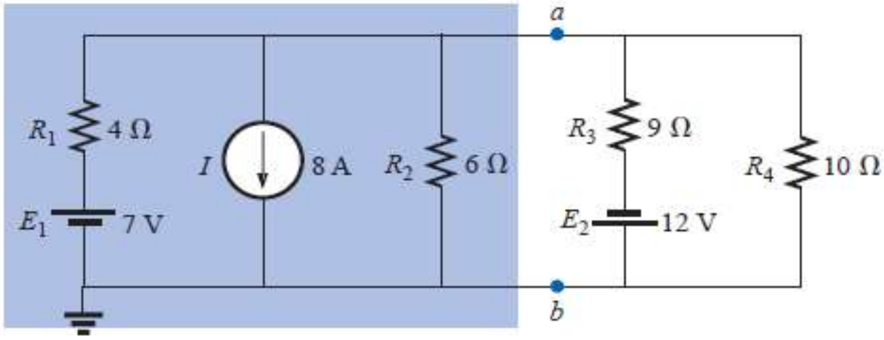


FIG. 9.71

Solution:

Steps 1 and 2: See Fig. 9.72.

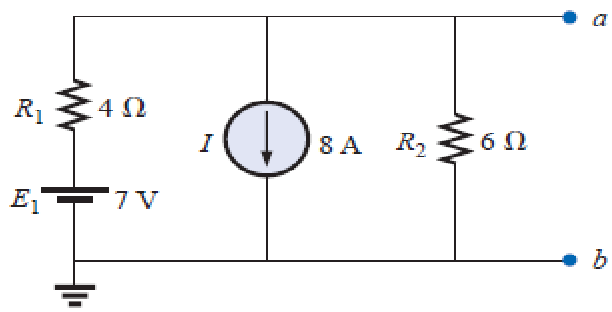


FIG. 9.72

Step 3 is shown in Fig. 9.73, and

$$R_N = R_1 \parallel R_2 = 4\ \Omega \parallel 6\ \Omega = \frac{(4\ \Omega)(6\ \Omega)}{4\ \Omega + 6\ \Omega} = \frac{24\ \Omega}{10} = \mathbf{2.4\ \Omega}$$

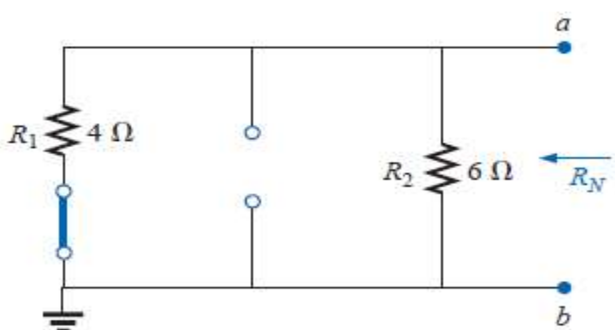


FIG. 9.73

Step 4: **(Using superposition) for the 7-V battery (Fig. 9.74)**

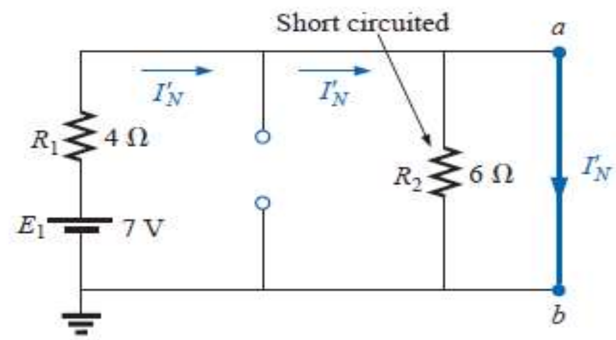


FIG. 9.74

$$I'_N = \frac{E_1}{R_1} = \frac{7\ \text{V}}{4\ \Omega} = 1.75\ \text{A}$$

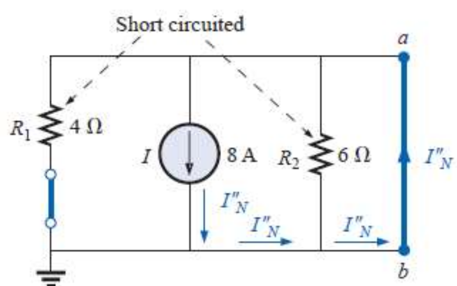


FIG. 9.75
Determining the contribution to I_N from the current source I.

For the 8-A source (Fig. 9.75), we find that both R_1 and R_2 have been "short circuited" by the direct connection between *a* and *b*, and

$$I''_N = I = 8\ \text{A}$$

The result is

$$I_N = I''_N - I'_N = 8\ \text{A} - 1.75\ \text{A} = \mathbf{6.25\ A}$$

Step 5: See Fig. 9.76.

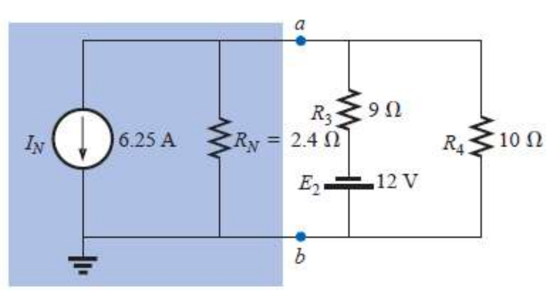
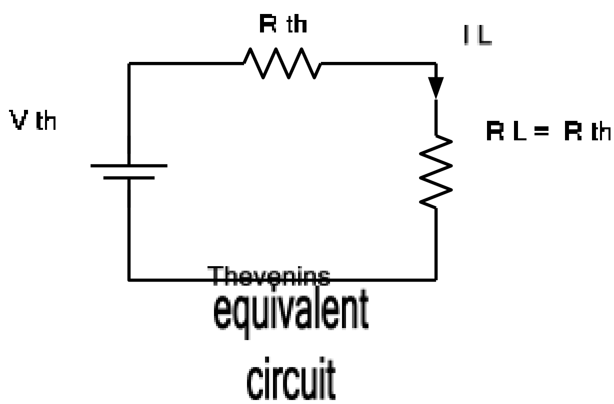


FIG. 9.76

Maximum power transfer theorem:

A resistor load will abstract maximum power from a network when the load resistance is equal to the resistance of the network as viewed from the output terminals with all voltage sources removed leaving behind their internal resistances and all current sources replaced by open circuit.



$R_L = R_{th}$

$$I_L = \frac{V_{th}}{R_{th} + R_L}$$

$$I_L = \frac{V_{th}}{2 R_{th}}$$

$P = (I_L)^2 \times R_{th}$

$$P_{max} = \frac{(V_{th})2}{4 (R_{th})^2} \times R_{th} \quad , \quad P\ max = \frac{(Vth)^2}{4\ Rth}$$

Example 2.115. *In the network shown in Fig. 2.231 (a), find the value of R_L such that maximum possible power will be transferred to R_L. Find also the value of the maximum power and the power supplied by source under these conditions.* **(Elect. Engg. Paper I Indian Engg. Services)**

Solution. We will remove R_L and find the equivalent Thevenin's source for the circuit to the left of terminals *A* and *B*. As seen from Fig. 2.231 (*b*) V_{th} equals the drop across the vertical resistor of 3Ω because no current flows through 2 Ω and 1 Ω resistors. Since 15 V drops across two series resistors of 3 Ω each, $V_{th} = 15/2 = 7/5$ V. Thevenin's resistance can be found by replacing 15 V source with a short-circuit. As seen from Fig. 2.231 (*b*), $R_{th} = 2 + (3 \,||\, 3) + 1 = 4.5\ \Omega$ Maximum power transfer to the load will take place when $R_L = R_{th} = 4.5\ \Omega$

