Chapter 2

Unit Impulse Signal and Fourier Transform

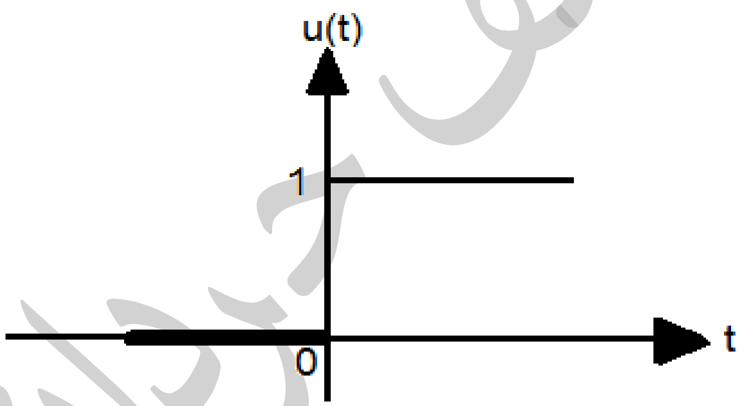
SINGULARITY FUNCTIONS

There are several singularity functions which play a vital role in the study of communication system . these singularity functions serve as basic building blocks for the construction of more complex signals .These singularity functions may be listed as ahead:

1.Unit step function .
2.Unit impulse function.
3.Ramp function.

1.Unit Step Function.

The unit step function is that type of singularity function which exits only for positive side and is zero for negative side . the unit step function is denoted by u(t) and my expressed as

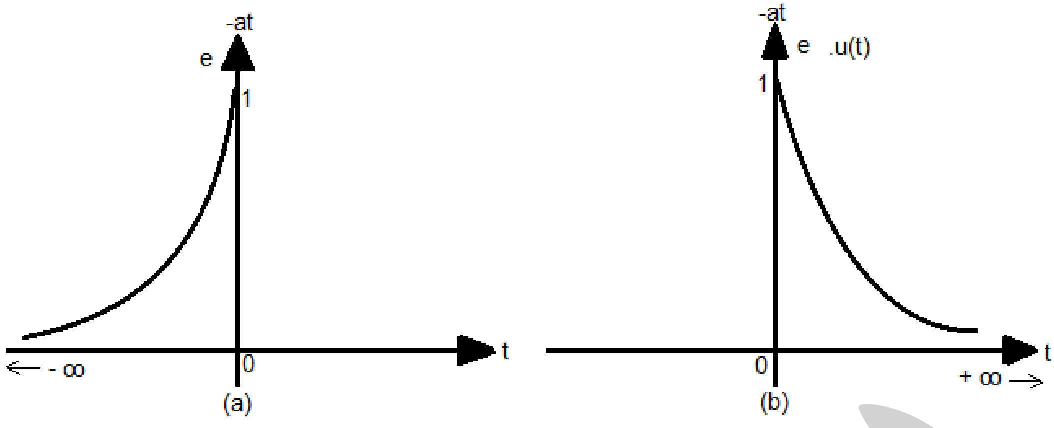


An unit-step function.

$$u(t) = \begin{cases} 0 \ for \ t < 0 \\ 1 \ for \ t > 0 \end{cases}$$

For example , the signal e^{-at} represents an exponential signal which starts at t=-∞.

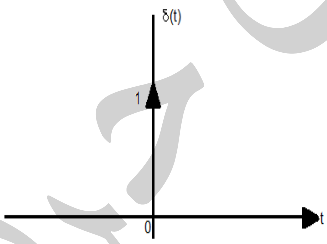
But if want this signal to start at t = 0 , it may be described as $e^{-at} . u(t)$.



(a) Exponential signal e^{-at} (b)exponential signal e^{-at} .u(t)

2.Unit Impulse Function

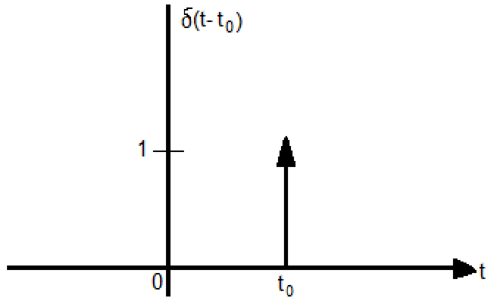
The unit impulse function is the most widely elementary function used in the analysis of communication system .
An unit – impulse function is denoted as ʒ(t) and may be expressed as
ʒ(t) = 0   t≠0



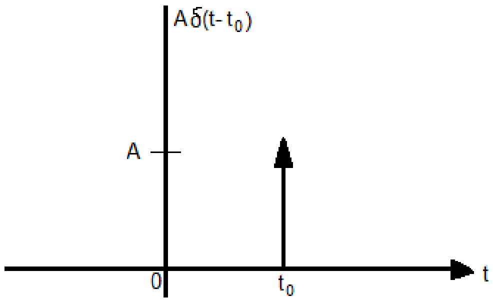
The unit-impulse function

and $\int_{-\infty}^{\infty} \text{ʒ}(t)dt = 1$

The unit –impulse function is also known Dirac-delta function.
If unit –impulses function is assumed in the form of a pulse , then following points may be observed about an unit – impulse function ;
a-The width of the pulse is zero . This means that the pulse exists only at t =0.
b-The height of the pulse goes to infinity .
c-The area under the pulse – curve is always unity.



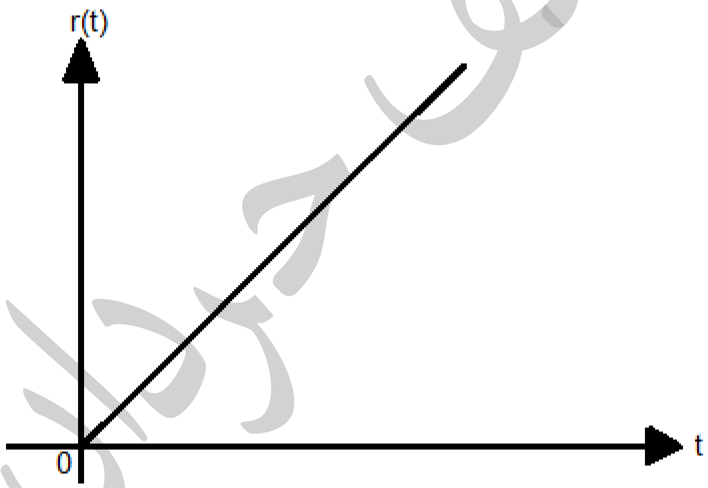
(a) Unit-impulse function centered at $t = t_0$.



(b) Impulse-function centered at $t = t_0$ and haning area A

3.Unit Ramp Function

A continuous –time unit –ramp function is that type of function which starts at t=0 and increase linearly with time t. It denoted by r(t).
Mathematically ,



(c) Continuous-time unit-ramp function

$$r(t) = \begin{cases} o\ for\ t < 0 \\ T\ for\ t > 0 \end{cases}$$

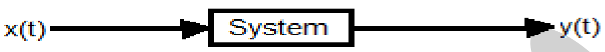
SYSTEM

A system may be defined as a set of elements or functional blocks which are connected together and an output in response to an input signal .The response output of the system depends upon transfer function of the system. Mathematically , the functional relationship between input and output may be written as

Y(t)= f[x(t)]

Symbolically, we may write: $x(t) \rightarrow y(t)$

This means that an input or excitation x(t) is producing an output or response y(t).