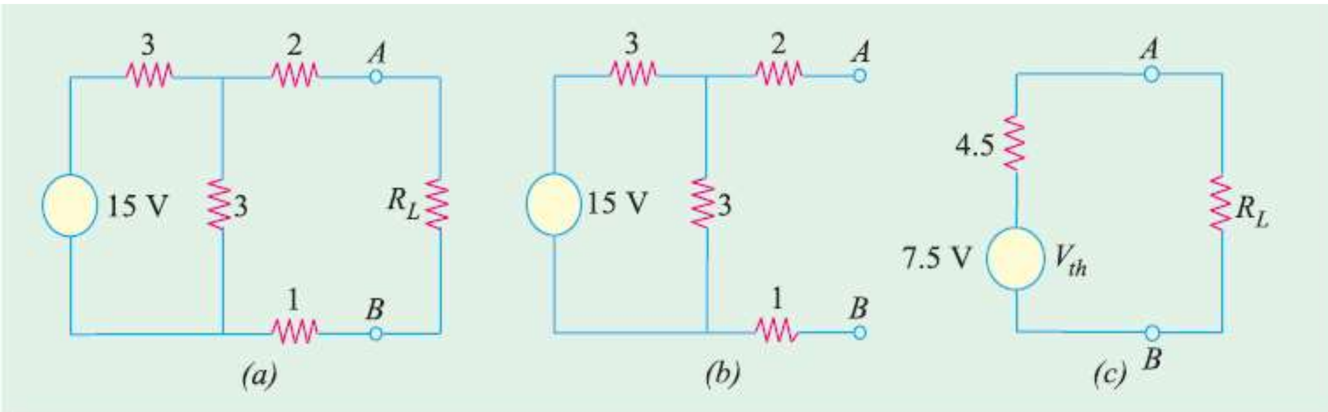


Fig. 2.231

Maximum power drawn by $R_L = V_{th}^2/4 \times R_L = 7.5^2/4 \times 4.5 = 3.125$ W.

Since same power in developed in R_{th}, power supplied by the source = 2 × 3.125 = 6.250 W.

Example 2.116. *In the circuit shown in Fig. 2.232 (a) obtain the condition from maximum power transfer to the load R_L. Hence determine the maximum power transferred.*

(Elect. Science-I Allahabad Univ. 1992)

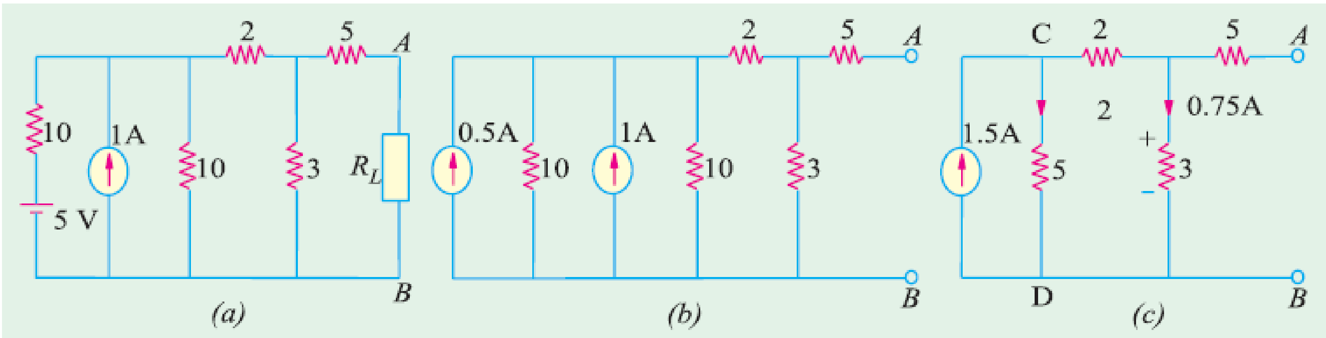


Fig. 2.232

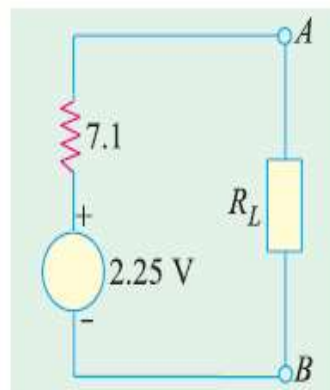


Fig. 2.233

Solution. We will find Thevenin's equivalent circuit to the left of trminals *A* and *B* for which purpose we will convert the battery source into a current source as shown in Fig. 2.232 (*b*). By combining the two current sources, we get the circuit of Fig. 2.232 (*c*). It would be seen that open circuit voltage V_{AB} equals the drop over 3 Ω resistance because there is no drop on the 5 Ω resistance connected to terminal *A*. Now, there are two parallel path across the current source each of resistance 5 Ω Hence, current through 3 Ω resistance equals 1.5/2 = 0.75 A. Therefore, $V_{AB} = V_{th} = 3 \times 0.75 = 2.25$ V with point *A* positive with respect to point *B*.

For finding R_{AB}, current source is replaced by an infinite resistance.

$$\therefore \quad R_{AB} = R_{th} = 5 + 3 \,||\, (2 + 5) = 7.1\ \Omega$$

The Thevenin's equivalent circuit alongwith R_L is shown in Fig. 2.233. As per Art. 2.30, the condition for *MPT* is that $R_L = 7.1\ \Omega$

Maximum power transferred = $V_{th}^2 /\ 4R_L = 2.25^2/4 \times 7.1 = 0.178$ W = 178 mW.

EXAMPLE 9.16 For the network of Fig. 9.86, determine the value of R for maximum power to R, and calculate the power delivered under these conditions.

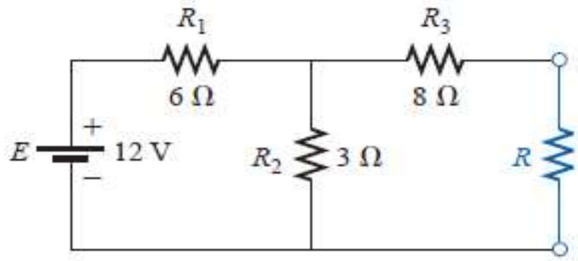


FIG. 9.86

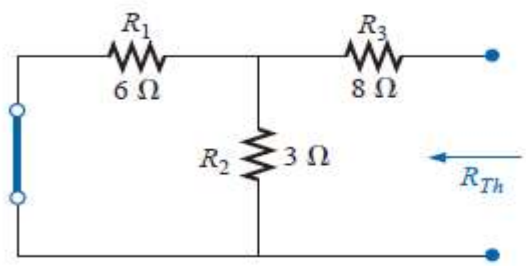


FIG. 9.87

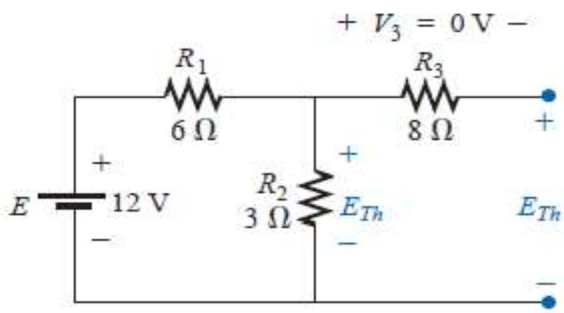


FIG. 9.88

Solution: See Fig. 9.87.

$$R_{Th} = R_3 + R_1 \parallel R_2 = 8\ \Omega + \frac{(6\ \Omega)(3\ \Omega)}{6\ \Omega + 3\ \Omega} = 8\ \Omega + 2\ \Omega$$

and
$$R = R_{Th} = \mathbf{10\ \Omega}$$

See Fig. 9.88.

$$E_{Th} = \frac{R_2 E}{R_2 + R_1} = \frac{(3\ \Omega)(12\ \text{V})}{3\ \Omega + 6\ \Omega} = \frac{36\ \text{V}}{9} = \mathbf{4\ V}$$

and, by Eq. (9.6),

$$P_{L_{max}} = \frac{E_{Th}^2}{4R_{Th}} = \frac{(4\ \text{V})^2}{4(10\ \Omega)} = \mathbf{0.4\ W}$$

Loop Current Method (Mesh Analysis)

Mesh analysis provides another general procedure for analyzing circuits, using mesh currents as the circuit variables. Using mesh currents instead of element currents as circuit variables is convenient and reduces the number of equations that must be solved simultaneously.

In Fig. 3.17, for example, paths *abefa* and *bcdeb* are meshes, but path *abcdefa* is not a mesh. The current through a mesh is known as *mesh current.* In mesh analysis, we are interested in applying KVL to find the mesh currents in a given circuit. In this section, we will apply mesh analysis to planar circuits that do not contain current sources. In the next section, we will consider circuits with current sources. In the mesh analysis of a circuit with n meshes, we take the following three steps.

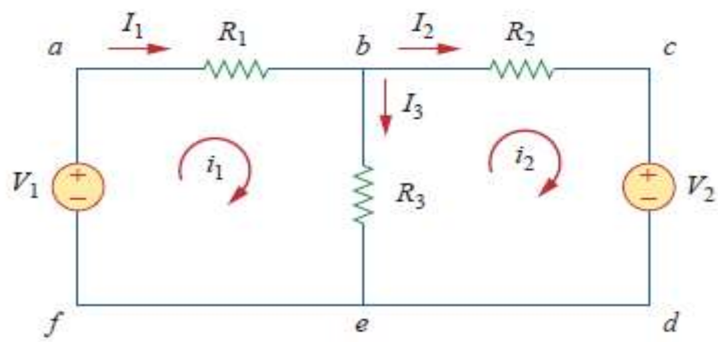


Figure 3.17
A circuit with two meshes.

Steps to Determine Mesh Currents:

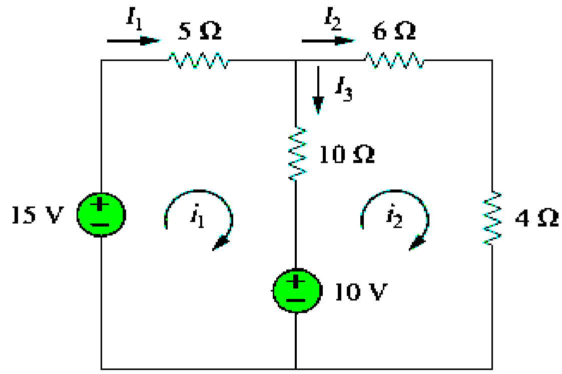
1. Assign mesh currents $i_1, i_2, \ldots, i_n$ to the n meshes.
2. Apply KVL to each of the n meshes. Use Ohm's law to express the voltages in terms of the mesh currents.
3. Solve the resulting n simultaneous equations to get the mesh currents.

Example1:-For the circuit below, find the branch currents I_1, I_2, and I_3 using mesh analysis.

$$-15 + 5i_1 + 10(i_1 - i_2) + 10 = 0$$

or

$$3i_1 - 2i_2 = 1$$



$$6i_2 + 4i_2 + 10(i_2 - i_1) - 10 = 0$$

or

$$i_1 = 2i_2 - 1$$

$$6i_2 - 3 - 2i_2 = 1 \quad \Longrightarrow \quad i_2 = 1 \text{ A}$$

$$i_1 = 2i_2 - 1 = 2 - 1 = 1 \text{ A. Thus,}$$

$$I_1 = i_1 = 1 \text{ A}, \qquad I_2 = i_2 = 1 \text{ A}, \qquad I_3 = i_1 - i_2 = 0$$

Second method:

$$\begin{bmatrix} 3 & -2 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

We obtain the determinants

$$\Delta = \begin{vmatrix} 3 & -2 \\ -1 & 2 \end{vmatrix} = 6 - 2 = 4$$

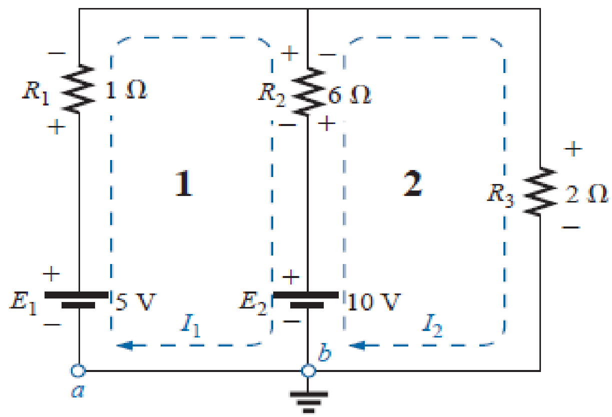
$$\Delta_1 = \begin{vmatrix} 1 & -2 \\ 1 & 2 \end{vmatrix} = 2 + 2 = 4, \qquad \Delta_2 = \begin{vmatrix} 3 & 1 \\ -1 & 1 \end{vmatrix} = 3 + 1 = 4$$

Thus,

$$i_1 = \frac{\Delta_1}{\Delta} = 1 \text{ A}, \qquad i_2 = \frac{\Delta_2}{\Delta} = 1 \text{ A}$$

as before.

EXAMPLE2:- Find the current through each branch of the network of Fig. using mesh analysis.



Solution:

Steps 1 and 2 are as indicated in the circuit. Note that the polarities of the 6 Ω resistors are different for each loop current.

Step 3: Kirchhoff's voltage law is applied around each closed loop in the clockwise direction:

loop 1: $+E_1 - V_1 - V_2 - E_2 = 0$ (clockwise starting at point a)

$$+5\text{ V} - (1\ \Omega)I_1 - (6\ \Omega)(I_1 - I_2) - 10\text{ V} = 0$$

I_2 flows through the 6-Ω resistor in the direction opposite to I_1.

loop 2: $E_2 - V_2 - V_3 = 0$ (clockwise starting at point b)

$$+10\text{ V} - (6\ \Omega)(I_2 - I_1) - (2\ \Omega)I_2 = 0$$

The equations are rewritten as

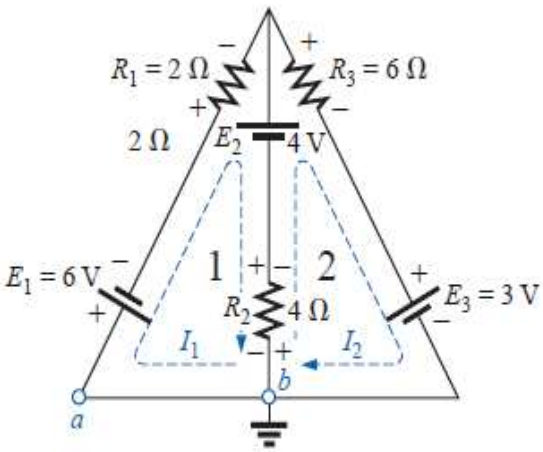
$$\left.\begin{array}{r} 5 - I_1 - 6I_1 + 6I_2 - 10 = 0 \\ 10 - 6I_2 + 6I_1 - 2I_2 = 0 \end{array}\right\} \begin{array}{r} -7I_1 + 6I_2 = 5 \\ +6I_1 - 8I_2 = -10 \end{array}$$

Step 4:

$$I_1 = \frac{\begin{vmatrix} 5 & 6 \\ -10 & -8 \end{vmatrix}}{\begin{vmatrix} -7 & 6 \\ 6 & -8 \end{vmatrix}} = \frac{-40 + 60}{56 - 36} = \frac{20}{20} = \mathbf{1\ A}$$

$$I_2 = \frac{\begin{vmatrix} -7 & 5 \\ 6 & -10 \end{vmatrix}}{20} = \frac{70 - 30}{20} = \frac{40}{20} = \mathbf{2\ A}$$

EXAMPLE 3:- Find the branch currents of the network of Fig. using mesh analysis.



Solution:

Steps 1 and 2 are as indicated in the circuit.