(a) Block diagram of a system

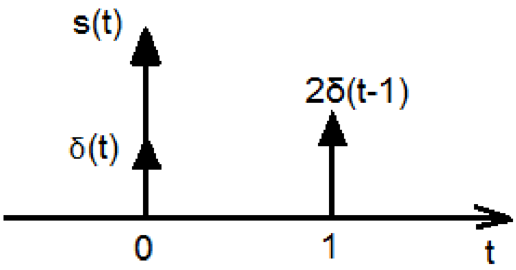
Delta-function-{δ(t)}- هي عبارة عن (pulse) ضيق إلى ما لانهاية مع (Amplitude) قيمتهُ أيضاً ما لانهاية.

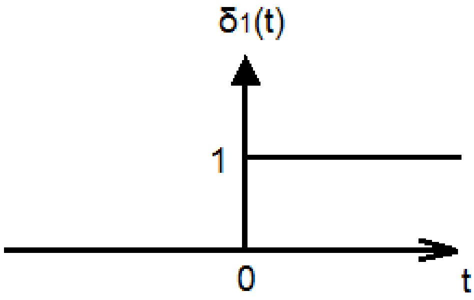
$\delta(t)=0, t\neq 0$, $\delta(t)=\infty, t=0$ argument=0

تكون اعلىٰ قيمة للدالة عندما يكون (t=0) أما مساحة الـ (pulse) تساوي واحد ($\delta_s(t)=1$).

$$\int_{-\infty}^{\infty} \delta(t)\, dt$$

تأخذ بالأعتبار إن هذا النوع من الإشارة لا يمكن إطلاقا تحقيقيه من الناحية الفيزياوية ولكن لغرض التحليل (Analysis) الرياضي للإشارات والمنظومات (systems).





للإشارة s(t)=δ(t)+2δ(t-1)

لنبضة واحدة

تكامل الدالة δ(t) يؤدي إلى ناتج بدون وحدة قياس لكن وحدة قياس δ(t) تكون عكس وحدة قياس (argument) على سبيل المثال دالة الوقت δ(t) تمتلك قياس (1/sec) أي (Hz).

الدالة δ1(t) تساوي صفراً إذا كانت قيمة (argument) سالبة, وتساوي (1) اذا كانت قيمة (argument) موجبة.

$$0, t < 0$$

$$\delta 1(t) = 1\backslash 2, t = 0$$

$$0, t > 0$$

δ (t) تستخدم للتعبير الرياضي للإشارات ذات (interval) المحدود. أبسط مثال على ذلك هو تكوين (pulse) ذو السعة(Amplitude) (A) و (interval) (T).

$$s(t) = A[\delta_1(t)] - [\delta_1(t-1)].$$

الأشارات التي تعكس تصرف القيم او الكميات الفيزياويه تسمى بالدوال الحقيقيه للوقت . لكن في عده حالات من الملائم ملاحظه (complex-signals) وابسط مثال على ذالك هي الاشارة (Complex-Harmonical signal)

$$S'(t) = A\exp[j(\omega t + \varphi) = A'_m \exp(j\omega t),$$

$$A'_m = A\exp(j\varphi) - \ complex - Amplitude\ of\ signal$$

Digital coding

Digital Coding yields a rugged signal highly immune to distortion and interference ,permitting long-distance communication through regenerative repeaters .

Coding provides of signal conversion performed to improve the quality of communication.

Coding was made possible through the use of large integrated circuits (LIC) and the applications of high –speed digital signal processing . This method allowed to increase productivity by more than 10 dB at much lower costs than other methods .For example ,methods of dragging the transmitter power or size of the antenna.

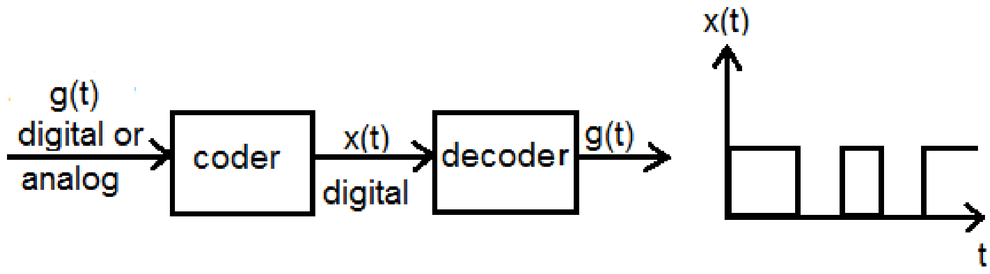
For any communication system, it is true that the received signal is different from the transmitted signal. This effect is the result of distortion , noise , interference and fading.

There are three most common tools for dealing with errors in the data transfer process.

-error detection codes.

-Error correcting codes (Forward Error Correction-FEC).

Protocols with automatic retransmission requests (Automatic Repeat Request-ARQ).



coding in digital signal

تستخدم (coding) لغرض جعل خواص الإرسال مثالية.

(coding) مرتبطة مع طرق تحسين مواصفات الإشارات الرقمية, والتي تؤدي بالنتيجة الى جعل الإشارة اقل تأثراً بالعوامل السلبية المؤثرة عليها كالضوضاء (noise), وذلك بإستخدام (bit) إضافية لغرض تحديد (error), الناجمة عن وجود (noise) في قناة الإتصال:

واحدة من تكنولوجيا تحديد (error) هو (automatic repeat request) (ARQ), ببساطة تقوم بالتعرف على الخطأ (error) ويطلب إعادة إرسال الإشارة مرة أخرى (مباشر, أمامي) التكنولوجيا الأخرى هي (Forward error correction FEC) يسمح بمعالجة الأخطاء اوتوماتيكياً.

Fourier transform

Trigonometric Fourier Series :-

A periodic function x(t) may by expressed in the form of Trigonometric Fourier Series comprising the following sine and cosine terms .

$x(t)= a_0 + a_1\cos\omega_\circ t + a_2 \cos 2\omega_\circ t+\ldots\ldots+ a_n \cos n\, \omega_\circ t +\ldots$

$b_1\sin \omega_\circ t+b_2 \sin 2\, \omega_\circ t+\ldots\ldots.+b_n \sin n\ \ \omega_\circ t+\ldots$

$x(t)=a_\circ + \sum_{n+1}^{\infty} \quad (an \cos n\, \omega^\circ t + bn \sin n\ \ \omega^\circ t). (t^\circ \ \le\ t\ \le\ t^\circ\ + T).$

Here $T=\frac{2\pi}{\omega^\circ}, an\ , bn\ coefficients.$

$\omega_\circ$ _ fundamental frequency

$2\omega_\circ$, 3 $\omega_\circ$,4$\omega_\circ$,……..harmonics at $\omega_\circ$.

$a_\circ = \frac{1}{T}\int_0^T x(t)dt, an = \frac{2}{T}\int_0^T x(t) \cos\ n\, \omega^\circ t\ dt$

$b_n=\frac{2}{T}\int_0^t x(t)\ \sin n\ \ \omega^\circ t\ dt\ .$

$X(\omega)=F[x(t)]=\int_{-\infty}^{\infty} x(t)e^{-j\omega t}\ dt$

$x(\omega)$ is the frequency domain representation of time domain function x(t).This means that we are converting a time domain signal into its frequency domain representation with help of Fourier Transform .

Inverse Fourier Transform

$F^{-1}[x(\omega)] = x(t)$.

$x(t)= F^{-1}[x(\omega)]=\frac{1}{2\pi}\int_{-\infty}^{\infty} x(\omega)e^{j\omega t}\ dt$

(periodical signal) ممكن أن نجزئه الى (sin) ذو ترددات مختلفة, وبالعكس يمكن جمع تلك الأجزاء لتكوين إشارة معينة ذات شكل ا هيئة مختلفة تماماً.

نلاحظ على الرسم (1.a) الدالة التالية

f(t)=2sint-sin2t

والتي تسمى مجموع (trigonometric functions) وبتعبير آخر (trigonometric series).

مكون من حدين. نضيف حد آخر ونكون متوالية جديدة متكونة من ثلاث حدود.

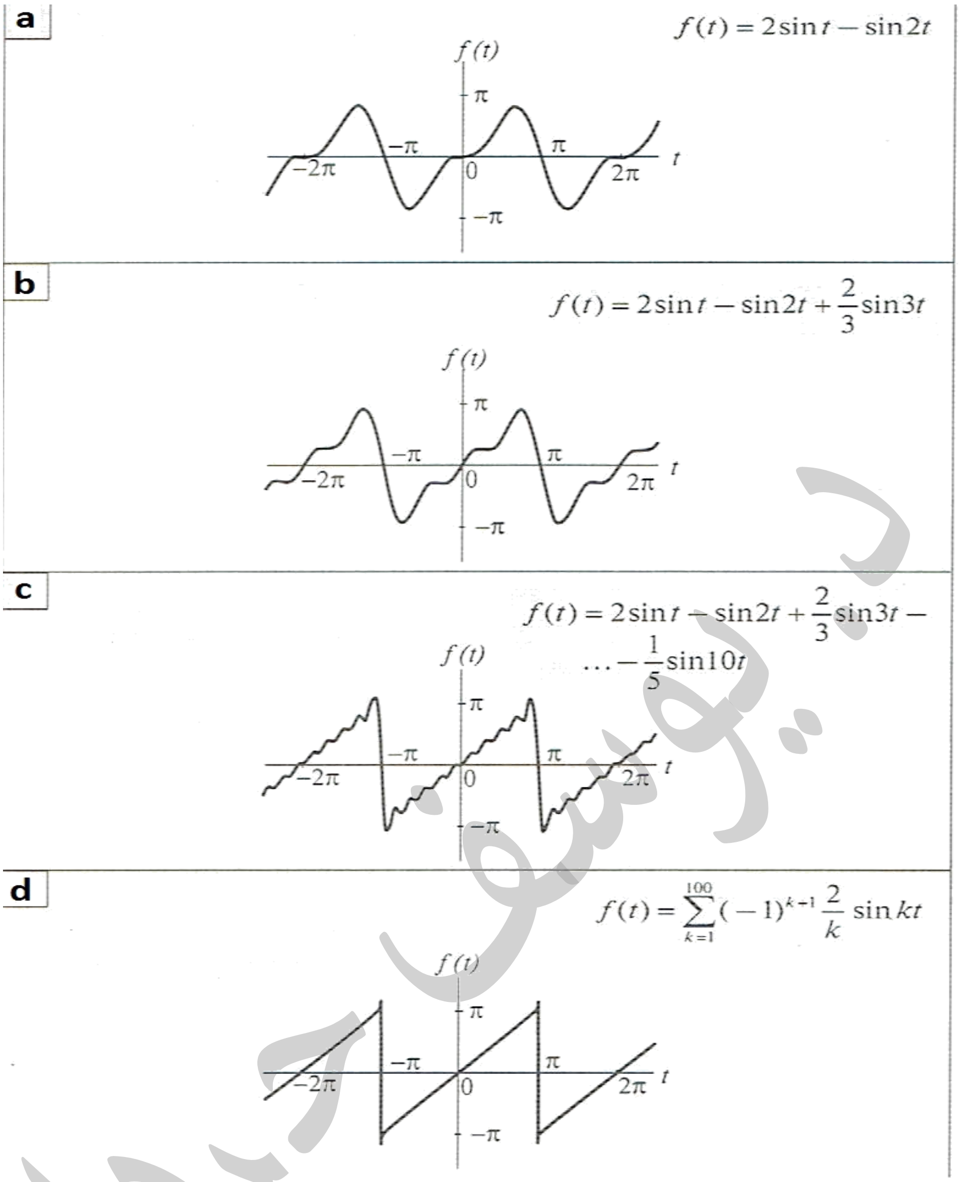
f(t)=2sint-sin2t+2\3sin3t

ويمكن ملاحظة الرسم (1.b). مرة أخرى نضيف عدة حدود نحصل على متوالية متكونة من عشرة حدود

f(t)=2sint-sin2t+2\3sin3t-1\2sin4t+2\5sin5t-1\3sin6t+2\7sin7t-1\4sin8t+2\9sin9t-1\5sin10t

Coefficients نرمز له (b_k), أما (k) فهو عدد صحيح. $b_k=(-1)^{k+1}.2\backslash k$

عندها يمكن كتابة المتوالية f(t) بالشكل التالي: $f(t)=\sum_{k=1}^{m} bk\ sinkt$



Sampling theorem:

(analog signal) و (discrete) مرتبطان بعملية تسمى (sampling process), هذه العملية يمكن تحقيقها من خلال عملية (sample-and-hold) في هذه الحالة آلة الخزن في (computer) [Transistor, capacitor] ,تكوَّن من (Analog signal) المرسلة (sample sequence).

نتيجة عملية (sampling process) ستكوَّن إشارة بشكل يسمى (pulse-amplitude modulation) (PAM). هذه التسمية جاءت من كون إن الإشارة على المخرج يمكن وصفها بأنها مجموعة من (pulses) والتي تحدد (samples). (input analog signal) ممكن استنباطه من (PAM) للـ(modulated signal) والذي يمر عبر (low-pass-filter). من الضروري جداً أن نعرف بان (modulated signal) كم من الدقة بحيث يكون متطابق مع الإشارة الداخلة. الإجابة على هذا السؤال سيكون من خلال نظرية (sampling theorem)

$T_s \leq 1\backslash 2f_m$

T_s - time interval of sampling

f_m – frequency of modulated signal

$f_s \geq f_m$ (Nyquist criterion)

$f_s = 2f_m$ (Nyquist rate)

EXAMPLE 1. An analog signal is expressed by the equation x(t)=3 cos (50πt) + 10 sin (300πt) – cos (100πt). Calculate the Nyquist rate for this signal.