Step 3: Kirchhoff's voltage law is applied around each closed loop:

loop 1: $-E_1 - I_1R_1 - E_2 - V_2 = 0$ (clockwise from point a)

$$-6\text{ V} - (2\ \Omega)I_1 - 4\text{ V} - (4\ \Omega)(I_1 - I_2) = 0$$

loop 2: $-V_2 + E_2 - V_3 - E_3 = 0$ (clockwise from point b)

$$-(4\ \Omega)(I_2 - I_1) + 4\text{ V} - (6\ \Omega)(I_2) - 3\text{ V} = 0$$

which are rewritten as

$$\left.\begin{aligned} -10 - 4I_1 - 2I_1 + 4I_2 &= 0 \\ +\ 1 + 4I_1 - 4I_2 - 6I_2 &= 0 \end{aligned}\right\} \begin{aligned} -6I_1 + 4I_2 &= +10 \\ +4I_1 - 10I_2 &= -1 \end{aligned}$$

or, by multiplying the top equation by -1, we obtain

$$\begin{aligned} 6I_1 - 4I_2 &= -10 \\ 4I_1 - 10I_2 &= -1 \end{aligned}$$

Step 4:

$$I_1 = \frac{\begin{vmatrix} -10 & -4 \\ -1 & -10 \end{vmatrix}}{\begin{vmatrix} 6 & -4 \\ 4 & -10 \end{vmatrix}} = \frac{100 - 4}{-60 + 16} = \frac{96}{-44} = \mathbf{-2.182\ A}$$

$$I_2 = \frac{\begin{vmatrix} 6 & -10 \\ 4 & -1 \end{vmatrix}}{-44} = \frac{-6 + 40}{-44} = \frac{34}{-44} = \mathbf{-0.773\ A}$$

Nonlinear direct current circuit:

There are, components of electrical circuits which do not obey Ohm's law; that is, their relationship between current and voltage (their I–V curve) is *nonlinear*. An example is the p-n junction diode (curve at right). As seen in the Fig.1, the current does not increase linearly with applied voltage for a diode.

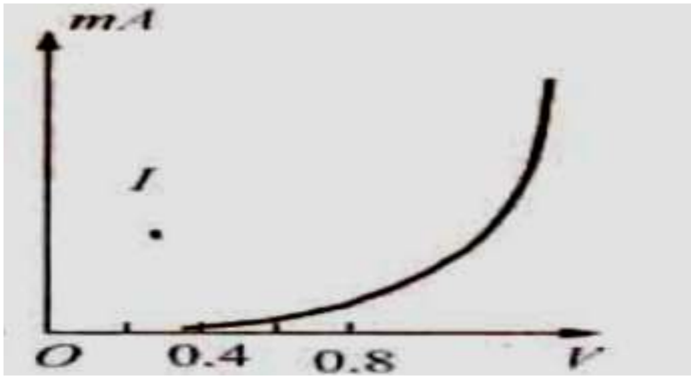


Fig. 1

One can determine a value of current (I) for a given value of applied voltage (V) from the curve, but not from Ohm's law, since the value of "resistance" is not constant as a function of applied voltage.

The source-free RC circuit:

Consider a series combination of a resistor and an initially charged capacitor, as shown in Fig. 2.

Our objective is to determine the circuit response, we assume to be the voltage *v(t)* across the capacitor. Since the capacitor is initially charged, we can assume that at time $t = 0$, the initial voltage is $v(0) = V_0$

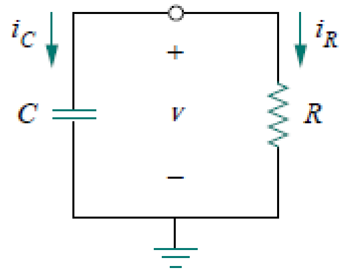


Fig. 2

Applying KCL at the top node of the circuit in Fig. 2,

$$i_c + i_R = 0$$

By definition, $i_c = C\frac{dv}{dt}$ and $i_R = \frac{v}{R}$. Thus,

$$C\frac{dv}{dt} + \frac{v}{R} = 0$$

$$\frac{dv}{v} = -\frac{1}{RC}\,dt$$

Integrating both sides, we get

$$\ln v = -\frac{t}{RC} + \ln A$$

where $\ln A$ is the integration constant. Thus,

$$\ln\frac{v}{A} = -\frac{t}{RC}$$

Taking powers of e produces

$$v(t) = Ae^{-t/RC}$$

But from the initial conditions, $v(0) = A = V_0$. Hence,

$$v(t) = V_0 e^{-t/RC}$$

Since the response is due to the initial energy stored and the physical characteristics of the circuit and not due to some external voltage or current source, it is called the *natural response* of the circuit.

The natural response is illustrated graphically in Fig. 3. Note that at $t = 0$, we have the correct initial condition. As t increases, the voltage decreases toward zero. The rapidity with which the voltage decreases is expressed in terms of the *time constant*, denoted by the lower case Greek letter tau, τ.

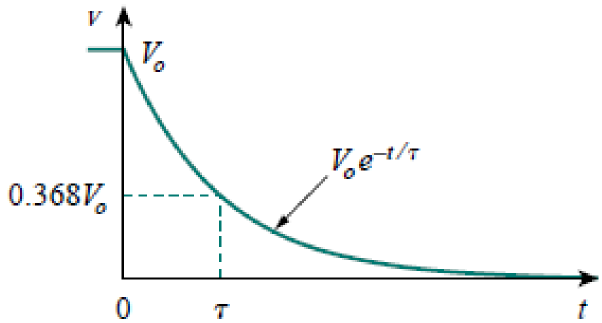


Fig. 3

This implies that at $t = \tau$,

$$V_0 e^{-\tau/RC} = V_0 e^{-1} = 0.368\, V_0$$

or

$$\tau = RC$$

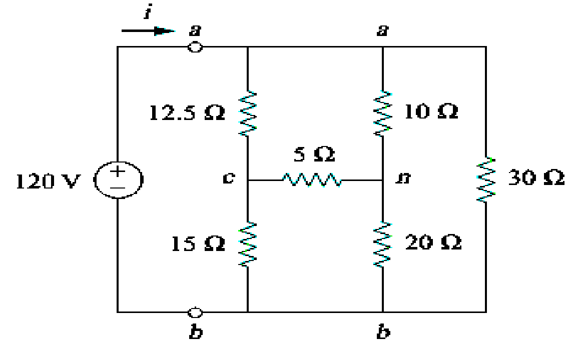
At any rate, whether the time constant is small or large, the circuit reaches steady state in five time constants. We can find the current $i_R(t)$,

$$i_R(t) = \frac{v(t)}{R} = \frac{V_0}{R} e^{-t/\tau}$$

The power dissipated in the resistor is

$$p(t) = v i_R = \frac{V_0^2}{R} e^{-2t/\tau}$$

Example 15: In Fig. 4, let $v_C(0) = 15$ V. Find v_C, v_x, and i_x for $t > 0$.



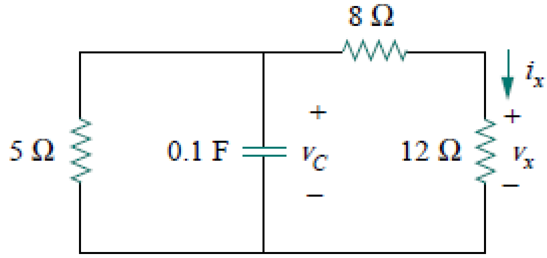


Fig.4

We find the equivalent resistance .

$$R_{eq} = (8+12)\,\|\,5 = \frac{20\times 5}{20+5} = 4\Omega$$

$$\tau = R_{eq}C = 4\times 0.1 = 0.4\ s$$

Thus,

$$v = v(0)e^{-t/\tau} = 15e^{-t/0.4}\ \text{V}, \qquad v_C = v = 15e^{-2.5t}\ \text{V}$$

From Fig. 4, we can use voltage division to get v_x; so

$$v_x = \frac{12}{12+8}v = 0.6(15e^{-2.5t}) = 9e^{-2.5t}\ \text{V}$$

Finally,

$$i_x = \frac{v_x}{12} = 0.75e^{-2.5t}\,\text{A}$$

Practice problem:

Refer to the circuit in Fig. 5. Let $v_C(0) = 30$ V. Determine v_C, v_x , and i_0 for $t \geq 0$.