Solution:- the given signal is expressed as

$x(t)=3 \cos 50\pi t + 10 \sin 300\pi t - \cos 100\pi t$

let three frequencies present be ω1, ω2 and ω3

so that the new equation for signal,

$x(t)= 3 \cos \omega_1 t + 10 \sin \omega_2 t - \cos \omega_3 t$

$\omega 1t= 50\pi t$; $\omega 1=50\pi$

or $2\pi f_1=50\pi$

f_1=25Hz

similarly for second factor and third factor

f_2=150Hz , f_3=50Hz

$f_s=2f_m$

where f_m=maximum frequency present in the signal

$f_s=2f_t = 2.150=300$Hz ans.

Example 2. Find the Nyquist rate and the Nyquist interval for the signal

$x(t)=1 /2\pi \cos(4000\pi t) \cos(1000\pi t)$

Solution : Given signal is

$x(t)=1/2\pi \cos (4000\pi t) \cos(1000\pi t)$

Or $x(t)= 1/4\pi[2\cos(4000\pi t) \cos(1000\pi t)]$

Or $x(t)=1/4\pi[\cos(4000\pi t + 1000\pi t)+\cos(4000\pi t-1000\pi t)$

$[\because 2\cos A \cos B= \cos (A+B)+\cos(A-B)]$

Or $x(t)=1/4\pi[\cos5000\pi t+\cos3000\pi t]$ …(1)

Let the two frequencies present in the signal be ω_1 and ω_2 so that the new equation for the signal will be

$x(t)=1/4\pi [\cos\omega_1 t + \cos\omega_2 t]$ …(2)

Comparing equation (1) and (2) , we have

$$\omega_1 t = 5000\pi t$$

or $2\pi f_1 t = 5000\pi t$

or $2 f_1 = 5000$

$\therefore$ $f_1 = 2500$ Hz

Similarly , for second factor

$$\omega_2 t = 3000\pi t$$

or $2\pi f_2 t = 3000\pi t$

or $2\pi f_2 = 3000$

$\therefore$ $f_2 = 1500$ Hz

Therefore, the maximum frequency present in x(t) is

$$f_1 = 2500 \text{ Hz}$$

Nyquist rate is given as

$$f_s = 2 f_m$$

where f_m = Maximum frequency present in the signal .

Here, $f_m = f_1 = 2500$ Hz

Therefore Nyquist rate

$$f_s = 2 f_m = 2 \times 2500 = 5000 \text{ Hz} = 5\text{kHz} \quad \text{Ans.}$$

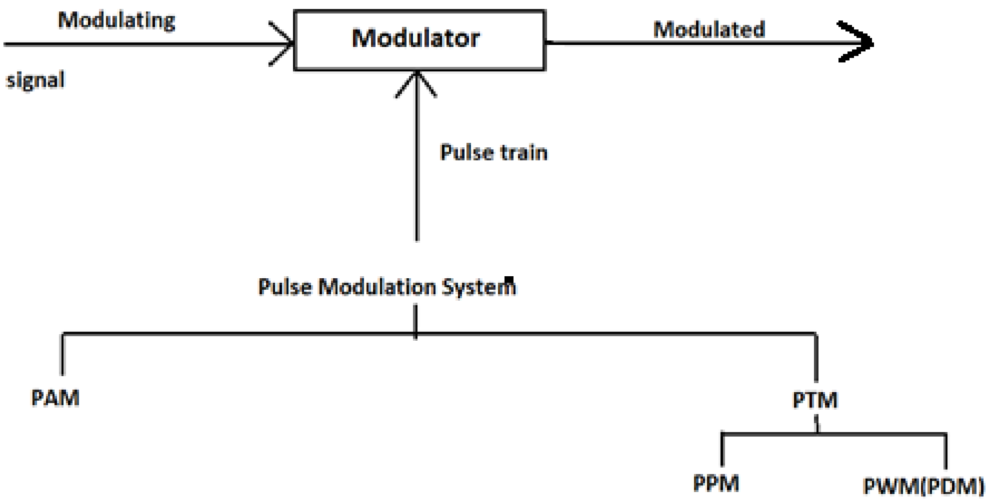
Nyquist interval is given as

$$T_s = 1/2f_m = 1/2 \times 2500 = 1/5000$$

Or $T_s = 0.2 \times 10^{-3}$ seconds = 0.2 m sec. Ans.

Analog Pulse Modulation Method

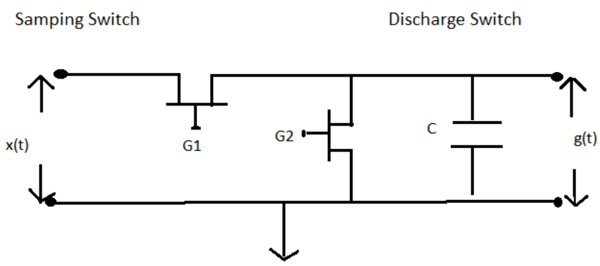
In analog modulation systems , some parameter of a sinusoidal carrier is varied according to the instantaneous value of the modulating signal . in pulse modulation methods , the carrier is no longer a continuous signal but consists of a pulse train . Some parameter of which is varied a cording to the instantaneous value of the modulating signal .



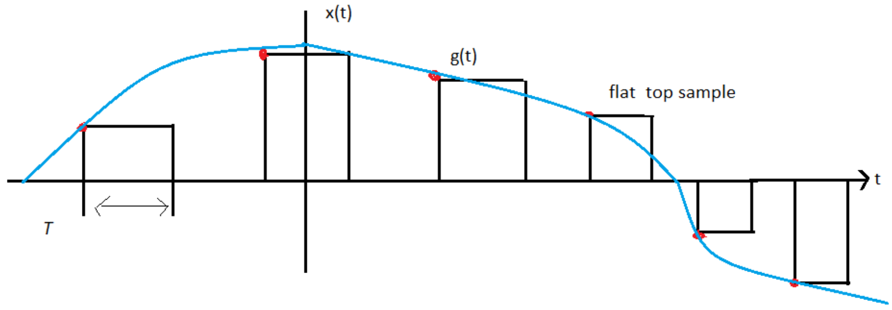
Pulse Amplitude Modulation (PAM)

Pulse Amplitude Modulation may by defined as that type of modulation in which the Amplitudes of pulses vary according to instantaneous value of the modulating or message is signal . In fact , the pulses in a PAM signal may be of flat top type or natural type or ideal type . out of these three pulse amplitude modulation methods , the flat top PAM is most popular and is widely used , because noise can be easily removed if the PAM pulse has flat Top.

A sample and hold circuit is used to produce Flat top sampled PAM.