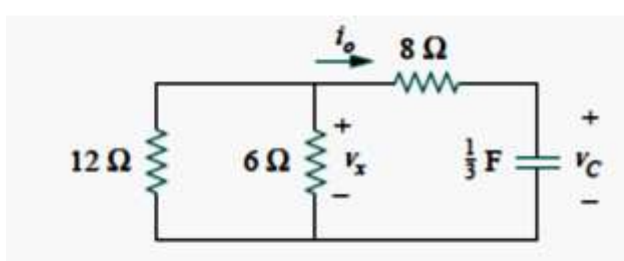


Fig. 5

Answer: $30e^{-0.25t}$ V, $10e^{-0.25t}$ V, $-2.5e^{-0.25t}$ A.

Example:

The switch in the circuit in Fig. 6 has been closed for a long time, and it is opened at t = 0. Find *v(t)* for $t \geq 0$. Calculate the initial energy stored in the capacitor.

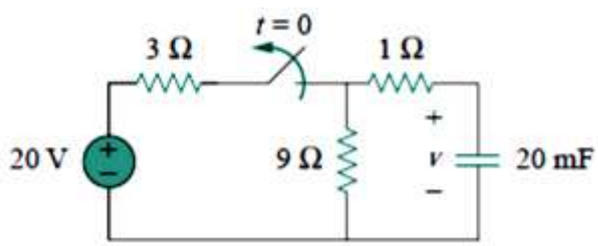


Fig.6

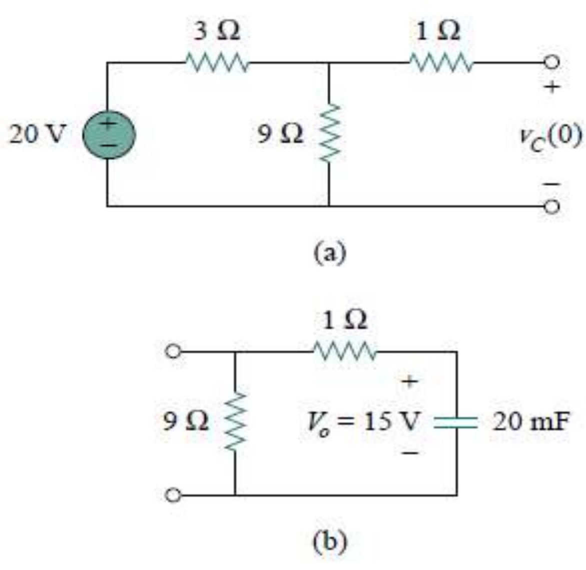


Fig. 7

For $t < 0$, and using voltage division for fig. 27 (a):

$$v_c(t) = \frac{9}{3+9} \times 20 = 15\ V \qquad t < 0$$

Hence $$vc(0) = V_0 = 15\ V$$

For$t > 0$, the switch is opened, and we have the *RC* circuit shown in Fig.7 (b)

$$R_{eq} = 1 + 9 = 10\ \Omega$$

The time constant is

$$\tau = R_{eq}C = 10 \times 20 \times 10^{-3} = 0.2\ s$$

Thus, the voltage across the capacitor for $t \geq 0$ is

$$v(t) = vC(0)e^{-t/\tau} = 15e^{-t/0.2}\ V$$

or

$$v(t) = 15e^{-5t}$$

The initial energy stored in the capacitor is

$$w_C(0) = \frac{1}{2}Cv_C^2(0) = \frac{1}{2} \times 20 \times 10^{-3} \times 15^2 = 2.25\ J$$

The source-free RL circuit:

Consider the series connection of a resistor and an inductor, as shown in Fig. 8.

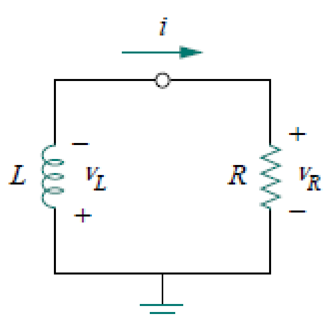


Fig. 8

Our goal is to determine the circuit response, which we will assume to be the current *i(t)* through the inductor. At $t = 0$, we assume that the inductor has an initial current I_0 , or $i(0) = I_0$

Applying KVL around the loop in Fig. 28,

$$v_L + v_R = 0$$

But $v_L = L\,di/dt$ and $v_R = iR$. Thus,

$$L\frac{di}{dt} + Ri = 0$$

or

$$\frac{di}{dt} + \frac{R}{L}i = 0$$

Rearranging terms and integrating gives

$$\int_{I_0}^{i(t)} \frac{di}{i} = -\int_0^t \frac{R}{L}dt$$

$$\ln i \Big|_{I_0}^{i(t)} = -\frac{Rt}{L}\Big|_0^t \quad \Longrightarrow \quad \ln i(t) - \ln I_0 = -\frac{Rt}{L} + 0$$

or

$$\ln\frac{i(t)}{I_0} = -\frac{Rt}{L}$$

Taking the powers of e, we have

$$i(t) = I_0 e^{-Rt/L}$$

This shows that the natural response of the *RL* circuit is an exponential decay of the initial current as shown in fig.9.

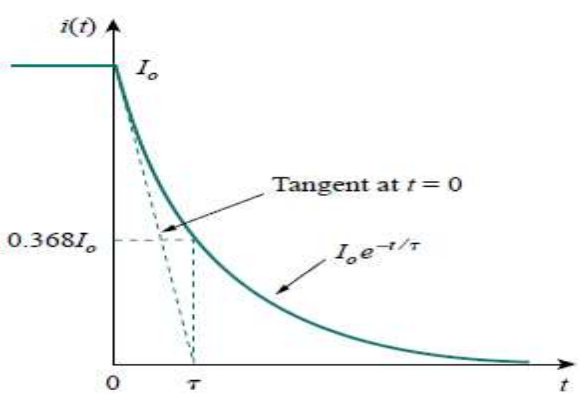


Fig. 9

The time constant of RL circuit is,

$$\tau = \frac{L}{R}$$

Hence $$i(t) = I_0 e^{-t/\tau}$$

So we can find the voltage across the resistor as

$$v_R(t) = i_R = I_0 e^{-t/\tau}$$

The power dissipated in the resistor is

$$p = v_R i = I_0^2 R e^{-2t/\tau}$$

The energy absorbed by the resistor is

$$w_R(t) = \frac{1}{2} L I_0^2 (1 - e^{-2t/\tau})$$

Note that as $t \to \infty$, $w_R(\infty) \to \frac{1}{2} L I_0^2$, which is the same as $w_L(0)$, the initial energy stored in the inductor as. Again, the energy initially stored in the inductor is eventually dissipated in the resistor.

Example:

The switch in the circuit of Fig. 10 has been closed for a long time. At $t = 0$, the switch is opened. Calculate $i(t)$ for $t > 0$.

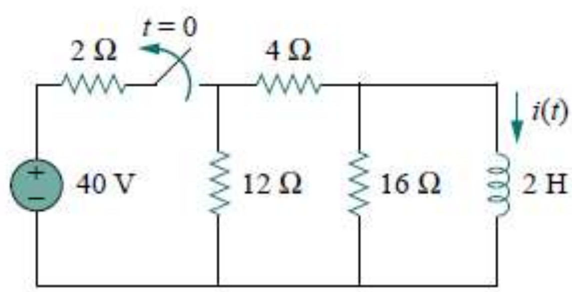


Fig.10

When $t < 0$, resulting circuit is shown in Fig. 11(a).

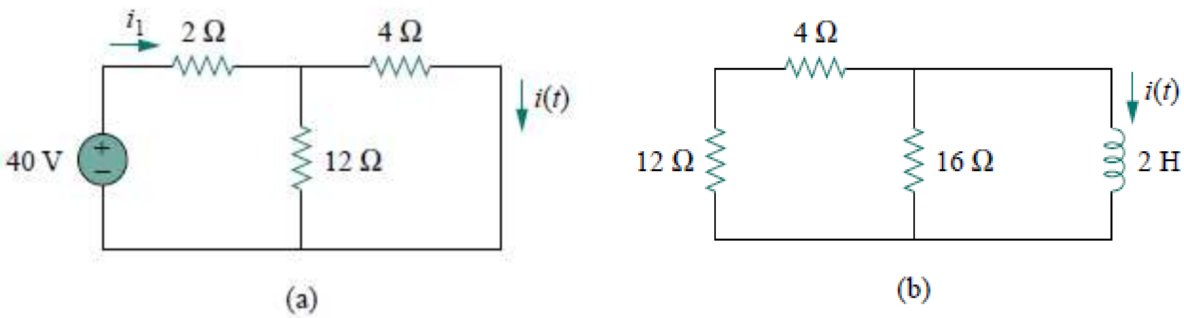


Fig. 11

we combine the 4Ω and 12Ω resistors in parallel to get

$$\frac{4 \times 12}{4 + 12} = 3\Omega$$

Hence $$i_1 = \frac{40}{2+3} = 8\ A$$

$$i(t) = \frac{8 \times 12}{12 + 4} = 6\text{ A} \qquad t < 0$$

When $t > 0$, the switch is open and the voltage source is disconnected. We now have the *RL* circuit in Fig. 11 (b). Combining the resistors, we have

$$R_{eq} = (4 + 12) \| 16 = 8\ \Omega$$

$$\tau = \frac{L}{R_{eq}} = \frac{2}{8} = \frac{1}{4}\ s$$

Thus $$i(t) = i(0)e^{-t/\tau} = 6e^{-4t} A$$

The current after 1/8 s is

$$i(1/8) = 6e^{-4\times\frac{1}{8}} = 3.64\ A$$

Practice problem:

For the circuit in Fig. 12, find *i(t)* for $t > 0$.

Answer: $2e^{-2t}$ A, $t > 0$.

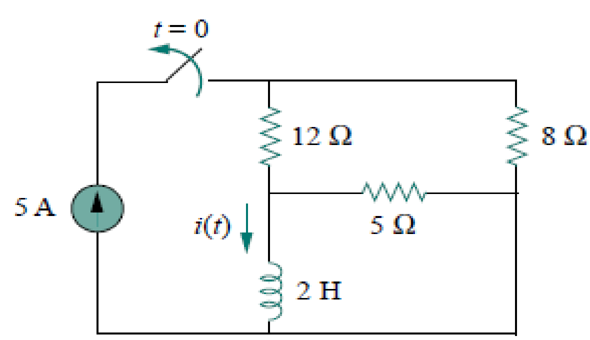
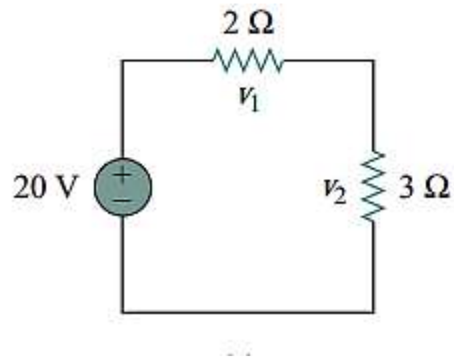


Fig.12

Example for Kirchhoff's lows

Example 1:

For the circuit in Fig. 1, find voltages *v*1 and *v*2.



From Ohm's law,

$$v_1 = 2i\ ,\quad v_2 = 3i$$

Applying KVL

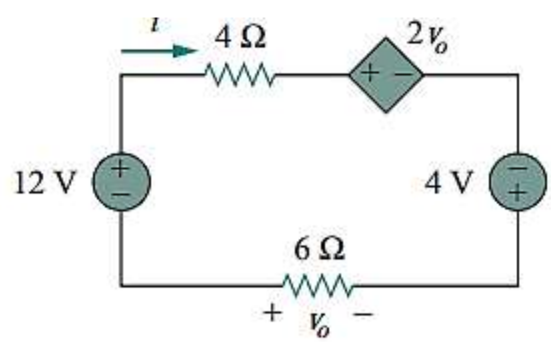
$-20 + 2i + 3i = 0 \Longrightarrow\ 5i = 20 \qquad \therefore\quad i = 4\ A$

Figure 1

$v_1 = 2 \times 4 = 8\ V,\ \ v_2 = 3 \times 4 = 12\ V$

Example 2:

Determine *vo* and *i* in the circuit shown in Fig. 2



Applying KVL:

$$-12 + 4i + 2V_o - 4 + 6i = 0$$

$$V_o = -6i$$

$-12 + 4i - 12i - 4 + 6i = 0$

Figure 2

$-16 - 2i = 0 \qquad \therefore i = -8\ A$

$and\ V_o = -6 \times -8 = 48$ V

Example 3:

Find current *io* and voltage *vo* in the circuit shown in Fig.3:

Applying KCL to node a, we obtain:

$0.5i_o + 3 = i_o$

$i_o - 0.5i_o = 3$

$i_o = 6\ A, \quad V_o = 4i_o = 24\ V$

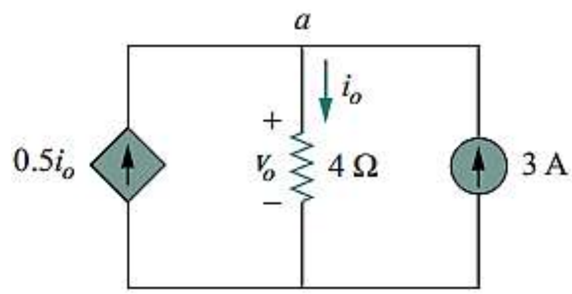


Figure 3

Example 4:

Find the currents and voltages in the circuit shown in Fig. 4

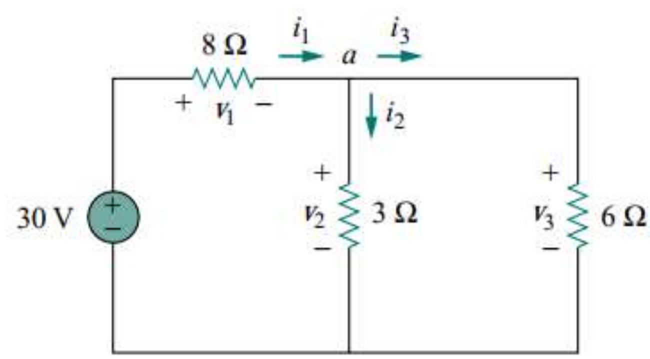


Figure 4

For loop 1

$v_1 = 8i_1, v_2 = 3i_2, \ v_3 = 6i_3$

$i_1 = i_2 + i_3$……………(1)

$$-30 + 8i_1 + 3i_2 = 0$$

$8i_1 = 30 - 3i_2 \ \ , i_1 = \frac{30-3i_2}{8}$……..(2)

For loop 2

$v_3 - v_2 = 0, \quad \therefore v_3 = v_2, \quad 6i_3 = 3i_2$

$\therefore i_3 = \frac{i_2}{2}$ …………..(3), now put (2) and (3) in (1):

$$\frac{30 - 3i_2}{8} = i_2 + \frac{i_2}{2} \qquad \therefore \ i_2 = 2\ A, \ \ i_3 = 1\ A \ \ \ i_1 = 2 + 1 = 3A$$

$v_1 = 24\,V, \quad v_2 = 6\,V \; and \; v_3 = 6\,V$

HW:

1- Find the currents and voltages in the circuit shown in Fig. 5

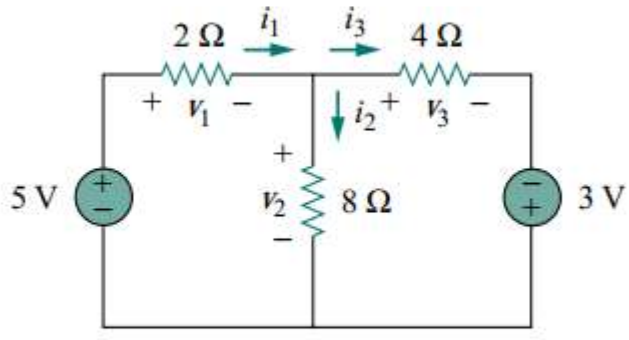


Figure 5

Answer: v1 = 3 V, v2 = 2 V, v3 = 5 V, i1 = 1.5 A, i2 = 0.25 A, i3 =1.25 A.