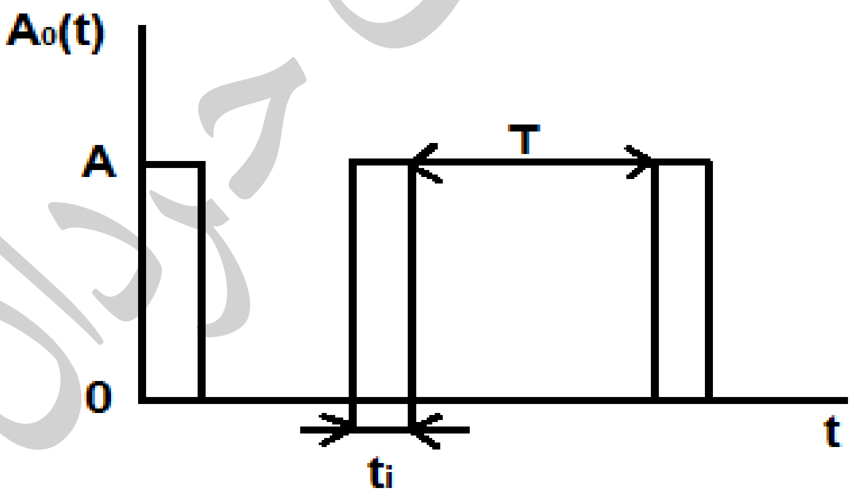
(a)Sample and hold circut generating flat top sampled PAM



(b) Waveforms of flat top sampled PAM.

The sampling switch is closed for a short duration by a short pulse applied to the gate G1 of the transistor. During this period , the capacitor C is quickly charged up to voltage equal to the instaneous sample value of the incoming signal x(t) . Now the sampling switch is and the capacitor C holds the charge.

The discharges switch in then closed be a pulse applied to gate G2 of the other transistor . Due to this , the capacitor is is discharged to zero .



v(A) – PULSE Amplitude

t_i – interval (pulse width)

T – period

Carrier wave:

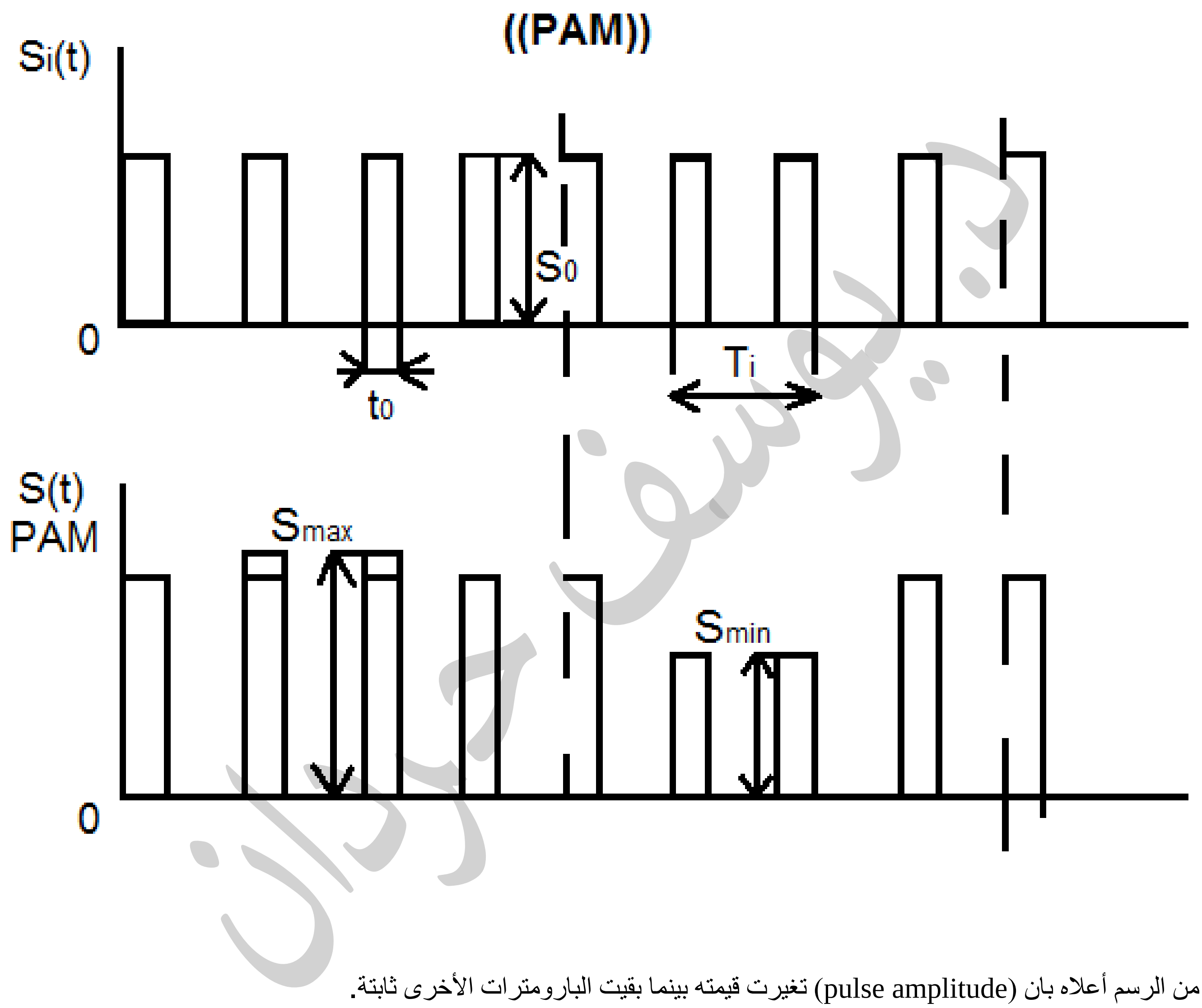
$s_i(t)=s_0 \sum_{k=-\infty}^{\infty}\{si(t)-to-kt2\}$

$s_i(t)$-one pulse function

s_0 – pulse amplitude

t_i – pulse interval

t_0 – phase



واضح من الرسم أعلاه بان (pulse amplitude) تغيرت قيمته بينما بقيت البارومترات الأخرى ثابتة.

Example: for a pulse –amplitude Modulation (PAM) transmission of voice signal having maximum frequency equal to f_m = 3KHz, calculate the transmission bandwidth. It is given that the sampling frequency fs=8KHz and the pulse duration Ꞇ=0.1 Ts.

Solution : we know that the sampling Ts is expressed as :

$$Ts = \frac{1}{fs} = \frac{1}{8 * 10^3}\ seconds$$

Ts = 0.125 * 10^{-3} m seconds ; Ts =125 µ seconds

Ꞇ = 0.1 Ts

Ꞇ = 0.1 * 125

=12.5 µ seconds

Now we know that the transmission bandwidth for PAM signal is expressed as

$$BW = \frac{1}{2Ꞇ}$$

$$BW = \frac{1}{2 * 12.5 * 10^{-6}} = \frac{1 * 10^{6}}{25}$$

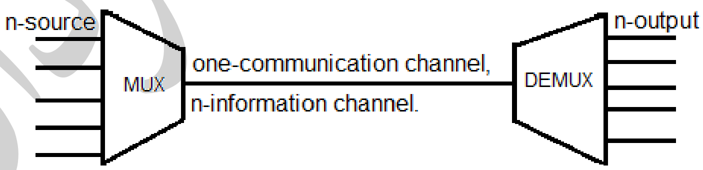
$$BW = 40\ KHz\ .Ans.$$

Time Division Multiplexing (TDM)

In PAM, PPM and PDM, the pulse is present for a short duration and for must of the time between the two pulses, no signal is present. This free space between the pulses can be occupied by pulses from other channels. This is known as Time Division Multiplexing (TDM).