2- Obtain v1 through v3 in the circuit of Fig. 6

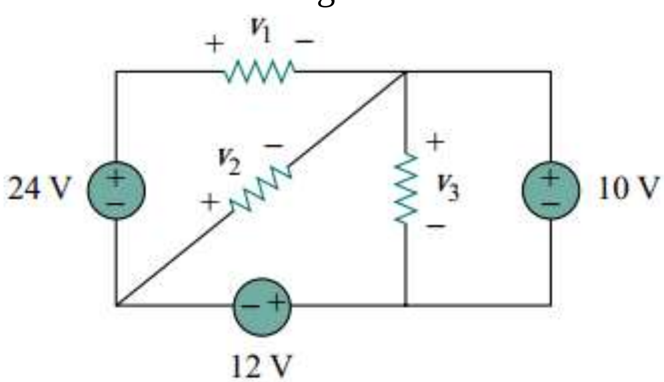


Figure 6

3- Determine v1 through v4 in the circuit in Fig. 7

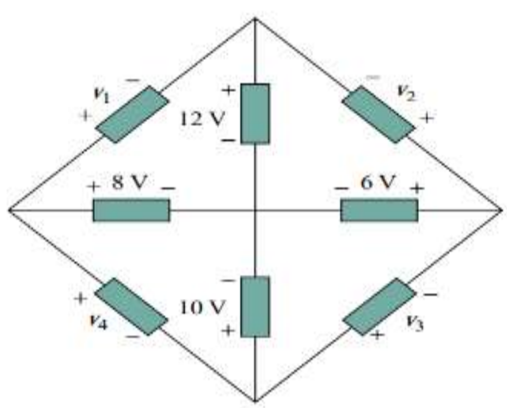


Figure 7

Superposition Theorem

Analyzing a circuit using superposition has one major disadvantage: it may very likely involve more work. If the circuit has three independent sources, we may have to analyze three simpler circuits each providing the contribution due to the respective individual source. However, superposition does help reduce a complex circuit to simpler circuits through **replacement of voltage sources by short circuits** and of **current sources by open circuits:**

Example: 1

Use the superposition theorem to find *v* in the circuit in Fig. 1.

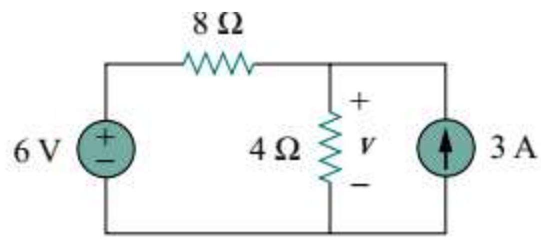


Figure 1

Let $v = v_1 + v_2$

To obtain *v*1, we set the current source by **open** as

Shown in Fig. 1b and apply KVL:

$-6 + 8i_1 + 4i_1 = 0$

$12i_1 = 6; \quad \therefore\ i_1 = 0.5\ A$

$v_1 = 4i_1 = 2V$

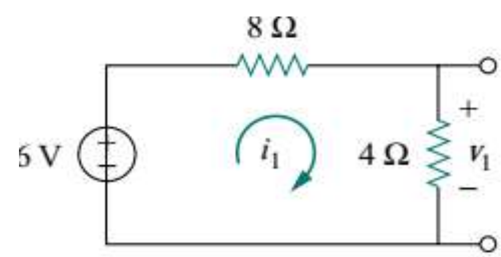


Figure 1 b

To get *v*2, we set the voltage source to **short circuits**, as in Fig. 1c. Using current division:

$$i_3 = \frac{8}{4+8}(3) = 2\text{ A}$$

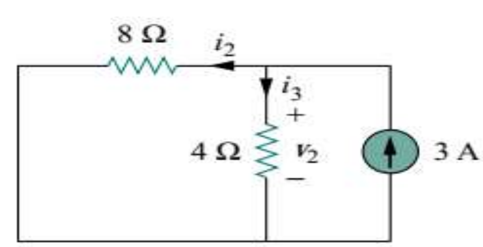
Hence,

$$v_2 = 4i_3 = 8\text{ V}$$

And we find

$$v = v_1 + v_2 = 2 + 8 = 10\text{ V}$$

fig.1C



H. W: Using the superposition theorem, find v_o in the circuit in Fig. 2.

Answer: 12 V

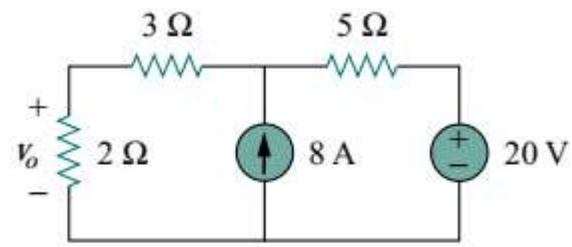


Figure 2

Calculate i_x of Fig. 3 using superposition.

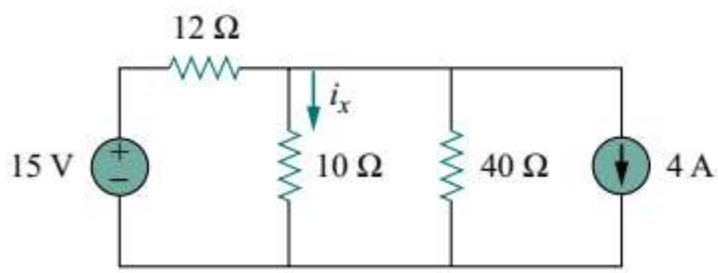


Figure 3.

EXAMPLE2: - Using superposition, determine the current through the 4- Ω resistor of Fig. 9.6.

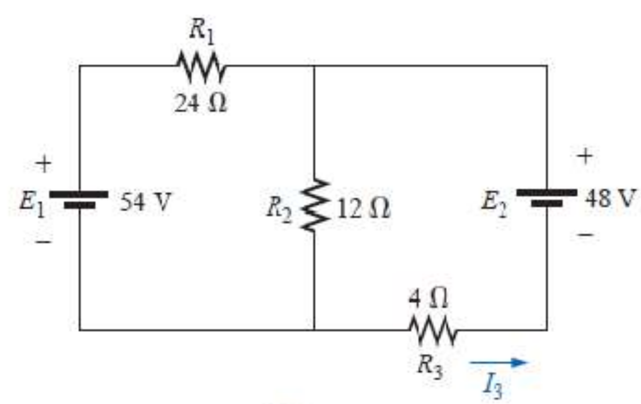


FIG. 9.6

Solution: Considering the effects of a 54-V source (Fig. 9.7):