Thus, time division multiplexing makes maximum utilization of the transmission channel.

لغرض زيادة كفاءة منظومات الاتصال, من المهم جداً إرسال عدة إشارات في آنٍ واحد بدلاً من الإشارة الواحدة, هذا المفهوم يعرف بـ (multiplexing).



(multiplexing)

في الرسم التالي موضح عملية (multiplexing), حيث المطلوب هو تجميع عدة إشارات $m_i(t)$, $_i$=1,n هذه الإشارات هي حاملة للمعلومات الرقمية, وفي أغلب الأحيان تسمى بالإشارات الرقمية. البيانات (data) القادمة من كل مصدر (source) تحفظ مؤقتاً في (buffer). قياس كل (buffer) او حجمه في الغالب يعادل 1 (bit) أو 1 (byte) أو (symbol). الـ (buffer) يقوم بتكوين (flow) أو (stream) من البيانات (data) $m_c(t)$. وعليه سرعة إرسال البيانات $m_c(t)$ يجب ان لا تكون اقل من مجموع سرع إرسال المصادر (digital signal). $M_c(t)$ يمكن ان يرسل بحالته الرقمية أو يحوَّل إلى (analog) عند مروره من خلال (modem).

الكادر (frame) هو (cycle of time interval) (ويكون واحد لكل مصدر (source))

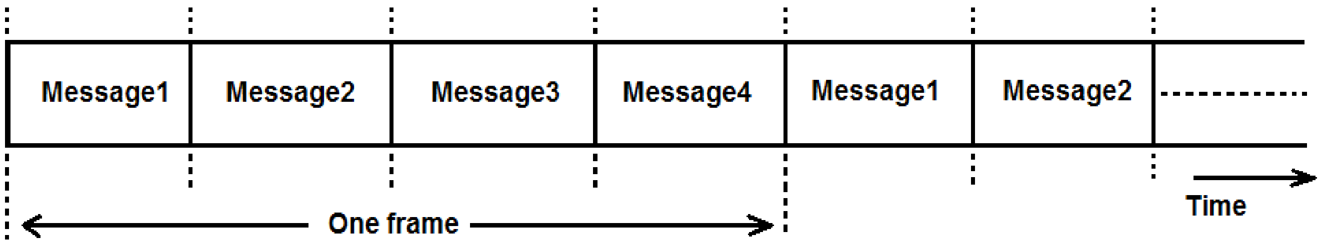
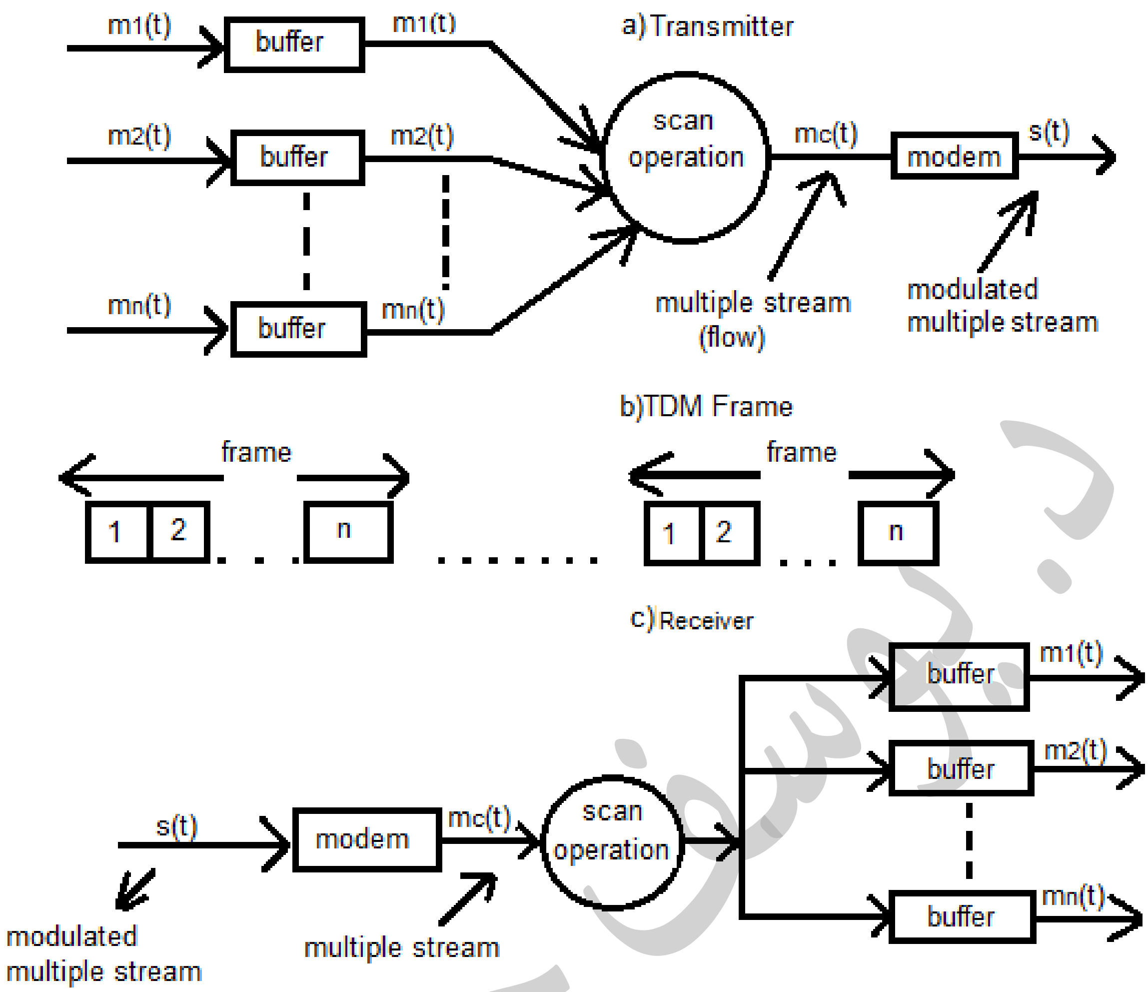


Illustration of TDM concept



((TDM System))

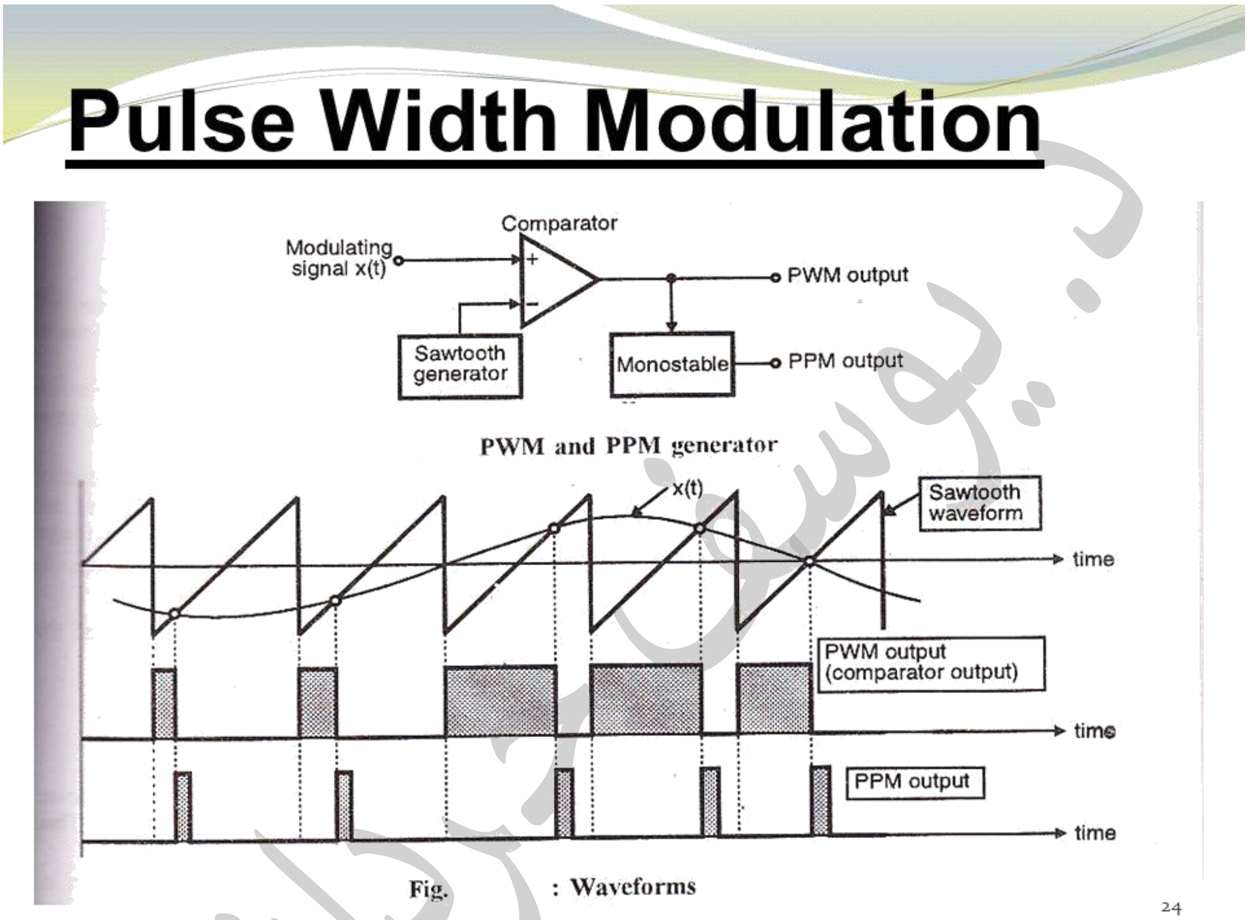
PULSE TIME MODULATION

In pulse time modulation , the signal to be transmitted is sampled as in pulse amplitude modulation (PAM). In pulse time modulation, amplitude.

Of pulse is held constant, whereas position of pulse or width of pulse is made proportional to the amplitude of signal at the sampling instant. There are two types of pulse time modulation, viz. Pulse width modulation (PWM) and Pulse Position Modulation (PPM). Because in both PWM and PPM, amplitude is held constant and does not carry any information, therefore amplitude limiters can be used. The amplitude limiters, similar to those used in FM, will clip off the portion of the signal corrupted by noise and hence provide a good degree of noise immunity.

Pulse Width Modulation

This is also known as pulse Duration Modulation (PDM). Three variations of pulse width modulation are possible. In one variation, the leading edge of the pulse is held constant and change in pulse width signal is measured width respect to the leading edge. In other variation, the tail edge is held constant and width respect to it, pulse width is measured. In the third variation, centre of the pulse is held constant and pulse width changes on either side of the centre of the pulse.



Advantages Of PWM

1. Unlike, PAM, noise is less, since in PWM, amplitude is held constant.
2. Signal and noise separation is very easy.
3. PWM communication does not require synchronization between transmitter and receiver.

Pulse Position Modulation

In this system, the amplitude and width of the pulses are kept constant, while the position of each pulse, with reference to the position of a reference pulse, is changed according to the instantaneous sampled value of the modulation signal . Thus , the transmitter has to send synchronizing pulses to keep the transmitter and receiver in synchronism As the amplitude and width of the pulses are constant . the transmitter handles constant power output , a definite advantage over the PWM . But the disadvantage of the PPM system is the need for transmitter –receiver synchronization . Pulse position modulation is obtained from pulse width modulation . Each trailing edge of the PWM pulse is a starting point in PWM and hence it is proportional to the instantaneous amplitude of the sampled modulating signal .

Advantages of PPM

1.Like PWM , in PPM , amplitude is held constant thus less noise interference .

2. Like PPM , signal and noise separation is very easy .

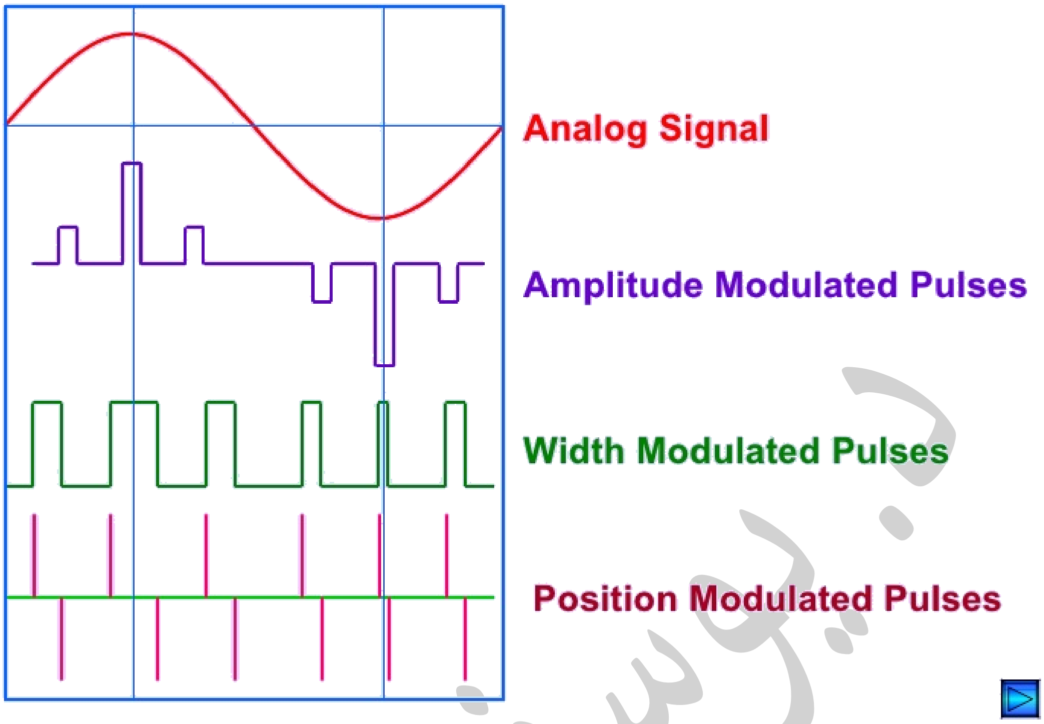
3. Because of the constant pulse widths and amplitudes , transmission power for each pulse is same.