RT = R1 + R2 // R3 = 24 + 12 // 4 = 24 +3 = 27 Ω

I = $\frac{E1}{RT} = \frac{54\,v}{27\,\Omega} = 2A$

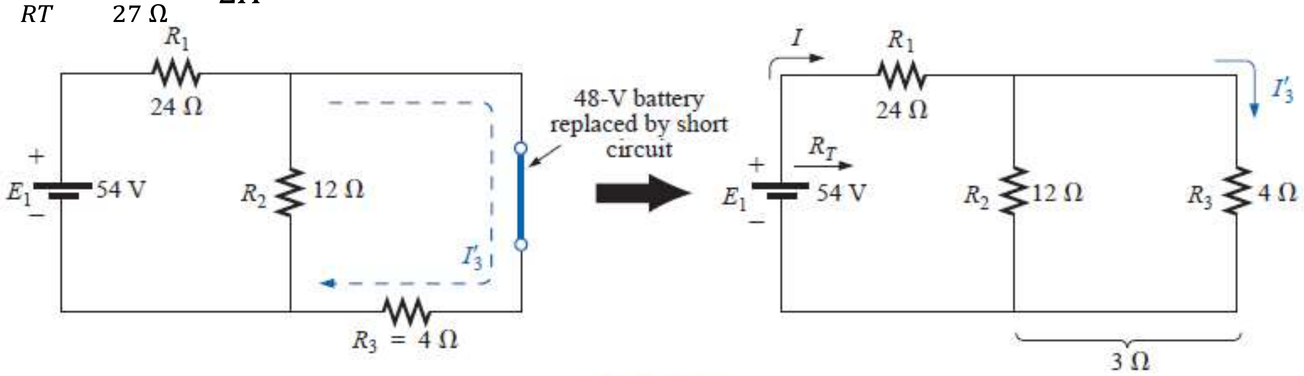


FIG. 9.7

The effect of E1 on the current I3.

Using the current divider rule,

I'3 = $\frac{R2\,I}{R2+R3} = \frac{12\Omega*2A}{12\Omega+4\Omega} = \frac{24\,A}{16} = 1.5\,A$

Considering the effects of the 48-V source (Fig. 9.8):

RT = R3 + R1 // R2 = 4 + 24 //12 = 4 + 8 = 12Ω.

I"3= $\frac{E2}{RT} = \frac{48v}{12\,\Omega} = 4\,A$

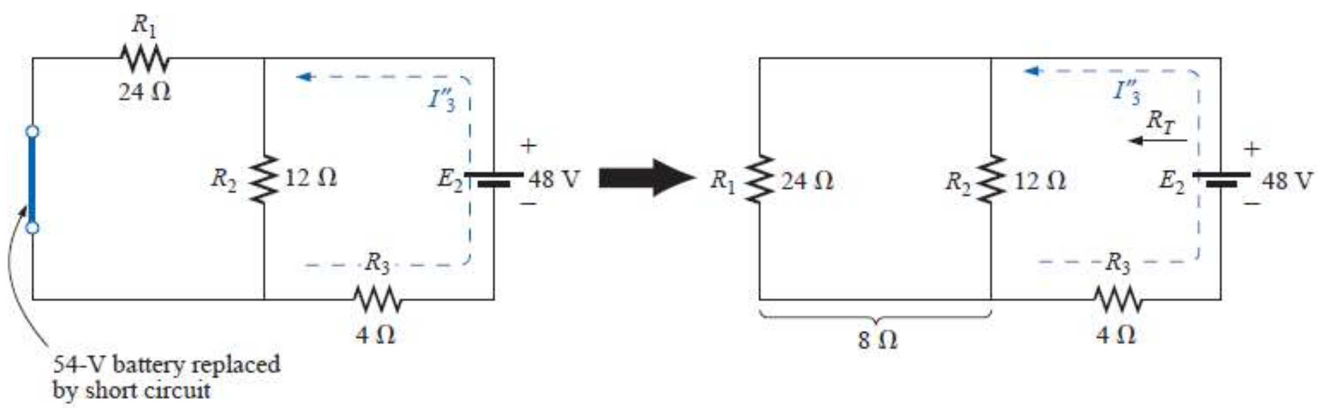


FIG. 9.8

The effect of E_2 on the current I_3.

The total current through the 4-Ω resistor (Fig. 9.9) is

I3 = I"3 – I'3 = 4A – 1.5 A = 2.5A   (direction of *I″*3)

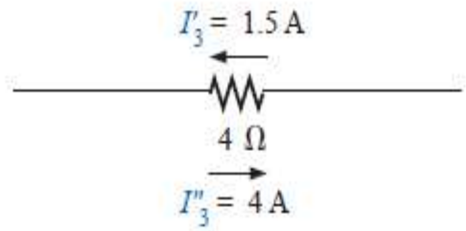


FIG. 9.9

The resultant current for I_3.

EXAMPLE 9.3

a. Using superposition, find the current through the 6-Ω resistor of the network of Fig. 9.10.

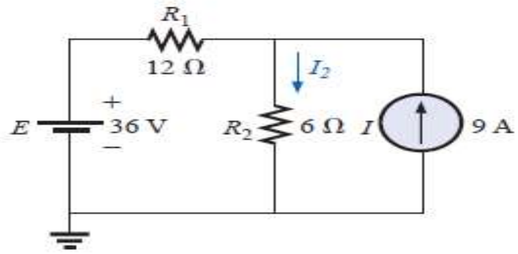


FIG. 9.10
Example 9.3.

b. Demonstrate that superposition is not applicable to power levels.

Solutions:

a. *Considering the effect of the 36-V source (Fig. 9.11):*

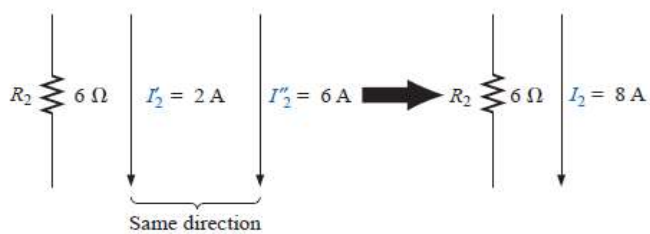
$$I'_2 = \frac{E}{R_T} = \frac{E}{R_1 + R_2} = \frac{36\text{ V}}{12\ \Omega + 6\ \Omega} = 2\text{ A}$$

Considering the effect of the 9-A source (Fig. 9.12):
Applying the current divider rule,

$$I''_2 = \frac{R_1 I}{R_1 + R_2} = \frac{(12\ \Omega)(9\text{ A})}{12\ \Omega + 6\ \Omega} = \frac{108\text{ A}}{18} = 6\text{ A}$$

The total current through the 6-Ω resistor (Fig. 9.13) is

$$I_2 = I'_2 + I''_2 = 2\text{ A} + 6\text{ A} = \mathbf{8\text{ A}}$$



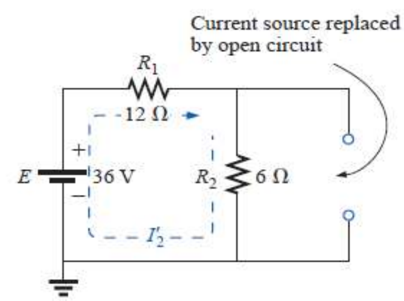
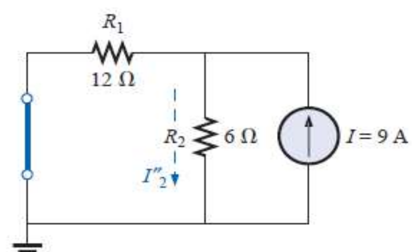


FIG. 9.11
The contribution of E to I_2.



b. The power to the 6-Ω resistor is

$$\text{Power} = I^2R = (8\text{ A})^2(6\ \Omega) = \mathbf{384\ W}$$

The calculated power to the 6-Ω resistor due to each source, *misusing* the principle of superposition, is

$$P_1 = (I'_2)^2R = (2\text{ A})^2(6\ \Omega) = 24\text{ W}$$

$$P_2 = (I''_2)^2R = (6\text{ A})^2(6\ \Omega) = 216\text{ W}$$

$$P_1 + P_2 = 240\text{ W} \neq 384\text{ W}$$

This results because 2 A + 6 A = 8 A, but

$$(2\text{ A})^2 + (6\text{ A})^2 \neq (8\text{ A})^2$$