Disadvantages of PPM

1.Synchronizaition between transmitter and receiver is required .

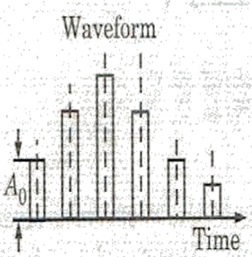
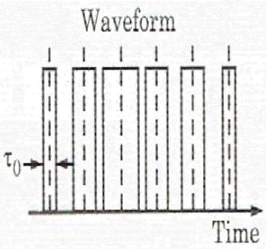
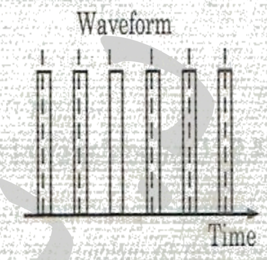
2. Large bandwidth is required as compared to PAM.

PAM, PWM and PPM at a glance:



PERFORMANCE COMPARISON OF VARIOUS PULSE ANALOG MODULATION METHODS

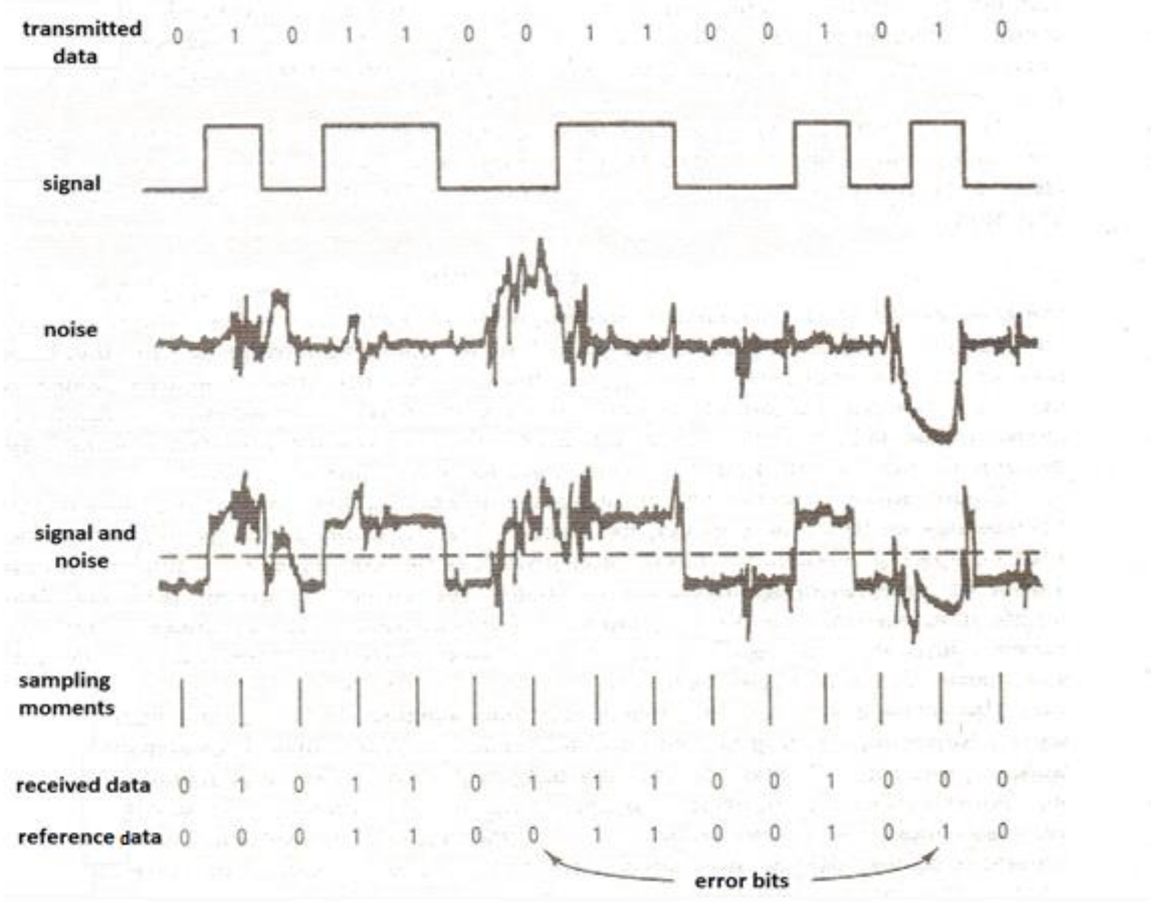
Table 5.2. Performance Comparison of PAM, PPM and PDM

S. No.	*Pulse Amplitude Modulation (PAM)*	*Pulse Width/Duration Modulation (PWM) or (PDM)*	*Pulse Position Modulation (PPM)*
1.	Waveform 	Waveform 	Waveform 
2.	Amplitude of the pulse is proportional to amplitude of modulating signal.	Width of the pulse is proportional to amplitude of modulating signal.	The relative position of the pulse is proportional to the amplitude of modulating signal.
3.	The bandwidth of the transmission channel depends on width of the pulse.	Bandwidth of transmission channel depends on rise time of the pulse.	Bandwidth of transmission channel depends on rising time of the pulse.
5.	The instantaneous power of the transmitter varies.	The instantaneous power of the transmitter varies.	The instantaneous power of the transmitter remains constant.
6.	Noise interference is high. System is complex	Noise, interference is minimum.	Noise, interference is minimum

S/N in analog pulse modulation

The higher the data rate , the greater the damage may be cause unwanted noise (Claude Shannon). The task of the detector is to recognize the received signal as accurately as possible with a given signal quality degradation during transmission .There are two reasons for the increase in the probability of error. The first is the effect of filtering in the transmitter , channel and receiver . The second reason for the increase in the probability of error is electrical noise generated by various sources, such as the galaxy and the atmosphere, impulse noise combinational noise, and also interference with signals from other sources. Whit proper precaution , most of the noise can be removed and reduce the effects of interference .At the time there is noise

, which cannot be eliminated ;it is noise caused by the thermal movement of electrons in any conductive medium .This movement generates thermal noise in amplifiers and communication channels ,which is superimposed on signal .These characteristics (Additive ,White ,Gaussian) have defined the accepted name for noise AWGN (additive ,white ,Gaussian noise) .



effects of noise on the digital signal.

SNR $_{db}$ =10 lg(signal power / noise power)

C= B $\log_2$ (1+ SNR)

Where C- channel capacity (bit/sec)

B- channel band width (Hz)

Example: Assume that the channel extends range from 3 to 4 MHz, a (SNR) is 24 db

B= 4-3= 1MHz

SNR db= 24 db = 10 lg (SNR)

SNR= 13.8

C=10^6 . ln (1+13.8)= 10^6. 4= 4 Mbit/sec